Kvantni materiali : od pasovne teorije do koreliranih elektronov

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Gradivo za predavanja pri specialističnem seminarju: "Izbrane teme iz teoretične fizike"

V Ljubljani, Feb 2018.

Redneck perspective on strong correlations

- a working-class white person from the southern US, especially a politically reactionary one.

Left-wing radicals vs. conservative

From : http://condensedconcepts.blogspot.si

- Left-wing:
  - emergence (Phil Anderson, Nobel ‘77)

Completely new conceptual structures and techniques are needed. We must go beyond Landau's mean-field theory and Fermi liquid theory: topological order, quasi-particles with fractional quantum numbers, AdS-CFT, quantum criticality, ...

To the barricades!

- Right-wing
  - atomistic details (Walter Kohn, Nobel prize 98)

. .. we have the conservatives who believe that the key ingredients are atomistic detail, good density functionals, perturbation theory, mean-field theory, and the random-phase approximation.

New concepts and methods are not really needed. They have a good system [just like capitalism].

In particular, we don't need a revolution, just bigger computers!

To balance Rok's part I will discuss more the redneck part

- Density functional theory
- Dynamical mean-field theory
  (a centrist thread - a bridge between left and right)
- The idea was also to bring the discussion in contact to recent research work ruthenates and on TaS_2 (next week)

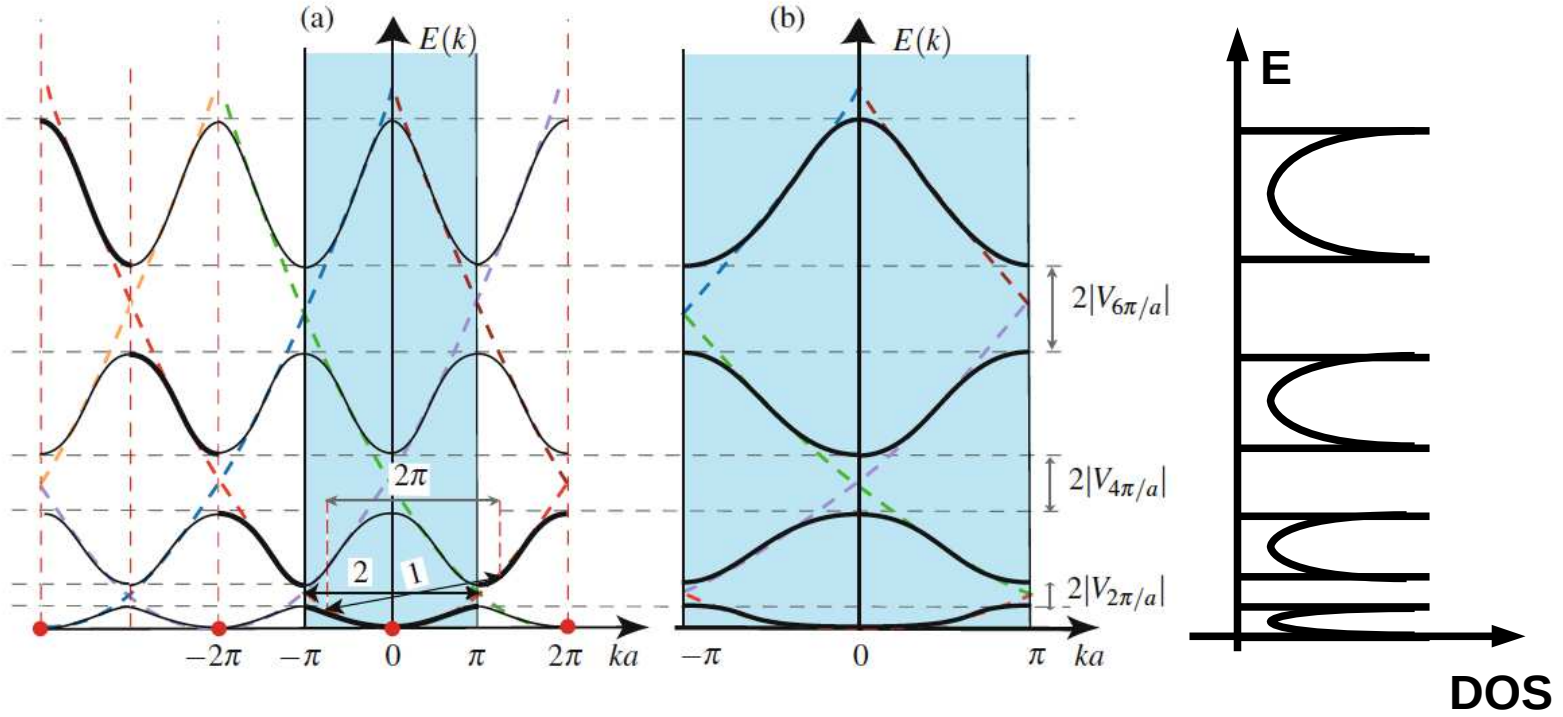
Some (10?) facts about SCES

(Initially, I wanted to make 10 – like 10 commandments from the bible. Then it was 7, like 7 things one should absolutely avoid on the first date. I ended up at only a few (tend to forget ...))

Fact 1: there is a thing called a DFT and it describes the bandstructure of many compounds well

Fact 1: there is a thing called DFT and it describes the *bandstructure* of many compounds *well*

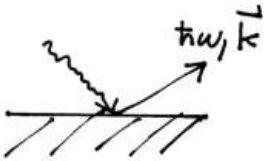
Bandstructure



- k-resolved spectrum $A(k,E)=\Sigma_{kn}\delta[E-E_n(k)]$; $DOS=\Sigma_k\delta[E-E(k)]$. DOS determines many properties of solid state

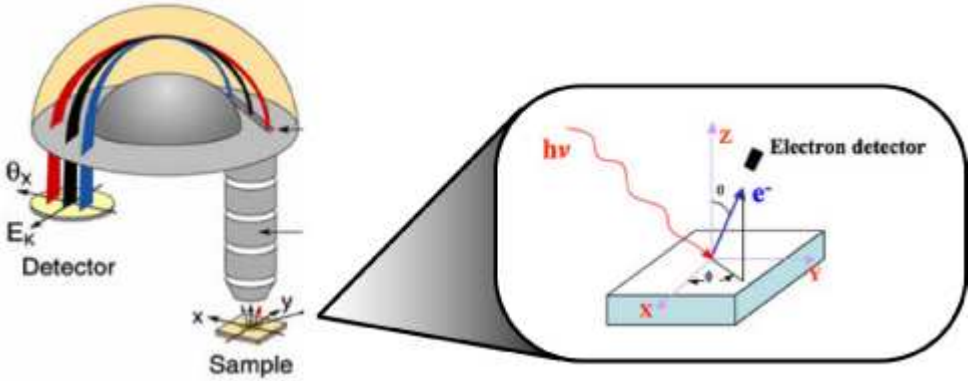
Well: Fact 2: Several experimental techniques exist that allow quantitative comparison with theory

Measuring spectra PES,ARPES



- Probes occupied one-electron states

  I(k,E)=f(E) A(k,E)
- PES : k-integrated, probes DOS
- ARPES : k-resolved, probes A(k,E)

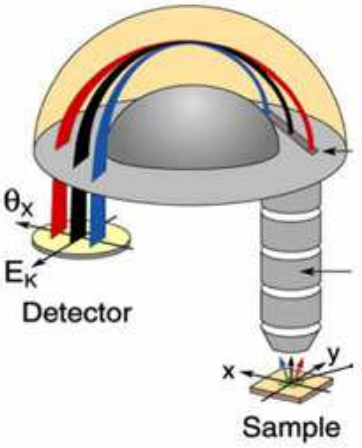


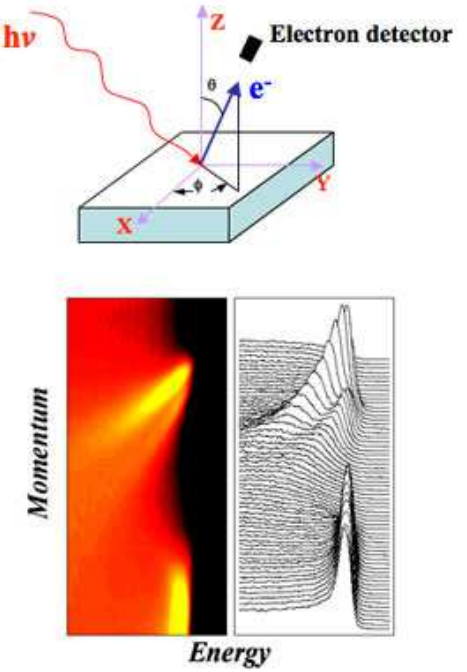
http://www.stanford.edu/group/photontheory/ARPES.html

ARPES

- Probes occupied part of spectral function
- Energy resolution

  10meV (sinhrotron)
  1meV(laser)





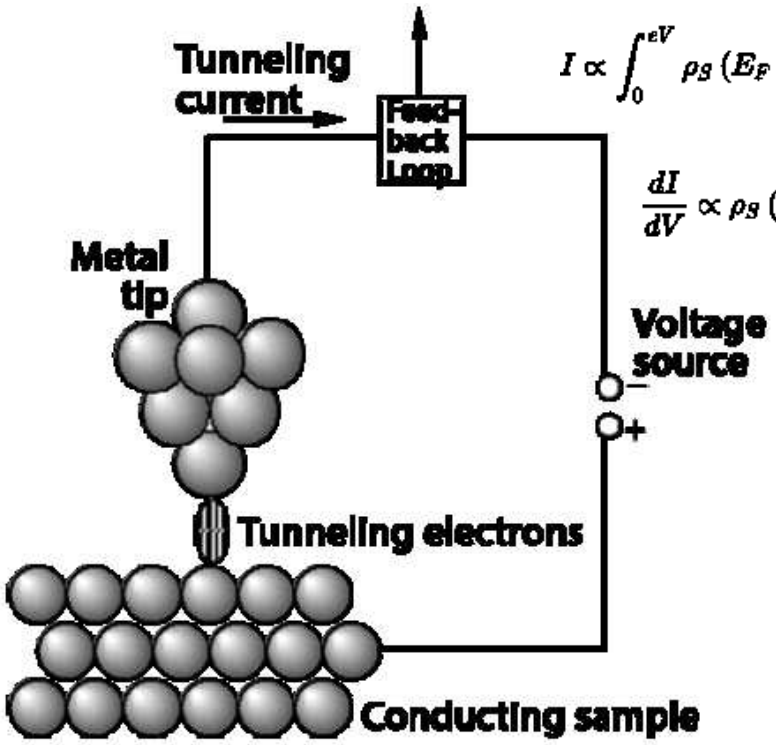
STS (measures ~DOS)

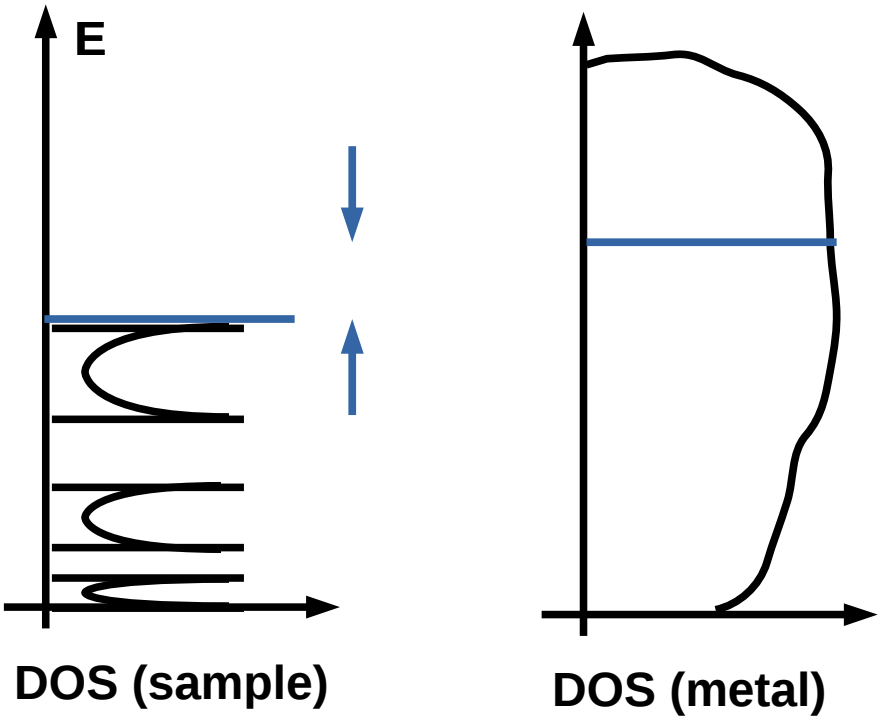
- STM :

$$I = \frac{4\pi e}{\hbar} \int_{-\infty}^{\infty} [f(E_F - eV + \epsilon) - f(E_F + \epsilon)]\, \rho_S(E_F - eV + \epsilon)\, \rho_T(E_F + \epsilon)\, |M_{\mu\nu}|^2\, d\epsilon\,, \qquad (1)$$

$$I \propto \int_0^{eV} \rho_S(E_F - eV + \epsilon)\, \rho_T(E_F + \epsilon)\, d\epsilon\,, \qquad (3)$$

$$\frac{dI}{dV} \propto \rho_S(E_F - eV)\,, \qquad (4)$$

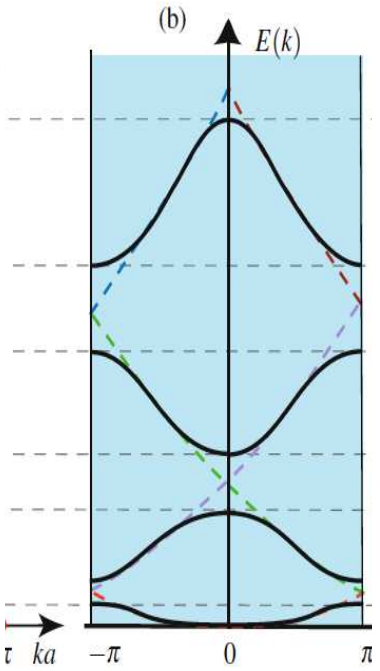




Optical conductivity

- Also gives information about DOS (more precisely, joint DOS)

$$\sigma(\Omega) = \sum_k \int d\omega v_k^2 A_k(\omega) A_k(\omega + \Omega) \frac{f(\omega) - f(\omega + \Omega)}{\Omega}$$



- Schroedinger equation for $\Psi(\vec{r}_1, \ldots, \vec{r}_N)$

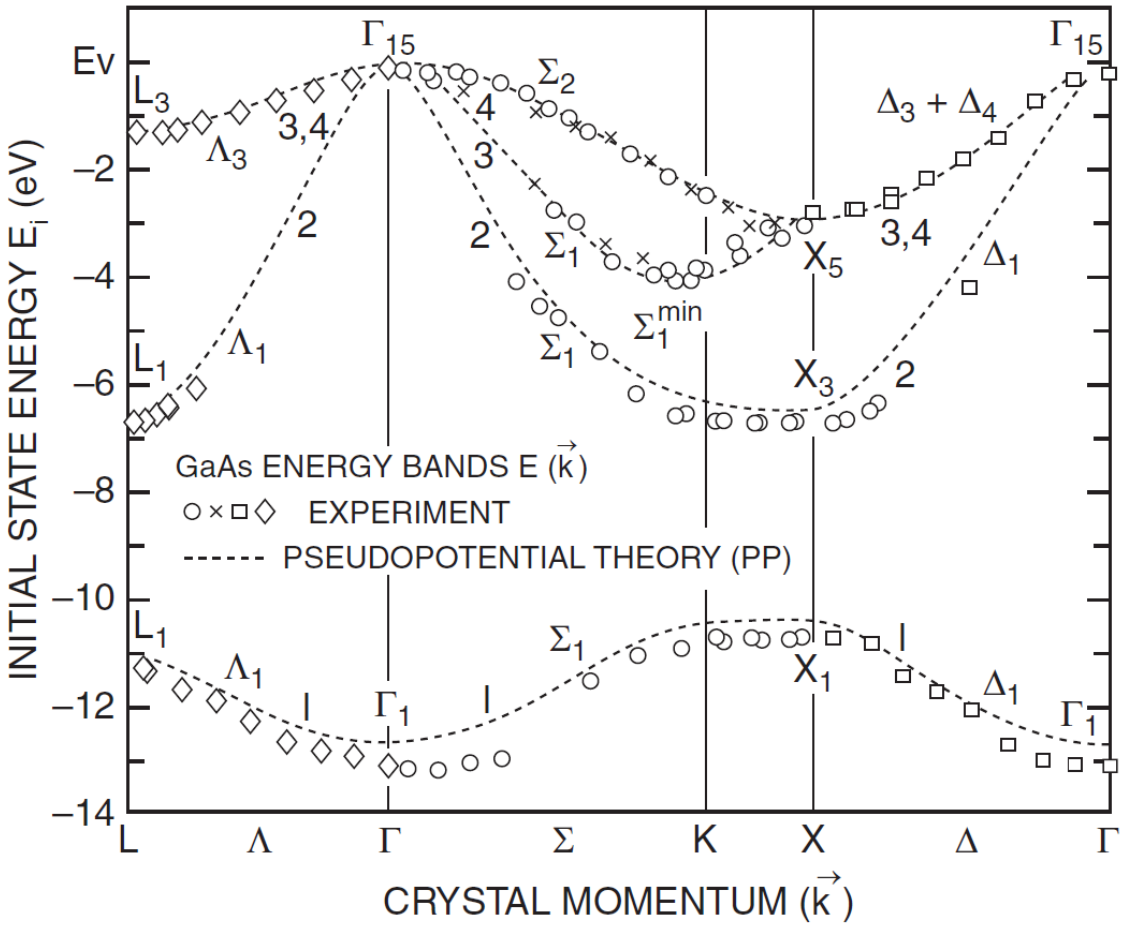
$$\hat{H}\Psi = \left[\hat{T} + \hat{V} + \hat{U}\right]\Psi = \left[\sum_i^N \left(-\frac{\hbar^2}{2m_i}\nabla_i^2\right) + \sum_i^N V(\vec{r}_i) + \sum_{i<j}^N U(\vec{r}_i, \vec{r}_j)\right]\Psi = E\Psi$$

$$n(\vec{r}) = N \int \mathrm{d}^3 r_2 \cdots \int \mathrm{d}^3 r_N \Psi^*(\vec{r}, \vec{r}_2, \ldots, \vec{r}_N)\Psi(\vec{r}, \vec{r}_2, \ldots, \vec{r}_N).$$

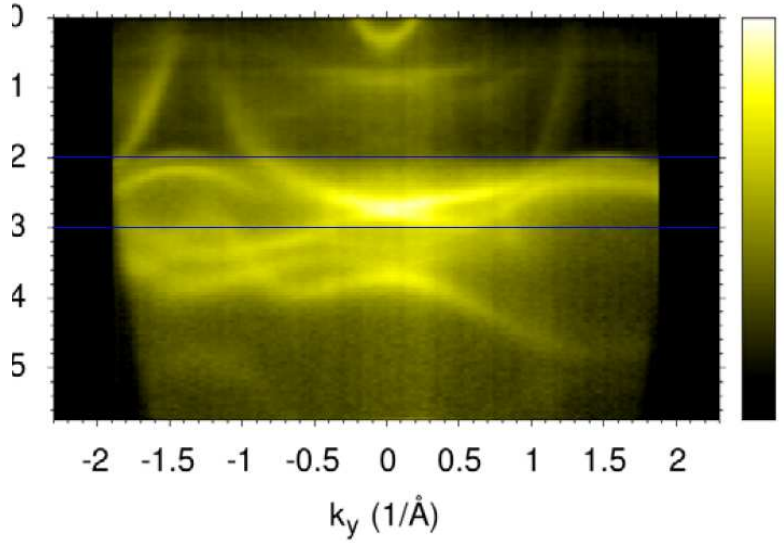
there is a bijective relation between V(r), n(r). V(r) defines the ground state density. The relation can be inverted. There is a unique functional

$$\Psi_0 = \Psi[n_0]$$

Fact 1: there is a thing called DFT and it describes the bandstructure of many compounds well

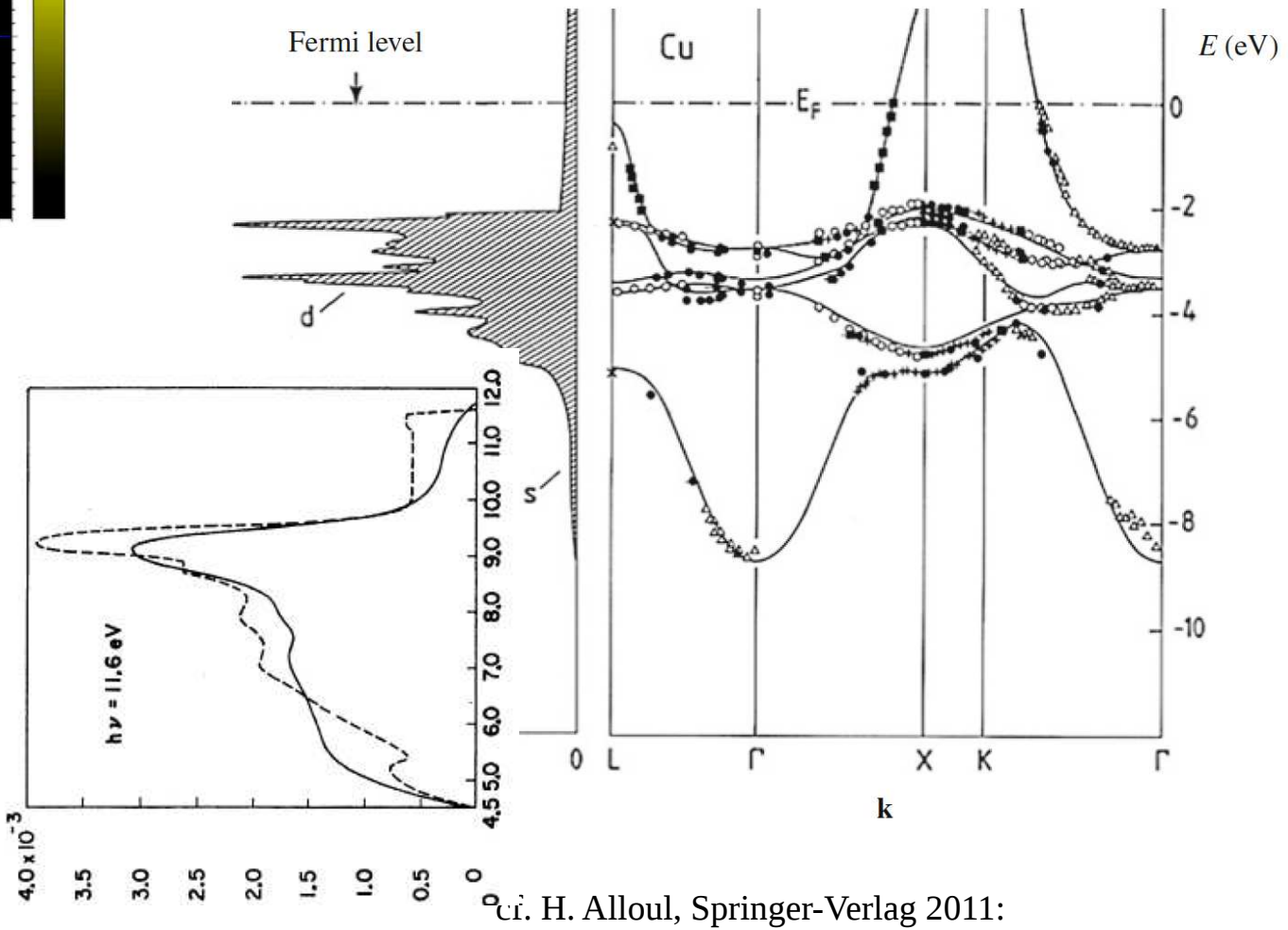


ARPES on Cu



New Journal of Physics **14** (2012) 043009

Symbols: peak of I(k,E)



- Sharp excitations over wide energy range
- Agrees with band theory

Cf. H. Alloul, Springer-Verlag 2011:

Courths, R., Hüfner, S.: Phys. Rep. **112**, 55 (1984)

Eckardt, H., Fritsche, L., Noffke, J.: J. Phys. F **14**, 97 (1984)

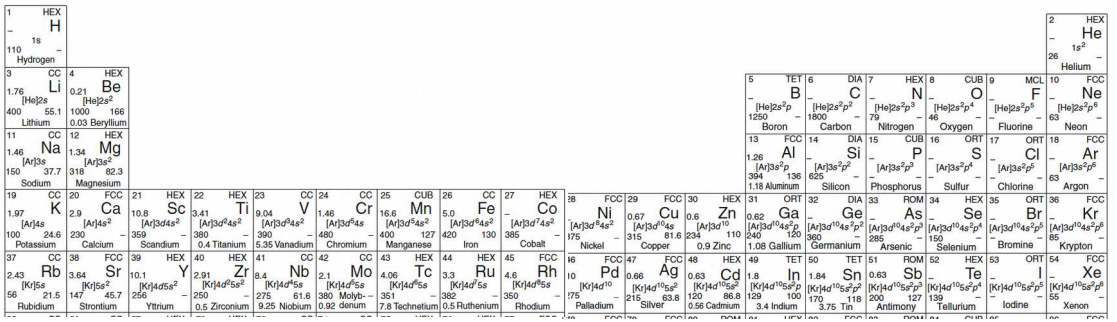
This DFT looks quite neat. What is it? (blackboard)

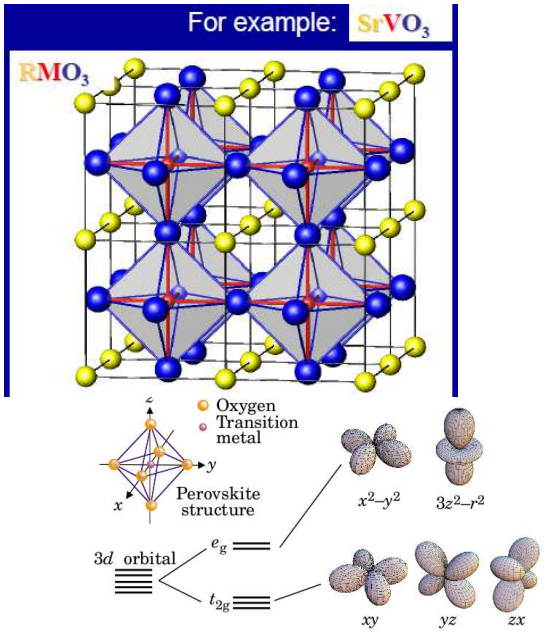
- Comments: information about exact energy of homogeneous electron gas is built in

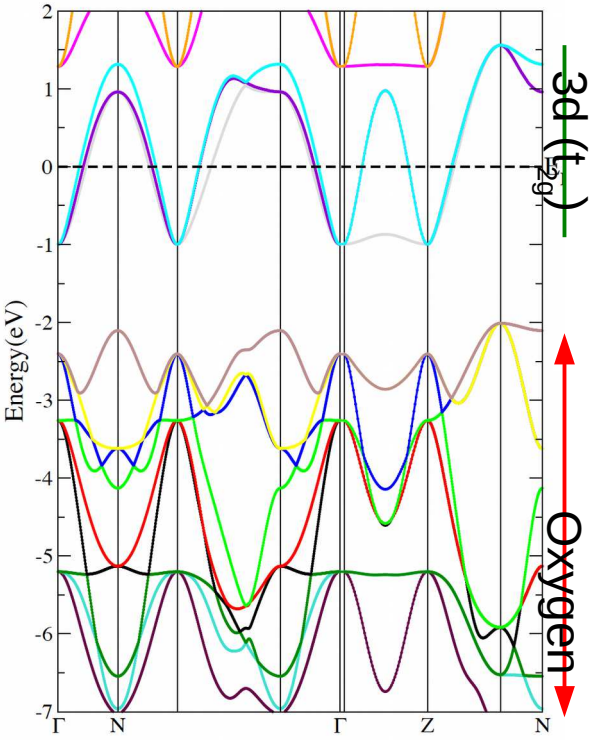
- LDA is ab-initio technique: you put in coordinate of atoms and you get E (can investigate which structures are stable)
- It describes state in terms of auxiliary Kohn-Sham states
- Energies of these eigenstates for some reason are close to the experiments for several cases (noone really knows why)

- The energies also have meaning: take $SrVO_3$

Transition metal oxides; structure and band-structure







An observation

- Bands quite narrow (2eV) in contrast to Cu (~10eV)

Part 2

When LDA breaks down:

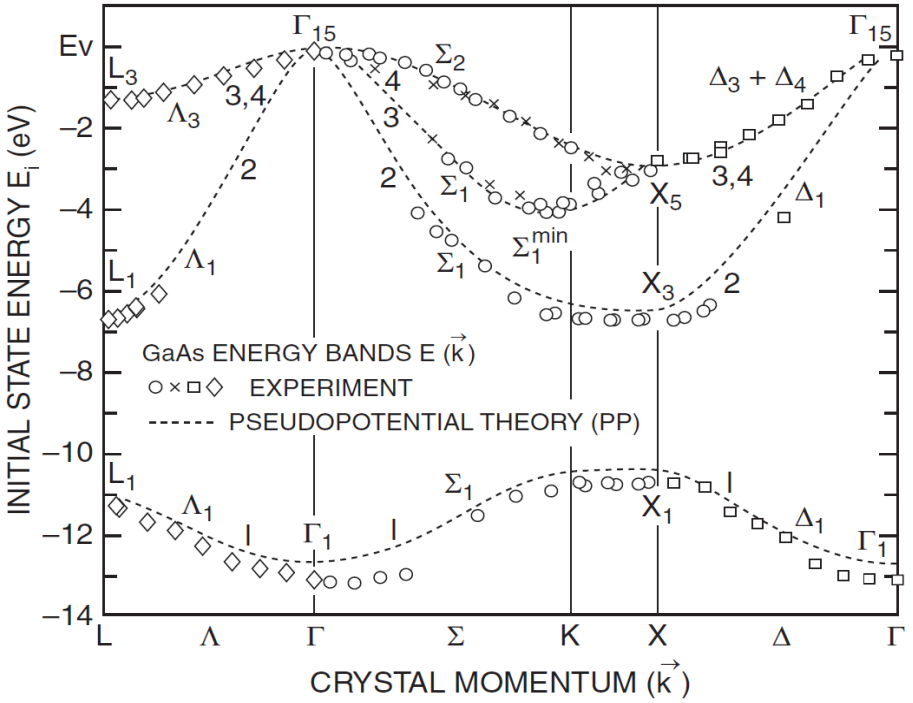
electronic correlations

Redneck perspective on strong correlations (part 2)

From past week

Fact 1. there is a thing called DFT(LDA) and it describes the bandstructure of many compounds well

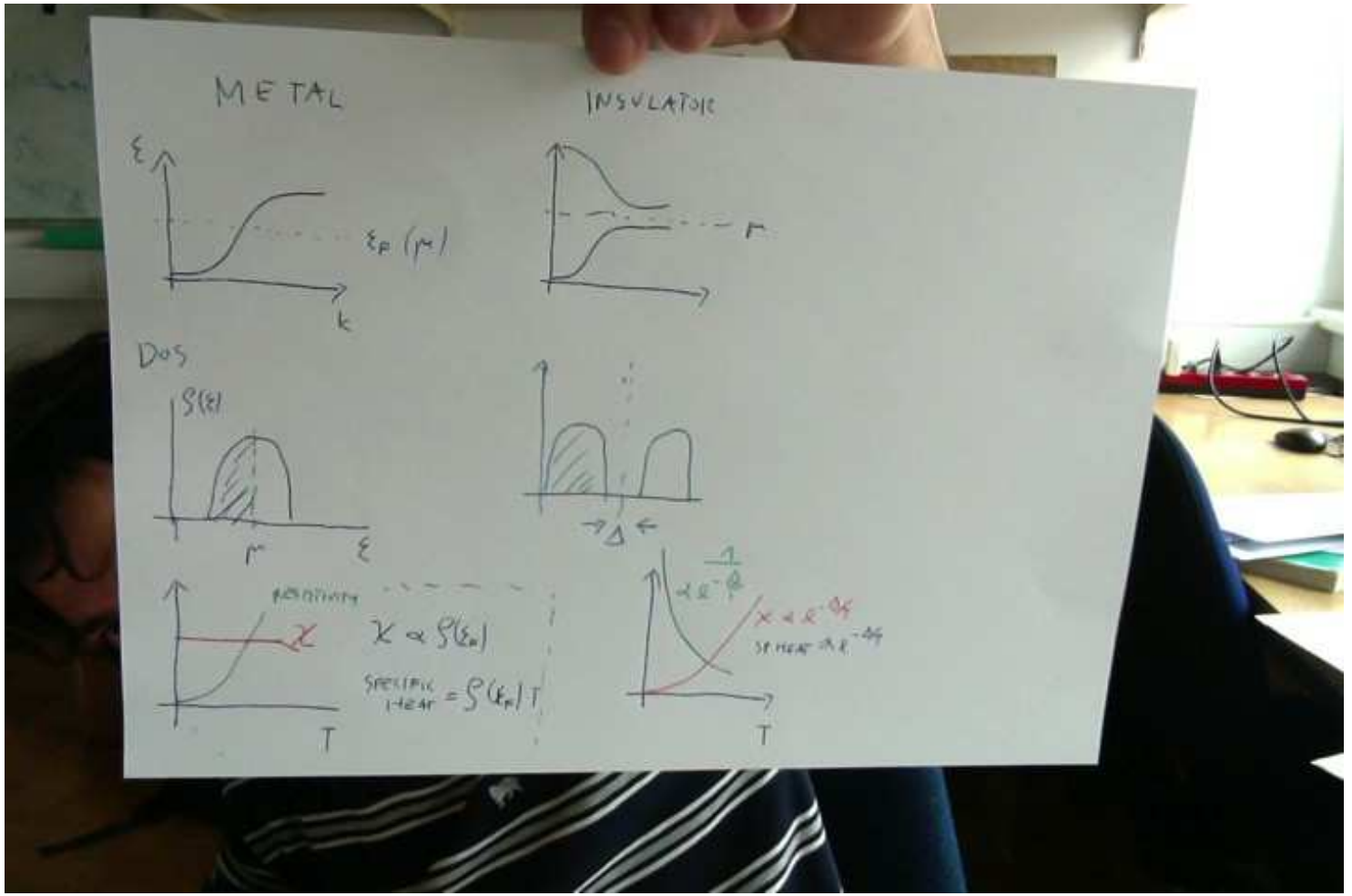
Fact 2. Several experimental techniques exist that allow quantitative comparison with theory



- LDA breaks down when interactions get strong
- Electronic correlations and Mott transition
- DMFT
- Sr_2RuO_4: Correlated metal with low coherence scale due to Hund's rule coupling
- Spin-liquid behavior in TaS_2

Resistivity, magnetic susceptibility, and specific heat : band-insulators vs. metals

Resistivity, magnetic susceptibility, and specific heat : band-insulators vs. metals



Note: band insulator needs even # of els. / band.

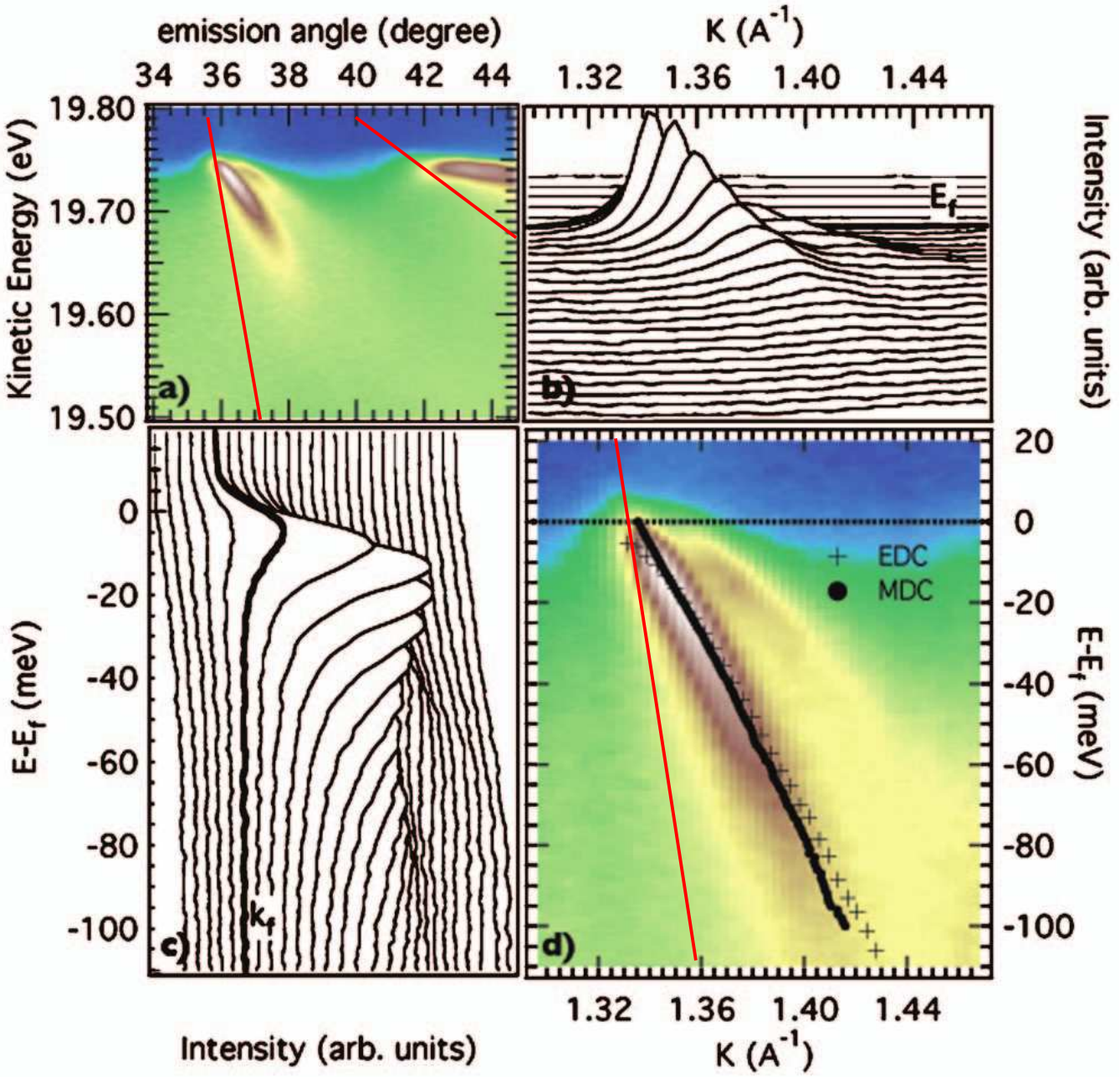
Breakdown of band picture: correlated metals and Mott insulators

Fact 3: strong Coulomb repulsion leads to a breakdown of band theory and occurrence of Mott insulator

- Partial breakdown: quasiparticles with large renormalizations compared to band-theory

- Complete breakdown: Mott insulator

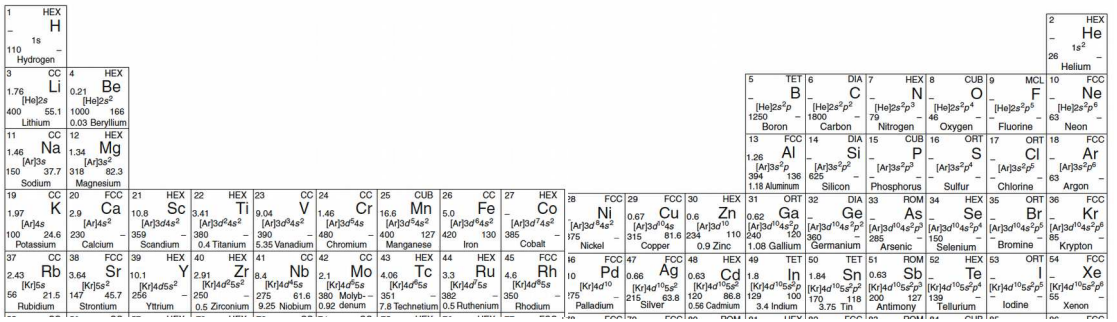
Partial breakdown (Sr_2RuO_4)

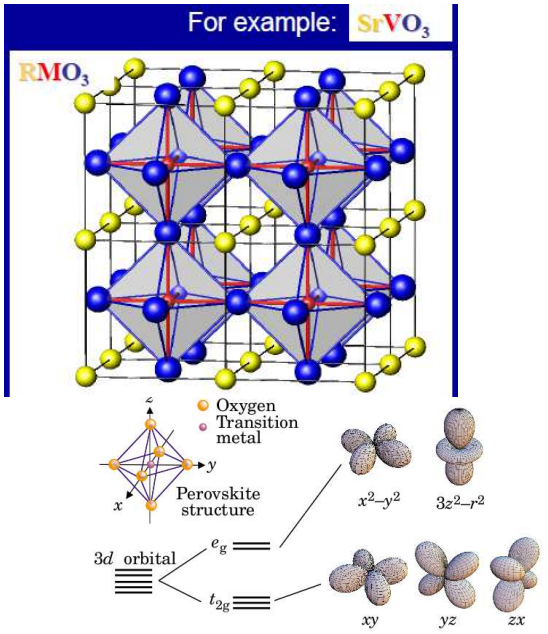


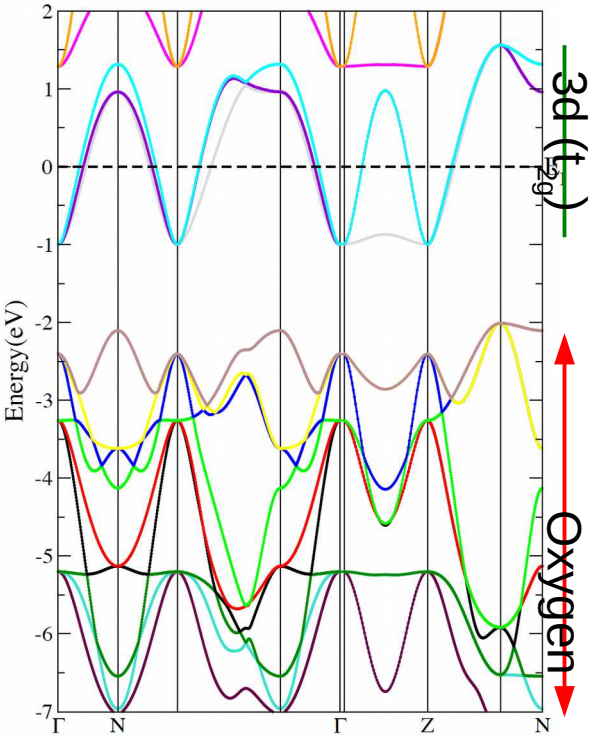
- Completely differ
  picture.
- Coherent excitat
  only up to a 0.1e
- Heavy quasiparti
  ($E=k^2/2m$, dE/dk

Red: band-theory
Strong renormaliza
tions

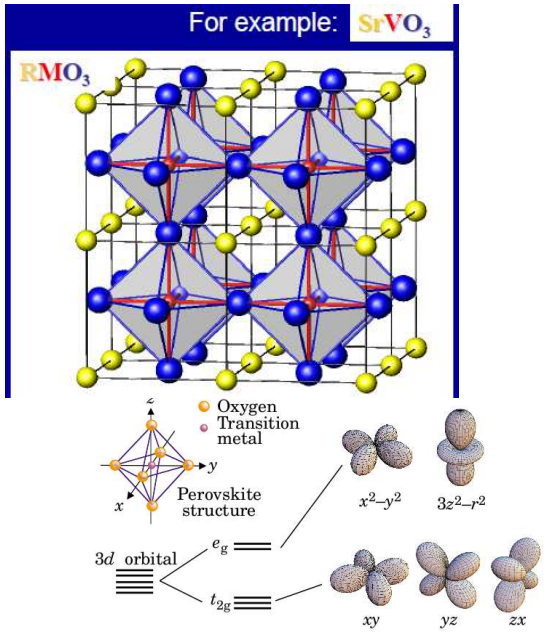
Transition metal oxides; structure and band-structure

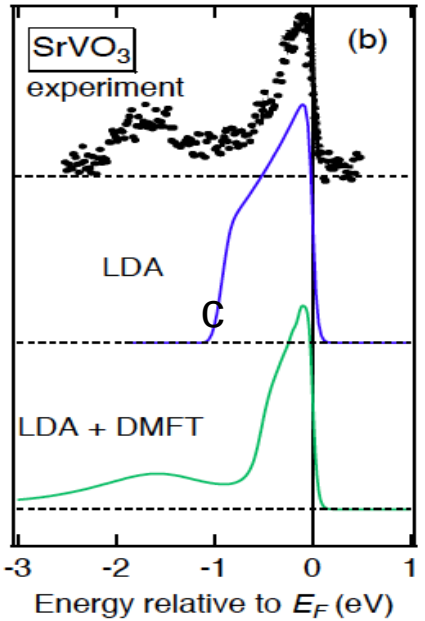


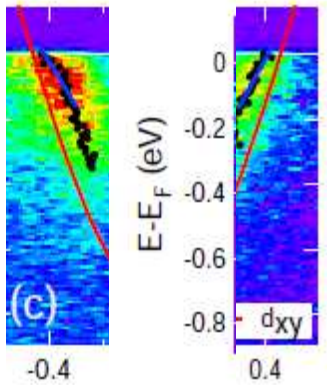




TMOs: structure and el. structure



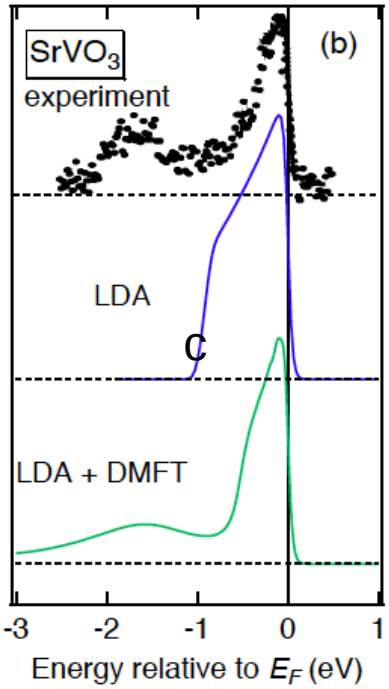


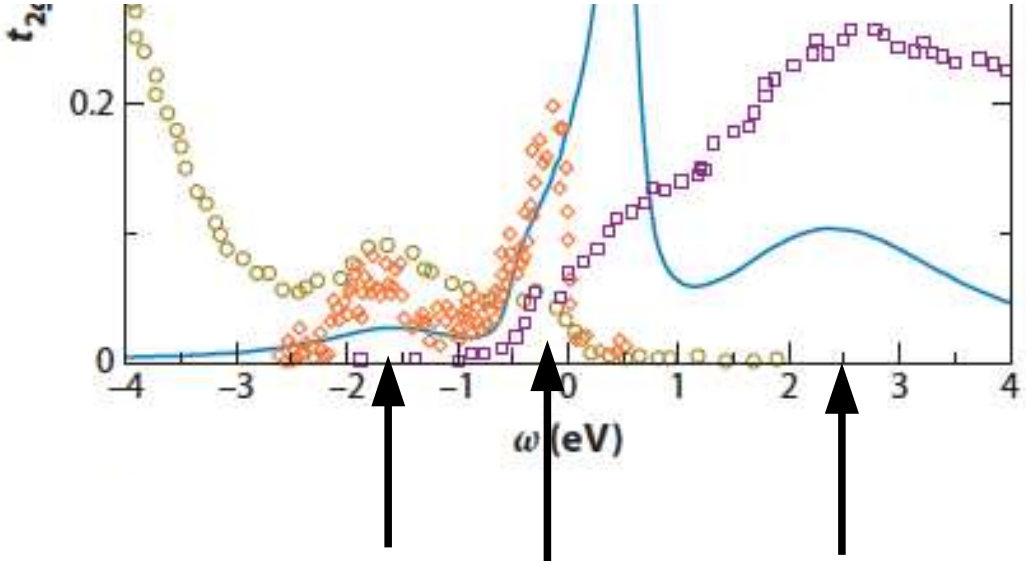


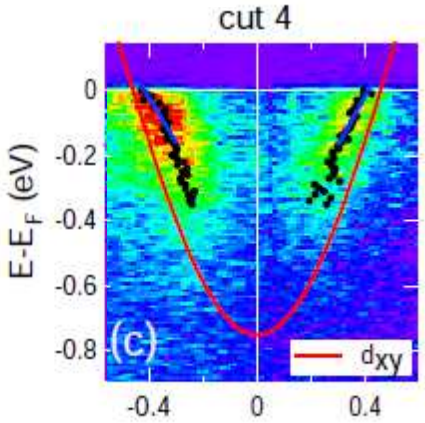
- PES: satellites (not present in LDA)
- ARPES: coherent excitations only up to a 0.5eV
- Renormalization of slope (mass) by 2

Yoshida, …, ZX Shen (2008)

TMOs: photoemission





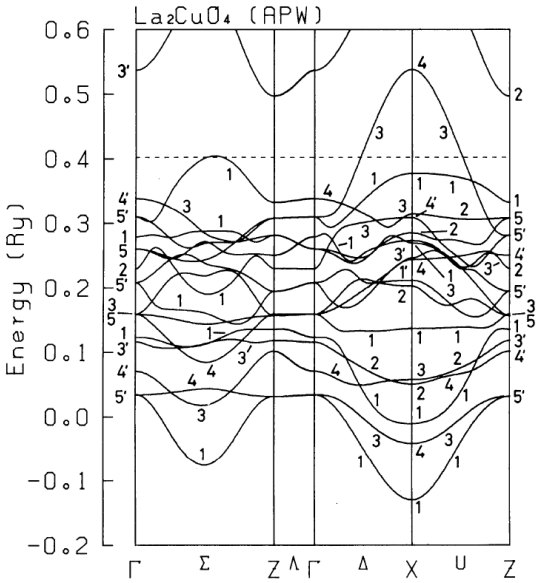


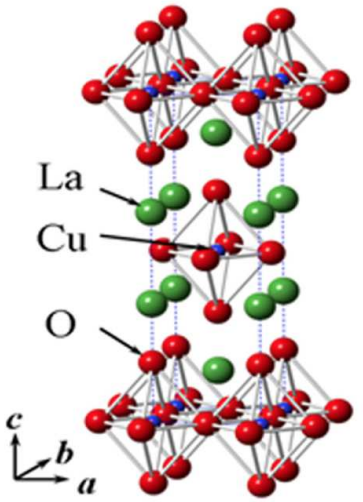
- PES:atomic satellites (not preser
- QP peak (narrowed by 2 from LD
- ARPES: coherent excitations only up to a 0.5eV

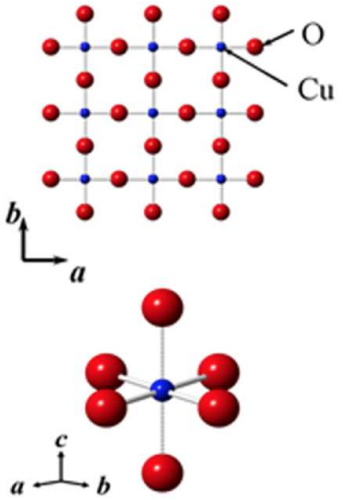
Yoshida, …, ZX Shen (2008)

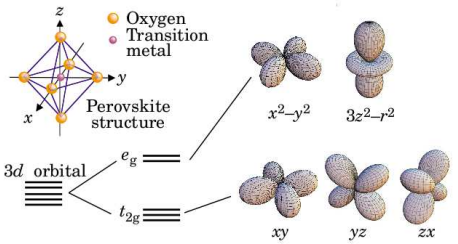
La_2CuO_4

- 9 els in Cu 3d orbitals: 1 hole in x^2-y^2



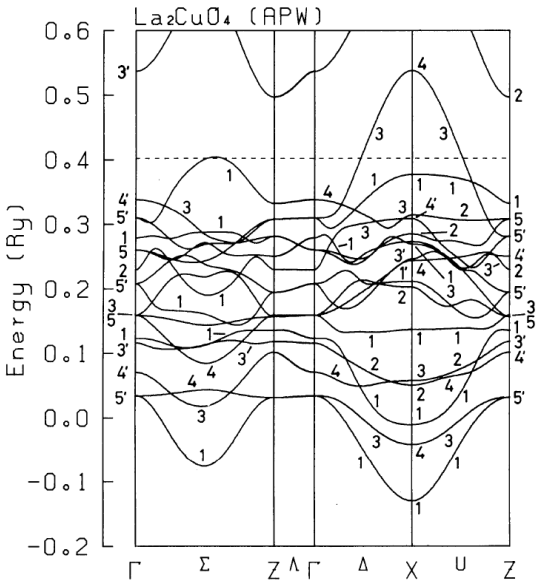


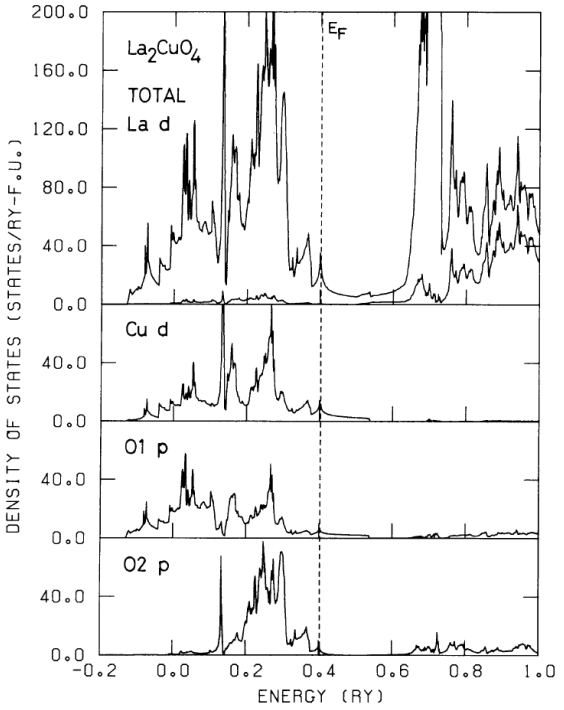


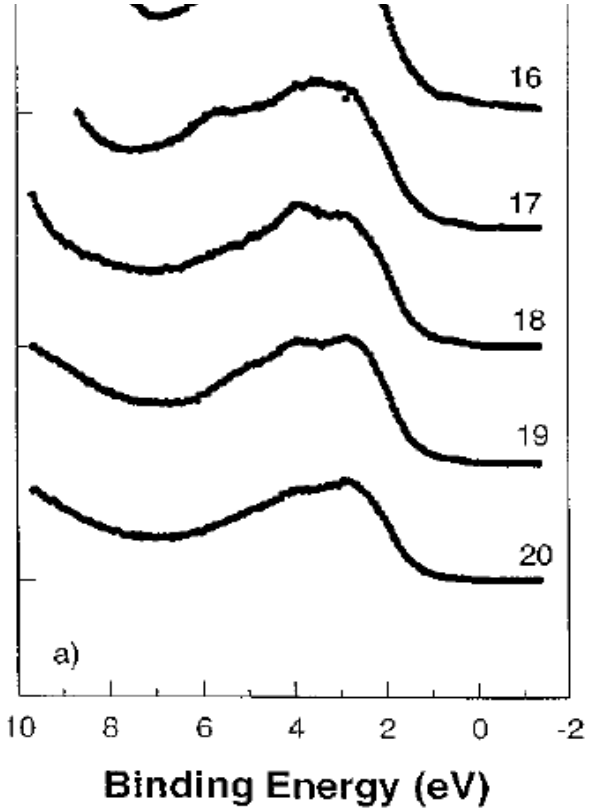


Mott transition: La_2CuO_4 is (Mott) insulator

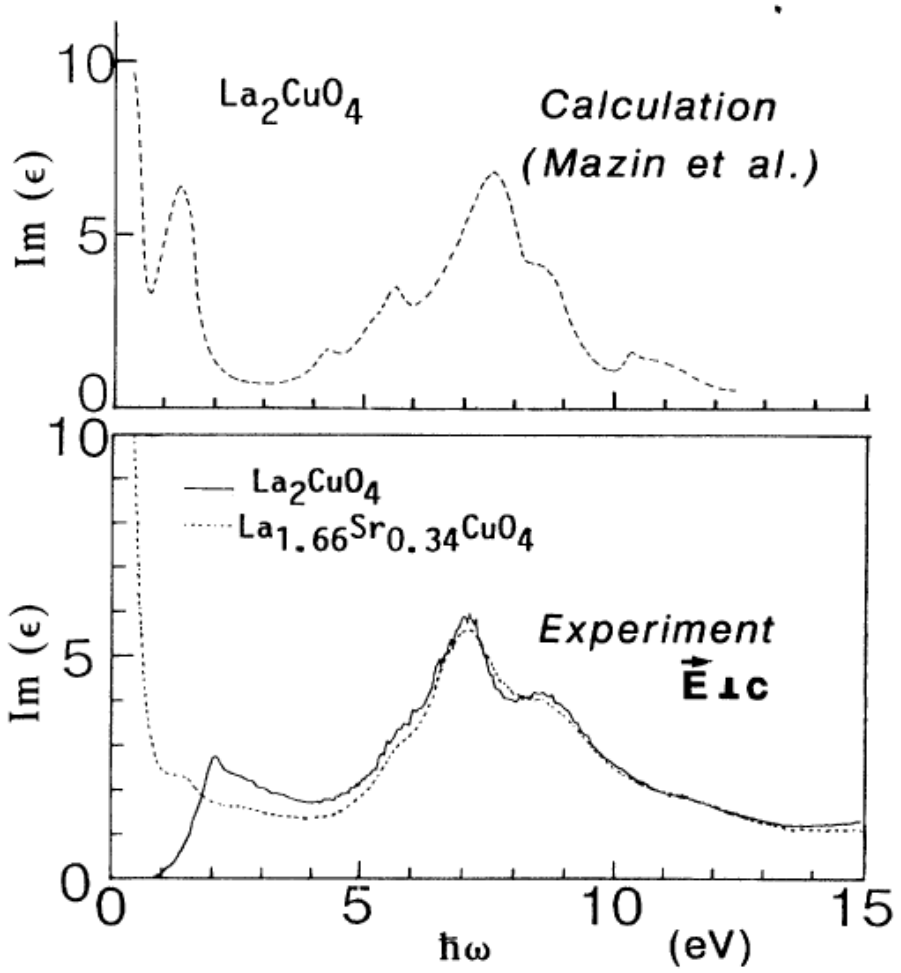
- PES shows a 2eV gap instead of a metal







Optical conductivity also shows gap

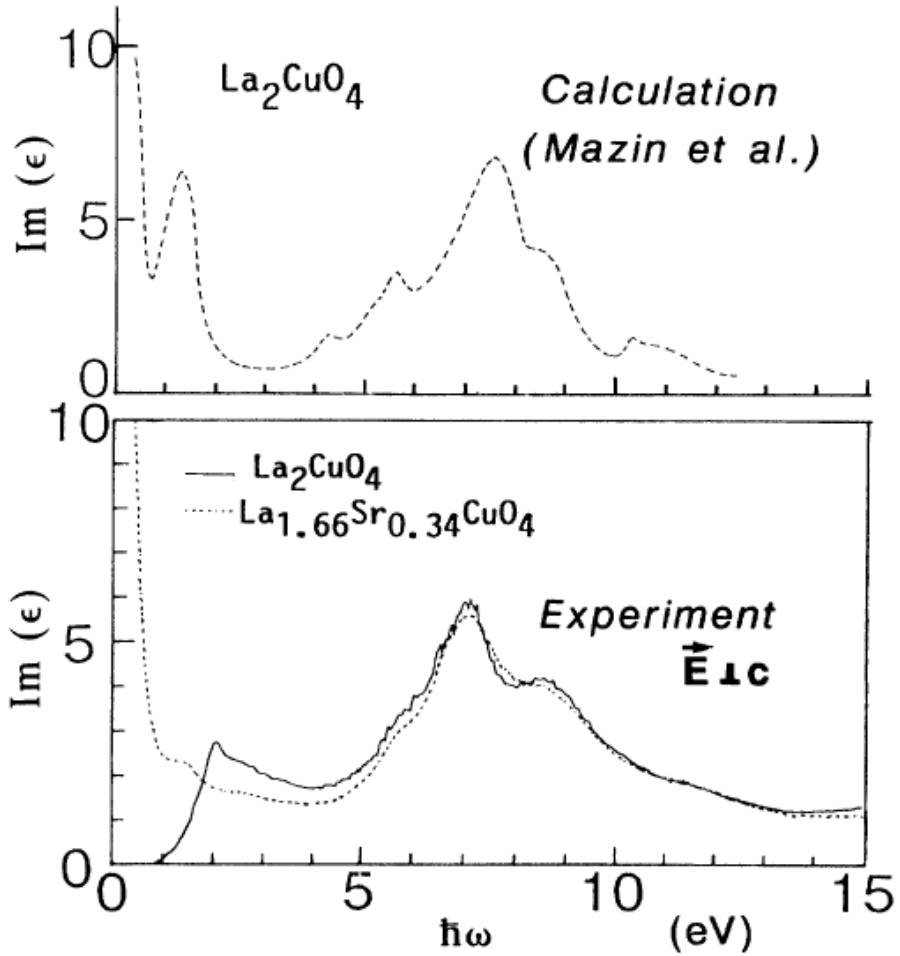


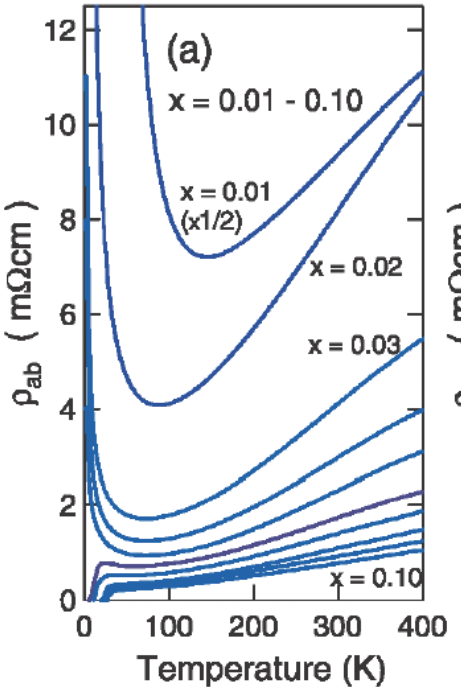
Uchida'91

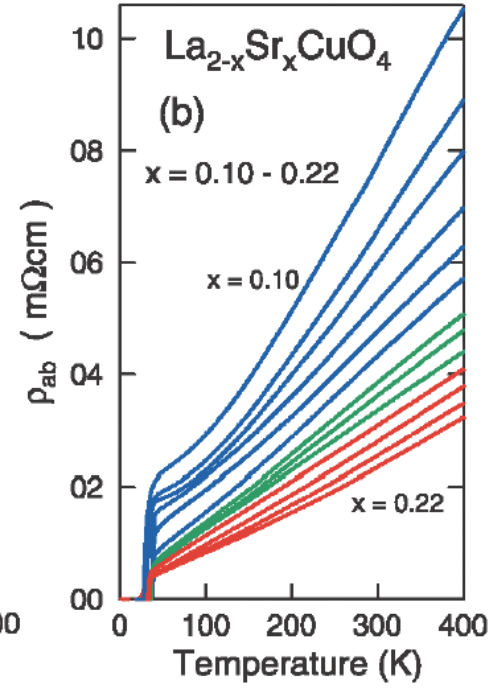
Notice overall LDA describes result well, only low energy spectral weight is missing

On doping, metallicity is restored; filling controlled Mott transition

- (Actually, more than that : doped LSCO is a high Tc superconductor!)

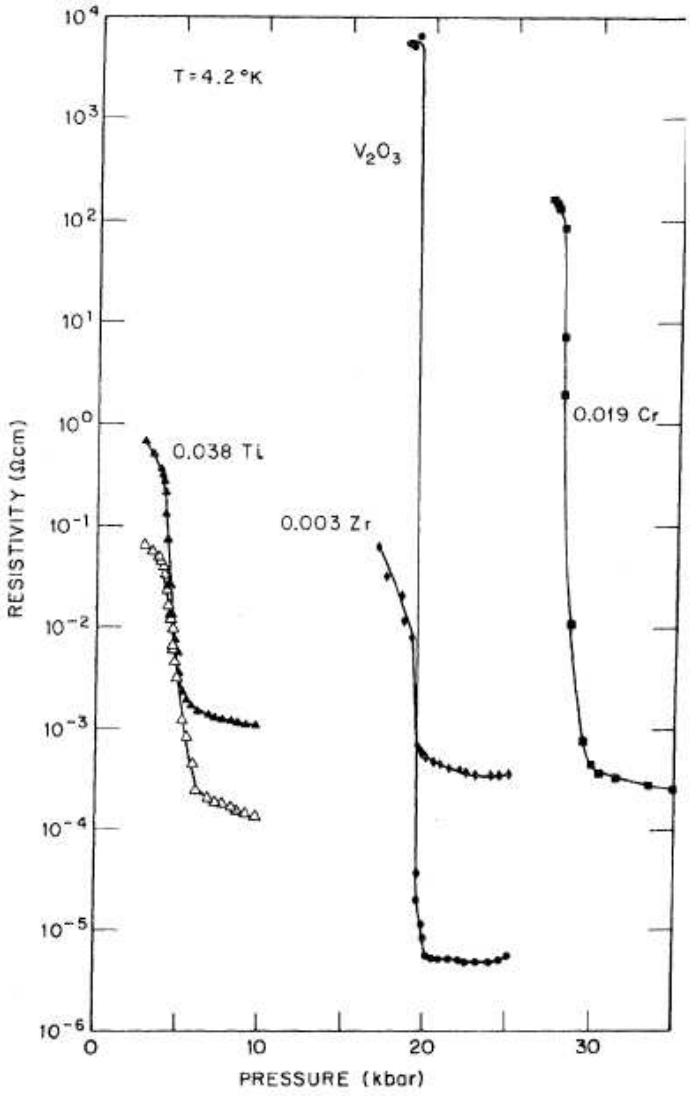






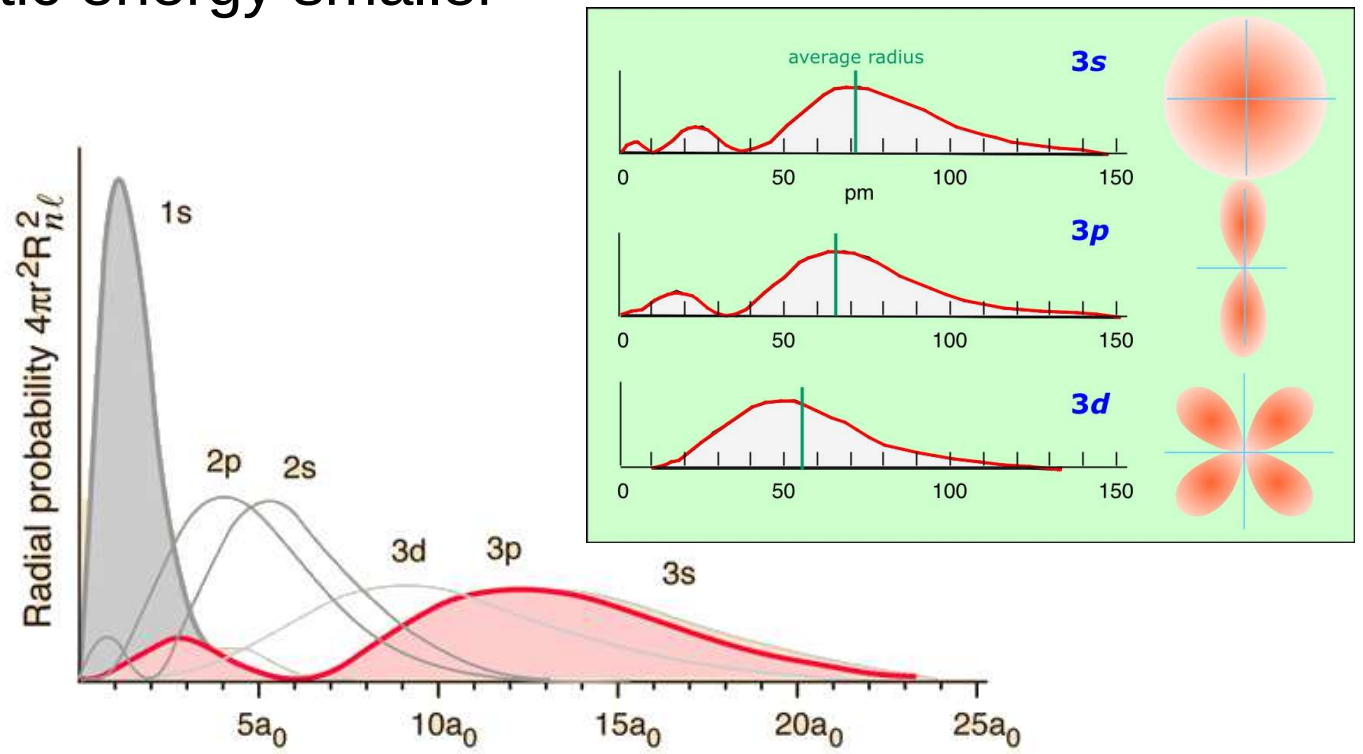
Ando'04

Band-width controlled driven Mott transition



Fact 4: orbitals matter!
Why 3d special?

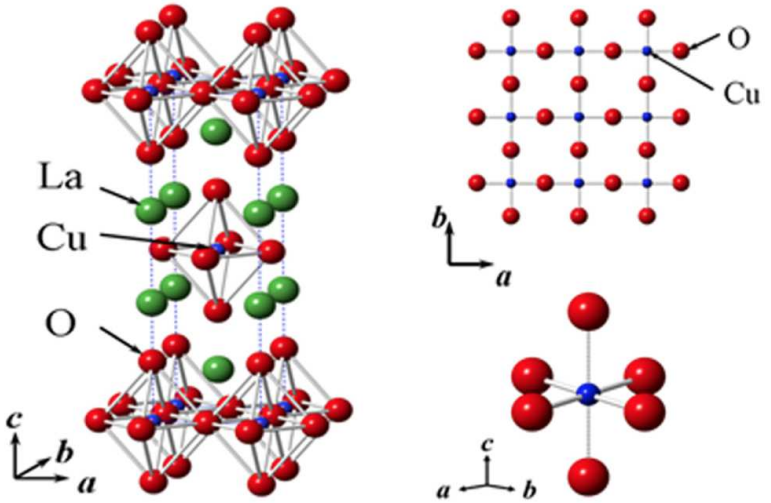
- 3d orbitals don't have nodes, reach further into core
- -> charge is screened less
- -> kinetic energy smaller



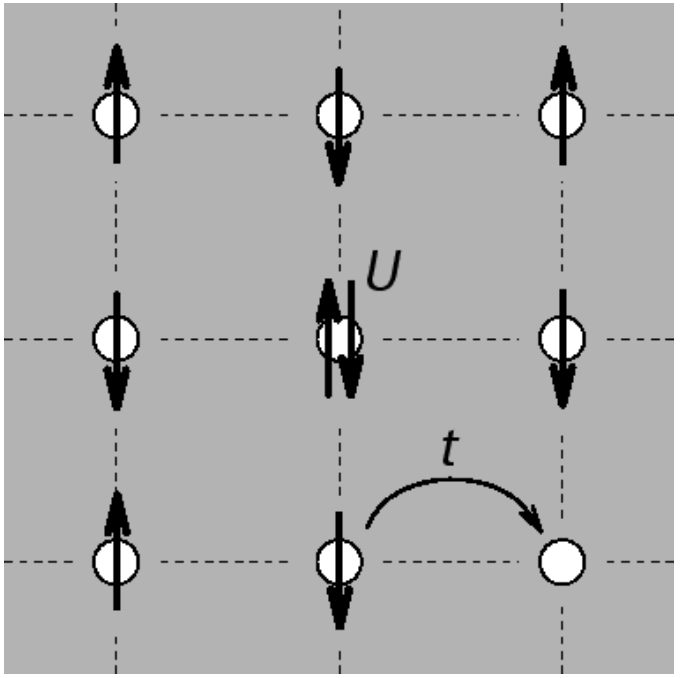
Values of interaction can also be calculated

- Atomic U_{at}(3d orbitals) ~$e^2/4\pi\varepsilon_0 r$ ~ 15 eV
- In oxides screened to U~5eV
- 4d orbitals ~2.5eV ; 5d orbitals 1.5eV

- One has an insulating state in a half-filled band
- Mott insulator
- Hubbard model: take band of x^2-y^2 orbitals and supplement it by Hubbard repulsion U
- LDA bands as a starting point
  for many body calculations



Hubbard model

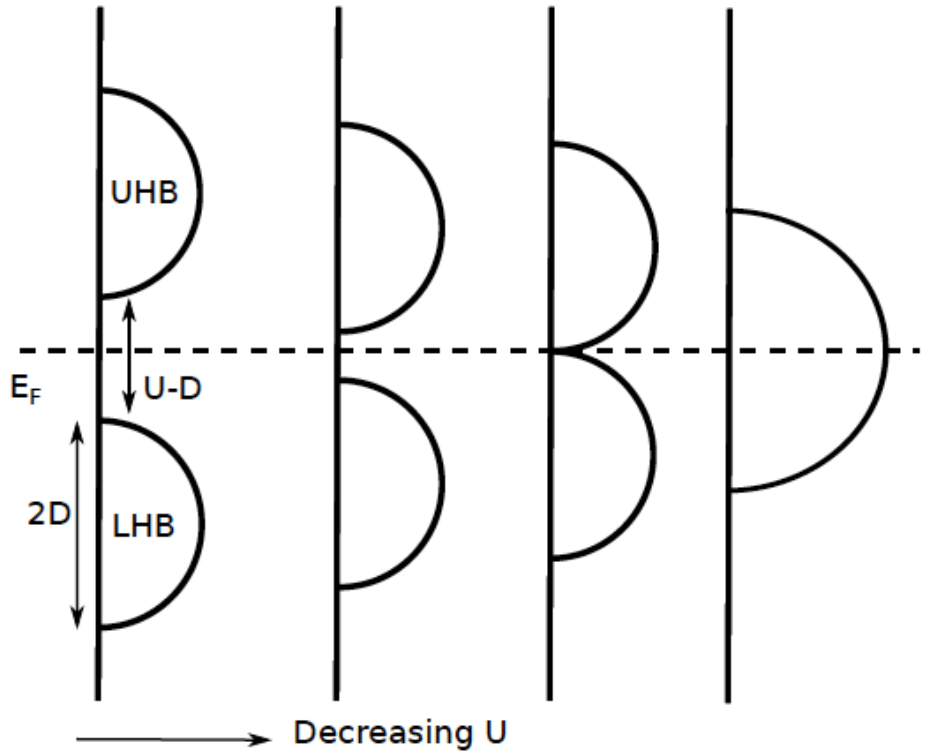


$$H = -t \sum_{<ij>\sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + \sum_{i} U n_{i\uparrow} n_{j\downarrow}$$

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{i} U n_{i\uparrow} n_{j\downarrow}$$

Mott picture of the Mott transition

- (atomic : blackboard)

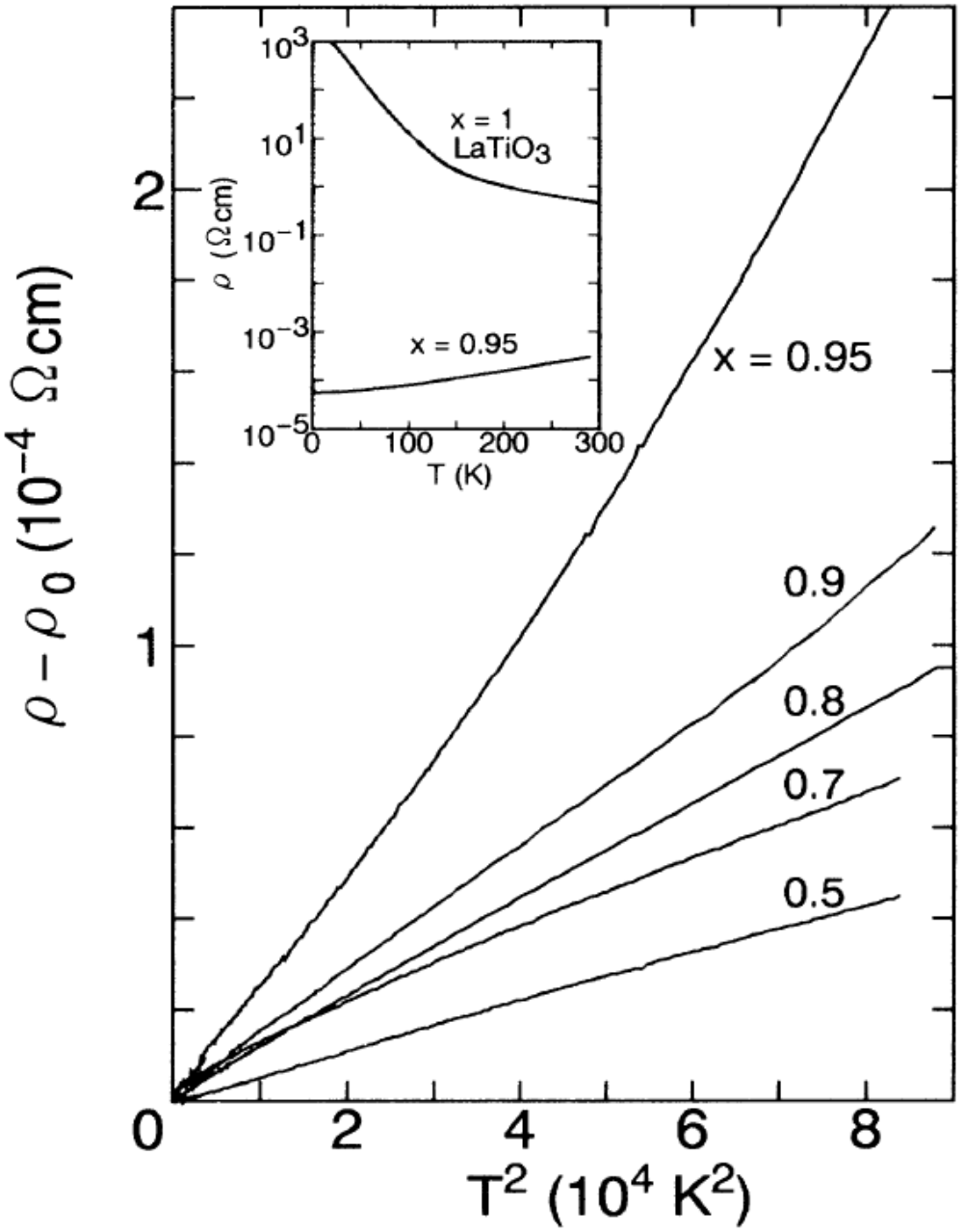


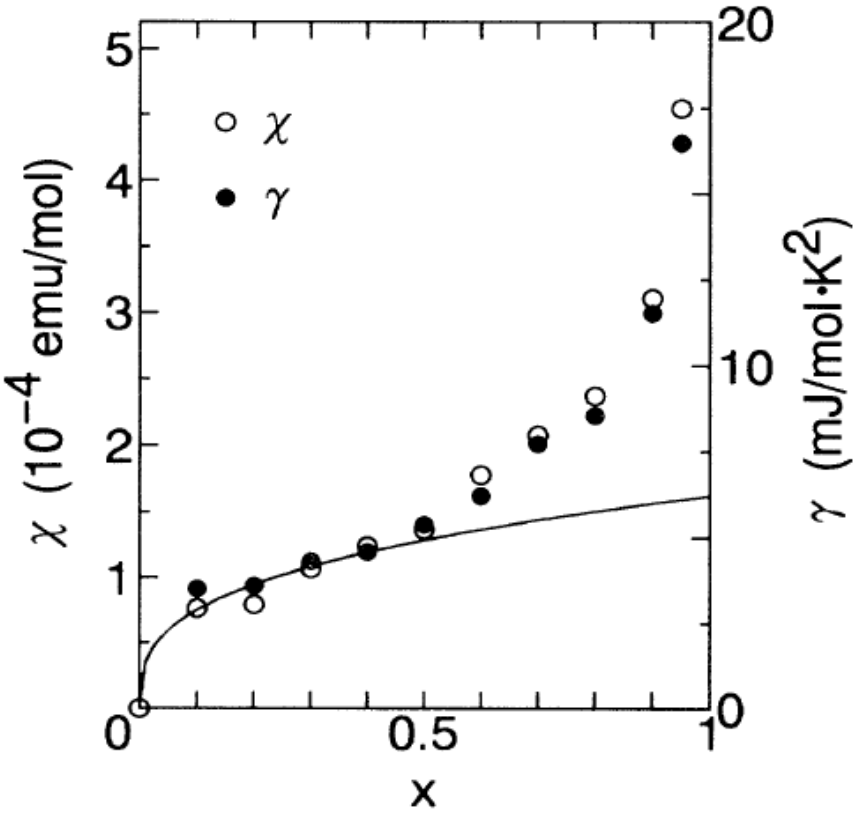
Fact 5 : Mott insulators are magnets

- localized electrons/site = spin
- Spins are coupled by exchange interaction -> Heisenberg model and magnetism
- Magnetic susceptibility rises as one cools down and magnetism arises

- Mott picture: adiabatic continuity with an insulator, but not with a metal:
- Does not described correctly doped Mott insulator

$Sr_{1-x}La_xTiO_3$

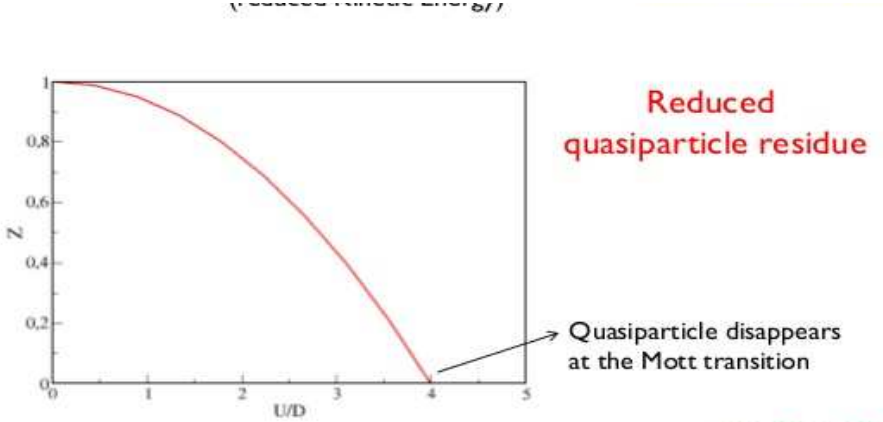


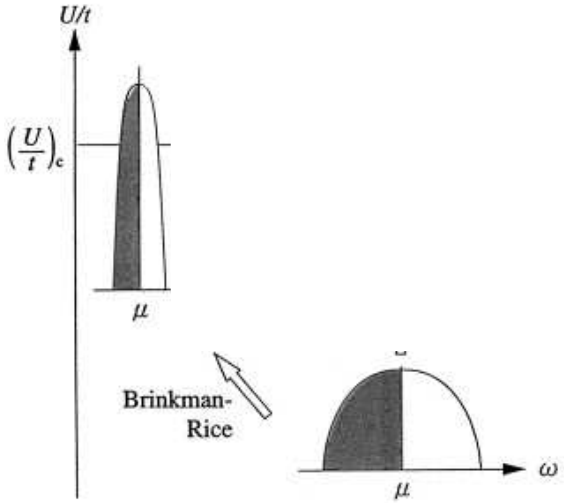


Brinkman-Rice picture

- Solving Hubbard model with a variational ansatz

$$|\Psi_G\rangle = g^{\hat{D}} |\mathrm{FG}\rangle$$
$$= \prod_{\boldsymbol{R}_i} [1 - (1-g)\hat{D}_i] |\mathrm{FG}\rangle ,$$

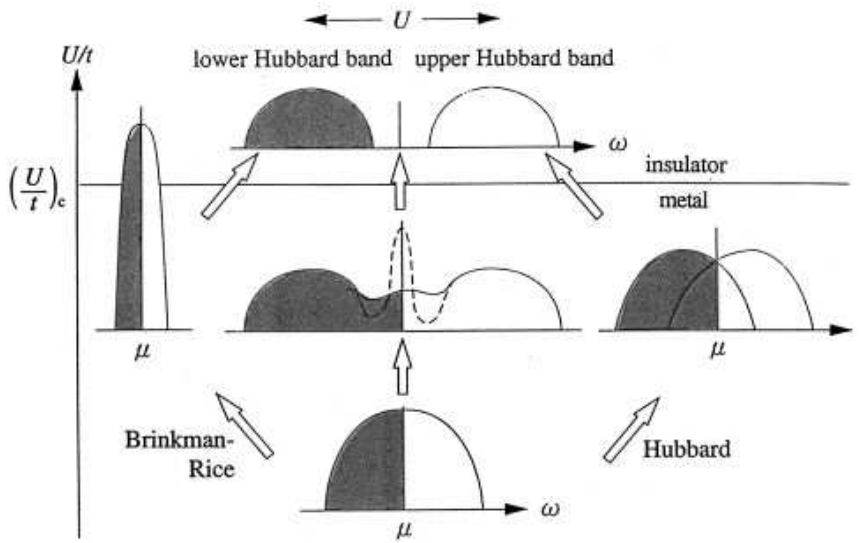


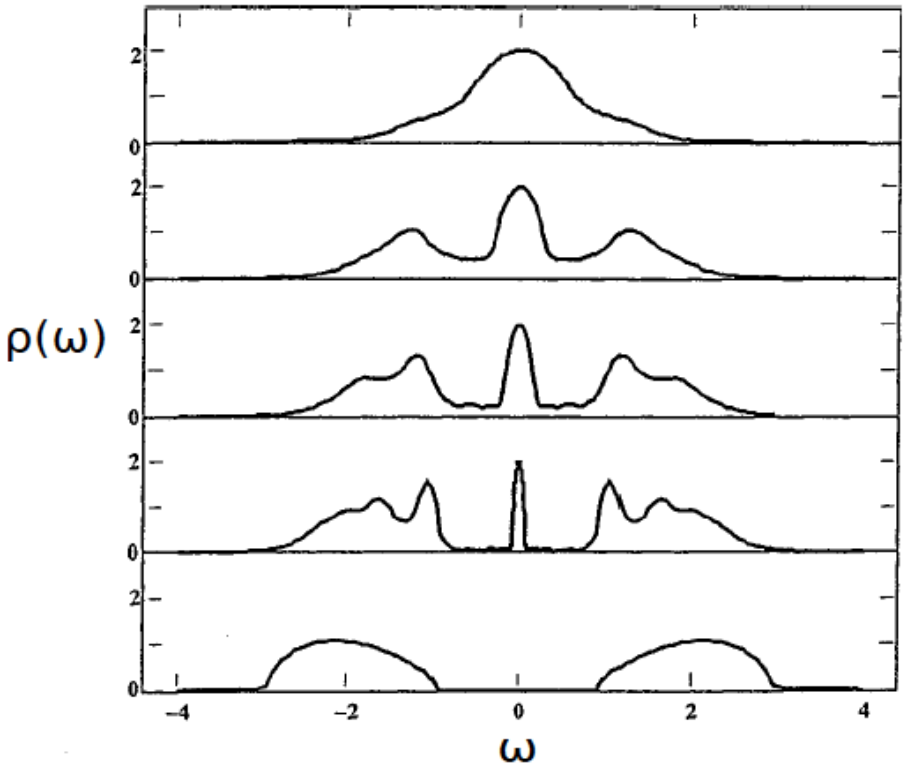


Doping driven: Z ~ δ

Fact 6: DOS of correlated metal consists of quasiparticle peak and Hubbard bands

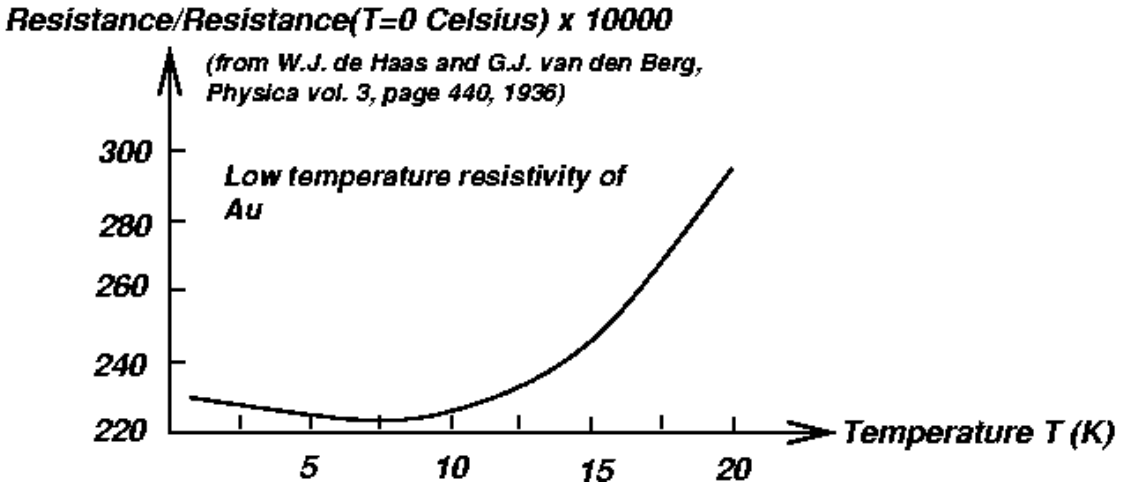
- Mott transition: on metallic side adiabatic continuity with a noninteracting gas (with reduced bandwidth by Z), on insulating with an atom.





DMFT

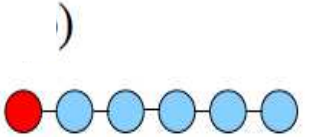
Kondo effect



Resistivity minimum:
associated to presence
of magnetic impurities

RG picture of Kondo: infrared slavery (like quarks)

- Anderson model -> Kondo model -> Kondo effect



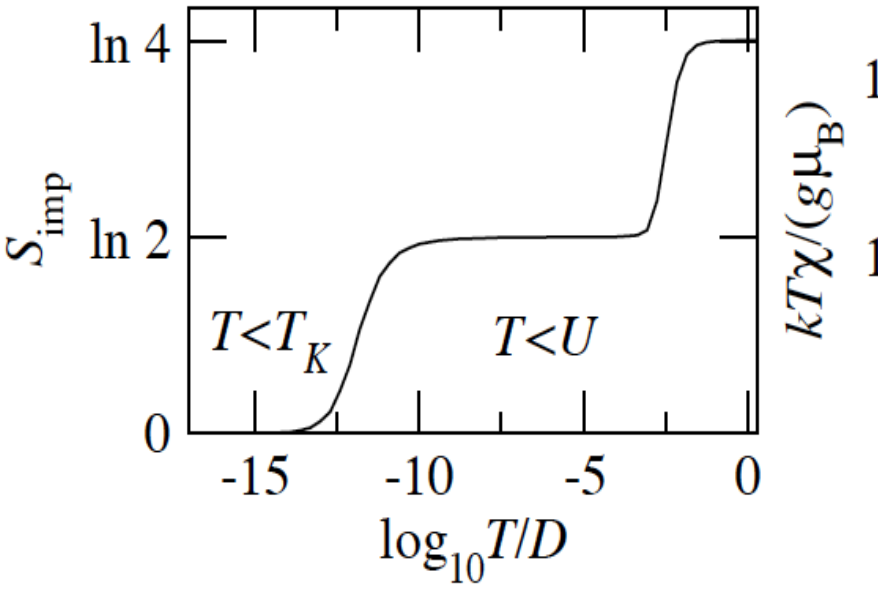
$$H = \sum_k \epsilon_k n_k + \sum_{k\sigma} \left(V_k c^\dagger_{k\sigma} d_\sigma + h.c. \right) + \epsilon n + U n_\uparrow n_\downarrow$$

$$H_{\text{Kondo}} = \sum_k \epsilon_k n_k + J \mathbf{S} \cdot \mathbf{s}$$

$$J(D') = \frac{J}{1/2 + J\rho \log(D'/D)}$$

$$T_K \sim D e^{-\frac{1}{2J\rho}}$$

NRG results

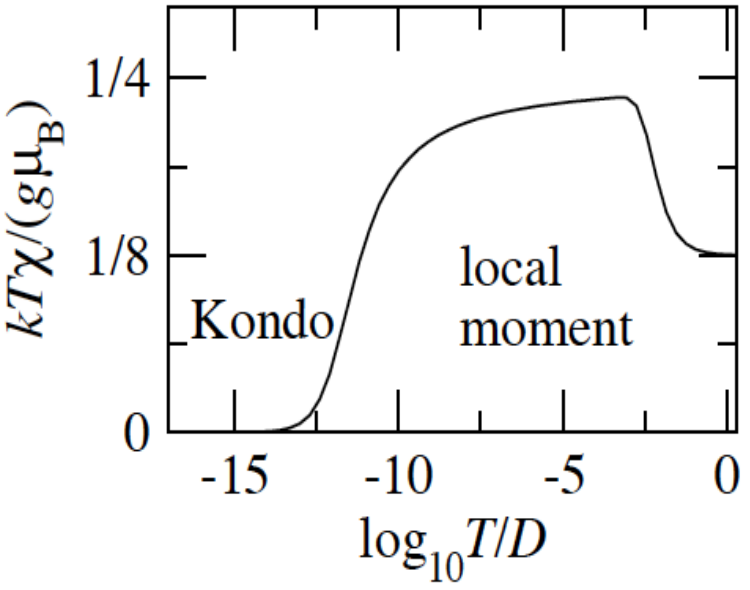


$$H = \sum_k \epsilon_k n_k + \sum_{k\sigma} \left(V_k c^\dagger_{k\sigma} d_\sigma + h.c. \right) + \epsilon n + U n_\uparrow n_\downarrow$$

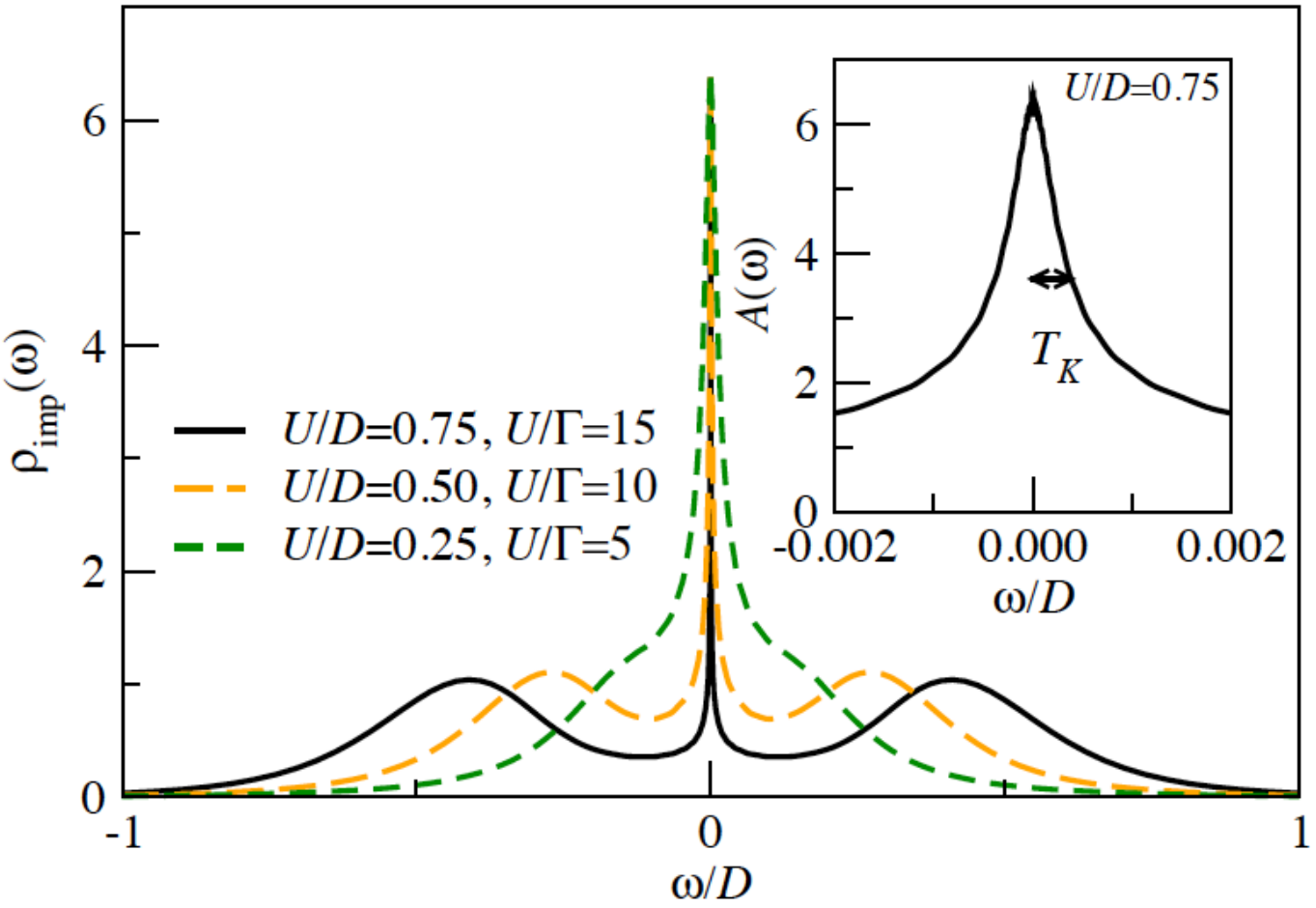
$$H_{\text{Kondo}} = \sum_k \epsilon_k n_k + J\mathbf{S}\cdot\mathbf{s}$$

$$J(D') = \frac{J}{1/2 + J\rho \log(D'/D)}$$

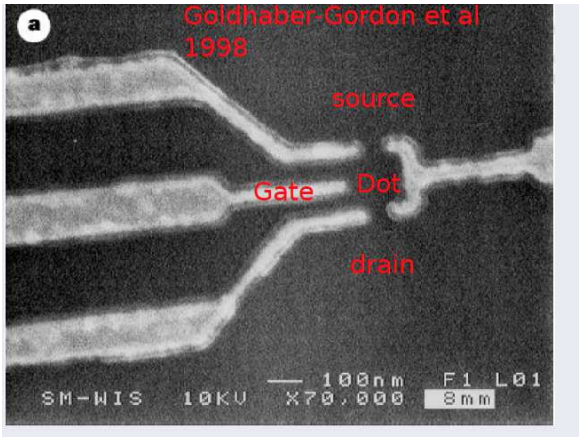
$$T_K \sim D e^{-\frac{1}{2J\rho}}$$

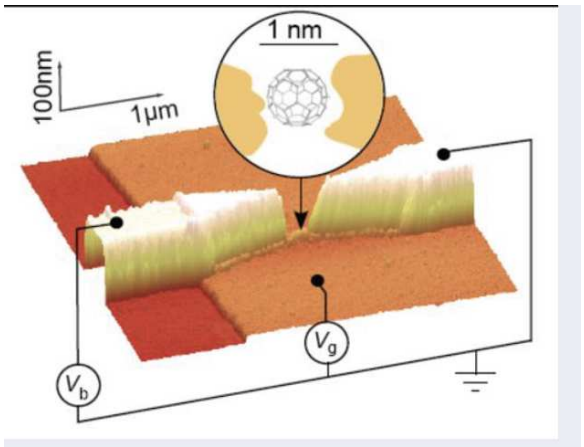


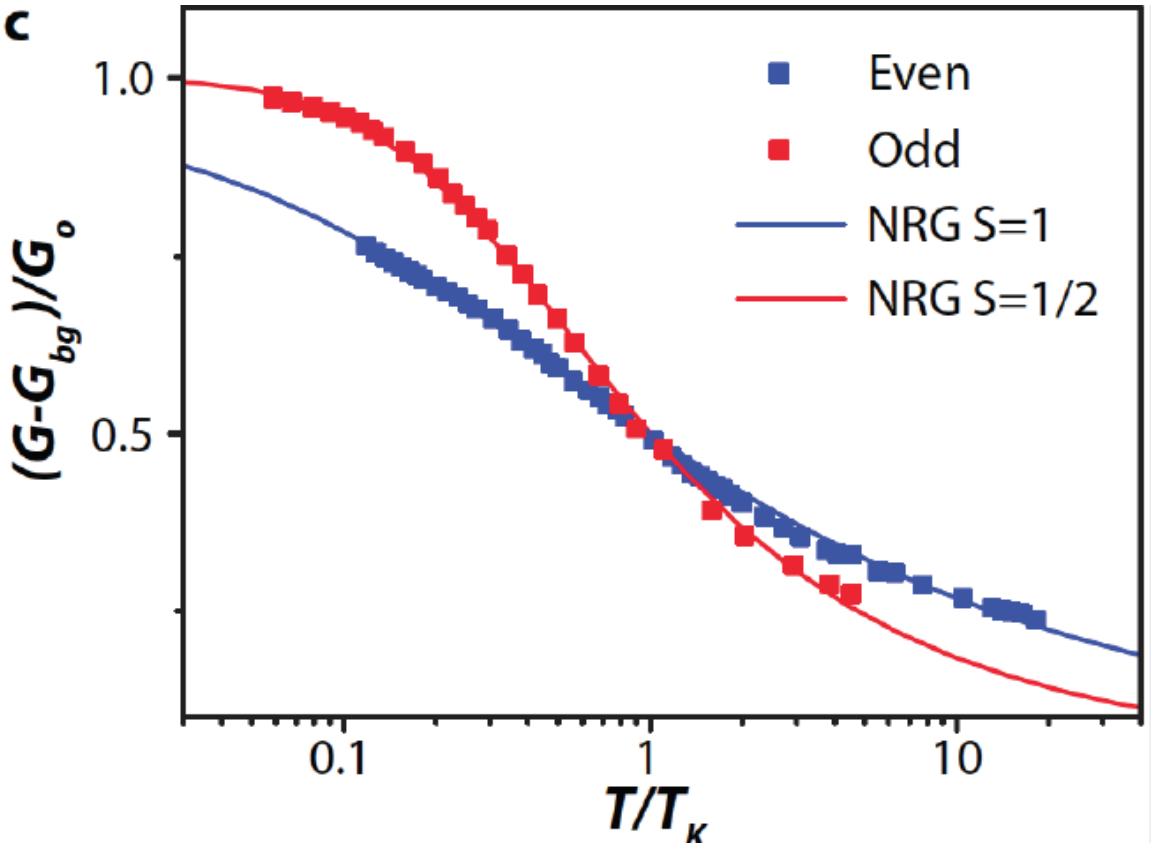
Spectral function of Anderson model

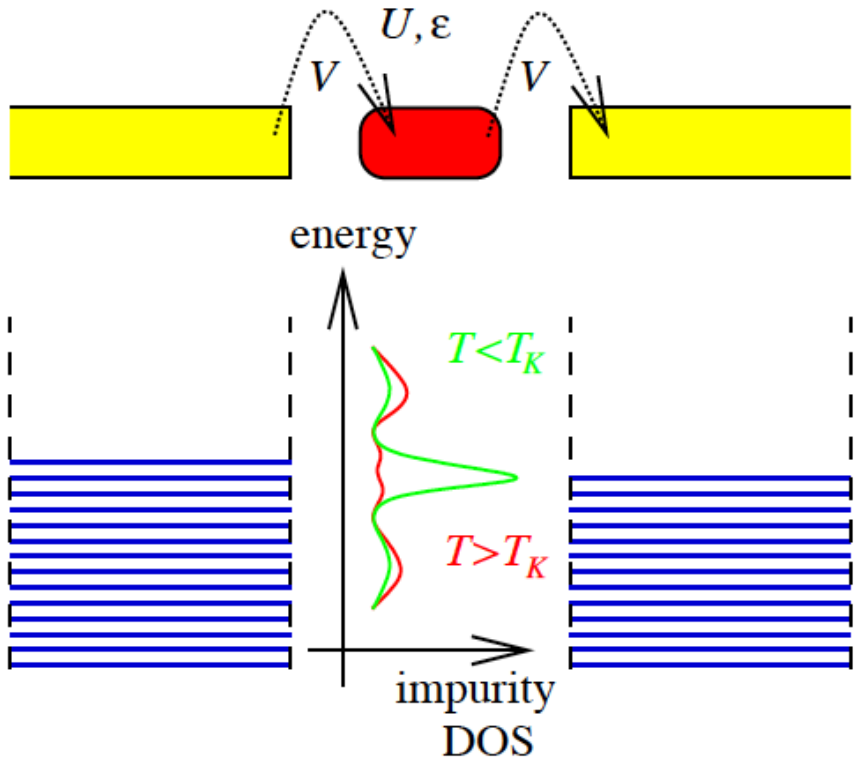


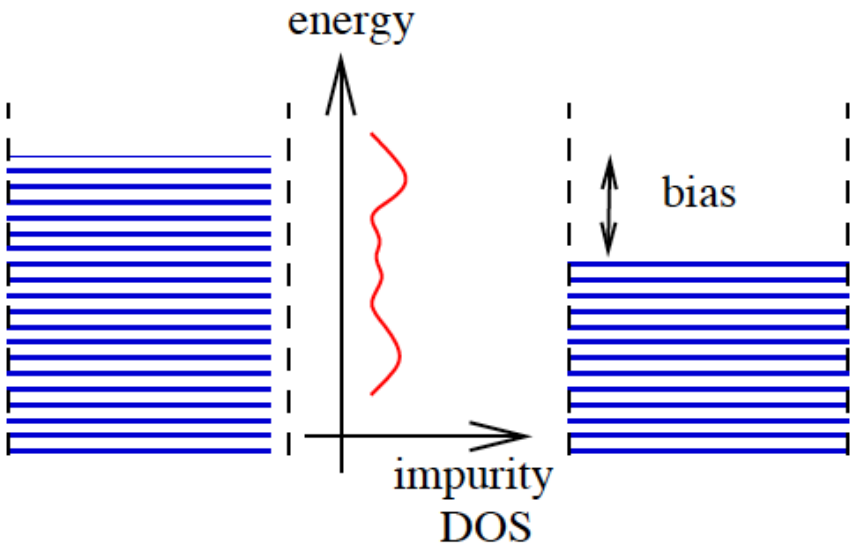
- Good numerics to calculate, tests in quantum dots and transmission through molecules



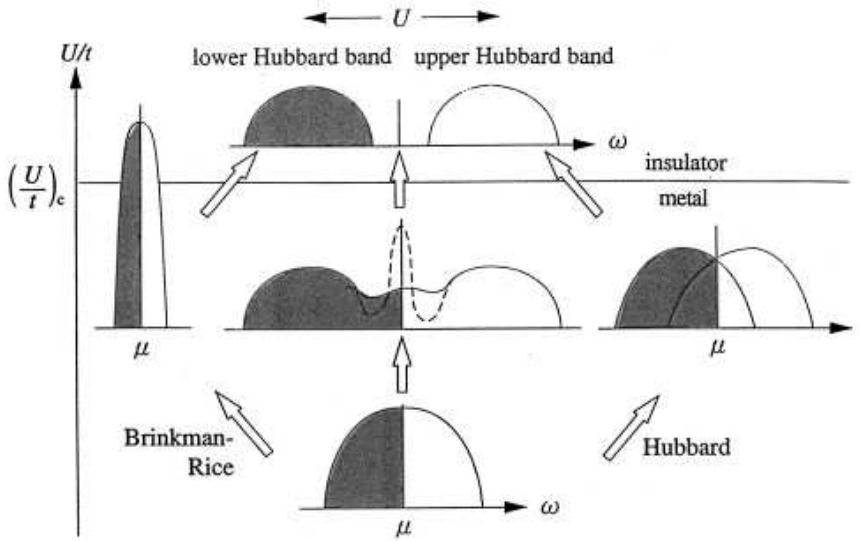




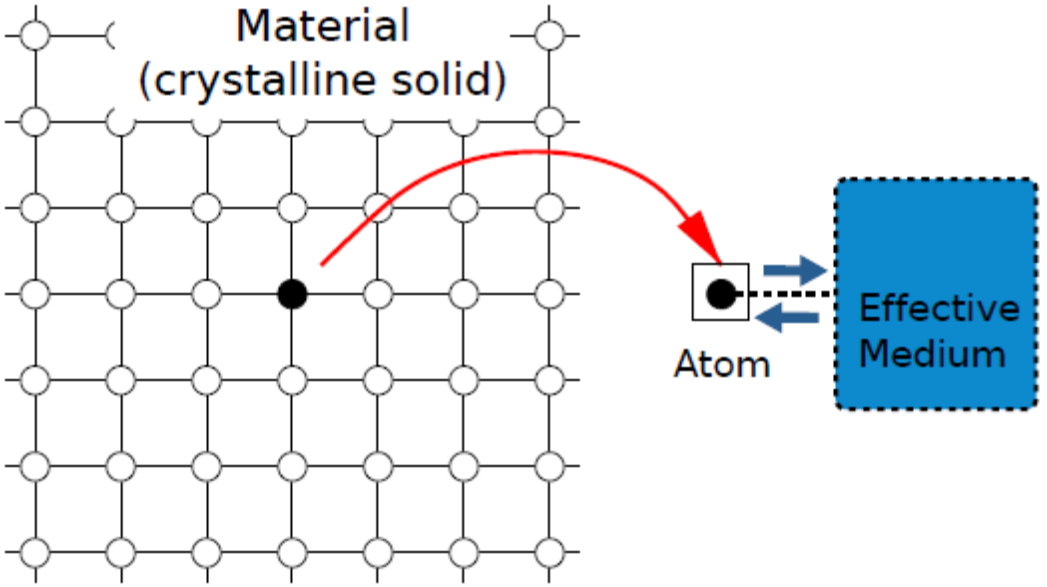

U, ε
V
V
energy
T<T_K
T>T_K
impurity
DOS


energy
bias
impurity
DOS

Back to bulk

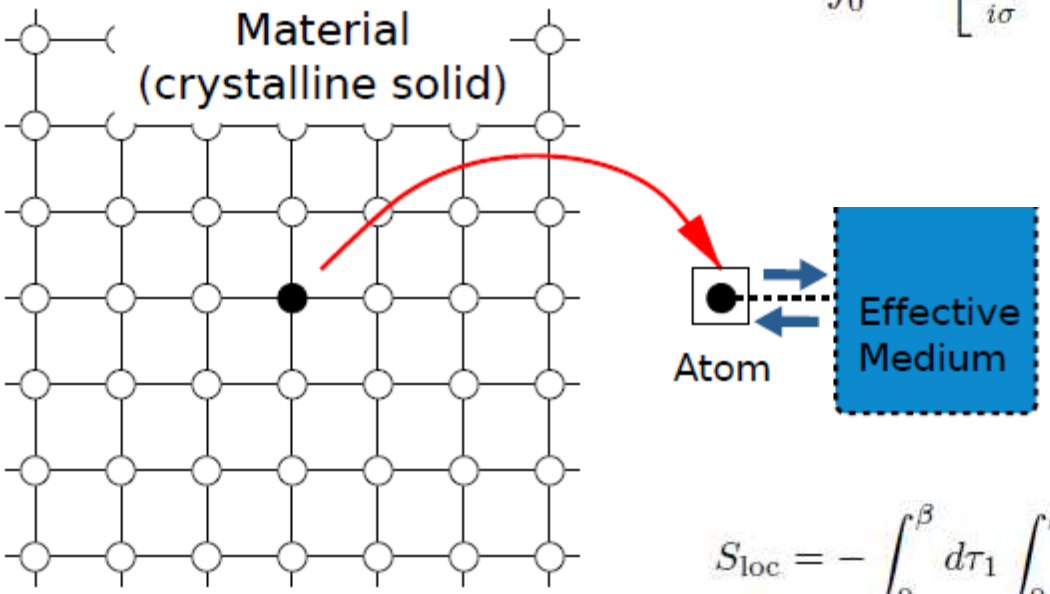

U
U/t
lower Hubbard band
upper Hubbard band
ω
insulator
metal
$\left(\frac{U}{t}\right)_c$
μ
μ
Brinkman-
Rice
Hubbard
ω
μ

Dynamical mean-field theory (DMFT)



Dynamical mean-field theory (DMFT)

$$S = \int_0^\beta d\tau \left[\sum_{i\sigma} c^*_{i\sigma}(\tau)(\frac{\partial}{\partial\tau} - \mu)c_{i\sigma}(\tau) + \sum_{ij\,\sigma} t_{ij} c^*_{i\sigma}(\tau) c_{j\sigma}(\tau) \right.$$
$$\left. + \sum_i U c^*_{i\uparrow}(\tau) c_{i\uparrow}(\tau) c^*_{i\downarrow}(\tau) c_{i\downarrow}(\tau) \right]$$



$$S_{\rm loc} = -\int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \sum_\sigma c^*_\sigma(\tau_1)\mathcal{G}^{-1}_\sigma(\tau_1 - \tau_2)c_\sigma(\tau_2)$$
$$+ U \int_0^\beta d\tau c^*_\uparrow(\tau) c_\uparrow(\tau) c^*_\downarrow(\tau) c_\downarrow(\tau)$$

In limit of large d, self energy becomes local (independent of k)

$$G_{\boldsymbol{k}\,\sigma}(i\omega_n) = \frac{1}{i\omega_n - \epsilon_{\boldsymbol{k}} + \mu - \Sigma_\sigma(i\omega_n)}.$$

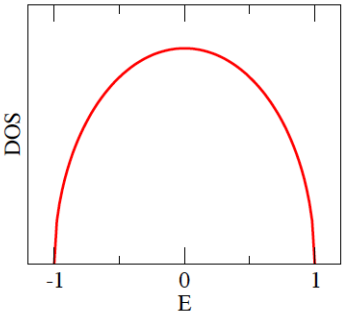
Georges et al. RMP'96

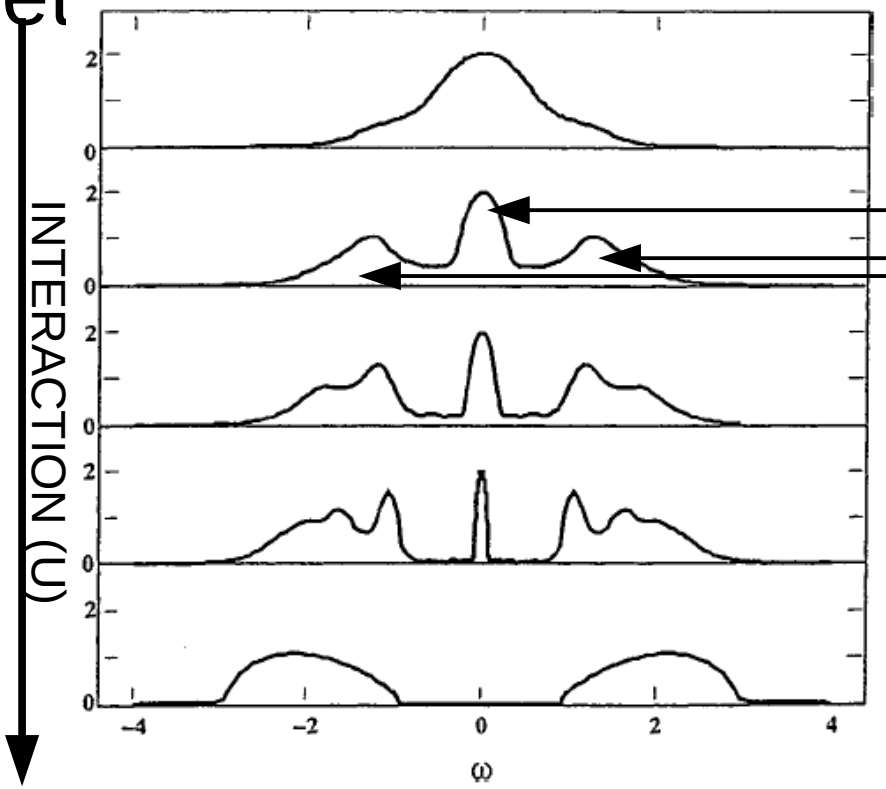
DMFT results

- single orbital Hubbard moc
- Metal-insulator transition

$$\mathcal{H}_t = -t \sum_{\langle ij \rangle} (c^\dagger_{i\sigma} c_{j\sigma} + \text{H.c.}),$$

$$\mathcal{H}_U = U \sum_i (n_{i\uparrow} - \tfrac{1}{2})(n_{i\downarrow} - \tfrac{1}{2}),$$



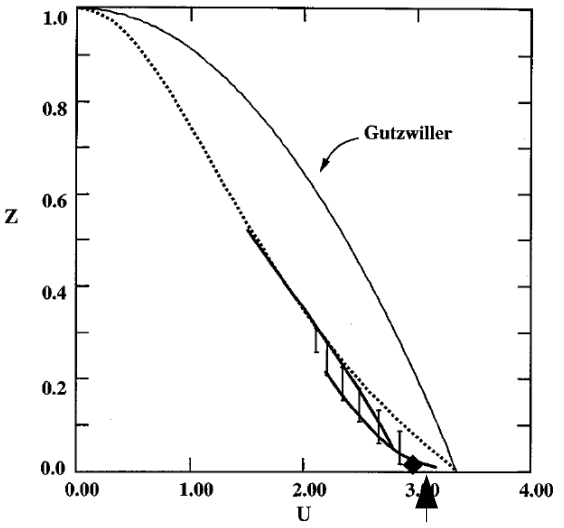


BAND-like

Quasiparticle band (weight Z)

Atomic satellites (weight 1-Z)

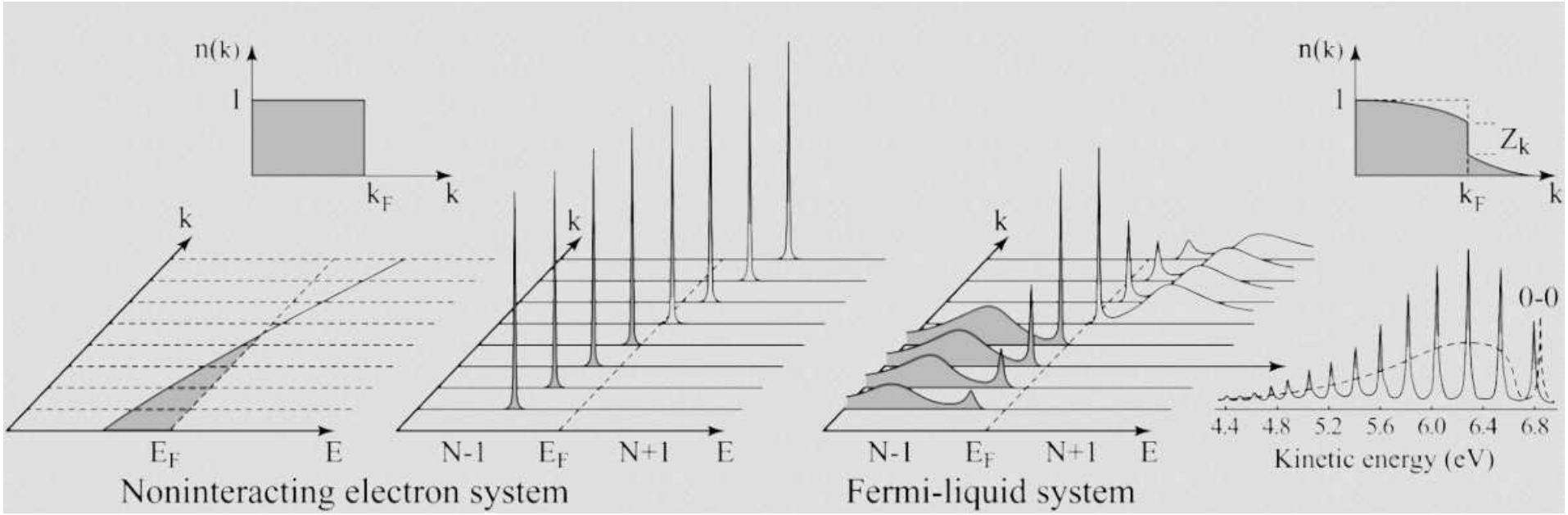
ATOMIC-like



U_c

Georges, Kotliar,Krauth,Rozenberg,RMP'96

ARPES

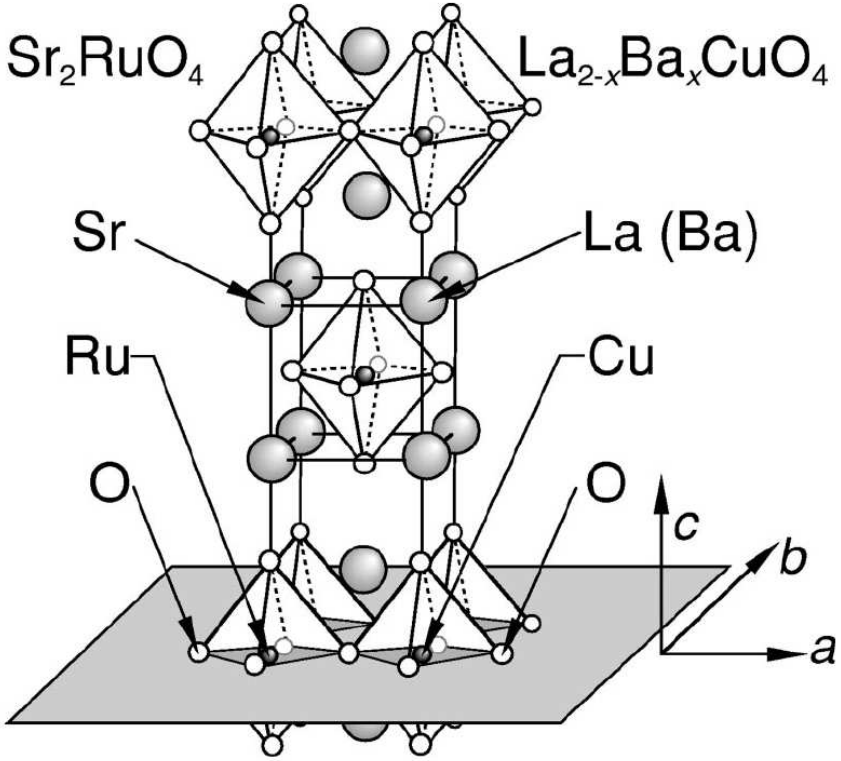

n(k)
1
k_F
k
k
E_F
E
Noninteracting electron system
k
N-1
E_F
N+1
E
n(k)
1
Z_k
k_F
k
k
N-1
E_F
N+1
E
Fermi-liquid system
0-0
4.4
4.8
5.2
5.6
6.0
6.4
6.8
Kinetic energy (eV)

LDA+DMFT

- Take LDA bands
- Describe interactions with DMFT (substracting mean-free part included in LDA).

Sr_2RuO_4

Sr_2RuO_4 properties



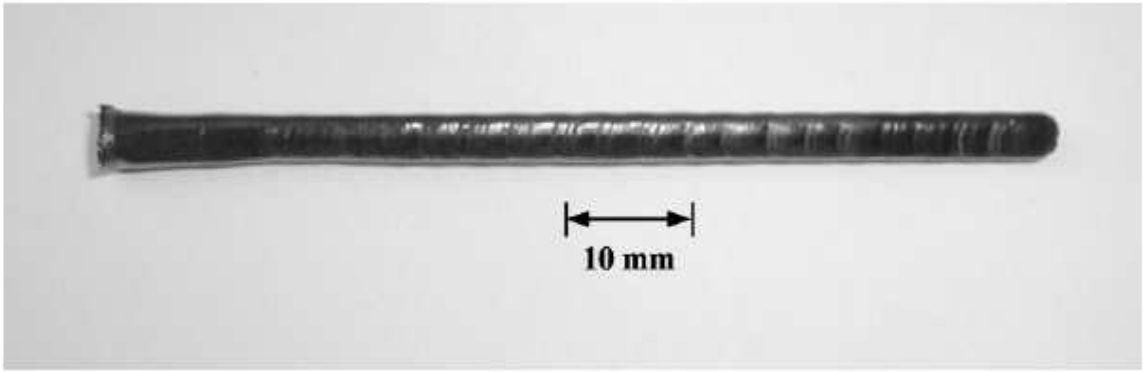
p-wave supercond.

T_c ~2K Maeno et al., Nature'94

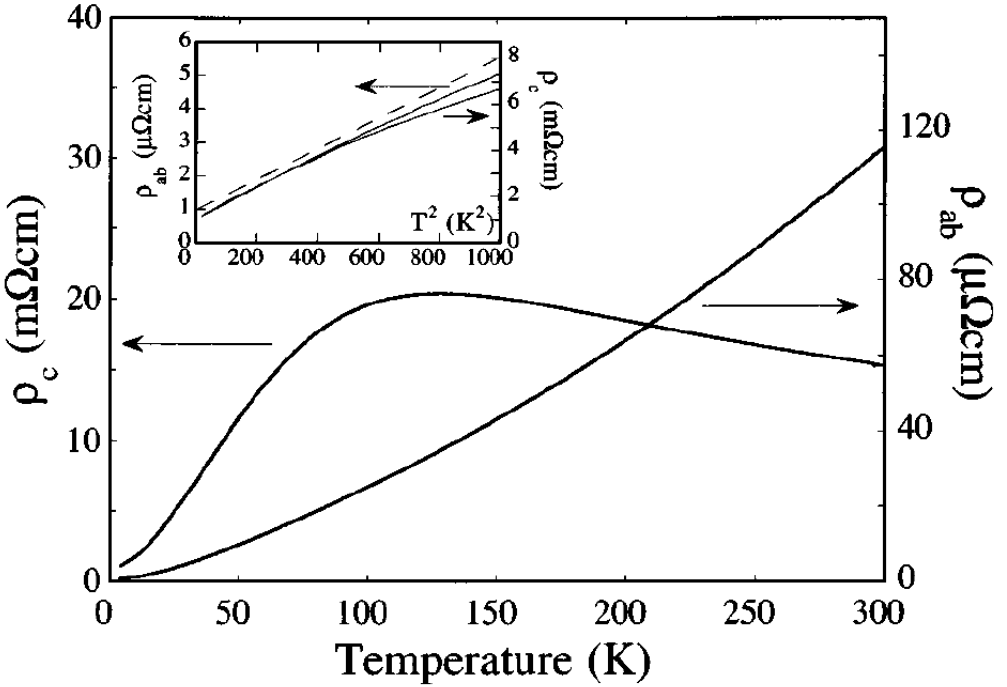
Rice and Sigrist , J.Phys.CM'95

Correlated metal: Fermi liquid, (m*/m~4)

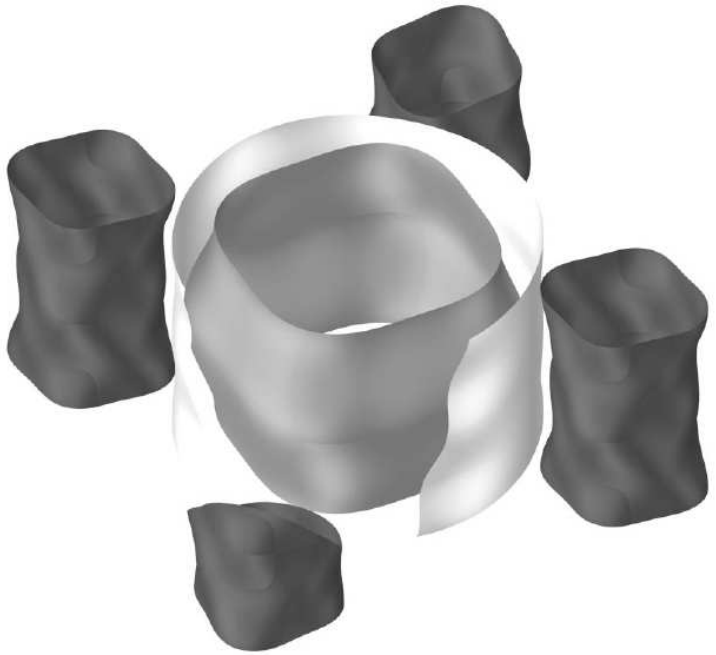
4 el. in Ru t_{2g} orbitals



Fermi liquid behavior below 20K
Fact 7: In Fermi liquids $\rho \sim T^2$

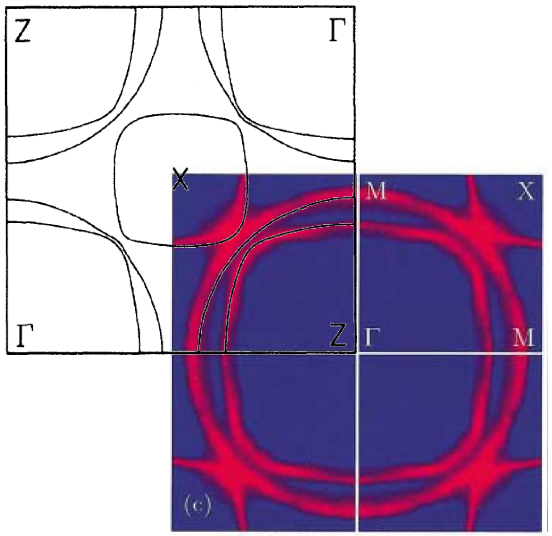


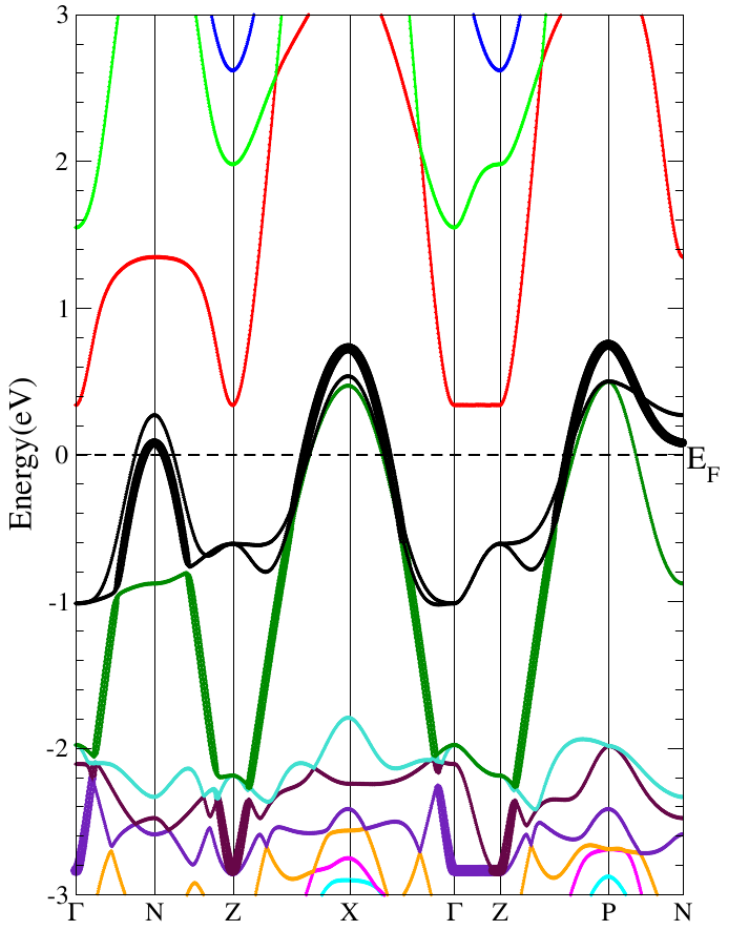
Fermi surfaces



Sr_2RuO_4: el. structure

LDA and experiments give very similar Fermi surfaces. Shape can be understood (blackboard)





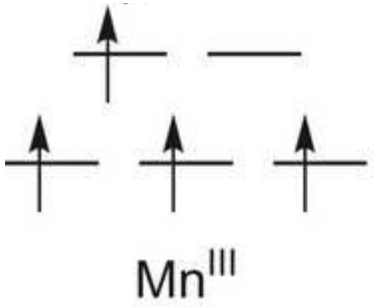
Strong renormalization.
~ 4

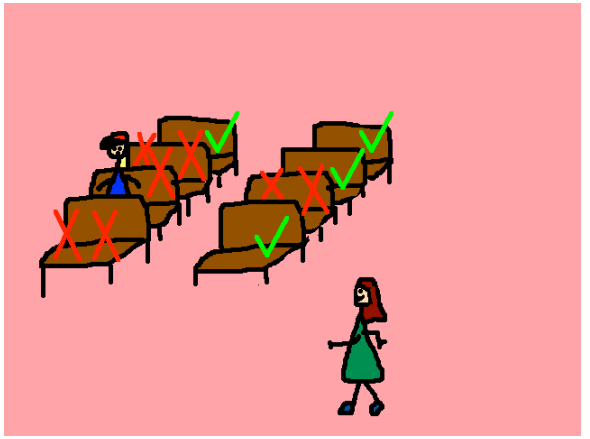
Band-width 3eV,
U~2.5eV.

($SrVO_3$ had renormalization 2, but U~5eV)

Going multi-orbital: new ingredient:Hund's rule coupling

- "Bus seat rule"



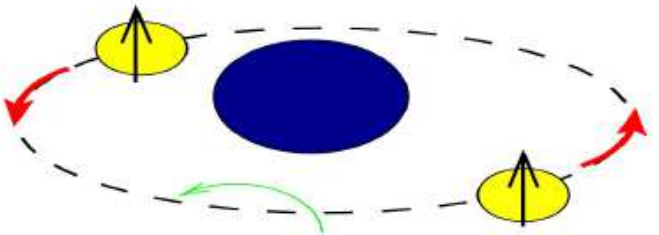


- Maximize 1st spin, 2nd orbital momentum, 3rd

$$U = \int d\mathbf{r}d\mathbf{r}' |\phi_m(\mathbf{r})|^2 V_c(\mathbf{r},\mathbf{r}') |\phi_m(\mathbf{r}')|^2,$$
$$U' = \int d\mathbf{r}d\mathbf{r}' |\phi_m(\mathbf{r})|^2 V_c(\mathbf{r},\mathbf{r}') |\phi_{m'}(\mathbf{r}')|^2, \quad \text{and}$$
$$J = \int d\mathbf{r}d\mathbf{r}' \phi_m(\mathbf{r})\phi_{m'}(\mathbf{r}) V_c(\mathbf{r},\mathbf{r}') \phi_m(\mathbf{r}')\phi_{m'}(\mathbf{r}').$$

$$H_K = U \sum_m \hat{n}_{m\uparrow}\hat{n}_{m\downarrow} + U' \sum_{m\neq m'} \hat{n}_{m\uparrow}\hat{n}_{m'\downarrow} + (U'-J) \sum_{m<m',\sigma} \hat{n}_{m\sigma}\hat{n}_{m'\sigma} +$$
$$-J \sum_{m\neq m'} d^\dagger_{m\uparrow} d_{m\downarrow} d^\dagger_{m'\downarrow} d_{m'\uparrow} + J \sum_{m\neq m'} d^\dagger_{m\uparrow} d^\dagger_{m\downarrow} d_{m'\downarrow} d_{m'\uparrow}.$$

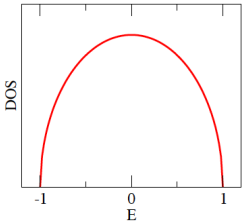
- Coulomb interaction: $V_c = 1/(r-r')$
- exchange



$$H_{t_{2g}} = (U-3J)\frac{\hat{N}(\hat{N}-1)}{2} - 2J\vec{S}^2 - \frac{J}{2}\vec{L}^2.$$

DMFT results: 3orbital Kanamori model

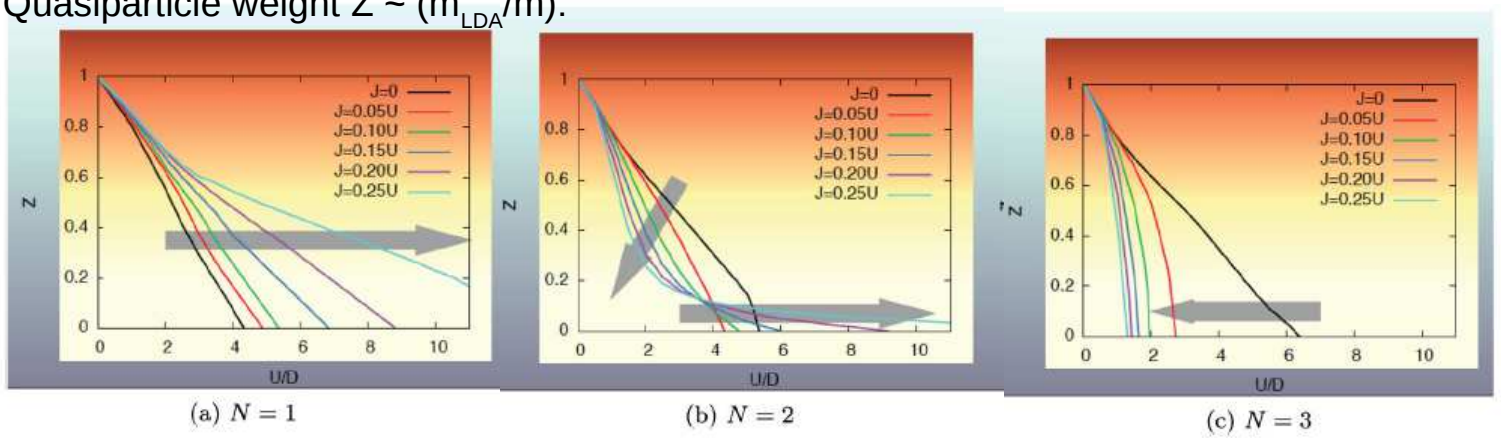
- $H=H_{band}+H_{atom}$



$$H_{t_{2g}} = (U-3J)\frac{\hat{N}(\hat{N}-1)}{2} - 2J\vec{S}^2 - \frac{J}{2}\vec{L}^2$$

- Large and filling-dependent effects of J!

Quasiparticle weight Z ~ (m_{LDA}/m):



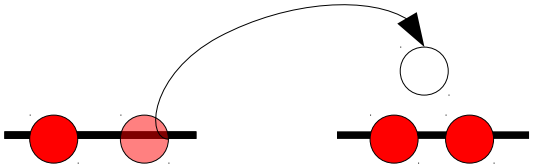
- 1. shift of U_c 2.additional effect on correlations
- At N=2, correlated state (a "Hund's metal") far from Mott insulator (for physical values of J)

L.de Medici, JM, and A. Georges,PRL'11

Why? : Two effects of J
1st modified atomic charge gap

- Effective interaction

  U_{eff}=E(N+1)+E(N-1) – 2 E(N)



  - U-3J away from half-filling (U_{eff} **diminished** by J)

  - U+(M-1)J at half filling (U_{eff} **increased** by J)

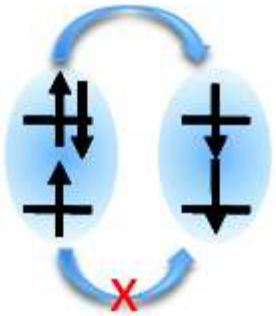
- Slater all d-states Hamiltonian (# of orbs. M=5)

	Effective Coulomb interaction U_{eff} for Hund's rule ground-state				
	Full Hamiltonian		Simple	Kanamori	Kanamori mean field
d^1	$F^0-\frac{8}{49}F^2-\frac{9}{441}F^4$	U_0-J_H-C	U_0-J_H	U'- J	U'- J
d^2	$F^0+\frac{1}{49}F^2-\frac{54}{441}F^4$	U_0-J_H+C	U_0-J_H	U'- J	U'- J
d^3	$F^0+\frac{1}{49}F^2-\frac{54}{441}F^4$	U_0-J_H+C	U_0-J_H	U'- J	U'- J
d^4	$F^0-\frac{8}{49}F^2-\frac{9}{441}F^4$	U_0-J_H-C	U_0-J_H	U'- J	U'- J
d^5	$F^0+\frac{14}{49}F^2+\frac{126}{441}F^4$	U_0+4J_H	U_0+4J_H	U+4J	U+4J
d^6	$F^0-\frac{8}{49}F^2-\frac{9}{441}F^4$	U_0-J_H-C	U_0-J_H	U'- J	U'- J
d^7	$F^0+\frac{1}{49}F^2-\frac{54}{441}F^4$	U_0-J_H+C	U_0-J_H	U'- J	U'- J
d^8	$F^0+\frac{1}{49}F^2-\frac{54}{441}F^4$	U_0-J_H+C	U_0-J_H	U'- J	U'- J
d^9	$F^0-\frac{8}{49}F^2-\frac{9}{441}F^4$	U_0			

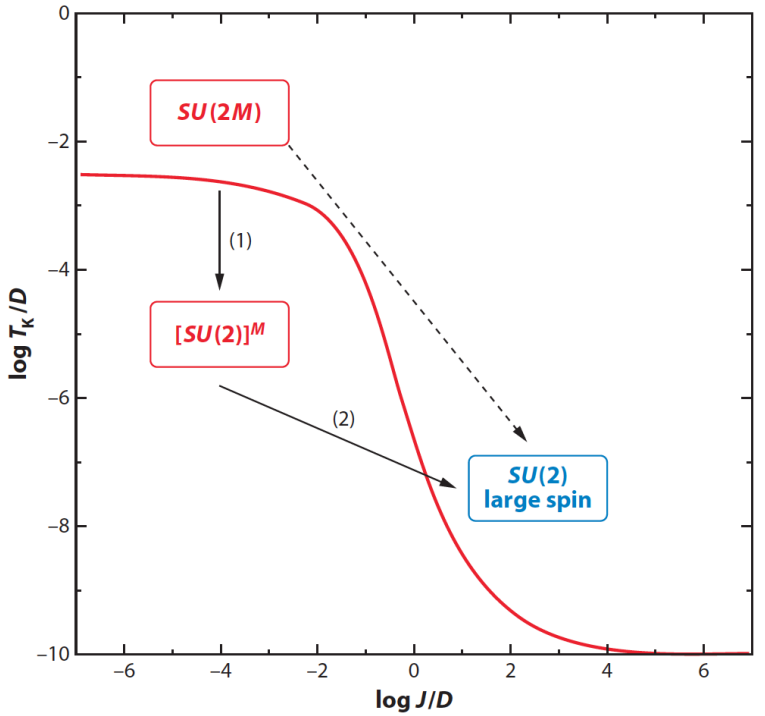
D.Van der Marel and G.Sawatzky PRB 37 (1988) 10674 [VdMS]

2nd effect: J suppresses tunneling

- J lowers atomic degeneracy, blocks orbital fluctuations, hence prohibits some of the hopping
- Example: Create a charge excitation in half-filled two orbital problem. J allows it to move only in one of two possible ways.



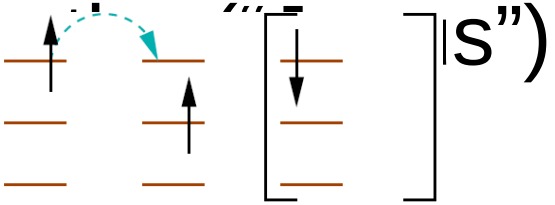
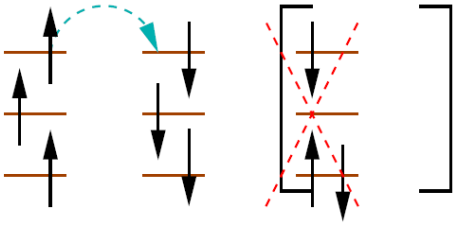
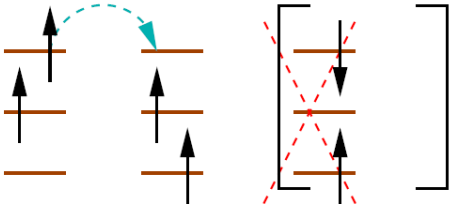
- Suppression of tunneling → Kondo temperature known for a long time in Kondo problem



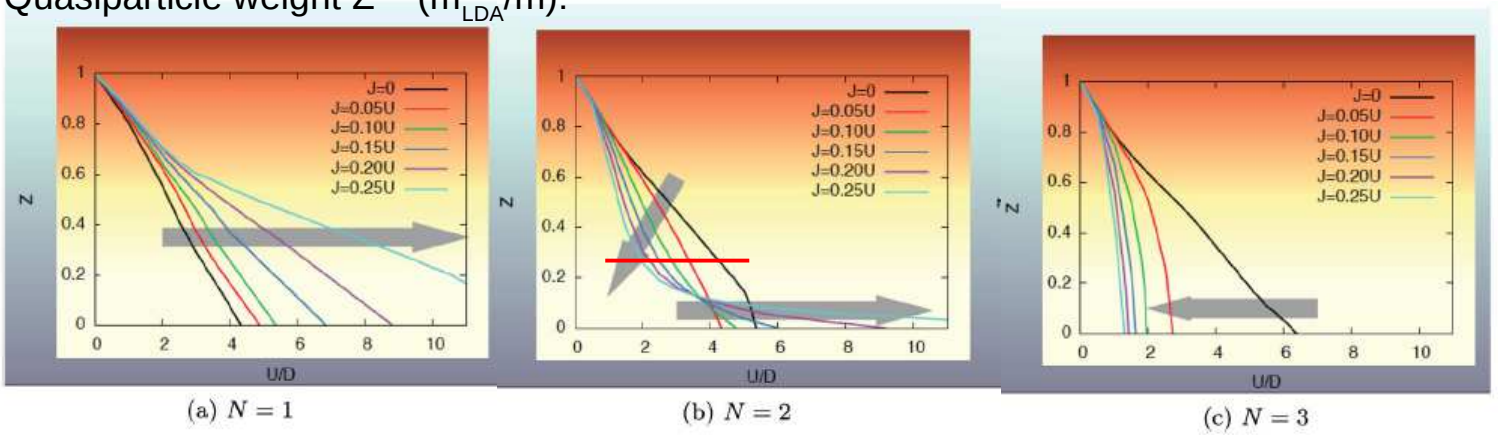
Yanase et al, J.Phys.Soc.Jpn. '97

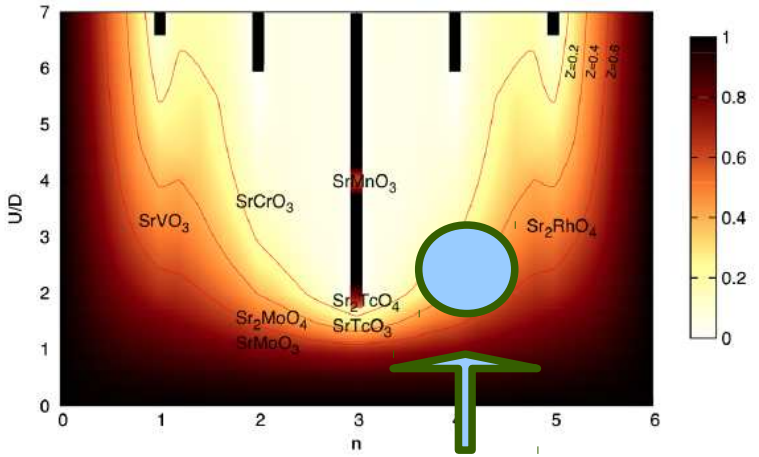
(Schrieffer 1967, Okada, Yosida 1973, Jayaprakash et al, 1981, Nevidomskiy-Coleman 2009).

A localized perspective

	single occ.	half-filled	
			
atomic deg.	6 (unchanged by J)	20 (supp → 4 by J)	15 (supp → 9 by J)
E of charge excit	diminished	increased	diminished
drives	metallic	insulating	incoherent metallic

Quasiparticle weight Z ~ (m_{LDA}/m):



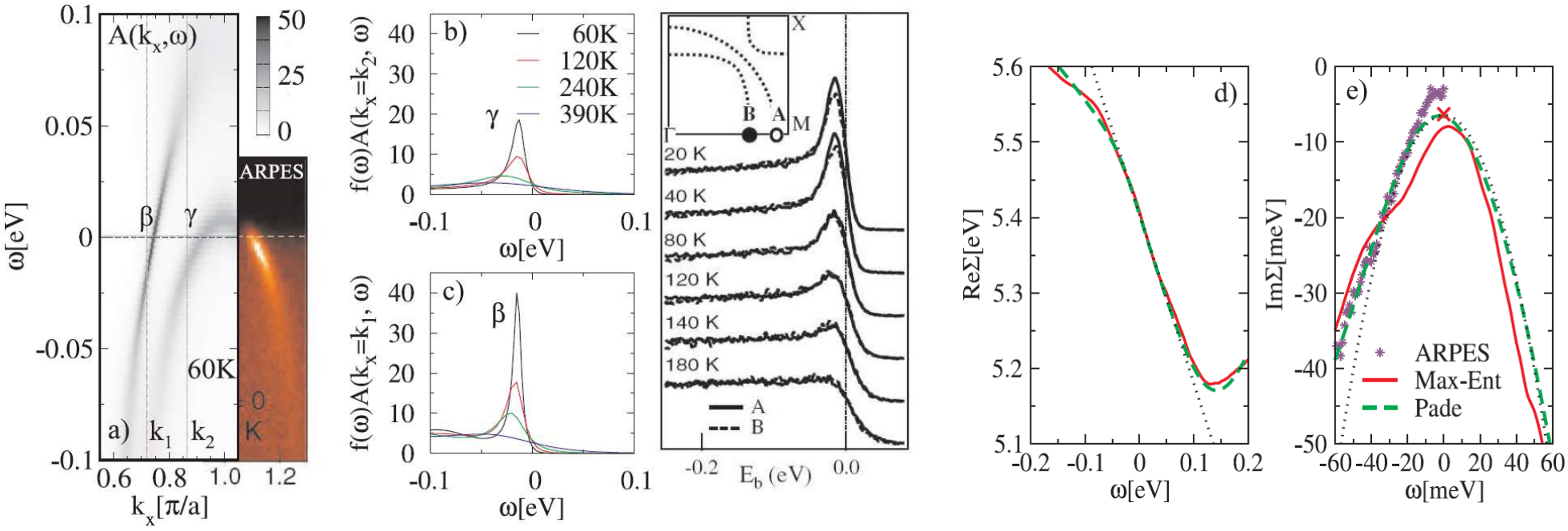


Correlated state, far from Mott, ruthenates and pnictides.

L.de Medici, JM, and A. Georges,PRL'11

Coherence-incoherence crossover in Sr_2RuO_4

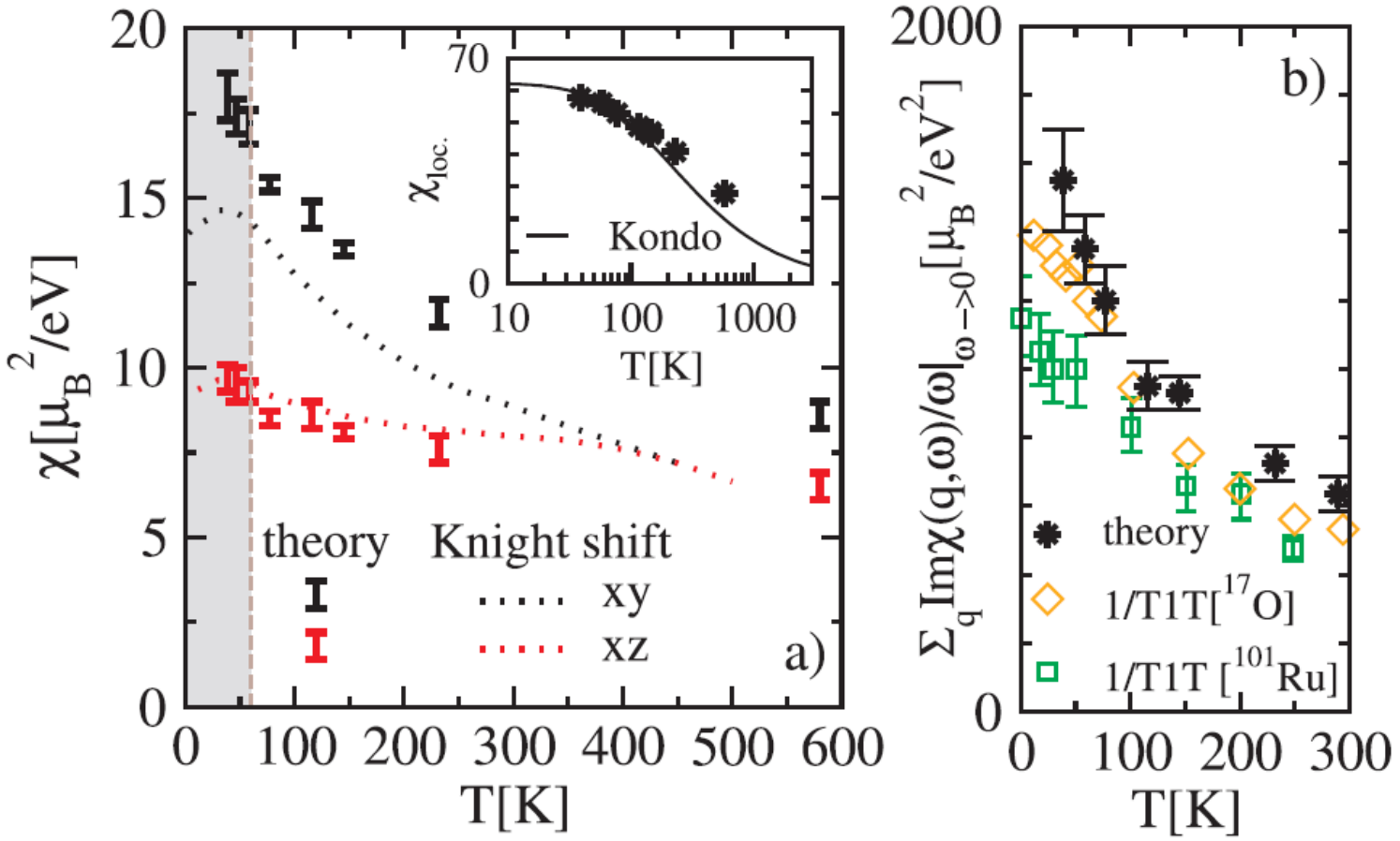
ARPES



Shen et al.,PRL'07 Wang et al.,PRL'04 Ingle et al.,PRB'05

Magnetic response

Pauli susceptibilty
$1/T_1 \sim T$

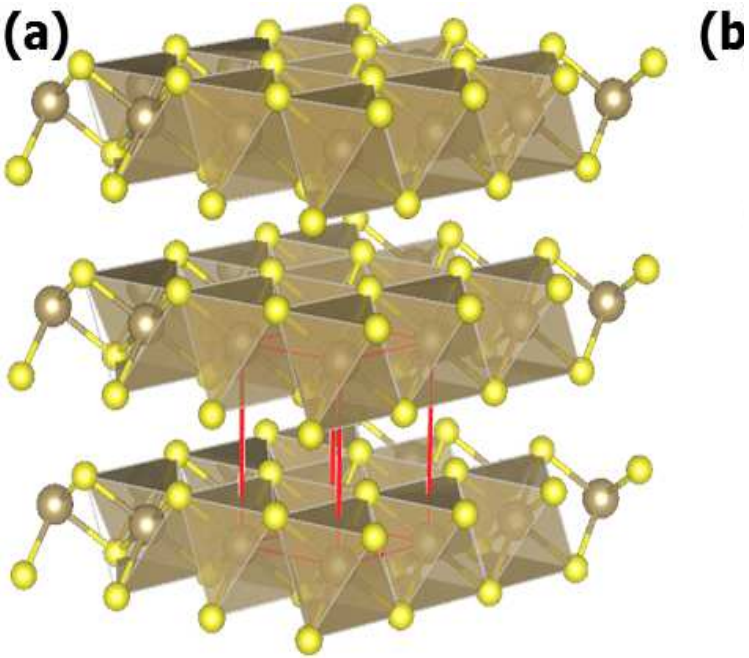


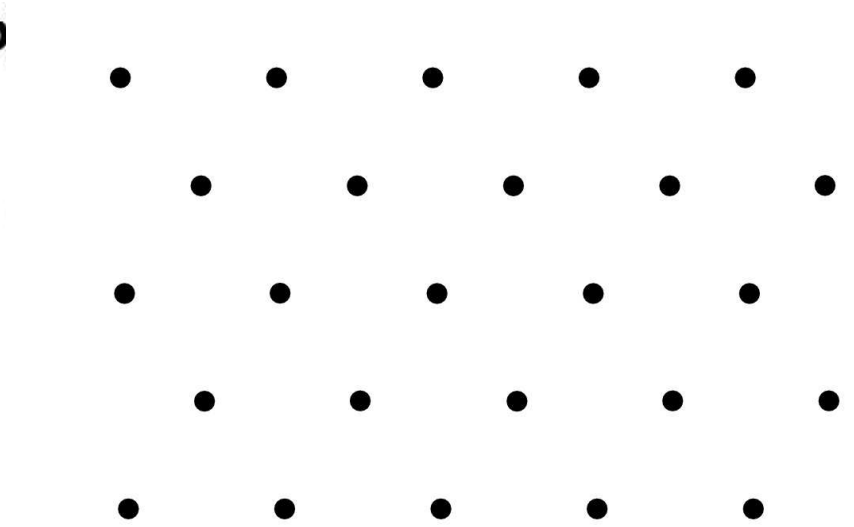
Fact 8: Many correlated electron systems are well described by LDA+DMFT

TaS_2

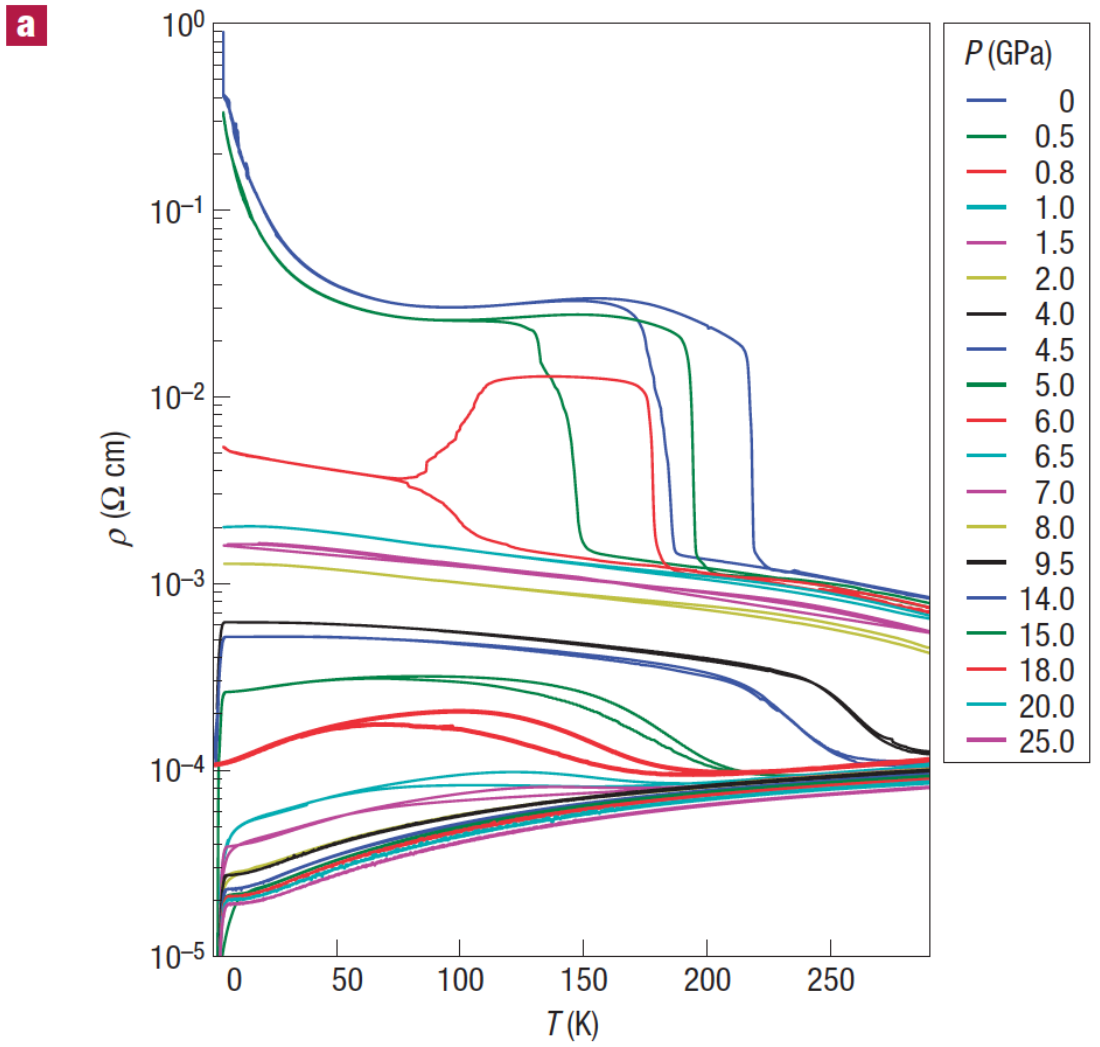
TaS_2

- TaS_2 one electron in Ta d-bands (local t_{2g} env.)
- Layers of triangular lattices
- 5-d electrons (small U)

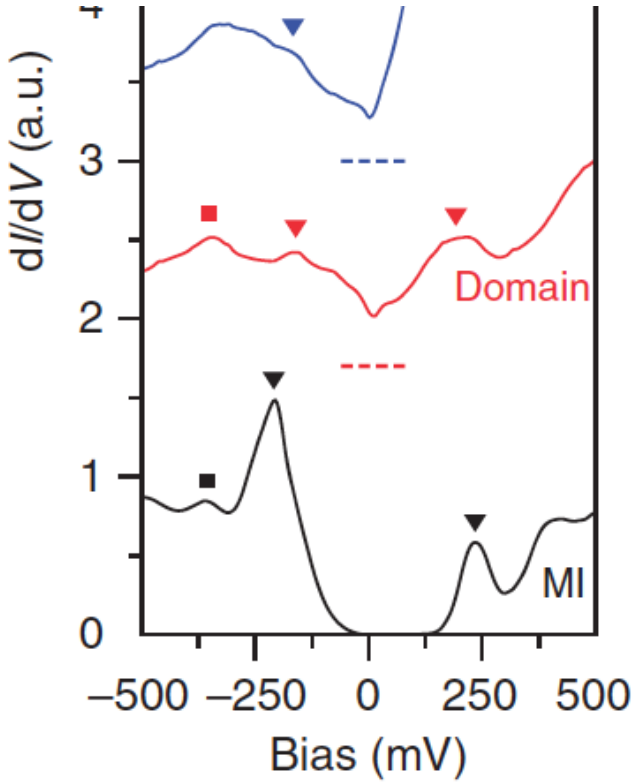




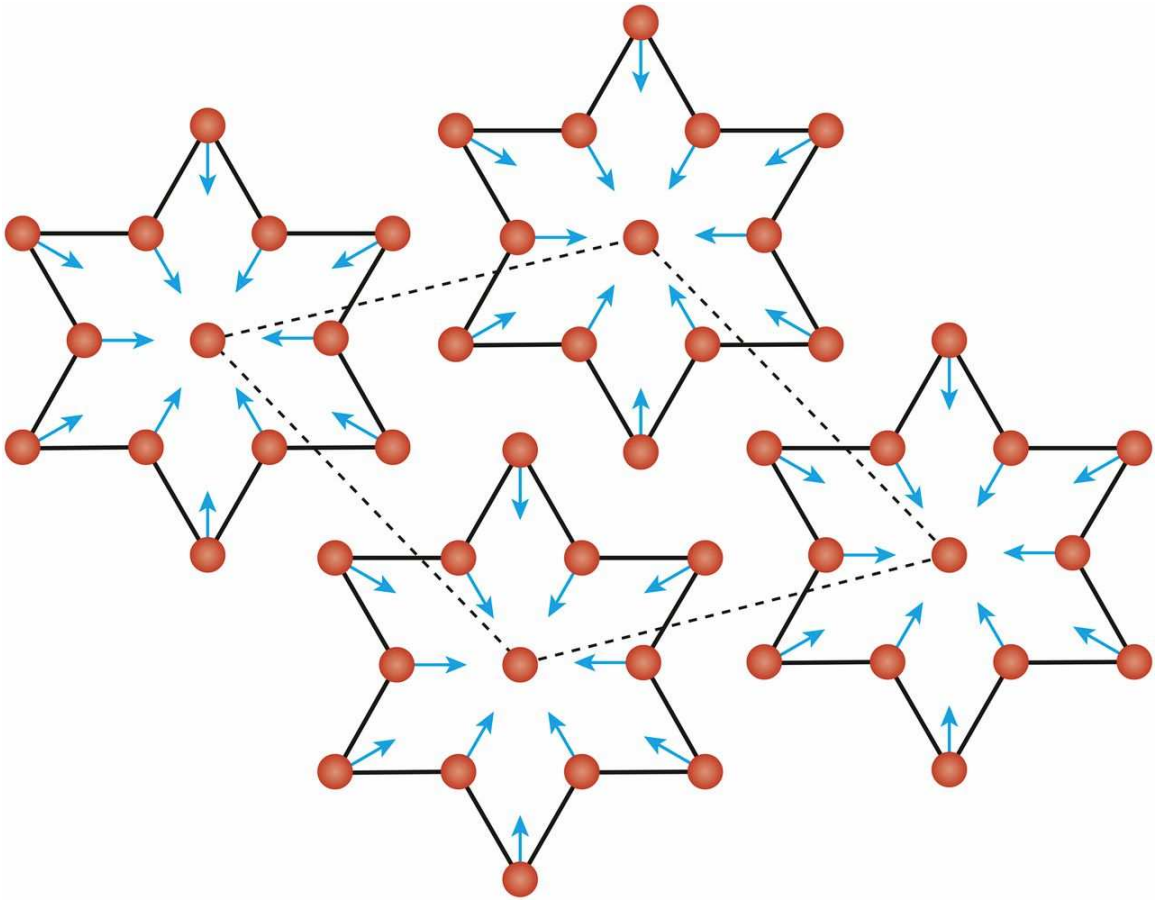
Mott transition



- STS shows insulating behavior (also optics)
- Small gap (0.1eV)

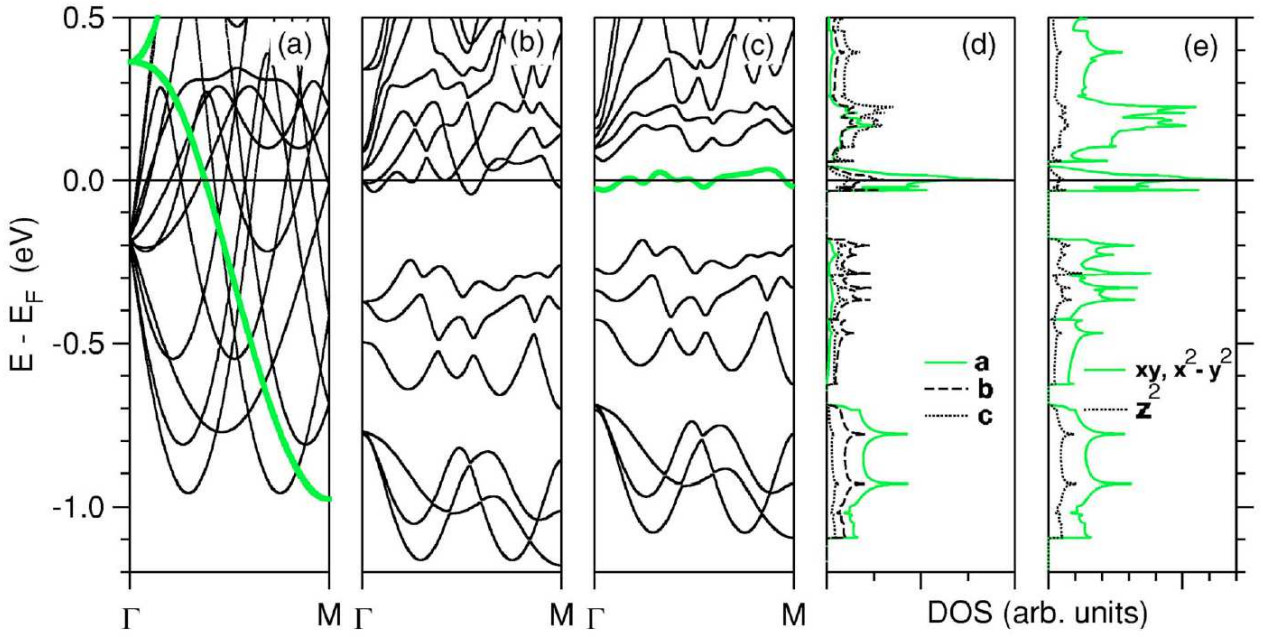


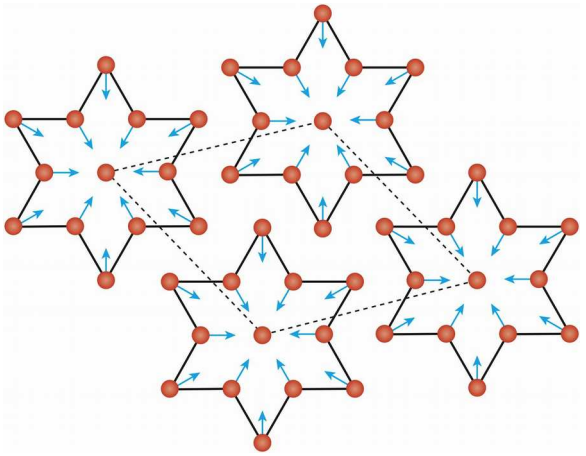
- Actually, a set of structural transitions. Mott state is realized in CDW at low-T.

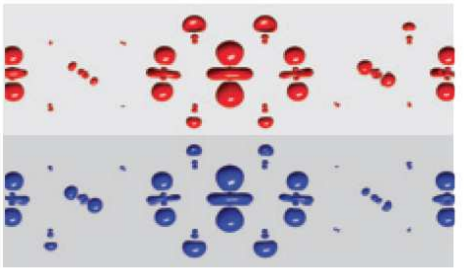


Narrow band close to FL following structural transition

- 1 band per cluster of 13 Ta atoms relevant at low energies



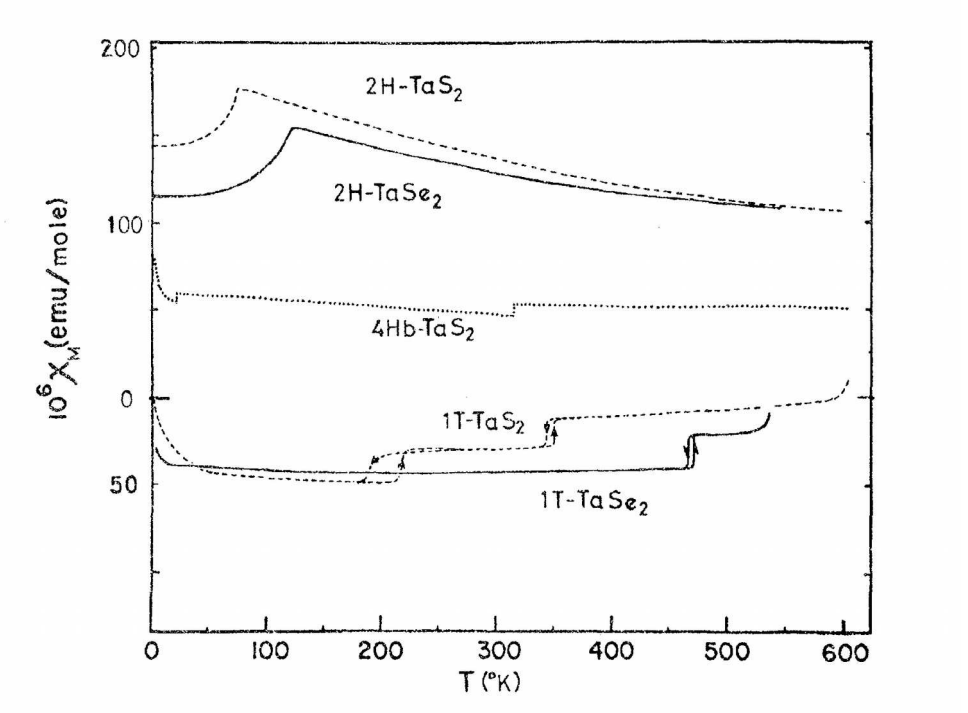




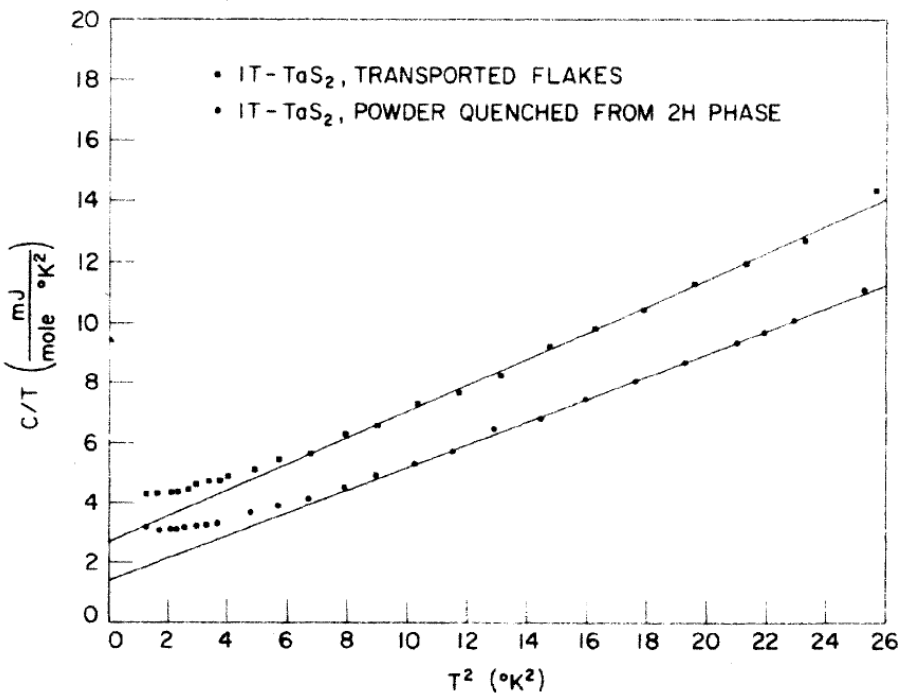
- If it is a Mott insulator, what do spins do ?

Mott insulator, but Pauli susceptibility?

- Pauli magnetic susceptibility

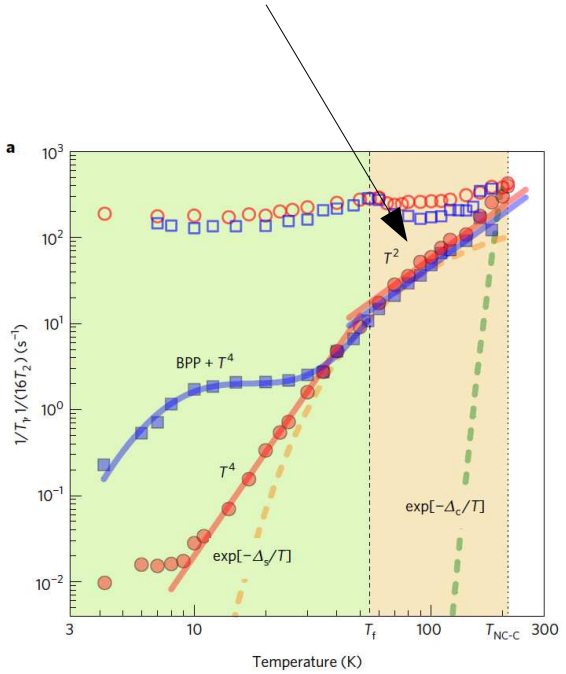


- Finite linear in T specific heat. Like in metals.



Collaboration with F5/F7

- Klanjsek et al, Nphys'17
- No magnetic order (muon spectroscopy)
- $1/T_1$ shows unusual T^2 dependence
- Spin liquid!



Summary

- Atomistic/ band-structure point of view on correlated electron systems
- LDA describes band-structure of many compounds well, but breaks down when U is strong
- Mott transition ; DMFT
- Sr_2RuO_4 : low coherence scale & Hund's coupling
- TaS_2 Mott insulator at small U; spin-liquid behavior

Fact 1: there is a thing called DFT and it describes the bandstructure of many compounds well

Fact 2: Several experimental techniques exist that allow quantitative comparison with theory

Fact 3: strong Coulomb repulsion leads to a breakdown of band theory and occurrence of Mott insulator

Fact 4: orbitals matter!

Fact 5: Mott insulators are magnets

Fact 6: DOS of correlated metals consists of quasiparticle peak and Hubbard bands

Fact 7: in Fermi liquids $\rho \sim T^2$

Fact 8: Many correlated electron systems are well described by DMFT (LDA+DMFT)

Thank you!