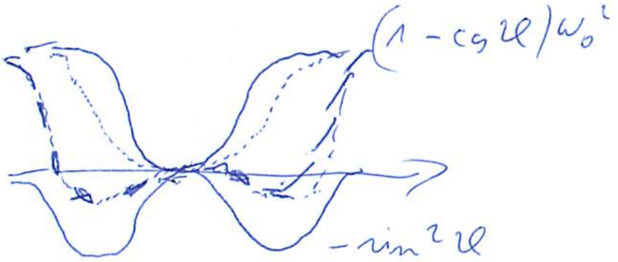
1.

$$L = \frac{1}{2} m r^2 \sin^2\vartheta\, \omega^2 + \frac{1}{2} m r^2 \dot{\vartheta}^2 - m g r (1-\cos\vartheta)$$

$$V_{eff}(\vartheta) = m g r (1-\cos\vartheta) - m r^2 \omega^2 \sin^2\vartheta$$

$$V_{eff}(\vartheta) = m r^2 \left[\omega_0^2 (1-\cos\vartheta) - \frac{\omega^2}{2} \sin^2\vartheta\right]$$



$\cdots\cdots$ $\omega < \omega_0$

$- - -$ $\omega > \omega_0$

ravnovesna lega: $\frac{dV_{eff}}{d\vartheta} = 0$

vpeljem

$$v = \frac{V_{eff}}{m r^2 \omega_0^2} \qquad \omega/\omega_0 = c$$

$$v = (1-\cos\vartheta) - \frac{c}{2}\sin^2\vartheta$$

$$\frac{dv}{d\vartheta} = \sin\vartheta - c\sin\vartheta\cos\vartheta = \sin\vartheta\,(1 - c\cos\vartheta)$$

za $c > 1$ dve rešitvi; $\vartheta > 0$ stabilna, $\vartheta = 0$ labilna

za $c < 1$ samo ena rešitev $\vartheta = 0$

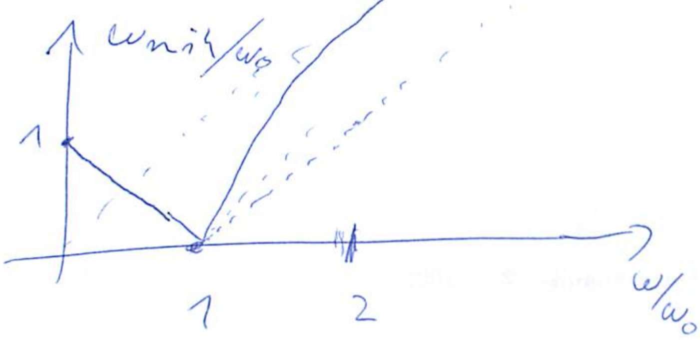
def.: ϑ_0 ... stabilna rešitev $\qquad \vartheta_0 = \begin{cases} 0 ; & c<1 \\ \arccos\frac{1}{c} ; & c>1 \end{cases}$

$$\frac{dv^2}{d\vartheta^2} = \cos\vartheta - c(\cos^2\vartheta - \sin^2\vartheta) = \cos\vartheta - c(2\cos^2\vartheta - 1)$$

$$d_0 = \frac{dv^2}{d\vartheta^2}\Big| = \begin{cases} 1 - c ; & c<1 \\ \frac{1}{c} - c\left(\frac{2}{c^2} - 1\right) = -\frac{1}{c} + c \end{cases}$$

Razvoj za $\tilde{\vartheta} = \vartheta_0 + \vartheta$ $\qquad L = \frac{1}{2} m r^2 \dot{\tilde{\vartheta}}^2 - \frac{m r^2 \omega_0^2}{2} d_0^2 \tilde{\vartheta}^2$

$$\omega_{min}^2 = \omega_0^2 d_0^2$$



$$\tilde{\vartheta} = \tilde{x}_1 \cos(\omega_{min} t)$$

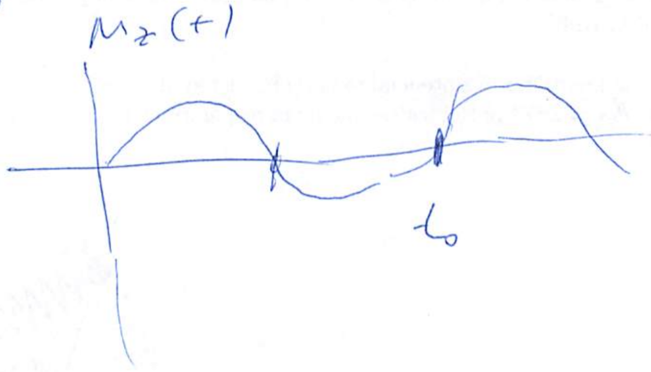
$$L_z = m r^2 \omega \sin^2\vartheta = m r^2 \omega \sin^2(\vartheta_0 + \tilde{\vartheta}(t))$$

$$\frac{dL_z}{dt} = M_z = m r^2 \omega \, 2 \sin(\vartheta_0 + \tilde{\vartheta}(t)) \cos(\vartheta_0 + \tilde{\vartheta}(t))$$

$$= m r^2 \omega \sin(2(\vartheta_0 + \tilde{\vartheta}(t))) \dot{\tilde{x}}$$

$$= m r^2 \omega \sin(2(\vartheta_0 + \tilde{\vartheta}(t))) \sin(\omega_{min} t) / \omega_{min}$$

; za $\vartheta_0 > 0$



$t_0 = \frac{2\pi}{\omega_{min}}$

za $\vartheta_0 = 0$

$$M_z = m r^2 \omega \sin(2 \omega_{min} t) \Rightarrow t_0 = \frac{\pi}{\omega_{min}}$$

2\. a) stran od ~~roba~~ roba:

~~za~~ V ... konst ohranjene količine

$$p_x = m v_x \,,\; p_y = m v_y \,,\; p_z = m v_z \,,\; E$$

gibanje po ravni črti

na robu: $V \neq V(y)$; $V \neq V(z)$

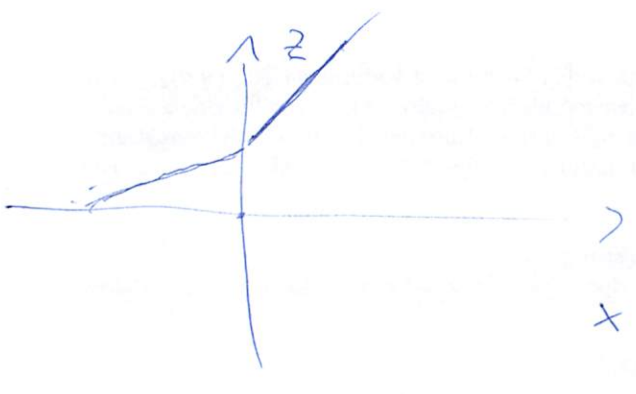
$\Rightarrow p_z, p_y$ ohranjene

$x<0$ $\quad m v_x, m v_y, m v_z, \frac{1}{2} m v^2 = E$

$x>0$ $\quad m v_x', m v_y, m v_z \quad \frac{1}{2} m(v_x'^2 + v_y^2 + v_z^2) + V_0$

$$= \frac{1}{2} m (v_x^2 + v_y^2 + v_z^2)$$

$$\frac{1}{2} m v_x'^2 + V_0 = \frac{1}{2} m v_x^2$$



za $V_0 > 0$

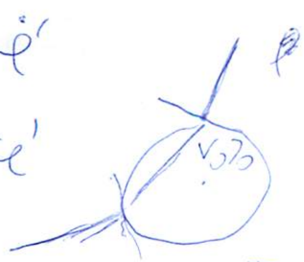
b) stran od roba: enako

na robu: $V \neq V(\varphi, \vartheta) \Rightarrow$ centralni potencial

gibanje v ravnini. Ohranja se vrtilna količina.

$$m r_0^2 \dot{\varphi} = m r_0^2 \dot{\varphi}'$$

$$\vec{v} = v_r \hat{e}_r + v_\varphi \hat{e}_\varphi \qquad v_\varphi = v_\varphi'$$



$$\frac{1}{2} m v_r^2 + V_0 = \frac{1}{2} m v_r'^2$$

(ali po premislek zračunirap.)