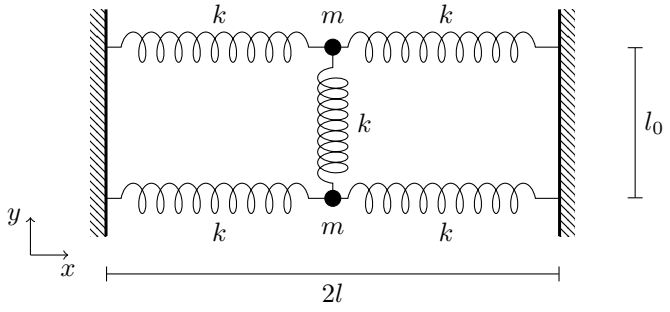
1B izpit iz Klasične mehanike, 5.6.2023

1. A satellite is orbiting around Earth in a circular orbit with $r = r_0$. At a certain instance an object hits the satellite so that only the direction of the velocity changes but its magnitude v_0 remains unchanged. In the new orbit the closest approach to the Earth is at $r = (2/5)r_0$. What is the magnitude of the velocity (in units of v_0) at that point? In terms of v/v_0 express also the angle between the new and old orbit at the point where the impact took place!

2. Consider the **planar oscillations** (i.e, in plane x, y, see sketch below) of two weights with mass m, suspended between two walls by four springs and coupled by a fifth spring. The distance between the walls is $2l$, and the distance between the weights in their equilibrium position is $l_0 < l$. The springs are identical, with spring constants equal to k, and their unstretched length is l_0 (the same as the distance between the weights in equilibrium). Neglect gravitational acceleration, $g = 0$.

a) Write the exact expression (without approximations) for the kinetic and elastic potential energy of the system!
b) Derive the expression for the potential energy in the approximation of small oscillations and write it in a matrix form. Use the Taylor expansion up to the 2nd order: $\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \ldots$
c) Calculate the normal modes and the corresponding natural frequencies! Sketch the motion of the weights for each mode! Hint: use the symmetries of the problem.



3. Investigate motion of a charged particle in a magnetic field using the Hamiltonian formalism. The particle is moving in a magnetic field that is in cylindrical coordinates given as $\mathbf{B} = B\hat{\mathbf{e}}_\phi$, where B is a constant.

a) Find a vector potential that corresponds to such magnetic field. Rotor in cylindrical coordinates reads

$$\boldsymbol{\nabla} \times \mathbf{A} = [(1/r)\partial A_z/\partial\phi - \partial A_\phi/\partial z]\hat{\mathbf{e}}_r + [\partial A_r/\partial z - \partial A_z/\partial r]\hat{\mathbf{e}}_\phi + (1/r)[\partial(rA_\phi)/\partial r - \partial A_r/\partial\phi]\hat{\mathbf{e}}_z.$$

b) Write down the Lagrangian in cylindrical coordiantes for a particle moving in such magnetic field!
c) From Lagrangian derive the Hamiltonian!
d) Write down the Hamiltonian equation of motion. Find the conserved quantitities!
e) Solve the equations of motion for a case where at $t = 0$ a particle at $r = r_0$ has initial velocity $\mathbf{v} = v_r\hat{\mathbf{e}}_r$!