KLM spirala

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rešitev in kriterij

a) T ; +

b) $\frac{1}{2} m v^2 = \frac{1}{2} m (r^2 \dot\varphi^2 + \dot r^2)$

vez: $r = a\varphi$ $\dot r = a\dot\varphi$

$T = \frac{1}{2} m a^2 \dot\varphi^2 (1+\varphi^2)$ 1/4

$\vec r, \dot{\vec r}$

$a\varphi\cos\varphi,\ a\varphi\sin\varphi$

$a\dot\varphi(\cos\varphi - \varphi\sin\varphi),\ a\dot\varphi(\sin\varphi + \varphi\cos\varphi)$

$a\dot\varphi,\ 0 = (\dot r, 0)$

c) $v_0^2 = a^2 \dot\varphi^2 (1+\varphi^2)$

$\frac{v_0}{a} dt = d\varphi \sqrt{1+\varphi^2} =$

$\frac{v_0}{a} t = \frac{1}{2}\left(\varphi\sqrt{1+\varphi^2} + \operatorname{arsinh}\varphi\right)$ 1/4

d) mali t (mali φ) : $\frac{v_0 t}{a} = \varphi$

veliki t (veliki φ) : $\frac{v_0 t}{a} = \frac{1}{2}\varphi^2$, $\varphi = \sqrt{\frac{2 v_0 t}{a}}$ 1/4

e) $L = m r^2 \dot\varphi = m a^2 \varphi^2 \dot\varphi$

$M = \dot L = m a^2 (2\varphi\dot\varphi^2 + \varphi^2\ddot\varphi)$

za male t $M = m v_0^2\, 2\frac{v_0 t}{a}$

razlaga $L = m r\, r\dot\varphi =$

$= m a\varphi\, \varphi a\dot\varphi \simeq m a \varphi^2 v_0$

$L \sim t^2,\ M \sim t$

za velike t   hitrost ima le φ komp.

$L \doteq m r\, a\varphi\dot\varphi = m r v_0 \propto \sqrt{t}$

$M \sim 1/\sqrt{t}$

vrtilna količina se povečuje, ker se povečuje r, kot $\sqrt{t}$

Skica

1/4 +

f) $L = \frac{1}{2} m r^2 (\dot\varphi + \dot\alpha)^2 + \frac{J}{2}\dot\alpha^2$
$+ \frac{1}{2} m a^2 \dot\varphi^2$

$r = a\varphi$
$\dot r = a\dot\varphi$

$r = a\varphi$ (φ zasuk glede na spiralo)

$$L = \frac{1}{2} m a^2 \varphi^2 (\dot\varphi + \dot\alpha)^2 + \frac{J}{2}\dot\alpha^2 + \frac{1}{2} m a^2 \dot\varphi^2$$

$L \neq L(\alpha)$

$\frac{\partial L}{\partial \dot\alpha} = \text{konst} = L_z$; z - komp. vrtilne količine

1/4

$$p_\alpha = \frac{\partial L}{\partial \dot\alpha} = m a^2 \varphi^2 (\dot\varphi + \dot\alpha) + J\dot\alpha$$

g) zač. pogoj : $r = 0$, $\dot\alpha = 0 \Rightarrow p_\alpha = 0$

za male t: r majhen

$$m r^2 \dot\varphi + (m r^2 + J)\dot\alpha = 0$$

$$\dot\alpha = -\frac{m r^2}{m r^2 + J}\dot\varphi \approx -\frac{m r^2}{J}\dot\varphi$$
$$\approx 0$$

($T \approx m r^2 \dot\varphi^2 \approx \text{konst}$ $\dot\varphi \approx 1/r$)

za velike t: r velik

$$\dot\varphi = -\dot\alpha \frac{(m r^2 + J)}{m r^2} \approx -\dot\alpha$$

položaj uteži gledano iz mirujočega sistema podaja $\varphi + \alpha$

$(\varphi + \alpha)^{\cdot} = 0$ $\varphi + \alpha = \text{konst.}$

1/4

gibanje v ravni črti, ki gre okoli izhodišče.

3. naloga

več načinov. Eden od njih:

$$L = \dot{q}^n f(q)$$

$$\dot{L} = \frac{\partial L}{\partial \dot{q}} \frac{d\dot{q}}{dt} + \underbrace{\frac{\partial L}{\partial q}}_{\Downarrow} \frac{dq}{dt}$$

Lagrangeove en.

$$= \frac{\partial L}{\partial \dot{q}} \frac{d\dot{q}}{dt} + \left(\frac{d}{dt}\frac{\partial L}{\partial \dot{q}}\right) \frac{dq}{dt}$$

$$= \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}} \dot{q}\right) = \frac{d}{dt}\left(n \dot{q}^n f(q)\right) = \frac{d}{dt}\left(n L\right)$$

za L oblike zgoraj

$\dot{L} = n\dot{L}$; če $n \neq 1$ ima rešitev le za $\dot{L} = 0$

QED.

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