# 2. Kolokvij 21'

1. $E = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} m r^2 \dot{\varphi}^2 - mg\, k/r^4$

ohranja se $E$ in $p_\varphi = m r^2 \dot{\varphi}$

$V_{eff} = \frac{p_\varphi^2}{2 m r^2} - mg\, k/r^4$


1/4

Kot za katerega zgrešimo je povezan s parametrom vpada.

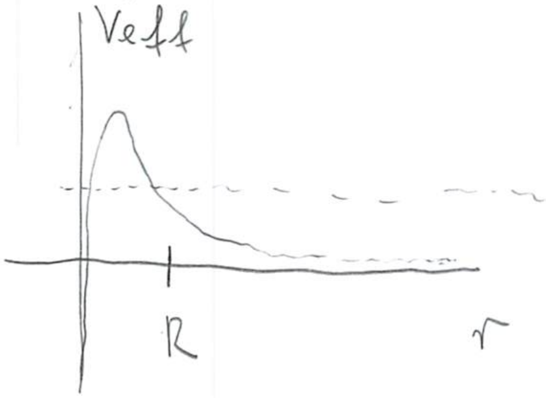
$\sin \gamma = b/l$


1/8

Največ kolikšen je lahko b, da zadenemo?

Dva režima!:

a) - za $^{\text{zelo}}$ blaga vzpetina ($k$ majhen) je maksimum $V_{eff}$ pri $r = r_0 < R$




$$\frac{1}{2} m v_0^2 = E$$

$$+ \frac{-mgk}{l^4} \longrightarrow 0$$

V tem primeru je pogoj, da žogica ravno pade v vdolbino $\dot{r} = 0 \,|_{r=R}$.

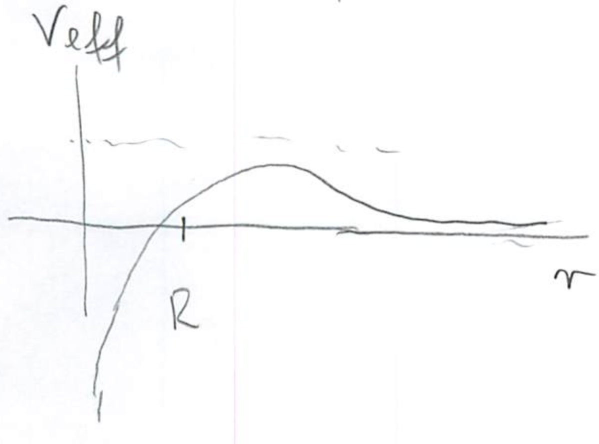
$$\Rightarrow \quad \frac{1}{2} m v_0^2 = \frac{m v_0^2}{2} \frac{b^2}{r^2} + \frac{1}{2} m \underset{\substack{\uparrow \\ 0}}{\dot{r}^2} - \frac{mgk}{r^4} \Bigg|_{r=R}$$

$$b^2 = R^2 \left( 1 + \frac{2gk}{R^4 v_0^2} \right)$$

1/4

$$\gamma = \arcsin \frac{R}{\cancel{b}\, l} \sqrt{1 + \frac{2gk}{r^4 v_0^2}}$$

b) - za močnejše vzpetina je maksimum $V_{eff}$ pri $r_0 > R$. V tem primeru je pogoj, da ravno pade




$$V_{eff}(r_0) = E$$

$$\frac{\partial V_{eff}}{\partial r} = -\frac{m v_0^2 b^2}{r^3} + \frac{4mgk}{r^5}$$

$$r_0^2 = \frac{4gk}{v_0^2 b^2}$$

$$V_{eff}(r_0) = \frac{m v_0^2}{2}\,\frac{b^4 v_0^2}{4gk} - \frac{mgk\, v_0^4 b^4}{16 g^2 k^2}$$

$$\| \quad E \quad \|$$

$$\frac{m v_0^2}{2} = \frac{m v_0^2}{2}\,\frac{b^4 v_0^2}{gk}\left(\frac{1}{4} - \frac{1}{8}\right) = \frac{m v_0^2}{2}\,\frac{b^4 v_0^2}{gk}\,\frac{1}{8}$$

$$b^4 = 8\,\frac{gk}{v_0^2}$$

1/4

$$\gamma = \arcsin b/l$$

- meja med režimoma a) in b) je, ka je $r_0 = R$

preverimo, da dobimo enak rezultat po obeh postopkih, če je $r_0 = R$!

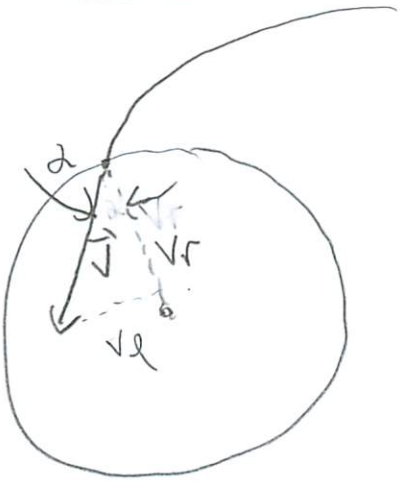
b): $b^4 = \frac{8gk}{v_0^2} = \frac{4gk}{v_0^2 b^2}\, 2b^2 = r_0^2 \cdot 2b^2$

$$\underline{b^2 = 2R^2}$$

a) $b^2 = R^2\left(1 + \frac{1}{2}\,\frac{4gk}{R^4}\,\frac{b^2}{v_0^2 b^2}\right) = R^2\left(1 + \frac{1}{2}\frac{b^2}{R^2}\right)$

$$\underline{b^2 = 2R^2}$$

vpadni kot : (Zn




$$\sin\alpha = \frac{v_\varphi}{v} = \frac{v_0\, b/R}{\sqrt{v_0^2 + \frac{2gK}{R^4}}}$$

$$v_\varphi = r\dot{\varphi} = \frac{r^2\dot{\varphi}}{r} = \frac{v_0 b}{r}$$

1/8

4/8

2. Kolokvij '21

3. 

$$\underline{\underline{V}} = k \begin{bmatrix} 1 & -1 & & \\ -1 & 2 & -1 & \\ & -1 & 2 & -1 \\ & & -1 & 1 \end{bmatrix} \qquad \underline{\underline{T}} = m \begin{bmatrix} 1 & & & \\ & M/m & & \\ & & M/m & \\ & & & 1 \end{bmatrix}$$

1/8

$$\omega^2 \underline{\underline{T}}\, \underline{a} = \underline{\underline{V}}\, \underline{a}$$

1) translacija $\quad a_1^T = (1,1,1,1) \; ; \; \omega_1^2 = 0$ 1/2

2) 

ta vzmet se ne raztegne

$$a_2^T = (-1, c, c, -1) \qquad a_2^T \underline{\underline{T}} a_1 = 0$$

$$\Downarrow$$

$$c = m/M$$

1/4 $\quad a_2^T = (-1, m/M, m/M, -1) \; ; \; \omega^2 = \frac{k}{m}\left(1 + \frac{m}{M}\right)$

3) $a_3^T = (1, b, -b, -1)$

$$\underline{\underline{V}} a_3 = k \begin{pmatrix} 1-b \\ 3b-1 \\ -3b+1 \\ -1-b \end{pmatrix} = m\omega^2 \begin{pmatrix} 1 \\ M/m\, b \\ M/m\,(-b) \\ 1 \end{pmatrix}$$ 1/4

$$\frac{3b-1}{1-b} = \frac{Mb}{m} \qquad b^2 \frac{M}{m} + b\left(3 - \frac{M}{m}\right) - 1$$ 1/4

$$\omega_{1/2}^2 = \frac{k}{m}\left(1 - b_{1/2}\right) \qquad b_{1/2} = \frac{M/m - 3 \pm \sqrt{9 + \frac{M^2}{m^2} - 6\frac{M}{m} + 4\frac{M}{m}}}{2M/m}$$