ANALITIČNA MEHANIKA

Rešene kolokvijske naloge

Urban Pompe
September 2002

Bralec naj vzame na znanje, da so ti zapiski zgolj produkt mojega učenja in priprav na kolokvije in izpite iz analitične, zato nikakor ne jamčim za pravilnost rešitev, ker pa se mi je zazdelo škoda te zapiske vreči v omaro in na njih pozabiti, sem jih dal na voljo naslednjim generacijam. Kljub temu upam, da bodo vedoželjnemu bralcu pomagali pri njegovem nabiranju znanj in veščin analitične mehanike. Bralec naj mi tudi ne zameri, ker se nisem kaj prida ukvarjal z numeričnimi izračuni, in ostalo računarijo, ki ne sodi najbolj v analitično mehaniko. Slike, ki pripadajo nalogam, so narisane pri rešitvah.

1. Po navpično postavljenem obroču (R=0.5m), ki se lahko vrti okoli navpične osi, drsi drobna utež. Zapiši Lagrangeove enačbe za opisan sistem in ugotovi, kje ima pri enakomernem vrtenju utež ravnovesno lego! Nadalje zapiši enačbe za majhno nihanje uteži in izračunaj frekvenco nihanja, če je utež v ravnovesni legi odklonjena za $\Theta_0 = 20°$. Vztrajnostna momenta obroča in uteži zanemari. Brez računanja opiši, kako nihanje vidi opazovalec, ki se vrti skupaj z obročem!

2. V poln lesen valj premera $2R$ in višine h je na razdalji d od osi vzporedno z osjo izvrtana luknja s premerom $2r$ ($d+r<R$). Valj postavimo na vodoravno ravnino, po kateri se lahko kotali brez trenja. S kolikšnim nihajnim časom zaniha valj, če ga narahlo sunemo iz mirovne lege? (Najprej poišči mirovno lego!)

3. Izračunaj Poissonove oklepaje med kartezičnimi komponentami vrtilne količine ($[L_x, L_z]$, $[L_y, L_z]$, $[L_z, L_z]$, $[L^2, L_z]$).

4. Homogen valj se lahko vrti okrog navpične osi. Na obod valja je pritjreno spiralno vodilo s hodom p $[cm/2\pi]$ po katerem brez trenja drsi drobna utež z maso m. V začetku utež miruje na vrhu valja, ko pa jo spustimo zaradi teže oddrsi navzdol. Zapiši Hamiltonovo funkcijo za opisan sistem, ter razreši ustrezne enačbe!

5. Denimo, da raketo izstrelimo tako, da jo najprej navpično dvignemo do željene višine ($h \approx 200km$), nato pa jo pospešimo v tangencialni smeri s tako histrostjo, da se vtiri v krožno orbito. Zaradi enostavnosti vzemimo še, da motorji delujejo najprej samo kratek čas ob navpični izstrelitvi (navpičen met) in nato še kratek čas pri vtirjanju v orbito (t.j. v točkah 1 in 2). Kolikšen del začentne mase rakete lahko na tak način spravimo v orbito? Hitrost izpušnih plinov glede na šobo raketnega motorja je $3km/s$. Upoštevaj $h<<R$, vrtenje Zemlje zanemari.

6. Bat z maso m, ki se prosto giblje vzdolž cilindra, je pritrjen na vztrajnik (z vztrajnostnim momentom J), kot prikazuje slika. Zapiši Lagrangeovo funkcijo in ustrezne enačbe. Upoštevaj $R<<l$. Kó si to opravil, v legi $\varphi = 0$ zapri cilinder, tako da se bat ne giblje več prosto, saj bodisi stiska ali razpenja zrak v cilindru. Kakšna je v tem primeru frekvenca nihanja za majhne odmike od ravnovesne lege? Vzemi, da se zrak v cilindru stiska in razpenja izotermno.

7. Z vrha vodoravno postavljene gladke polkrogle s polmerom R zdrsne majhna utež. Z metodo Lagrangeovih multiplikatorjev izračunaj v kateri točki se utež odlepi od površine polkrogle!

8. Na stolpu v Torontu je vrteča restavracija. Zaradi okvare se je restavracija začela vrteti nekoliko hitreje kot običajno, tako da se obrne dvakrat v minuti. Natakar, ki mimo nas nese juho gostu, mora uporabiti vso svojo spretnost, da juhe ne razlije. Kako je glede na gladino juhe, ki je na naši mizi, nagnjena gladina juhe, ki jo nosi natakar? Natakar do gosta hodi radialno navzven s hitrostjo $1m/s$, naša miza pa je od središča oddaljena $20m$.

9. Kegljač na ledu cilja s svojim kegljem enak mirujoč kegelj (polmera R). Kegljač je ugotovil, da pri vsakem metu zadene, vendar je napaka smeri meta znotraj te omejitve povsem slučajna. V kolikšnem odstotku metov se vrženi kegelj odkloni med 20 in 30 stopinjami levo od prvotne smeri?

10. Po ravnem vodilu, ki se vrti s konstantno kotno hitrostjo okoli osi, pravokotno nanj, se giblje brez trenja kroglica mase m. Kroglica je pripeta na vodilo z vzmetjo dolžine l in koeficientom k. Zapiši gibalne enačbe v neinercialnem koordinatnem sistemu.

11. Jojo visi na idealni vrvici, tako da brez trenja niha med popolnoma raztegnjeno vrvico in najvišjo lego. Zapiši nihajni čas kot funkcijo višinske razlike med obema legama (ta je mnogo večja od polmera vretena). Predpostavi tudi, da je vrvica ves čas navpična.

12. Dve kroglici z masama m_1 in m_2 sta povezani z gibko, a neraztegljivo vrvico dolžine l. Prva kroglica drsi po gladki plošči, druga pa visi na drugem koncu vrvice, ki je speljana skozi majhno luknjico v plošči. Trenje vrvi je zanemarljivo. Sistem je na začetku v ravnovesju (prva kroglica enakomerno kroži okoli luknjice po krogu s polmerom r_0, druga pa miruje na drugem koncu vrvice). Sistem zmotimo tako, da drugo, visečo kroglico rahlo potegnemo v navpični smeri. Izračunaj razmerje med frekvencama radialnega gibanja in kroženja prve kroglice v tako perturbirani obliki!

13. Napiši Lagranžijan za matematično nihalo, ki je postavljeno na vrteči se Zemlji na geografski širini Θ. Upoštevaj, da so odmiki od ravnovesne lege majhni. Lagranžijan naj bo zapisan v koordinatah, ki mirujejo glede na Zemljo.

14. V skledo, ki ima obliko polkrogle s polmerom R, položimo kroglico z maso m in polmerom r. Zapiši gibalne enačbe, ter jih reši za primer ravninskega mjhnega nihanja. Kako se spreminja frekvenca nihanja, če večamo polmer kroglice?

15. Opazujemo simetrično vrtavko, ki smo jo na vodoravni podlagi zavrteli tako, da je bila v začetku simetrijska os postavljena navpično, hkrati pa vrtavka ni opletala ($\vartheta = 0$, $\dot{\vartheta} = 0$). Zaradi majhnega trenja v dotikališču se vrtavka počasi upočasnjuje in začne v nekem trenutku opletati (precedirati in nutirati). Ugotovi, pri kateri kotni hitrosti rotacije okoli simetrijske osi se to zgodi! Namig: pomagaj si z gibalno enačbo kot smo jo zapisali s spremenljivko $u = \cos\vartheta$.

16. V kroglasti skledi s polmerom R brez trenja drsi palica dolžine l in mase m ($l<R$). Z metodo Lagrangeovih multiplikatorjev ugotovi, s kolikšno silo pri danem kotu Θ deluje skleda na palico, če le-ta v skledi prosto ravninsko niha. Namig: zapiši sistem Lagrangeovih enačb in izrazi ustrezni množitelj. Enačb ne rešuj.

17. Po ravni cesti vozi enakomerna kolona petdesetih avtomobilov z medsebojno varnostno razdaljo $50m$. Vzemimo, da je zaviranje ali pospeševanje avtomobila v koloni sorazmerno z odstopanjem od varnostne razdalje, zato lahko kolono simuliramo kot linearno verigo mas (vozil), v kateri so vozila med seboj povezana z enakimi vzmetmi. Parametre verige določi iz naslednjih podatkov: če voznik v prvem avtomobilu zagleda oviro in zmanjša hitrost se pojavi »zgoščina« dolžine približno petih vozil. Zgoščina se širi vzdolž verige in voznik v zadnjem vozilu se odzove $100s$ kasneje.

(V zgornjem primeru je analogija verige vozil in verige mas vprašljiva, ker npr. i-ta masa pospešuje tudi, če je $(i-1)$-ta v ravnovesni razdalji $50m$, $(i+1)$-ta pa npr. le $30m$ za i-to, medtem, ko pri vozilih to ne velja. Vsak voznik zavira in pospešuje le odvisno od voznika spredaj in ne tudi tistega zadaj. – zgolj moja opomba)

18. Tri uteži enakih mas so povezane s tremi enakimi vzmetmi v enakostranični trikotnik. Vsaka od uteži se brez trenja giblje po žlebu, ki vodi v smeri proti težišču trikotnika. Izračunaj lastne frekvence in pripadajoče nihajne načine (le-te skicijraj) opisanega sistema.

19. Pet uteži z enakimi masami razporedimo v obliki križa in jih povežemo z enakimi listnatimi vzmetmi. Uteži se lahko gibljejo samo v navpični smeri, sila vzmeti pa je sorazmerna z višinsko razliko med dvema utežema. Kakšna so lastna nihanja in ustrezne lastne frekvence. Namig: lastnih vektorjev ne računaj ampak jih poskusi uganiti.

20. S stropa visijo tri vzmeti. Njihova pritrdišča so na isti premici na mdsebojnih razdaljah a. Na vzmeteh visita v vodoravni legi dve palici mase m in dolžine a. Zunanja konca palic sta pritrjena na zunanjih dveh vzmeteh, notranja pa sta povezana s kratko vrvico, ki je na sredini pritrjena na srednjo vzmet. Konstanta srednje vzmeti je $2k$, zunanjih dveh pa k. Zunanji konec ene od palic lahko sunemo v navpični smeri. Opiši nadaljnje gibanje palic.

21. Majhna kroglica visi na lahki vrvici z dolžino $10cm$. Vrvico v pritrdišču enakomerno sukamo s krožilno frekvenco ω. Poišči stabilno ravnovesno lego, če je frekvenca $\omega = 1Hz$ in če je $\omega = 30Hz$. Za oba primera izračunaj tudi frekvenco nihanja okoli ravnovesne lege za male odmike od ravnovesne lege.

22. Opazujemo dvojno matematično nihalo, sestavljeno iz dveh enakih mas m, ki sta obešeni zaporedno na dveh enako dolgih vrvicah z dolžino l. Za majhne ravninske odmike (obe kroglici se gibljeta v isti ravnini) določi lastne frekvence in lastne vektorje.

23. S stropa sta na razdalji a gibko vpeti dve lahki, neraztegljivi, enako dolgi žici (z dolžino l). V spodnji krajišči je gibko vpeta tanka toga prečka dolžine a in mase m. Zapiši Lagranžijan za gibanje prečke v približku majhnih odmikov iz mirovne lege in reši ustrezne gibalne enačbe. Navodilo: bodi pazljiv pri izbiri smiselnih generaliziranih koordinat.

24. Za škripčevje na sliki zapiši Lagrangeove enačbe in jih reši. Kakšen pa je rezultat, če upoštevano silo trenja v ležaju? Škripca sta valja.

25. Tri mase ($m_1 = 2M$, $m_2 = 6M$, $m_3 = 3M$) so povezane s škripčevjem kot kaže slika. Ob tem je srednja masa m_2 pritrjena še ob podlago z vzmetjo s koeficientom k. Mase na začetku mirujejo, nato pa jih spustimo. Kako se gibljejo, če lahko mase koles škripčevja zanemarimo? Kako pa, če imajo kolesa škripcev maso M in polmer R. Vrvica, ki povezuje mase, je lahka, škripci se vrtijo brez trenja. S

3606 26. Dve matematični nihali z masama m_1 in m_2 in enako dolžino l sta povezani z lahko vzmetjo s koeficientom k. Kakšne so lastne frekvence in lastni nihajni načini nihanja takega sistema?

27. Na vrtiljaku, ki se vrti okoli horizontalne osi s konstantno kotno hitrostjo ω, je na razdalji a od osišča pritrjeno majhno nihalo dolžine l, ki lahko niha le v ravnini vrtenja. Napiši diferencialno enačbo za nihanje tega nihala z majhnimi odmiki, če je $\omega^2 >> g/a$ (smer vrvice nihala je vedno skoraj radialna)

28. Kroglica brez trenja drsi po plašču navpično postavljenega stožca s kotom ob vrhu $\alpha = 2\pi/3$. Za krožno gibanje z obhodnim časom $T = 0.5s$ določi razdaljo kroglice od osi. Kakšna je frekvenca malih nihanj okoli ravnovesne lege?

29. Metrska palica mase m=400g se vrti brez trenja v horizontalni ravnini okoli navpične osi, ki gre skozi eno izmed krajišč palice. Po palici se brez trenja giblje prstan mase m=200g. Prstan pritrdimo na palico z vrvjo v razdalji 10cm od osi. Palico zavrtimo, tako da se vrti s kotno hitrostjo $\omega = 5Hz$ in prežgemo vrvico. Kako se giblje prstan?

30. Na cilinder polmera 10cm in mase 2kg je z lahkim 10cm dolgim drogom pritrjena krogla polmera 5cm in mase 500g. Na drugi strani je s pomočjo vrvi, ki je navita na valj, pritrjena utež z maso 1kg. Zapiši Lagrangeovo funkcijo, izpelji enačbe ter poišči prekvence nihanja okoli ravnovesne lege.

31. V zabaviščnem parku imajo vrtiljak, ki je narejen tako, da so na robu enakomerno vrtečega obroča še kletke s sedeži, ki se na razdalji r enakomerno vrtijo s kotno hitrostjo ω glede na vrteči obroč. Zapiši pospešek kot funkcijo časa, to je izrazi komponento pospeška, ki jo čutijo veseljaki v smeri osišča kletke in v smeri pravokotno na smer osišča.

32. Europa je eden od satelitov, ki krožijo okrog Jupitra. Z vesoljskim avtomobilom se vozimo po površini tega satelita. Kolikšna sila (in v kateri smeri) deluje na gume našega avtomobila, če se po tem vrtečem se satelitu avto giblje vzdolž poldnevnikov, in kolikšna, če se peljemo vzdolž vzporednikov? Privzemi, da je Europa idealno okrogle oblike, ter da lahko vsakršen vpliv Jupitra, Sonca in ostalih planetov zanemariš. Europa, ki ima polmer 1520km in gostoto $3g/cm^3$, se enkrat zavrti glede na oddaljene zvezde v 3,55 dneva.

33. Navpično postavljen vijak se brez trenja vrti v naoljeni matici. Izberi generalizirane koordinate, zapiši Lagrangeove enačbe in Hamiltonove enačbe. Zapiši in reši Lagrangeove enačbe še, če upoštevaš trenje (ki je sorazmerno s kotno hitrostjo).

34. Iz dveh enakih matematičnih nihal sestavimo nihalo tako, da ju v navpični legi na polovici dožine spnemo z nenapeto vzmetjo. Vsako od nihal je sestavljeno iz lahkega droga in kroglice z maso m. Ena izmed kroglic je potopljena v viskozno tekočino in se giblje tako, da velja linearni zakon upora. Poišči lastne frekvence in lastna nihanja sistema!

35. Molekulo HF si lahko predstavljamo kot dve krogli z masama $1u$ oz. $18{,}9u$, kjer je u atomska masna enota ($u = 1.66 \cdot 10^{-27}\,kg$), ki sta povezani z vzmetjo s konstanto $k=50N/m$. Določi osnovne frekvence nihanja takega sistema.

36. Za škripčevje na sliki zapiši Lagrangeovo funkcijo in gibalne enačbe. V začetnem trenutku škripčevje miruje, težišči uteži in gibljivega škripca pa sta na isti višini. Določi položaj uteži $1s$ kasneje, če je masa uteži $100g$, masa škripca $200g$, njegov polmer pa $10cm$.

37. Iz dveh metrskih drogov z masama $1kg$ in $3kg$ sestavimo nihalo tako, da je eno izmed krajišč lažjega droga vrtljivo okoli fiksne navpične osi, težji drog pa se vrti okoli drugega krajišča. Poišči lastne frekvence in lastna nihanja sistema.

38. Drsališče posebne vrste dobimo tako, da zmrznemo vodo v posodi, ki se enakomerno vrti okoli navpične osi s kotno hitrostjo ω. Kako se v taki posodi giblje drsalec, ki je imel v začetnem trenutku glede na led hitrost v_0 v tangencialni smeri in je bil za R oddaljen od osi? Rešitev zapiši v inercialnem in neinercialnem sistemu.

39. Matematično nihalo je obešeno na klado mase M, ki brez trenja drsi po ravni podlagi. Določi generalizirane koordinate, zapiši Lagrangeovo funkcijo in gibalne enačbe, ter za majhna nihanja izračunaj lastne frekvence sistema! Matematično nihalo sestavljata kroglica mase m in lahek drog dolžine l.

40. Vztrajnik s pravokotno prečko se brez zdrsavanja kotali poravni podlagi, nagnjeni za kot ε glede na vodoravno ravnino. Zapiši gibalne enačbe in jih reši za primer majhnega nihanja okoli ravnovesne lege! Namig: uporabi Lagrangeov formalizem podobno kot v primeru vrtavke!

26.06 41. Delec z maso m je z vzmetema k_1 in k_2 pritrjen na vzporedni steni, ki sta na medsebojni razdalji a. Dolžina neobremenjenih vzmeti je enaka 0. Zapiši Lagrangeovo funkcijo za enodimenzionalno gibanje delca. Zapiši še Hamiltonovo funkcijo in gibalne enačbe in jih reši.

42. Odpirač za steklenice s plutovinastim zamaškom ima v zgornjem delu navoj s hodom p (obhoda na centimeter). Po navoju se giblje krilata matica z vztrajnostnim momentom J. Matico spustimo z vrha navoja in opazujemo njeno prosto vrtenje, dokler h pod vrhom ne udari ob spodnji del odpirača in se v hipu ustavi. Koliko časa potrebuje za to? Kolikšna sta sunek sile in sunek navora ob trku? Odpirač držimo ves čas navpično, trenje zanemari.

43. Vzmetno nihalo, sestavljeno iz dveh enakih mas m ter vzmeti s konstanto k, vstavimo v dolgo, ravno cev vzdolž katere lahko le-to drsi brez trenja. Cev s konstantno kotno hitrostjo ω vrtimo okoli pravokotne osi. Zapiši in reši gibalne enačbe za opisan sistem! Nihalo umirimo in ga kar se da natančno vstavimo v os vrtenja (os zgrešimo za majhno razdaljo ξ). Opiši možne načine gibanja nihala v cevi! Privzemi, da sta masi točkasti, neraztegnjena vzmet pa ima dolžino nič.

44. Na tanko pravokotno ploščo dolžine a in širine b pritrdimo os, ki se ujema z diagonalo plošče. Os nato z ležaji pritrdimo na podlago in ploščo zavrtimo s konstantno kotno hitrostjo ω. Izračunaj sili, ki obremenjujeta ležaja!

DN6 45. Cev dolžine l vrtimo v vodoravni ravnini s kotno hitrostjo ω okoli navpične osi, ki je na polovici cevi. Kroglica, ki jo sunemo vzdolž cevi s hitrostjo v_0 iz središča navzven, se giblje po cevi brez trenja. Z uporabo Lagrangeovega formalizma izračunaj, po kolikšnem času kroglica zapusti cev! Izračunaj tudi izstopno hitrost in pod katerim kotom odleti iz cevi!

$\vec{R} = \vec{p} \times \vec{l} - m l \frac{\vec{r}}{r}$

46. Za gibanje v potencialnem polju $V = k/r$ je Laplace-Runge-Lenzov vektor $\bar{R}$ konstanta gibanja. Izračunaj Poissonov oklepaj $[R,H]$. Pokaži, da je povprečna vrednost tega Poissonovega oklepaja različna od 0 in kaže v smeri, ki je pravokotna na $\bar{R}$.

47. Kolesar pelje po mokri cesti. Na gumi se naredi tanek film vode, ki se, medtem ko se kolo vrti, oblikuje v kapljice. Ko je kapljica dovolj velika, odleti iz gume s tangencialno hitrostjo, ki je enaka hitrosti gume ob trenutku odlepljenja.

(a) Napiši enačbo trajektorije v sistemu zunanjega opazovalca (ki miruje ob cesti) za kapljico, ki se odlepi na višini h ($h<2R$) nad cesto!

(b) Zapiši enačbo te trajektorije še v sistemu kolesarja!

(c) Kako visoko nad tlemi se odlepi kapljica, ki doseže največjo višino? Kolikšno višino doseže ta kapljica in kje je os kolesa, ko je ta kapljica najvišje?

(d) Pokaži, da letijo v sistemu zunanjega opazovalca kapljice samo naprej!

Polmer gume je R, kolesarjeva hitrost pa v.

DN5 48. Slamica z maso m leži na prazni polkrožni skledi. S koncem se dotika notranje površine sklede, oprta pa je tudi na njen rob, saj je daljša od dveh premerov. Poišči ravnovesno lego slamice, če med slamico in skledo ni trenja. Zapiši Lagranžijan za tak sistem! Dolžina slamice je enaka trikratnemu polmeru sklede.
Navodilo: V ravnovesni legi ima potencialna energija minimum. Kinetično energijo zapiši kot vsoto translacijske energije težišča in rotacijske okrog njega. Za generalizirano koordinato vzemi kot med slamico in navpičnico.

49. Nabit delec se giblje v polju točkastega magnetnega monopola. Kako se giblje delec? Opiši gibanje delca še za poseben primer, ko je začetna hitrost delca pravokotna na zveznico med delcem in monopolom!

DN 6

50. V $2m$ dolgi cevi, ki se vrti v vodoravni ravnini okoli svojega središča, se brez trenja giblje delec z maso 1g. Delec nosi naboj $8\cdot 10^{-5} As$, nanj pa z električno silo deluje homogeno električno polje jakosti $1kV/m$. Polje je vzporedno z vodoravno ravnino, palica pa se zavrti desetkrat v sekundi. Zapiši Lagrangeovo funkcijo in gibalne enačbe! Po kolikšnem času delec odleti iz palice, če je na začetku miroval v središču, palica pa je bila takrat usmerjena pravokotno na smer električnega polja.

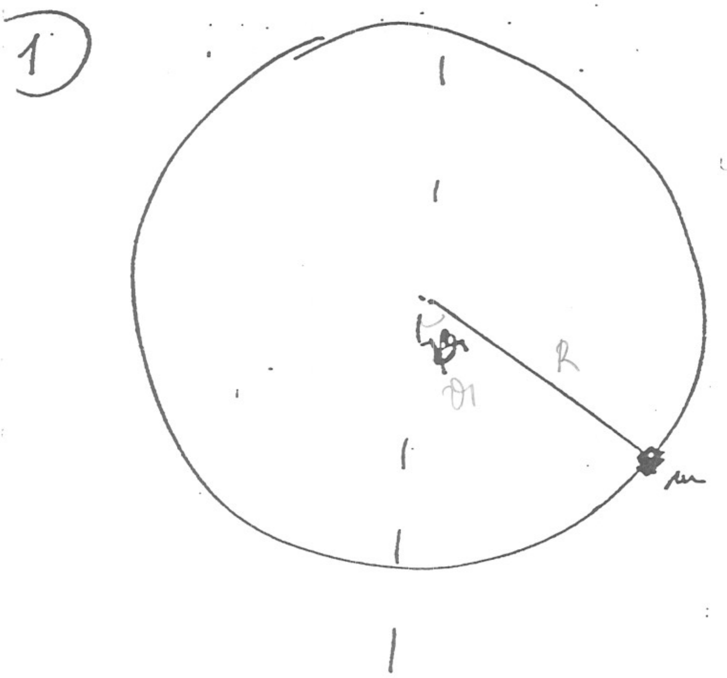
51. Drobna utež z maso m, ki drsi po vodoravni podlagi, je z vrvico privezana na količek s polmerom d. Utež poženemo tako, da je vrvica vseskozi napeta in se med gibanjem uteži navija na količek. Zapiši Lagranževo funkcijo za opisani sistem in pokaži, da je enaka Hamiltonovi funkciji! Reši enačbe gibanja! Navodilo: kljub temu, da se vrvica navija na količek, privzemi, da se njeno pritrdišče nahaja v središču količka.

DN 1

52. V jašek z globino h spustimo kamen. V točko, kjer je kamen treščil v dno jaška, posvetimo z laserjem, ki je nameščen v točki, od koder smo kamen spustili. Nato iz iste točke spustimo še en kamen (ki leti po isti trajektoriji kot prvi). Na kateri globini se kamen najbolj oddalji od laserskega žarka? Poskus izvajamo na geografski širini Θ. Upoštevaj vrtenje Zemlje!

jašek · kamen : začetek

1)



$V = -mgR\cos\vartheta$

$T = \frac{m}{2}R^2\dot\vartheta^2 + \frac{m}{2}\omega r^2 \Big\} \Rightarrow L = T - V = \frac{m}{2}R^2\left(\dot\vartheta^2 + \omega^2\sin^2\vartheta\right) + mgR\cos\vartheta$

uler – Lagrange:

$\ddot\vartheta - \omega^2\sin\vartheta\cos\vartheta + \frac{g}{R}\sin\vartheta = 0$

avnovesje $\quad \ddot\vartheta = 0 \quad \Rightarrow \omega^2\sin\vartheta\cos\vartheta = \frac{g}{R}\sin\vartheta$

Kako ugotovimo, katere lege so stabilne?

. $\vartheta_1 = 0$

2. $\vartheta_2 \Rightarrow \cos\vartheta_2 = \frac{g}{\omega^2 R}$

ala nihanja $\quad \vartheta = \vartheta_0 + \varphi$

$\sin\vartheta = \sin\vartheta_0 + \varphi\cos\vartheta_0$

$\cos\vartheta = \cos\vartheta_0 - \varphi\sin\vartheta_0$

$\sin\vartheta\cos\vartheta = \frac{1}{2}\sin 2\vartheta_0 + \varphi\cos 2\vartheta_0$

NAROBE

$\Rightarrow \ddot\varphi - \frac{\omega^2}{2}\left(\sin 2\vartheta_0 + \varphi\cos 2\vartheta_0\right) + \frac{g}{R}\left(\sin\vartheta_0 + \varphi\cos\vartheta_0\right) = 0$

$\ddot\varphi + \varphi\underbrace{\left[\frac{g}{R}\cos\vartheta_0 - \omega^2\cos 2\vartheta_0\right]}_{D^2} = C$

$\Omega^2 > 0 \qquad \frac{g}{R}\cos\vartheta_0 > \omega^2 \cos 2\vartheta_0 = \omega^2(\cos^2\vartheta_0 - \sin^2\vartheta_0) = \omega^2(2\cos^2\vartheta_0 -$

1. Za $\vartheta_1 = 0 \quad \Rightarrow \quad \frac{g}{R} > \omega^2$

Torej: za $\omega^2 < \frac{g}{R}$ je ravnovesna lega ena sama pri ~~$\vartheta=0$~~ $\vartheta = 0$ in je stabilna.

Frekvenca: $\underline{\Omega_1^2 = \frac{g}{R} - \omega^2}$

2. Za $\cos\vartheta_2 = \frac{g}{\omega^2 R}$

$\Rightarrow \Omega^2 > 0$

$$\frac{g}{\cancel{R}} \cdot \frac{g}{\cancel{\omega^2 R}} > \cancel{\omega^2}\left(\frac{2g^2 - \omega^4 R}{\cancel{\omega^4 R^2}}\right)$$

$\omega^2 R > g$

$\omega^2 > \frac{g}{R}$

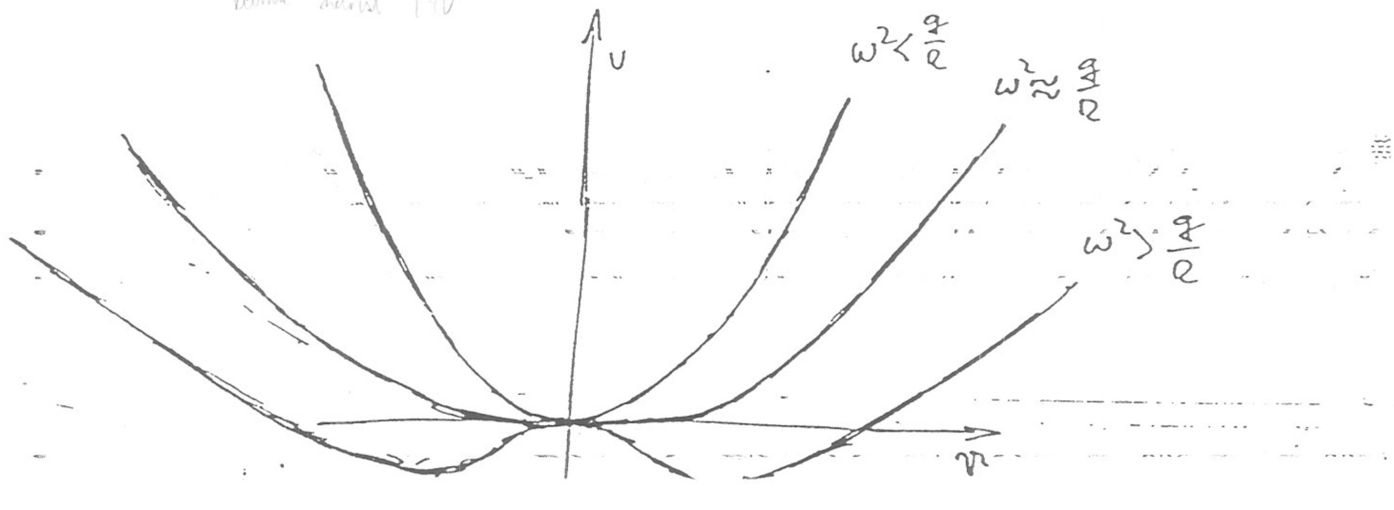
$\Rightarrow$ za $\omega^2 > \frac{g}{R}$ so ravnovesne lege tri: $\vartheta_2, -\vartheta_2, \vartheta_1$, vendar je pri ϑ_1 to labilna ravnovesna lega, pri ϑ_2 in $-\vartheta_2$ pa stabilna.

Frekvenca: $\Omega_2^2 = \frac{g}{R}\cdot\frac{g}{\omega^2 R} - \cancel{\omega^2}\left(\frac{2g^2 - \omega^4 R^2}{\cancel{\omega^4} R^2 \omega^2}\right) = \frac{\omega^4 R^2 - g}{R^2\omega^2}$

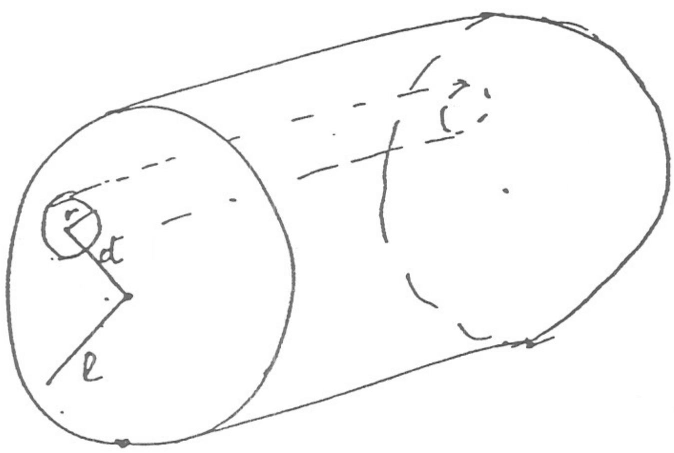
$$\underline{\underline{= \omega^2 - \frac{g^2}{R^2\omega^2} = \Omega_2^2}}$$

Potencialna jama se z ω spreminja tako:

doma nariši $T(\vartheta)$



c) valj z luknjo:



Valj: $M = \pi R^2 h \rho$, $J = \frac{MR^2}{2}$

luknja: $m = \pi r^2 h \rho$, $j = \frac{mr^2}{2}$

Valj: $x_1 = R\varphi$ $\quad \dot{x}_1 = R\dot{\varphi}$

$y_1 = 0$ $\quad \dot{y}_1 = 0$

$= \frac{1}{2}(J + MR^2)\dot{\varphi}^2$

luknja: $x_2 = x_1 + d\sin\varphi$

$y_2 = y_1 + d\cos\varphi$

$\cdot \frac{1}{2} m(\dot{x}_2^2 + \dot{y}_2^2) + \frac{1}{2} j \dot{\varphi}^2$

$\dot{x}_2 = \dot{x}_1 + \dot{\varphi} d\cos\varphi$

$= -mg y_2 = -mgd\cos\varphi$

$\dot{y}_2 = \dot{y}_1 - \dot{\varphi} d\sin\varphi$

$= \dot{x}_1^2 + \dot{\varphi}^2 d^2 \cancel{\cos^2\varphi} + 2\dot{x}_1\dot{\varphi} d\cos\varphi + \cancel{\dot{y}_1^2} \cancel{+ \dot{\varphi}^2 d^2 \sin^2\varphi} - \cancel{2\dot{y}_1 \dot{\varphi} d\sin\varphi + \frac{1}{2} d}$

stevem $\dot{x}_1 = R\dot{\varphi}$: $\quad \Rightarrow -T_2 = \left[\frac{1}{2} m \underbrace{\left(R^2 + d^2 + 2Rd\cos\varphi\right)} + \frac{1}{2} j\right]\dot{\varphi}^2$

(ravno kosinusni izraz za razdaljo od dotikališča do središča luknje)

$L = T - V = T_1 + T_2 - V = \frac{1}{2}\underbrace{\left(J + MR^2 + j + m(R^2 + d^2 + 2Rd\cos\varphi)\right)}_{A(\varphi)}\dot{\varphi}^2 + mgd\cos\varphi$

$A(\varphi) \leftarrow$ predstavlja ~~[illegible]~~ efektivni vztr. moment pri vrtenju okrog dotikališča

-Lagrange:

$\ddot{\varphi} + 2mRd\sin\varphi\, \dot{\varphi}^2 + mgd\sin\varphi = 0$

vna lega: $\ddot{\varphi} = 0$, $\dot{\varphi} = 0$ $\to \sin\varphi = 0 \to \varphi = 0, \pi$

i kotu: $\sin\varphi = \varphi$, $\cos\varphi = 0$, $\varphi = \varphi_0 + \alpha$

mi: za $\varphi_0 = \pi \to$ labilna lega

za $\varphi_0 = 0 \to$ stabilna lega

$\Rightarrow$ za $\varphi_0 = 0$:

$$A(0)\ddot{\alpha} + 2mld\alpha\dot{\alpha}^2 + mgd\alpha = 0$$

Če nastavim $\alpha = \alpha_0 \sin\omega t \Rightarrow \dot{\alpha} = \omega\alpha_0 \sin\alpha_0$

Vidim, da je $2mld\alpha\dot{\alpha}^2 = 2mld\,\alpha_0^3\,\omega^2 \sin^2\omega t$, torej III. reda in ga zanemarim. Ostane:

$$A(0)\ddot{\alpha} + mgd\alpha = 0$$

$$\ddot{\alpha} + \Omega^2\alpha = 0 \qquad \underline{\underline{\Omega^2 = \frac{mgd}{A(0)}}}$$

) Poissonovi oklepaji:

$[p_u, p_r]$ in $[x_m, x_e]$

$$[l_i, l_j] = [\varepsilon_{imn} x_m p_n, \varepsilon_{jqr} x_q p_r] = \varepsilon_{imn} \varepsilon_{jqr} (-x_m p_r [x_q, p_n] + x_q p_n [x_m, p_r])$$

$$= \varepsilon_{imn} \varepsilon_{jqr} \delta_{mr} x_q p_n - \varepsilon_{imn} \varepsilon_{jqr} \delta_{qn} x_m p_r$$

$$= \varepsilon_{imn} \varepsilon_{jqm} x_q p_n - \varepsilon_{imn} \varepsilon_{jnr} x_m p_r = (*)$$

(Upoštevam $\varepsilon_{imn} \varepsilon_{jqm} = \delta_{ij} \delta_{nq} - \delta_{iq} \delta_{jn}$)

$$(*) = (-\delta_{ij} \delta_{nq} + \delta_{iq} \delta_{jn}) x_q p_n + (\delta_{ij} \delta_{mr} - \delta_{ir} \delta_{jm}) x_m p_r$$

$$= -\delta_{ij} x_n p_n + x_i p_j + \delta_{ij} x_r p_r - p_i x_j =$$

1. če $i = j \Rightarrow \quad = 0$
2. če $i \neq j \Rightarrow \quad = x_i p_j - x_j p_i = l_k$

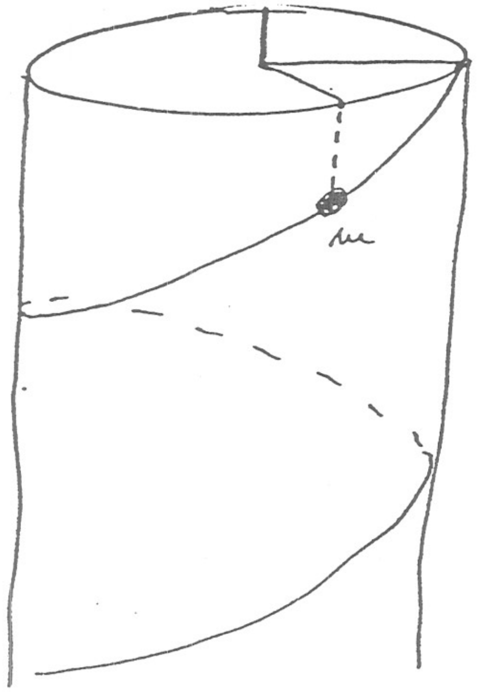
$$\rightarrow \underline{\underline{[l_i, l_j] = \varepsilon_{ijk} l_k}}$$

$$[L_z, \vec{L}^2] = [L_z, L_x^2 + L_y^2 + L_z^2] = [L_z, L_x^2] + [L_z, L_y^2] + 0$$

$$= L_x [L_z, L_x] + [L_z, L_x] L_x + L_y [L_z, L_y] + [L_z, L_y] L_y =$$

$$= 2L_x \cdot L_y - 2L_y L_x = \underline{\underline{0 = [L_i, \vec{L}^2]}}$$

4)



φ - položaj valja v našem sistemu

ϑ - lega mase glede na valj!

J_1 - vztr. moment valja

$J_2 = m r^2$

$$T = \underbrace{\frac{J_1}{2}\dot\varphi^2}_{\text{vrtenje valja}} + \underbrace{\frac{J_2}{2}\left(\dot\vartheta^2 + 2\dot\vartheta\dot\varphi + \dot\varphi^2\right)}_{\text{vrtenje mase}} + \underbrace{\frac{m}{2}p^2\dot\vartheta^2}_{\text{gibanje mase navzdol}}$$

$V = m g p \vartheta$

$L = T - V$

ciklična $\quad p_\varphi = J_1\dot\varphi + J_2(\dot\vartheta + \dot\varphi)$ ← vrtilna količina

$$\dot\varphi = \frac{p_\varphi - J_2\dot\vartheta}{J_1 + J_2}$$

Ker je vseeno, ali se sistem na začetku ob $t=0$ vrti ali ne (sila radialna na m je pravokotna na gibanje), lahko vzamem $\underline{p_\varphi = 0}$. Označim $\frac{J_2}{J_1+J_2} = A$ $\quad \dot\varphi = -A\dot\vartheta$

$\Rightarrow \varphi = -A\vartheta \; (+C)$

$\ddot\varphi = -A\ddot\vartheta$

$$T = \frac{J_1}{2}A^2\dot\vartheta^2 + \frac{J_2}{2}\left(\dot\vartheta^2 - 2A\dot\vartheta^2 + A^2\dot\vartheta^2\right) + \frac{m}{2}p^2\dot\vartheta^2$$

$$L = T - V = \dot\vartheta^2 \underbrace{\left(\frac{J_1 A^2}{2} + \frac{J_2(1-A)^2}{2} + \frac{m p^2}{2}\right)}_{c/2} - m g p \vartheta$$

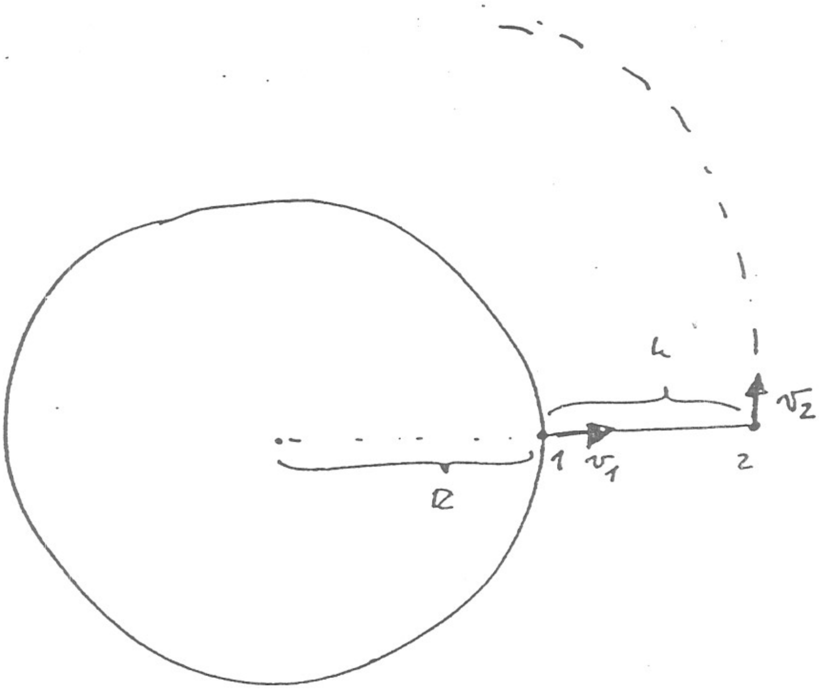
Euler - Lagrange:

$c\ddot\vartheta + m g p = 0$

$$\ddot\vartheta = -\frac{m g p}{c} \qquad \dot\vartheta = -\frac{m g p}{c}t \; (+\dot\vartheta_0)$$

$$\vartheta = -\frac{m g p}{2c}t^2 \; (+\dot\vartheta_0 t + \vartheta_0)$$

5)



čka 2 : Vodoraven pospešek

končna hitrost satelita (m_0): $v_2 = 8$ km/s ($= \sqrt{gr}$, $r = R+h$)

masa drugega balasta:

$m_0 v_2 = m_{B2} v_0$ ($v_0 = 3$ km/s)

$\Rightarrow m_0 = m_{B2} \cdot \frac{v_0}{v_2} = m_{B2} \cdot R_2$ ($R_2 = \frac{3}{8}$)

$$\frac{m_0}{m_0 + m_{B2}} = \frac{m_0}{m_0 + \frac{m_0}{R_2}} = \frac{1}{1+\frac{1}{R_2}} = \frac{3}{11}$$

($\doteq 27.3\%$ torej ostane po vodoravnem pospešku)

čka 1 : Navpični met

hitrost koristnega tovora: (m_0'): $v_1 = \sqrt{2gh} = 2$ km/s

masa prvega balasta: $= R_1$

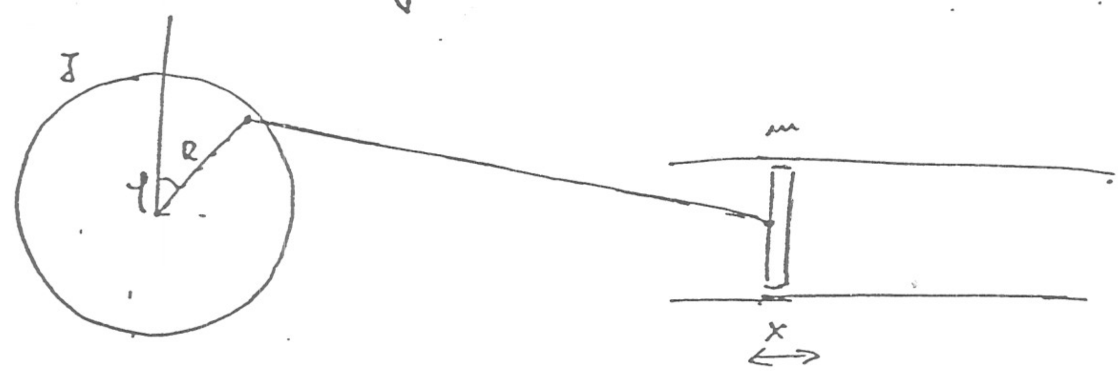
$m_0' v_1 = m_{B2} v_0 \rightarrow m_0' = m_{B2} \cdot \frac{v_0}{v_2} = m_{B2} \cdot \frac{3}{2}$

$\frac{m_0'}{m_0' + \dots} = \frac{\cancel{m_0'}}{\dots} = \frac{1}{\dots} = \frac{3}{5}$ ← Tak procent začetne mase spravimo v točko ...

Koristni tovor na raketi sme torej tehtati

$60\% \cdot 27.3\% = \frac{9}{55} = 16\%$ (ob predpostavki da je vse ostalo balast)

6) Bat in vztrajnik:



$V=0, \quad T=\frac{1}{2}J\dot{\varphi}^2+\frac{1}{2}m\dot{x}^2$

$x = R\sin\varphi$

$\dot{x} = R\dot{\varphi}\cos\varphi$

$\Rightarrow T = \frac{1}{2}\left[J + mR^2\cos^2\varphi\right]\dot{\varphi}^2$

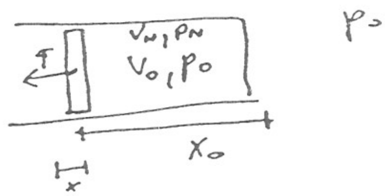
Euler-Lagrange:

$$\frac{d}{dt}\left(\left[J+mR^2\cos^2\varphi\right]\dot{\varphi}\right) + \dot{\varphi}^2 mR^2\cos\varphi\sin\varphi = 0$$

$$(J+mR^2\cos^2\varphi)\ddot{\varphi} - \cancel{2}mR^2\cos\varphi\sin\varphi\,\dot{\varphi}^2 + \cancel{mR^2\cos\varphi\sin\varphi\,\dot{\varphi}^2} = 0$$

$$\left(\frac{J}{mR^2}+\cos^2\varphi\right)\ddot{\varphi} - \cos\varphi\sin\varphi\,\dot{\varphi}^2 = 0$$

Ko bat zapremo:



$$F = \frac{1}{S}\left(p_0 - \frac{V_0 p_0}{V_0 + xS}\right) = \frac{p_0}{S} - \frac{p_0/S}{\left(1+\frac{x}{x_0}\right)} = \frac{p_0}{S} - \frac{p_0}{S}\left(1-\frac{x}{x_0}\right) = \frac{p_0}{S}\cdot\frac{x}{x_0}$$
$$= \frac{p_0}{V_0}\cdot x$$

[illegible] sila vedno deluje nasproti x, zato $F = -\frac{p_0}{V_0}x$

Torej $F = -\frac{p_0}{V_0}\cdot R\varphi$ za male kote.

Upoštevam $x = R\varphi$ $\quad \sin\varphi = \varphi$

$\dot{x} = R\dot{\varphi}$ $\quad \cos\varphi = 1$

$$\left(\frac{J}{mR^2}+1\right)\ddot{\varphi} - \varphi\dot{\varphi}^2 = -\frac{p_0}{V_0}R\varphi$$

$$\left(\frac{J}{mR^2}+1\right)\ddot{\varphi} + \frac{p_0}{V_0}R\,\varphi$$

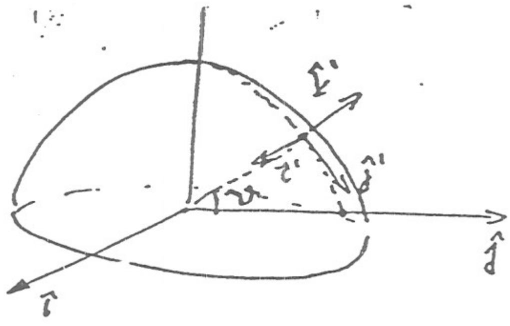
$$\ddot{\varphi} + \underbrace{\frac{p_0 R}{V_0\left(\frac{J}{mR^2}+1\right)}}_{\omega^2}\varphi = 0$$

Ocenimo $\varphi = \varphi_0 \sin\omega t$

$\dot{\varphi} = \omega\varphi_0\cos\omega t$

$$\underbrace{\varphi\dot{\varphi}^2 = \omega^2 \underline{\varphi_0^3} \cos^2\omega t}_{\text{III. red, zato lahko brez slabe vesti zanemarimo}}$$

(7)



Naj utež drsi v ravnini $\hat{k}\hat{j}$.

Potem:

$$\hat{i}' = \hat{i}$$
$$\hat{j}' = \sin\vartheta\,\hat{j} - \cos\vartheta\,\hat{k}$$
$$\hat{k}' = \sin\vartheta\,\hat{k} + \cos\vartheta\,\hat{j}$$

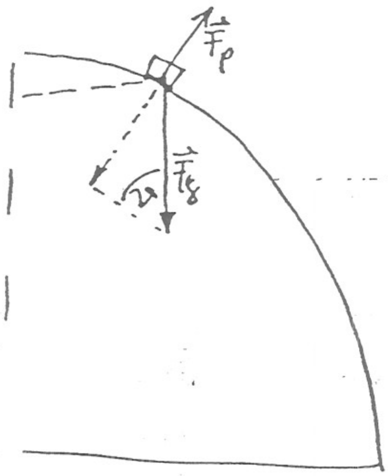
$$\tilde{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin\vartheta & \cos\vartheta \\ 0 & -\cos\vartheta & \sin\vartheta \end{bmatrix} \qquad \Rightarrow R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin\vartheta & -\cos\vartheta \\ 0 & \cos\vartheta & \sin\vartheta \end{bmatrix}$$

$$\dot{R} = \dot{\vartheta}\begin{bmatrix} 0 & 0 & 0 \\ 0 & \cos\vartheta & \sin\vartheta \\ 0 & -\sin\vartheta & \cos\vartheta \end{bmatrix}$$

$$\Rightarrow \dot{R}\tilde{R} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\vec{\omega} = -\dot{\vartheta}\,\hat{i}$$
$$\vec{r} = R\,\hat{k}'$$
$$\dot{\vec{r}} = \dot{\vartheta} R\,\hat{j}'$$
$$\ddot{\vec{r}} = -R\dot{\vartheta}^2\hat{k}' + R\ddot{\vartheta}\,\hat{j}'$$

Zunanji sili, ki delujeta na utež: teža, podlaga:



Pospešek pri kroženju: $\vec{a}_r = -\omega^2\vec{r}$, $\omega = \frac{v}{r}$

$$\Rightarrow \vec{a}_r = -\frac{v^2}{r^2}\vec{r}$$

V mejni točki velja $\cancel{m}\vec{a}_r = -F_g \sin\vartheta \cdot \frac{\vec{r}}{r}$

$$+\frac{v^2}{r^2}\cdot\cancel{\frac{\vec{r}}{r}} = +\cancel{F}_g \sin\vartheta\cdot\cancel{\frac{\vec{r}}{r}}$$

$$\frac{\cancel{m}v^2}{2} = \cancel{m}gh$$

$$v^2 = 2g(R - R\sin\vartheta) \qquad 2\cancel{g}(R - R\sin\vartheta) = R\cancel{g}\sin\vartheta$$

$$\Rightarrow 2R = 3R\sin\vartheta$$

$$\Rightarrow \underline{\underline{\vartheta = \arcsin\frac{2}{3}}}$$

se | z multiplikatorjem:

$T = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\vartheta}^2)$

$V = mgr\cos\vartheta$

$L = T - V$

vez: $r = R$
$\dot{r} = 0$

Euler-Lagrange:

1. ϑ: $\frac{d}{dt}(mr^2\dot{\vartheta}) - mgr\sin\vartheta = 0$

$mr^2\ddot{\vartheta} + 2mr\dot{r}\dot{\vartheta} - mgr\sin\vartheta = 0$

$\frac{\partial L}{\partial \dot{q}} = p_q$

2. r: $\frac{d}{dt}(m\dot{r}) + mg\cos\vartheta - \dot{\vartheta}^2 mr = \lambda$

$m\ddot{r} + mg\cos\vartheta - mr\dot{\vartheta}^2 = \lambda$ (1)

To je izražava $\lambda(\vartheta, \dot{\vartheta})$. Če želim $\lambda(\vartheta)$, moram najti $\dot{\vartheta}(\vartheta)$.

$H = T + V = \frac{1}{2} m (\dot{r}^2 + r^2\dot{\vartheta}^2) + mgr\cos\vartheta = mgr$

Upoštevam $r = R$, $\dot{r} = 0$ $\Rightarrow$ $H = \frac{m}{2} R^2 \dot{\vartheta}^2 + mgR\cos\vartheta = mgR$

$\Rightarrow \frac{m}{2} R\dot{\vartheta}^2 = mg(1-\cos\vartheta)$ $\Rightarrow \dot{\vartheta}^2 = \frac{2g}{R}(1-\cos\vartheta)$ (2)

(1) & (2) $\Rightarrow$ $mg\cos\vartheta - mR\cdot\frac{2g}{R}(1-\cos\vartheta) = \lambda$ (Upoštevam $r = R$, $\dot{r} = 0$ v (1))

$mg\cos\vartheta - 2mg + 2mg\cos\vartheta = \lambda$

$3mg\cos\vartheta - 2mg = \lambda$

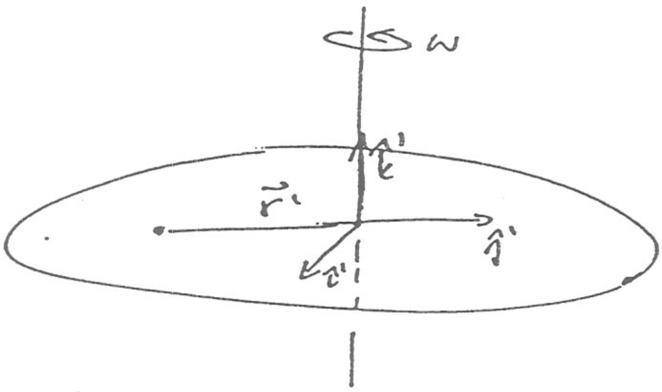
Ko se odlepi: $\lambda = 0$ $\Rightarrow$ $3\cos\vartheta = 2$, $\vartheta = \arccos\frac{2}{3}$

(Rezultat je enak tistemu na prejšnji strani, ker sem vzel drugačna kota ϑ)

$L = T - V + \lambda(R - r)$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r}$ $\quad \vartheta:$ $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\vartheta}}\right) - \frac{\partial L}{\partial \vartheta}$

5) Vrteče restavracije v Torontu:



$$\vec{a} = \cancel{\vec{A}}^{\,0} + \cancel{\vec{a}_{rel}}^{\,0} + 2\vec{\omega}\times\vec{v}_{rel} + \vec{\omega}\times\vec{\omega}\times\vec{r}\,' + \cancel{\dot{\vec{\omega}}\times\vec{r}\,'}^{\,0}$$

gost za mizo: $\vec{a} = \vec{\omega}\times\vec{\omega}\times\vec{r}\,' \Rightarrow a_r = \omega^2 r$

gladina:

$$\vec{a}\,' = g\hat{k}' + \omega^2 r\,\hat{j}'$$

$$\vec{a}_1\,' = \begin{bmatrix} 0 \\ \omega^2 r \\ g \end{bmatrix}$$

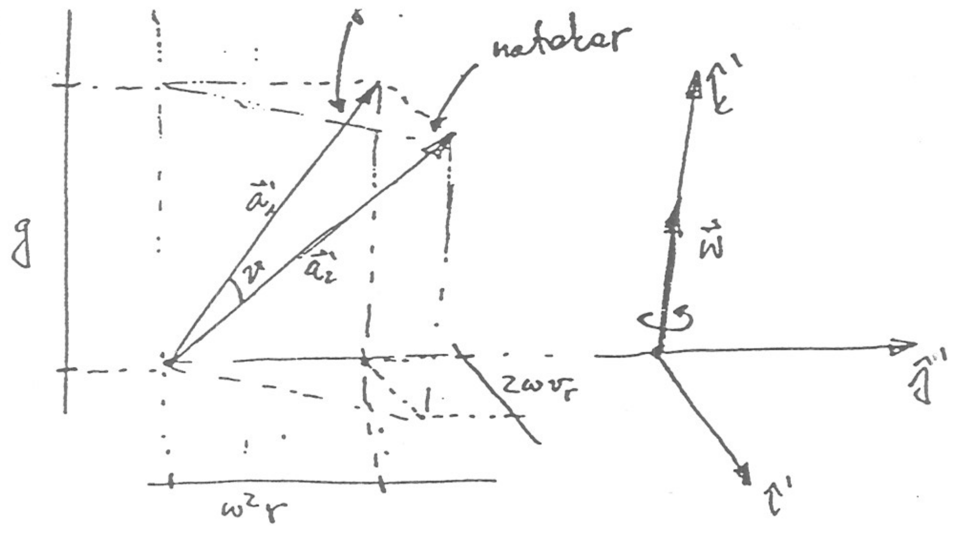
Natakar:

$$\vec{a} = 2\vec{\omega}\times\vec{v}_{rel} + \vec{\omega}\times\vec{\omega}\times\vec{r}$$

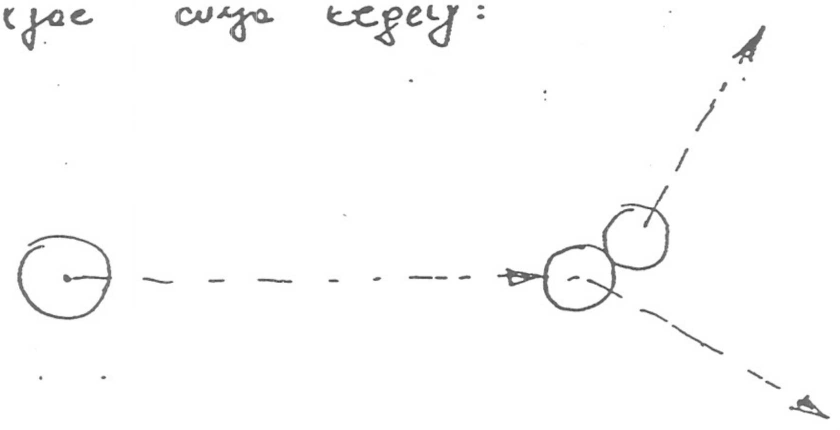
$$\Rightarrow \vec{a}_2\,' = \begin{bmatrix} 2\omega v \\ \omega^2 r \\ g \end{bmatrix}$$

Gladini sta pravokotni na vektorje $\vec{a}_1\,'$ (gost) in $\vec{a}_2\,'$ (natakar). Kot med normalama:

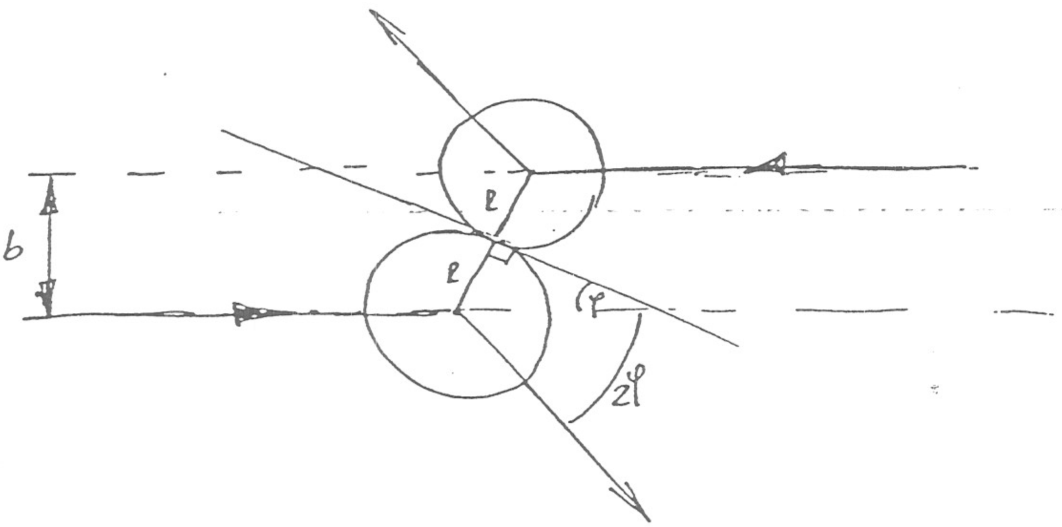
$$\cos\vartheta = \frac{\vec{a}_1\,'\vec{a}_2\,'}{a_1' a_2'} = \frac{\omega^4 r^2 + g^2}{\sqrt{\omega^4 r^2 + g^2}\cdot\sqrt{\omega^4 r^2 + g^2 + 4\omega^2 v^2}}$$


natoker
g
$\vec{a}_1$
$\vec{a}_2$
v
$2\omega v_r$
$\omega^2 r$
$\vec{k}'$
$\vec{w}$
$\vec{j}'$
$\vec{i}'$

7) Kegljač cilja kegelj:



Proučimo trk v težiščnem sistemu:



$b = 2R\cos\varphi$

Hitrosti pred trkom: $\vec{v}_{10}' = v_0(1,0)$, $\vec{v}_{20}' = v_0(-1,0)$

Hitrosti po trku: $\vec{v}_{11}' = v_0(\cos 2\varphi, \sin 2\varphi)$, $\vec{v}_{21}' = -v_0(\cos 2\varphi, \sin 2\varphi)$

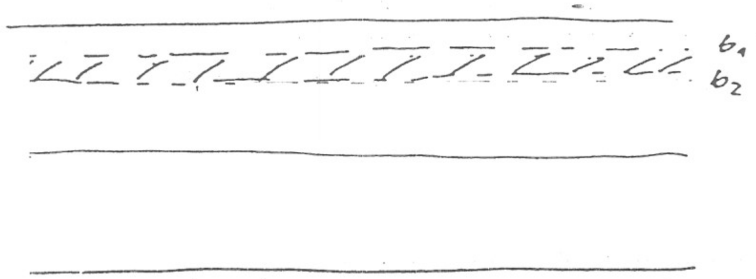
Nazaj v laboratorijski sistem: $(+\,v_0(1,0))$

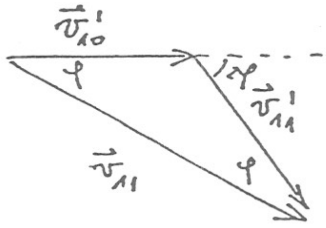
$\vec{v}_{10} = 2v_0(1,0)$ $\quad \vec{v}_{20} = (0,0)$

$\vec{v}_{11} = v_0(\cos 2\varphi + 1, \sin 2\varphi)$, $\vec{v}_{21} = v_0(-\cos 2\varphi + 1, -\sin 2\varphi)$

edaj me zanimata b_1 in b_2: pri b_1 → odklon 20° levo

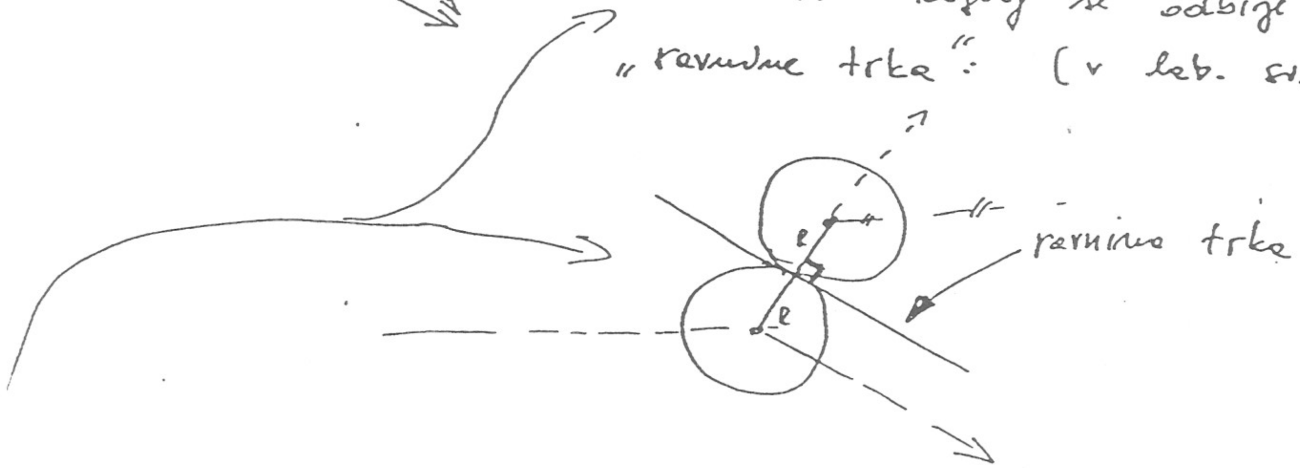
pri b_2 → odklon 30° levo





Vidimo, da $\alpha = \varphi$ (α-odklon keglja v lab. sistemu)

Zanimivo - kegelj se odbije v smeri „ravnine trka": (v lab. sistemu)



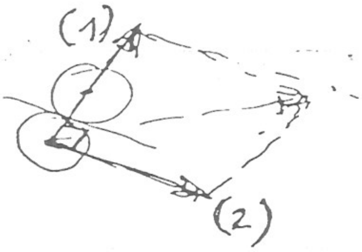
$$\Rightarrow \varphi_1 = \arccos\frac{b_1}{2R} = 20^\circ \qquad b_1 = 2R\cos 20^\circ$$

$$\varphi_2 = \arccos\frac{b_2}{2R} = 30^\circ \qquad b_2 = 2R\cos 30^\circ$$

$$\frac{b_1 - b_2}{4R} = \frac{1}{2}\left[\cos 20^\circ - \cos 30^\circ\right] = \underline{3{,}68\%}$$

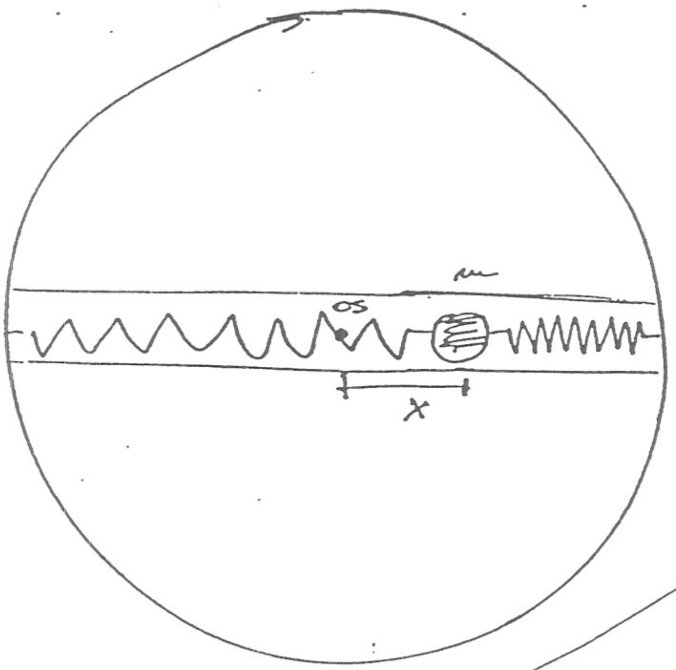
Ta polovica je zaradi odklanjanja samo levo.

(To je v bistvu pričakovano z malo drugačnim razmislekom. Spodnji kegelj nosi gib. kol. v teh dveh smereh:



V smeri (1) jo popolnoma preda zgornjemu keglju, v smeri (2) pa ostane nespremenjena)

(10)



(To lahko naredimo, saj mi vrtimo vodilo z ω in tako $\vec{L} \neq$ const)

Centrifugalno silo spravim v potencial:

$V_1 = -m\omega^2 \frac{x^2}{2}$ (sila deluje navzven)

Še vzmet: $V_2 = +\frac{kx^2}{2}$ (sila deluje navznoter)

$\Rightarrow$ $V = -\frac{mx^2}{2}\left(-\frac{k}{m} + \omega^2\right)$

$T = \frac{m\dot{x}^2}{2}$

$L = T - V = \frac{m}{2}\left(\dot{x}^2 + x^2\left(-\frac{k}{m} + \omega^2\right)\right) = 0$

E.L. : $\Rightarrow$ $\ddot{x} - x\left(-\frac{k}{m} + \omega^2\right) = 0$

(1.) $-\frac{k}{m} + \omega^2 = \omega_1^2 > 0$ (šibka vzmet)

$$\boxed{x = Ae^{\omega t} + Be^{-\omega t}}$$

$x(0) = x_0 = A + B$

$\dot{x}(0) = v_0 = A\omega e^{\omega t} - B\omega e^{-\omega t}$

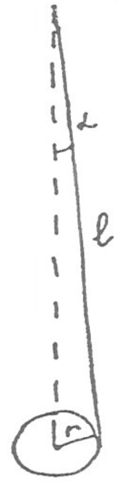
$x_0 = A + B$

$\frac{v_0}{\omega} = A - B$

$\Rightarrow$ $A = \frac{x_0 + \frac{v_0}{\omega}}{2}$, $B = \frac{x_0 - \frac{v_0}{\omega}}{2}$ (x_0, v_0 $\rightarrow$ začetna lega in hitrost)

Imamo eno ravnovesno lego, $x=0$, ki pa je labilna in utež prej ali slej zdrsi ob rob.

) Jojo



$\alpha \doteq 0$

$l = l_0 - r\varphi$, kjer l_0 = raztegnjena vrvica (ob predpostavki, da se kot ~~zmanjšuje~~ povečuje, ko Jojo navijamo)

zanima me čas od $l=0$ do $l=l_0$

oz. od $\varphi = +\varphi_0 = \frac{l_0}{r}$ do $\varphi_* = 0$

$$T = \frac{1}{2}J\dot{\varphi}^2 + \frac{1}{2}m\dot{l}^2 = \frac{1}{2}\underbrace{(J+mr^2)}_{J'}\dot{\varphi}^2$$

φ_0 - kot za katerega je jojo navit na začetku

$$V = -mgl = -mg(l_0 - r\varphi)$$

$$L = T - V = \frac{1}{2}J'\dot{\varphi}^2 - mgr\varphi \qquad (+mgl_0)$$

Euler-Lagrange: $J'\ddot{\varphi} + mgr = 0 \Rightarrow \ddot{\varphi} = -\frac{mgr}{J'}$

$$\Rightarrow \varphi = -\frac{mgr}{2J'}t^2 + \varphi_0$$

$$\varphi = 0 \Rightarrow \varphi_0 = \frac{mgr}{2J'}t^2$$

$$t_0 = \sqrt{\frac{2J'\varphi_0}{mgr}} = \sqrt{\frac{2J'l_0}{mgr}}$$

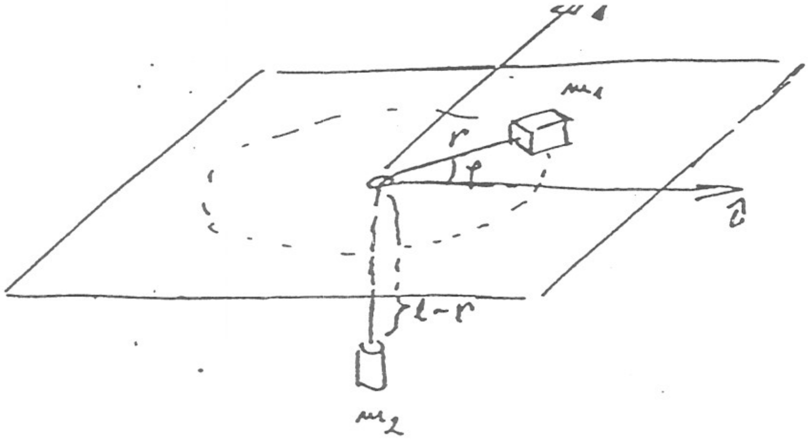
To je čas, ki ga jojo potrebuje iz $l=0$ do $l=l_0$.

l_0 - višinska razlika med amplitudo (max.) in mirovno lego (min)

Nihajni čas $\tau = 4t_0$

(Upoštevam, da se enkrat navije levo in enkrat desno,
...)

12)



$x = r\cos\varphi$
$y = r\sin\varphi$

$\dot{x} = \dot{r}\cos\varphi - \dot{\varphi}\sin\varphi\, r$
$\dot{y} = \dot{r}\sin\varphi + \dot{\varphi}\cos\varphi\, r$

$$T = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} = \frac{m_1}{2}\left(\dot{r}^2 + \dot{\varphi}^2 r^2\right) + \frac{m_2}{2}\dot{r}^2$$

$$V = -m_2 g(l-r)$$

$$L = T - V = \frac{m_1}{2}(\dot{r}^2 + \dot{\varphi}^2 r^2) + \frac{m_2}{2}\dot{r}^2 + m_2 g(l-r)$$

$$p_\varphi = m_1 \dot{\varphi} r^2 \quad (\varphi \text{ ciklična})$$

E. L. (po r):

$$M\ddot{r} = m_1 \dot{\varphi}^2 r + m_2 g \qquad (M = m_1 + m_2)$$

$$M\ddot{r} = \frac{p_\varphi^2}{m_1 r^3} + m_2 g$$

Iščemo $\ddot{r} = 0$

$$\frac{1}{m_1 r^3} = \frac{m_2 g}{p_\varphi^2} \Rightarrow \underline{\underline{r^3 = \frac{p_\varphi^2}{g m_1 m_2}}}$$

ravnovesje

jhni odmiki $r = r_0 + x$

$$-\frac{p_\varphi^2}{m_1(r_0+x)^3} + m_2 g = 0$$

Upoštevam $(r_0+x)^3 \doteq r_0^3 + 3r_0^2 x$ in $\frac{1}{1+x} \doteq 1 - x$

$$M\ddot{x} - \frac{p_\varphi^2}{m_1(r_0^3 + 3r_0^2 x)} + m_2 g = 0$$

$$M\ddot{x} - \frac{p_\varphi^2}{m_1 r_0^3}\left(1 - \frac{3x}{r_0}\right) + m_2 g = 0$$

$$\Longrightarrow \quad M\ddot{x} + \frac{3\rho\dot{\varphi}}{m_1 r_0^4} \cdot x + C = 0$$

$$\Longrightarrow \quad \Omega^2 - \frac{1}{M} \cdot \frac{3\rho\dot{\varphi}^2}{m_1 r_0^4} = \frac{1}{M} \cdot \frac{3 m_1^{\cancel{2}} \dot{\varphi}^2 \cancel{r_0^4}}{\cancel{m_1} \cancel{r_0^4}}$$

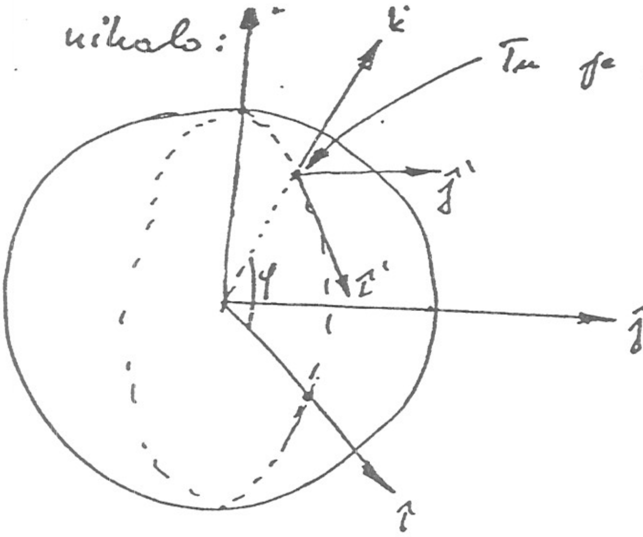
$$\Longrightarrow \quad \Omega^2 = \frac{3 m_1}{m_1 + m_2}\, \omega^2$$

$\Big(\omega = \dot{\varphi}$ ← kroženje

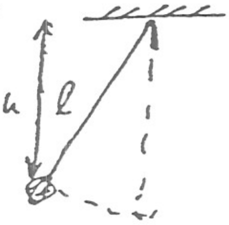
Ω ← nihanje

Foucaultovo nihalo:

3)



Ta je nihalo:



(pri $\varphi = \frac{\pi}{2}$: $\hat{i}' = \hat{i}$, $\hat{j}' = \hat{j}$, $\hat{k}' = \hat{k}$)

$T = \frac{1}{2} m \vec{v}^{\,2}$

$V = -mgh$

$\vec{v} = \vec{V} + \vec{v}_{rel} + (\vec{\omega} \times \vec{r}\,')$

lahko zanemarimo, saj je nekaj velikostnih redov manjši od ostalih

$\vec{v}^{\,2} = \vec{V}^2 + \vec{v}_r^{\,2} + \cancel{(\vec{\omega} \times \vec{r}\,')^2} + 2\vec{V}\vec{v}_r + 2\vec{v}_r(\vec{\omega} \times \vec{r}\,') + 2\vec{V}(\vec{\omega} \times \vec{r}\,')$

Ta člen uniči E.L., saj je const.

Ta člen bodo uničile E.L. enačbe (člen oblike $c \cdot \dot{q}_k$)

popravek k $\vec{g}$, zato bi, če bi bil ta člen zraven, to upošteval tam

to ni pomembno

$\frac{2}{m} T = C + \vec{v}_r^{\,2} + 2\vec{v}_r(\vec{\omega} \times \vec{r}\,')$

$\vec{v}_r = \begin{bmatrix} \dot{x} \\ \dot{y} \\ 0 \end{bmatrix}$

$\vec{v}_r(\vec{\omega} \times \vec{r}\,') = \omega \begin{vmatrix} \dot{x}' & \dot{y}' & 0 \\ -\cos\varphi & 0 & \sin\varphi \\ x' & y' & -l \end{vmatrix}$

$= \omega\left[(-\dot{x}'y' + x'\dot{y}')\sin\varphi - \dot{y}'\cos\varphi \cdot l\right]$

$z: \; x'^2 + y'^2 + z'^2 = l \;\Rightarrow\; z = l\sqrt{1 - x'^2 - y'^2} = l - \frac{x'^2 - y'^2}{2l}$

$V = -mg\left(l - \frac{x'^2 + y'^2}{2l}\right) = C_1 + \frac{x'^2 + y'^2}{2l} \cdot mg$

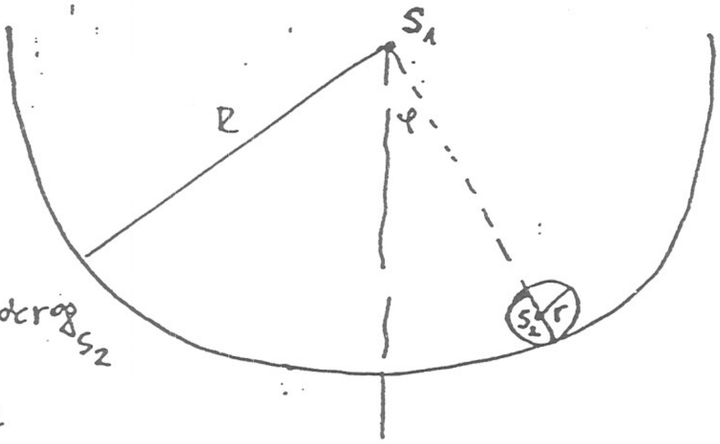
To uniči E.L. (člen $c \cdot \dot{q}_k$)

$L = T - V = \frac{m}{2}\left[\dot{x}'^2 + \dot{y}'^2 + 2\omega\sin\varphi(x'\dot{y}' - \dot{x}'y') - 2\dot{y}'\cos\varphi\, l\omega - \frac{g}{l}(x'^2 + y'^2)\right] + C_2$

E.L.: x': $\ddot{x}' - 2\omega\sin\varphi\, \dot{y}' + \omega_0^2 x' = 0$

y': $\ddot{y}' + 2\omega\sin\varphi\, \dot{x}' + \omega_0^2 y' = 0$

(4)



$\ell = R - r$

- vrtenje okoli S_1
- vrtenje okrog S_2

$= T_1 + T_2$

α – kot zasuka kroglice

$\varphi R = \alpha r \Rightarrow \alpha = \varphi \frac{R}{r}$

$\dot\alpha = \dot\varphi \frac{R}{r}$

$\Rightarrow T = \frac{1}{2} m \ell^2 \dot\varphi^2 + \frac{1}{2} J \dot\alpha^2$

$= \frac{1}{2} m \ell^2 \dot\varphi^2 + \underbrace{\frac{1}{2}\frac{2mr^2}{5}\cdot\frac{R^2}{r^2}\cdot\dot\varphi^2}$

Zanimivo: rotacijske okoli lastne osi kinetične energije ni odvisna od r. Manjša je kroglica => hitreje se vrti in $T_2(r) = \text{const}$.

$= \frac{1}{2}\underbrace{\left(m\ell^2 + \frac{2mR^2}{5}\right)}_{J'}\dot\varphi^2$

$= -mg\ell\cos\varphi$

$T - V = \frac{1}{2} J' \dot\varphi^2 + mg\ell\cos\varphi$

ler – Lagrange: $J'\ddot\varphi + mg\ell\sin\varphi = 0$

koti: $\sin\varphi \approx \varphi$

$\ddot\varphi + \frac{mg\ell}{J'}\varphi = 0 \quad \Rightarrow \quad \omega^2 = \frac{mg\ell}{J'} = \frac{mg(R-r)}{m(R-r)^2 + \frac{2mR^2}{5}}$

z $r \to 0 \Rightarrow \omega^2 = \frac{gR}{R^2 + \underbrace{\frac{2}{5}R^2}} = \frac{5g}{7R}$

(Rezultat je tak zato, ker ima tudi neskončno majhna kroglica rotacijsko energijo, saj $\dot\alpha = \infty$, tako da $T_2 = \text{const}$.)

ne upoštevamo rotacijske energije lice, pride znani rezultat $\omega^2 = \frac{g}{R}$

Če pa r povečujemo:

$$\omega^2 = \frac{g(R-r)}{(R-r)^2 + \frac{2R^2}{5}}$$

Vidim $\omega^2(r) \xrightarrow{r\to R} 0$ (razumljivo) $\Rightarrow \dot{\varphi} = \text{const}$

$\rightarrow$ enakomerno vrtenje krogle v skledi ✓

$$\left(\frac{a}{b}\right)' = \frac{a'b + b'a}{b^2}$$

Izračunajmo ekstrem:

$$\frac{\partial \omega^2}{\partial r} = \frac{-1(\quad) + 2(R-r)^2}{(\cdots\cdots)^2} = 0$$

$$\Rightarrow \cancel{2}(R-r)^{2} = \cancel{(R-r)^2} + \frac{2R^2}{5}$$

$$(R-r)^2 = \frac{2R^2}{5}$$

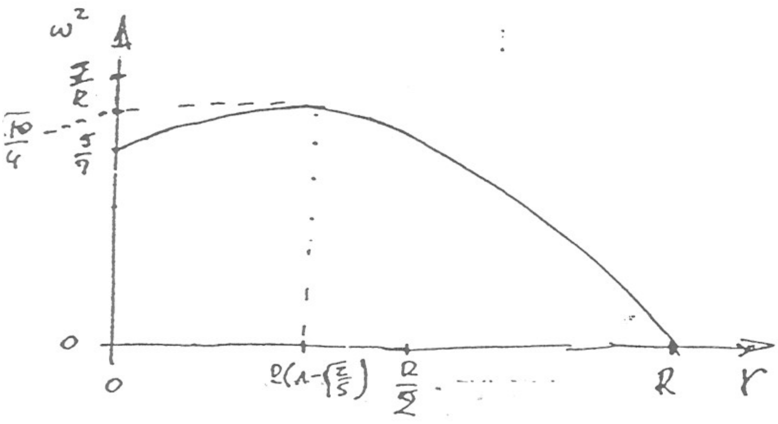
$$\Rightarrow \underline{\underline{r = R\left(1-\sqrt{\tfrac{2}{5}}\right)}}$$

$$\Rightarrow R-r = \cancel{R}-\cancel{R}+R\sqrt{\tfrac{5}{2}}$$

Tu doseže ω^2 maksimum, potem pa pade proti 0, ko $r \to R$

$$\omega^2_{ext} = \frac{gR\sqrt{\frac{2}{5}}}{R^2\cdot\frac{2}{5} + R^2\cdot\frac{2}{5}} = \frac{g}{R}\cdot\frac{\sqrt{5}}{4}\sqrt{\frac{2}{5}} = \underline{\underline{\frac{g}{R}\cdot\frac{\sqrt{10}}{4}}}$$

Pribl. potek $\omega^2(r)$:



5) Vrtavka začne opletati:

$$p(\mu) = (h - e\mu)(1-\mu^2) - (a - b\mu)^2$$

$p_\phi = J \cdot a$

$p_\varphi = J \cdot b$

Dokler je vrtavka navpična: $\mu = 1 = \cos\vartheta$

$p(1) \rightarrow a = b$

$h = \frac{2H}{\tilde{J}} - \frac{\tilde{J}}{J} b^2$

Odvod polinoma v ničli mora biti nič:

$e = \frac{2mgl}{\tilde{J}}$

$$\frac{dp}{d\mu}(\mu=1) = -(h-e) + 2a(b-a) \Rightarrow \underline{\underline{h=e}}$$

Zapišem nov polinom:

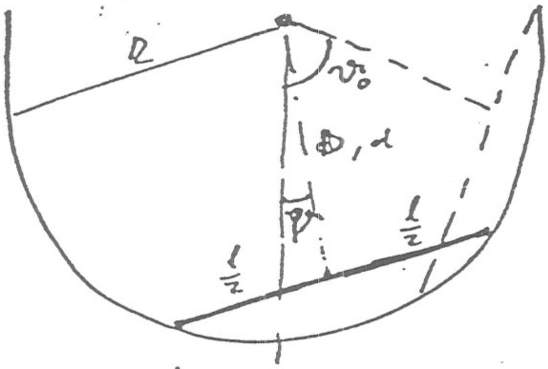
$$p(\mu) = e(1-\mu)^2(1+\mu) - a^2(1-\mu)^2 = (1-\mu)^2\left[e + e\mu - a^2\right]$$

$\mu_{1,2} = 1, \quad \mu_3 = \frac{a^2}{e} - 1$

pogoj $\mu_3 = 1 \quad \rightarrow \quad \frac{a^2}{e} = 2 \qquad a = \frac{p_\phi}{J} = \frac{J'}{J}\omega_z$

$$\Rightarrow a = \sqrt{2e} \quad \Rightarrow \quad \omega_z = \frac{J}{J'}\sqrt{2e} = \frac{J}{J'}\cdot\sqrt{2\cdot\frac{2mgl}{\tilde{J}}} = \underline{\underline{\frac{2J}{J'}\sqrt{\frac{mgl}{\tilde{J}}}}}$$

6)



$\to$ multiplikator

$$J = \left(\frac{ml^2}{12} + md^2\right)$$

$$J' = \frac{ml^2}{12} \qquad J = J' + J_s$$

$$J_s = md^2$$

$$T = \frac{1}{2} J \dot{\vartheta}^2 + \frac{1}{2} m \dot{d}^2$$

$$V = -mgd\cos\vartheta$$

$$= T - V = \frac{1}{2} m\dot{d}^2 + \frac{1}{2} md^2 \dot{\vartheta}^2 + \frac{1}{2} J' \dot{\vartheta}^2 + mgd\cos\vartheta$$

1.) $\vartheta: \frac{d}{dt}\left(md^2\dot{\vartheta} + J'\dot{\vartheta}\right) + mgd\sin\vartheta = 0$

$$2md\dot{d}\dot{\vartheta} + J'\ddot{\vartheta} + mgd\sin\vartheta = 0$$

.) $d: \; m\ddot{d} - md\dot{\vartheta}^2 - mg\cos\vartheta = \lambda$

Upoštevam $d = D = const \Rightarrow -mD\dot{\vartheta}^2 - mg\cos\vartheta = \lambda$

Zanima me $\dot{\vartheta}(\vartheta)$.

$$H = T + V = \frac{1}{2} mD^2\dot{\vartheta}^2 + \frac{1}{2}\frac{J'}{m}\dot{\vartheta}^2 - mgD\cos\vartheta = -mgd\cos\vartheta_0$$

$$\frac{1}{2}\left(D^2 + \frac{l^2}{12}\right)\dot{\vartheta}^2 = gD(\cos\vartheta - \cos\vartheta_0)$$

$$\Rightarrow \dot{\vartheta}^2(\vartheta) = \frac{gD(\cos\vartheta - \cos\vartheta_0)}{\frac{1}{2}\left(d^2 + \frac{l^2}{12}\right)}$$

$$\Rightarrow \lambda = -mD\frac{2gD(\cos\vartheta - \cos\vartheta_0)}{\left(d^2 + \frac{l^2}{12}\right)} - mg\cos\vartheta =$$

$$= mg\,\cancel{\cos\vartheta}\left(-\frac{2J_s}{J}(\cos\vartheta - \cos\vartheta_0) - \cos\vartheta\right)$$

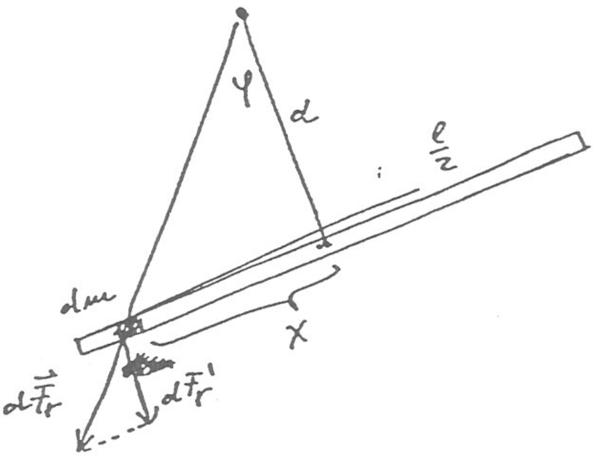
$$= \boxed{-mg\cos\vartheta\left(\frac{2J_s}{J}\left(1 - \frac{\cos\vartheta_0}{\cos\vartheta}\right) + 1\right)}$$

To je sila na skledo.

Preverim s klasično fiziko 1:

$$F = F_g' + F_r' = mg\cos\vartheta + F_r'$$

$$d\vec{F}_r = m\cdot\vec{a}$$



$$dm = \rho\, dx$$

$$\rho = \frac{m}{l}$$

$$F = m r \omega^2$$

$$dF_r = \rho\cdot dx\cdot\sqrt{d^2+x^2}\;\omega^2$$

$$dF_r' = dF_r\cdot\cos\varphi = dF_r\cdot\frac{d}{\sqrt{d^2+x^2}} = \rho\, dx\cdot d\cdot\omega^2$$

$\Rightarrow F_r' = m\cdot d\cdot\omega^2$ → (zanimiv rezultat – sila je enaka, kot če bi imel točkasto maso na sredini palice)

$$\Rightarrow F = mg\cos\vartheta + m d\dot{\vartheta}^2$$

$$H = \frac{1}{2}J\dot{\vartheta}^2 - mgd\cos\vartheta = -mgd\cos\vartheta_0$$

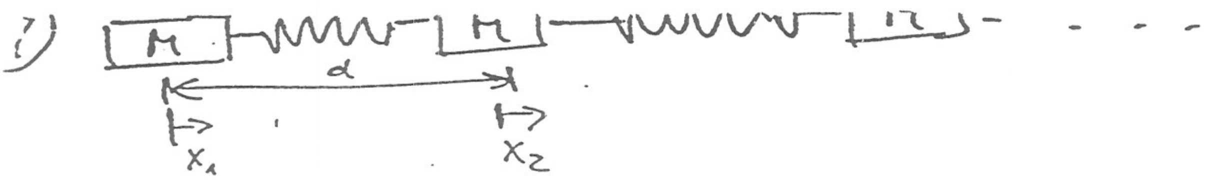
$$\rightarrow \dot{\vartheta} = 2g\frac{md}{J}(\cos\vartheta - \cos\vartheta_0)$$

$$\Rightarrow F = mg\cos\vartheta + 2gm\frac{J_s}{J}(\cos\vartheta - \cos\vartheta_0)$$

$$F = mg\cos\vartheta\left[1 + \frac{2J_s}{J}\left(1 - \frac{\cos\vartheta_0}{\cos\vartheta}\right)\right]$$ dobim isto

$(F = -\lambda)$

1)



$$T = \frac{m\dot{x}_1^2}{2} + \frac{m\dot{x}_2^2}{2} + \ldots + \frac{m\dot{x}_n^2}{2}$$

$$V = \frac{k}{2}(x_1-x_2)^2 + \frac{k}{2}(x_2-x_3)^2 + \ldots + \frac{k}{2}(x_{n-1}-x_n)^2$$

$$T = \frac{1}{2}\dot{x}^T \underline{\underline{T}}\,\dot{x} \;,\quad V = \frac{1}{2}x^T \underline{\underline{V}}\,x$$

$$\underline{\underline{T}} = \underline{\underline{I}}\cdot m$$

$$\underline{\underline{V}} = k\cdot \overbrace{\begin{bmatrix} 1 & -1 & & & 0 \\ -1 & 2 & -1 & & \\ & -1 & \ddots & -1 & \\ & & -1 & 2 & -1 \\ 0 & & & -1 & 1 \end{bmatrix}}^{\underline{\underline{V}}'}$$

$\underline{a} \to$ vektor odmikov

nastavek: $a_i = a_0 \sin(\varphi + i\alpha)$

$$\det(\underline{\underline{V}} - \omega^2 \underline{\underline{T}}) = 0 \Rightarrow \omega^2 \underline{\underline{T}}\,\underline{a} = \underline{\underline{V}}\,\underline{a} \Rightarrow \underline{\underline{V}}\,\underline{a} = \omega^2 m \underline{a}$$

$$\underline{\underline{V}}'\underline{a} = \left(\frac{\omega}{\omega_0}\right)^2 \underline{a} = \Omega^2 \underline{a} \;;\quad \omega_0^2 = \frac{k}{m}$$

valovni vektor

$i\alpha = iKd$

$$-a_i + 2a_i - a_i = \Omega^2 a_i$$

$$\cancel{\sin(\varphi+i\alpha)}\cos\alpha - \cancel{\cos(\varphi+i\alpha)\sin\alpha} - 2\cancel{\sin(\varphi+i\alpha)} + \cancel{\sin(\varphi+i\alpha)}\cos\alpha + \cancel{\cos(\varphi+i\alpha)\sin} = -\Omega^2 \cancel{\sin(\varphi+i\alpha)}$$

$$\Rightarrow 2\cos\alpha - 2 = -\Omega^2$$

$$\to \omega = \omega_0\sqrt{2(1-\cos(Kd))}$$

$$v = \frac{\partial\omega}{\partial K} = \omega_0 d\cdot\frac{\cancel{2}\sin(Kd)}{\cancel{2}\sqrt{2(1-\cos(Kd))}}$$

Oceniti moram K:

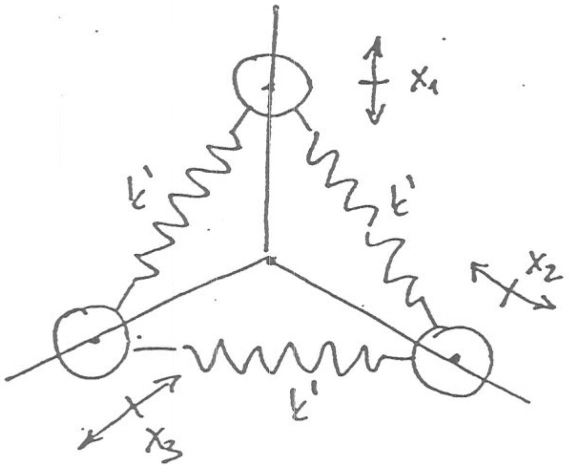
5 avtomobilov v zgoščini → v valovanju bo zgoščina dolžine „redčine" s prav tako 5 avtomobili, torej se faza obrne za 2π ravno na $10d$ $\Rightarrow k = \frac{2\pi}{10d} = \frac{\pi}{5d}$

$$\Rightarrow v = \omega_0 d\cdot\frac{\sin\frac{\pi}{5}}{\sqrt{2(1-\cos\frac{\pi}{5})}}$$

Vem $v = \frac{L}{\Delta t} = \frac{50d}{\Delta t}$, od tod ω_0:

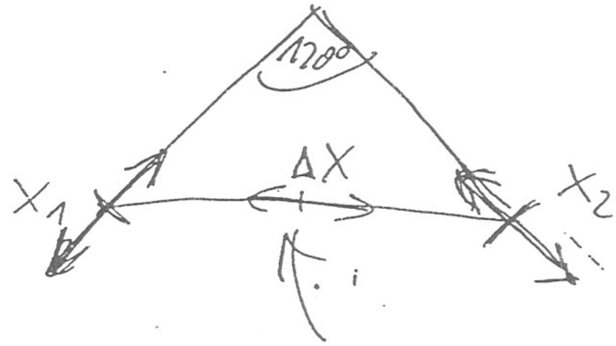
$$\omega_0 = \frac{v}{d}\cdot\frac{\sqrt{2(1-\cos\pi/5)}}{\pi/\ldots}$$

(18) Tri mase:



$k' \cos 30° \cdot x_1 = k x_1$

$\Rightarrow k = k' \cdot \frac{\sqrt{3}}{2}$



$$T = \frac{1}{2} m \sum_i \dot{x}_i^2$$

$$V = \frac{1}{2} k \left[(x_1 + x_2)^2 + (x_2 + x_3)^2 + (x_3 + x_1)^2 \right]$$

$$\Delta X = X_1^2 + X_2^2 - 2\cos(120°)\, x_1 x_2 = X_1^2 + x_2^2 + X_1 X_2$$

$$T = \frac{1}{2} \dot{x}^T \underline{\underline{T}} \dot{x}$$

$$V = \frac{1}{2} x^T \underline{\underline{V}} x$$

$$\Rightarrow \underline{\underline{T}} = m \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \underline{\underline{V}} = k \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\frac{k}{m} = \omega_0^2, \quad \omega = \Omega \omega_0$$

$$\det\left[\underline{\underline{V}} - \omega^2 \underline{\underline{T}}\right] = 0 \qquad \det \begin{bmatrix} 2-\Omega^2 & 1 & 1 \\ 1 & 2-\Omega^2 & 1 \\ 1 & 1 & 2-\Omega^2 \end{bmatrix} = 0$$

$$2^3 - 3 \cdot 2^2 \Omega^2 + 3 \cdot 2 {\Omega^2}^2 - {\Omega^2}^3 - 6 + 3 \cdot \Omega^2 + 2 = 0$$

$${\Omega^2}^3 - 6 {\Omega^2}^2 + 9 \Omega^2 - 4 = 0$$

$$(\Omega^2 - 4)(\Omega^2 - 1)^2 = 0$$

$$\underline{\Omega_1^2 = 4}$$

$$\underline{\underline{\Omega_{2,3}^2 = 1}}$$

$\Rightarrow$ lastni vektorji:

$\Omega_1^2 = 4$:

$$\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \quad \Rightarrow \quad x = y = z$$

$$\eta_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

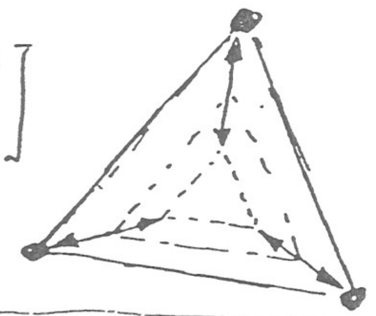
$\Omega_{2,3}^2 = 1$:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \quad \Rightarrow \quad x + y + z = 0$$

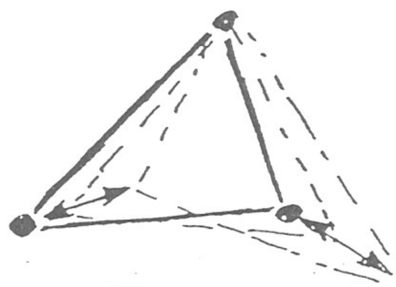
Dva nekolinearna vektorja:

npr $\eta_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ in $\eta_3 = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -1 \end{bmatrix}$

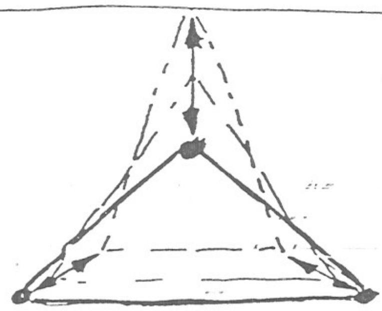
1. $\Omega_1 = 2$, $\eta_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$



2. $\Omega_2 = 1$, $\eta_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

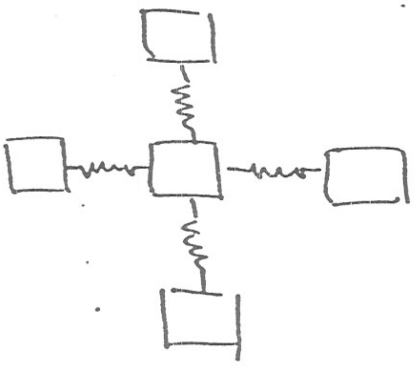


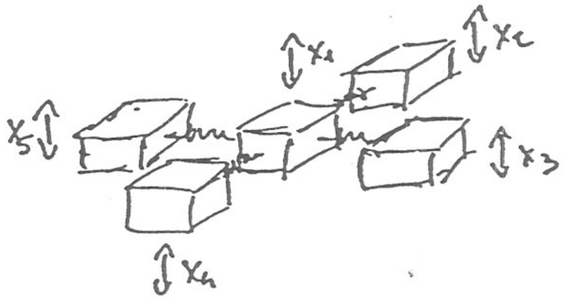
3. $\Omega_3 = 1$, $\eta_3 = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -1 \end{bmatrix}$



Pri teh dveh nihajnih načinih težišče ni pri miru, kar je posledica privzetka da se mase gibljejo po fiksnem žlebu (vodilu)

19) Pet uteži :





$$T = \frac{m}{2}\left(\dot{x}_1^2 + \dot{x}_2^2 + \dot{x}_3^2 + \dot{x}_4^2 + \dot{x}_5^2\right)$$

$$V = \frac{k}{2}\left((x_1-x_2)^2 + (x_1-x_3)^2 + (x_1-x_4)^2 + (x_1-x_5)^2\right)$$

$$T = \frac{1}{2}\dot{x}^T \underline{\underline{T}}\,\dot{x} \Rightarrow \underline{\underline{T}} = m\,\overbrace{\underline{\underline{I}}}^{\underline{\underline{T}}'}$$

$$V = \frac{1}{2} x^T \underline{\underline{V}}\, x \Rightarrow \underline{\underline{V}} = k\cdot\underbrace{\begin{bmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\underline{\underline{V}}'}$$

$$\det\left[\omega^2 \underline{\underline{T}} - \underline{\underline{V}}\right] = 0$$

$$\Rightarrow \det\left[\omega^2 \cdot m\,\underline{\underline{T}}' - k\,\underline{\underline{V}}'\right] \qquad \omega^2 \cdot \frac{m}{k} = \Omega^2, \quad \frac{k}{m} = \omega_0^2$$

$$\Rightarrow \frac{\omega^2}{\omega_0^2} = \Omega^2$$

$$\det\begin{bmatrix} 4-\Omega^2 & -1 & -1 & -1 & -1 \\ -1 & 1-\Omega^2 & 0 & 0 & 0 \\ -1 & 0 & 1-\Omega^2 & 0 & 0 \\ -1 & 0 & 0 & 1-\Omega^2 & 0 \\ -1 & 0 & 0 & 0 & 1-\Omega^2 \end{bmatrix} = 0 \quad \rightarrow \Omega_1^2 = 1$$

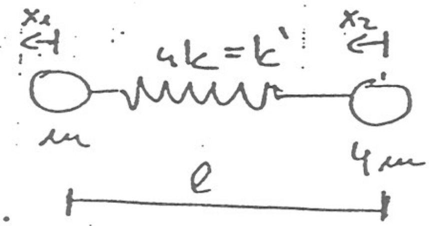
$$\Omega_1^2 = 1$$

stni vektorji so:

$$\underline{\eta_1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \quad \underline{\eta_2} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad \underline{\eta_3} = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \quad \underline{\eta_4} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$$\eta_5 = \begin{bmatrix} 4 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$$

... vektor, ki v bistvu predstavlja toko nihanje :



Določimo frekvenco:

Iz ohranitve težišča vidimo $4x_1 = -x_2$. Delček vzmeti na 4/5 njene dolžine torej miruje in problem je sedaj tak:

Velja $k = \frac{\alpha}{l}$ (dvakrat daljša vzmet → 2x manjši k)

$4k = \frac{\alpha}{l} \Rightarrow \alpha = 4kl$ za levo stran

$k = \frac{\alpha}{4/5\, l} = \frac{\cancel{4}k\cancel{l}5}{\cancel{l}\cdot\cancel{4}} = 5k \Rightarrow \omega = 5\frac{k}{m}$

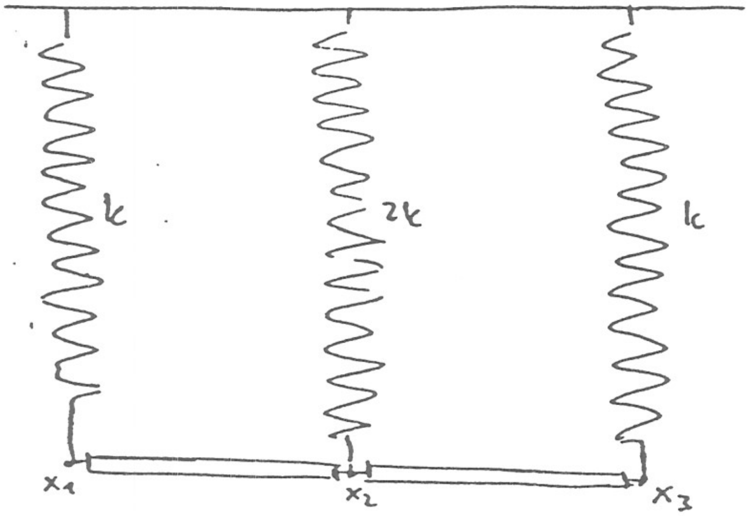
→ se ujema. Dobro!

Preizkus še za desno:

$k = \frac{\alpha}{1/5\, l} = \frac{4kl\cdot 5}{l} = 20k \Rightarrow \omega = \frac{20k}{4m} = 5\frac{k}{m}$

imamo torej še $\Omega_2^2 = 5\cdot\frac{k}{m}$. Tej lastni vrednosti pripada zadnji lastni vektor $\underline{v}_5 = \begin{bmatrix} 4 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$

20



$J = \frac{ma^2}{12}$

$$T = \underbrace{\frac{m}{2}\left(\frac{\dot x_1 + \dot x_2}{2}\right)^2 + \frac{m}{2}\left(\frac{\dot x_2 + \dot x_3}{2}\right)^2}_{\text{gibanje težišč}} + \underbrace{\frac{J}{2}\left(\frac{\dot x_1 - \dot x_2}{a}\right)^2 + \frac{J}{2}\left(\frac{\dot x_2 - \dot x_3}{a}\right)^2}_{\text{vrtenje okoli težišč}}$$

$$= \frac{m}{8}\left(\dot x_1^2 + 2\dot x_2^2 + \dot x_3^2 + 2\dot x_1 \dot x_2 + 2\dot x_2 \dot x_3\right)$$

$$+ \frac{m}{24}\left(\dot x_1^2 + 2\dot x_2^2 + \dot x_3^2 - 2\dot x_1 \dot x_2 - 2\dot x_2 \dot x_3\right)$$

$$T = \frac{m}{2}\left(\frac{\dot x_1^2}{3} + \frac{2\dot x_2^2}{3} + \frac{\dot x_3^2}{3} + \frac{\dot x_1 \dot x_2}{3} + \frac{\dot x_2 \dot x_3}{3}\right)$$

$$V = \frac{k}{2}\left(x_1^2 + 2x_2^2 + x_3^2\right)$$

$$\Rightarrow \quad \underline{\underline{T}} = m \begin{bmatrix} \frac{1}{3} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{1}{3} \end{bmatrix} ; \quad \underline{\underline{V}} = k \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Omega = \frac{\omega}{\omega_0}, \quad \omega_0^2 = \frac{k}{m}$

$$\det\left[\underline{\underline{V}} - \Omega^2 \underline{\underline{T}}\right] = 0 \Rightarrow \det \begin{bmatrix} 1 - \frac{\Omega^2}{3} & -\frac{\Omega^2}{6} & 0 \\ -\frac{\Omega^2}{6} & 2\left(1 - \frac{\Omega^2}{3}\right) & -\frac{\Omega^2}{6} \\ 0 & -\frac{\Omega^2}{6} & 1 - \frac{\Omega^2}{3} \end{bmatrix} = 0$$

$$2\left(1 - \frac{\Omega^2}{3}\right)^3 - \left(1 - \frac{\Omega^2}{3}\right)\left(\frac{\Omega^2}{6}\right)^2 - \cancel{\left(1 - \frac{\Omega^2}{3}\right)\left(\frac{\Omega^2}{6}\right)^2} = 0$$

$\Omega_2 = 3$

2.) $\left(1-\frac{\Omega^2}{3}\right)^2 - \left(\frac{\Omega^2}{6}\right)^2 = 0$

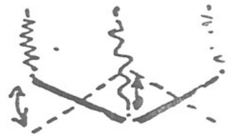
$\Rightarrow \Omega^4 - 8\Omega^2 + 12 = 0$, $(\Omega^2-2)(\Omega^2-6) = 0 \Rightarrow \Omega_1^2 = 2,\ \Omega_3^2 = 6$

Lastne vrednosti imam. Še lastne vektorje:

1. $\Omega_2^2 = 3;$ $\begin{bmatrix} 0 & -\frac{1}{2} & 0 \\ -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & -\frac{1}{2} & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \Rightarrow \begin{matrix} x_2 = 0 \\ x_1 = -x_3 \end{matrix}$ $\quad \underline{\eta}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

2. $\Omega_1^2 = 2;$ $\frac{1}{3}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \Rightarrow \begin{matrix} x_1 = x_2 \\ x_2 = x_3 \end{matrix}$ $\quad \underline{\eta}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

3. $\Omega_3^2 = 6;$ $-\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \Rightarrow \begin{matrix} x_1 = -x_2 \\ x_2 = -x_3 \end{matrix}$ $\quad \underline{\eta}_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$



imam vse tri lastne nastne nihanje in pripadajoče frekvence. Splošna rešitev:

$$\underline{\eta}(t) = A_1 \underline{\eta}_1 \cos(\omega_1 t + \phi_1) + A_2 \underline{\eta}_2 \cos(\omega_2 t + \phi_2) + A_3 \underline{\eta}_3 \cos(\omega_3 t + \phi_3)$$

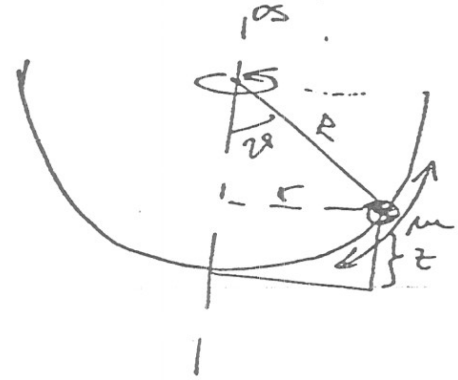
kjer A_1, A_2, A_3 – amplitude pripadajočih nihanj

ϕ_1, ϕ_2, ϕ_3 – fazni zamiki – II –

$\omega_i = \Omega_i \omega_0 = \Omega_i \sqrt{\frac{k}{m}}$, $i = 1, 2, 3$

21) vice na vrvici, ki jo sučemo:

Problem je enak, kot da imamo koravdo na polkrožni žici, ki jo vrtimo:



$V = -mgR\cos\vartheta$

$v_r = \omega r$

$\underline{V = -mgR\cos\vartheta}$

$$T = \underbrace{\frac{m}{2}R^2\dot\vartheta^2}_{\text{gibanje po žici}} + \underbrace{\frac{m}{2}\omega^2 r^2}_{\text{kroženje}} = \underline{\frac{m}{2}R^2\left(\dot\vartheta^2 + \omega^2\sin^2\vartheta\right) = T}$$

$L = T - V$

Euler-Lagrange: $\cancel{mR^2}\ddot\vartheta - \cancel{mR^2}\omega^2\sin\vartheta\cos\vartheta + \cancel{m}\frac{g}{R}\cancel{R^2}\sin\vartheta = 0$

Ravnovesje $\ddot\vartheta = 0 \Rightarrow \omega^2\sin\vartheta\cos\vartheta = \frac{g}{R}\sin\vartheta$

$\vartheta = 0$ – labilna ravnovesna lega za velike ω, stabilna za majhne

$\vartheta \Rightarrow \cos\vartheta_0 = \frac{g}{\omega^2 R}$ – stabilna ravnovesna lega za velike ω.

Za nihanje: $\vartheta = \vartheta_0 + \varphi$

$\sin\vartheta = \sin\vartheta_0 + \varphi\cos\vartheta_0$

$\cos\vartheta = \cos\vartheta_0 - \varphi\sin\vartheta_0$ drugi red $\varphi, \varphi^2 \doteq 0$

$\sin\vartheta\cos\vartheta = \frac{1}{2}\sin 2\vartheta_0\cdot(1 - \cancel{\varphi^2}) + \cos 2\vartheta_0\cdot\varphi$

$$\Rightarrow \ddot{\varphi} - \frac{\omega^2}{2}\left(\sin 2\vartheta_0 + \varphi\cos 2\vartheta_0\right) + \frac{g}{l}\left(\sin\vartheta_0 + \varphi\cos\vartheta_0\right) = 0$$

$$\ddot{\varphi} + \varphi\underbrace{\left[\frac{g}{l}\cos\vartheta_0 - \omega^2\cos 2\vartheta_0\right]}_{\Omega^2} = C$$

$\Omega^2 > 0$: $\frac{g}{l}\cos\vartheta_0 > \omega^2\cos 2\vartheta_0 = \omega^2(\cos^2\vartheta_0 - \sin^2\vartheta_0) = \omega^2(2\cos^2\vartheta_0 - 1)$

za $\vartheta_0 = 0 \Rightarrow \underline{\frac{g}{l} > \omega^2}$

za $\cos\vartheta_0 = \frac{g}{\omega^2 l}$

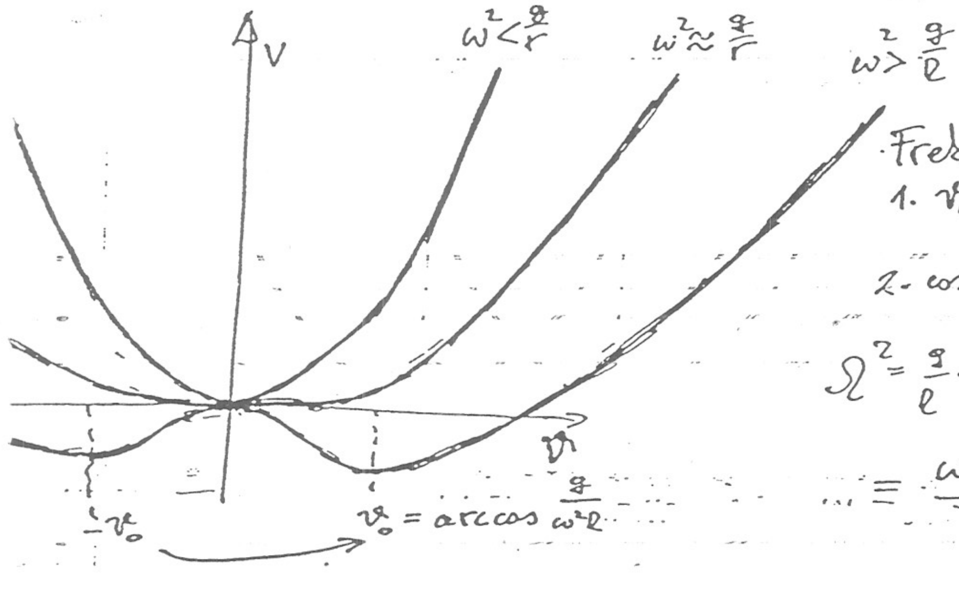
$$\Rightarrow \frac{g}{\cancel{l}}\cdot\frac{g}{\cancel{\omega^2 l}} > \cancel{\omega^2}\left(\frac{2\cdot g^2 - \omega^4 l^2}{\cancel{\omega^4 l^2}}\right)$$

$$g^2 > 2g^2 - \omega^4 l^2$$

$$\omega^4 l^2 > g^2$$

$$\underline{\omega^2 l > g}$$

Potencialni lonec spreminja obliko z ω. Za $\omega^2 < \frac{g}{l}$ imamo le eno ravnovesno lego. Ta je stabilna in sicer $\vartheta_0 = 0$. Za $\omega^2 > \frac{g}{l}$ imamo dve ravnovesni legi, labilno pri $\vartheta_0 = 0$ in stabilno pri $\cos\vartheta_1 = \frac{g}{\omega^2 l}$. (in še eno pri $\vartheta_2 = -\vartheta_1$)



Frekvence:

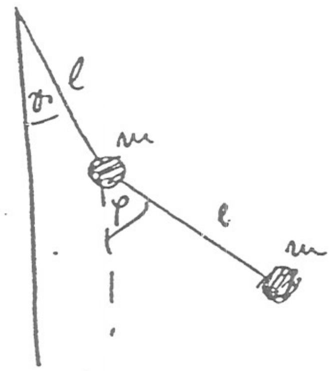
1. $\vartheta_0 = 0 \Rightarrow \underline{\Omega_1^2 = \frac{g}{l} - \omega^2}$

2. $\cos\vartheta_0 = \frac{g}{\omega^2 l} \Rightarrow$

$$\Omega^2 = \frac{g}{l}\cdot\frac{g}{\omega^2 l} - \cancel{\omega^2}\left(\frac{2g^2 - \omega^4 l^2}{\cancel{\omega^4} l^2\omega^2}\right)$$

$$\ldots = \frac{\omega^4 l^2 - g^2}{l^2\omega^2} = \underline{\omega^2 - \frac{g^2}{l^2\omega^2} = \Omega_2^2}$$

22) Sistem uteži:



$$\vec{r}_1 = l(\sin\vartheta, -\cos\vartheta)$$
$$\vec{r}_2 = l(\sin\vartheta + \sin\varphi, -\cos\vartheta - \cos\varphi)$$

$$\dot{\vec{r}}_1 = l\dot\vartheta(\cos\vartheta, \sin\vartheta)$$
$$\dot{\vec{r}}_2 = l(\dot\vartheta\cos\vartheta + \dot\varphi\cos\varphi, \dot\vartheta\sin\vartheta + \dot\varphi\sin\varphi)$$

$$T = T_1 + T_2$$

$$T_1 = \frac{1}{2}m\dot{\vec{r}}_1^{\,2} = \frac{1}{2}ml^2\dot\vartheta^2$$

$$T_2 = \frac{1}{2}m\dot{\vec{r}}_2^{\,2} = \frac{1}{2}ml^2\Big(\dot\vartheta^2\cancel{\cos^2\vartheta} + \dot\varphi^2\cancel{\cos^2\varphi} + 2\dot\vartheta\dot\varphi\cos\vartheta\cos\varphi$$
$$\cancel{+\dot\vartheta^2\sin^2\vartheta} + \cancel{\dot\varphi^2\sin^2\varphi} + 2\dot\vartheta\dot\varphi\sin\vartheta\sin\varphi\Big)$$
$$= \frac{1}{2}ml^2\left(\dot\vartheta^2 + \dot\varphi^2 + 2\dot\vartheta\dot\varphi\cos(\vartheta-\varphi)\right)$$

$$T = \frac{1}{2}ml^2\left(2\dot\vartheta^2 + \dot\varphi^2 + 2\dot\vartheta\dot\varphi\cos(\vartheta-\varphi)\right)$$

$$V = -mgl(2\cos\vartheta + \cos\varphi)$$

mali odmiki: $T = \frac{1}{2}ml^2(2\dot\vartheta^2 + \dot\varphi^2 + 2\dot\vartheta\dot\varphi)$

$$V = -mgl\left(2\left(1-\frac{\vartheta^2}{2}\right) + 1 - \frac{\varphi^2}{2}\right) = -mgl\left(3 - \left(2\frac{\vartheta^2}{2} + \frac{\varphi^2}{2}\right)\right)$$

$$V_1 = \frac{mgl}{2}(2\vartheta^2 + \varphi^2)$$

$$\underline{\underline{T}} = ml^2\begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix} \qquad \underline{\underline{V}} = mgl\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix} \qquad \frac{g}{l} = \omega_0^2$$

$$\frac{\omega}{\omega_0} = \Omega$$

$$\det[\omega^2\underline{\underline{T}} - \underline{\underline{V}}] = 0 \quad\Rightarrow\quad \det\begin{bmatrix}2(1-\Omega^2) & -\Omega^2\\ -\Omega^2 & 1-\Omega^2\end{bmatrix}$$

$$2(1-\Omega^2)^2 - \Omega^4 = 0$$
$$\Omega^4 - 4\Omega^2 + 2 = 0 \quad\Rightarrow\quad \Omega^2_{1,2} = 2 \pm \sqrt{2}$$

1.) $\Omega_1^2 = 2+\sqrt{2}$: $\quad \omega_1' = \Omega_1 \omega_0 = \sqrt{(2+\sqrt{2})}\sqrt{\frac{g}{l}}$

$$\begin{bmatrix} -2(1+\sqrt{2}) & -(2+\sqrt{2}) \\ -2+\sqrt{2} & -(1+\sqrt{2}) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \quad \Rightarrow \quad \underline{x_2 = -x_1\sqrt{2}}$$

$$\eta_1 = \begin{bmatrix} 1 \\ -\sqrt{2} \end{bmatrix} \cdot \frac{1}{\sqrt{3m}\, l}$$

2.) $\Omega_2^2 = 2-\sqrt{2}$ $\quad \omega_2' = \Omega_2 \omega_0 = \sqrt{(2-\sqrt{2})}\sqrt{\frac{g}{l}}$

$$\begin{bmatrix} 2(-1+\sqrt{2}) & -2+\sqrt{2} \\ -2+\sqrt{2} & -1+\sqrt{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \quad \Rightarrow \quad x_2 = x_1\sqrt{2}$$

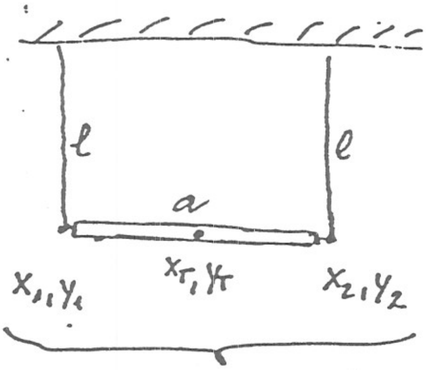
$$\eta_2 = \begin{bmatrix} 1 \\ \sqrt{2} \end{bmatrix} \cdot \frac{1}{\sqrt{3m}\, l}$$

1.) Ω_1^2 : $\boxed{\begin{aligned} x_1 &= x_{10} \sin \omega_1' t \\ x_2 &= -\sqrt{2}\, x_{10} \sin \omega_1' t \end{aligned}}$

2.) Ω_2^2 : $\boxed{\begin{aligned} x_1 &= x_{10} \sin \omega_2' t \\ x_2 &= \sqrt{2}\, x_{10} \sin \omega_2' t \end{aligned}}$

obe možna nihajna načina

(23) Nihanje prečke na vrvicah:



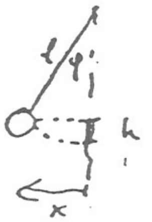
te koordinate se nanašajo na vodoravno ravnino zaradi majhnih odmikov velja $y_1 = y_T = y_2$ ← vez.

$$T = \frac{1}{2} m (\dot{x}_T^2 + \dot{y}_T^2) + \frac{1}{2} J \left(\frac{\dot{x}_1 - \dot{x}_2}{a}\right)^2 \qquad J = \frac{ma^2}{12}$$

$$T = \frac{1}{2} m \left[\frac{1}{4}(\dot{x}_1^2 + \dot{x}_2^2 + 2\dot{x}_1\dot{x}_2) + \frac{1}{4}(\dot{y}_1^2 + \dot{y}_2^2 + 2\dot{y}_1\dot{y}_2) + \frac{1}{6}(\dot{x}_1^2 + \dot{x}_2^2 - 2\dot{x}_1\dot{x}_2)\right]$$

$$\Rightarrow \underline{T = \frac{1}{2} m \left[\frac{\dot{x}_1^2}{3} + \frac{\dot{x}_2^2}{3} + \frac{2\dot{x}_1\dot{x}_2}{6} + \dot{y}^2\right]}$$

V:



$$h = \cancel{mg}\ l - l\cos\varphi = l - l\left(1 - \frac{\varphi^2}{2}\right) = l\frac{\varphi^2}{2}$$

$$\varphi = \frac{l}{x} \quad \Rightarrow \quad h = \frac{x^2}{2l}$$

$$V_y = mg\frac{y^2}{2l}$$

$$V_x = mg \cdot \frac{1}{2l} \cdot \left(\frac{x_1^2 + x_2^2}{2}\right) = mg\frac{x_1^2 + x_2^2}{4l}$$

Označim $\frac{g}{l} = \omega_0^2$

$\Omega^2 = \frac{\omega^2}{\omega_0^2}$

$$V = \frac{1}{2} \frac{mg}{l}\left[y^2 + \frac{x_1^2}{2} + \frac{x_2^2}{2}\right]$$

$$T = \frac{1}{2}\dot{x}^T \underline{\underline{T}}\, \dot{x}, \quad V = \frac{1}{2} x^T \underline{\underline{V}}\, x$$

$$\underline{\underline{T}} = m \cdot \underbrace{\begin{bmatrix} \frac{1}{3} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{T'} ; \quad \underline{\underline{V}} = \frac{mg}{l} \underbrace{\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\underline{V'}}$$

$$\det\left[\underline{\underline{V}} - \omega^2 \underline{\underline{I}}\right] = 0 \quad \Rightarrow \quad \det\left[\underline{\underline{V}}' - \Omega^2 \underline{\underline{I}}'\right] = 0$$

$$\begin{bmatrix} \frac{1}{2} - \frac{\Omega^2}{3} & -\frac{\Omega^2}{6} & 0 \\ -\frac{\Omega^2}{6} & \frac{1}{2} - \frac{\Omega^2}{3} & 0 \\ 0 & 0 & 1-\Omega^2 \end{bmatrix}$$

(ker ni nobenega sklopitvenega člena med x_1 ali x_2 in y (vsi so 0), vidimo da je nihanje v y smeri neodvisno od nihanja v x smeri in bi ga lahko obravnavali ločeno)

$$\left(\frac{1}{2} - \frac{\Omega^2}{3}\right)^2 (1-\Omega^2) - (1-\Omega)\left(\frac{\Omega^2}{6}\right)^2 = 0$$

1. $\Omega_1^2 = 1 \Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad \eta_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $\quad (\Omega_1^2 = 1)$

$$\left(\frac{1}{2} - \frac{\Omega^2}{3}\right)^2 - \left(\frac{\Omega^2}{6}\right) = 0$$

$$9 + 4\Omega^4 - 12\Omega^2 - \Omega^4 = 0$$

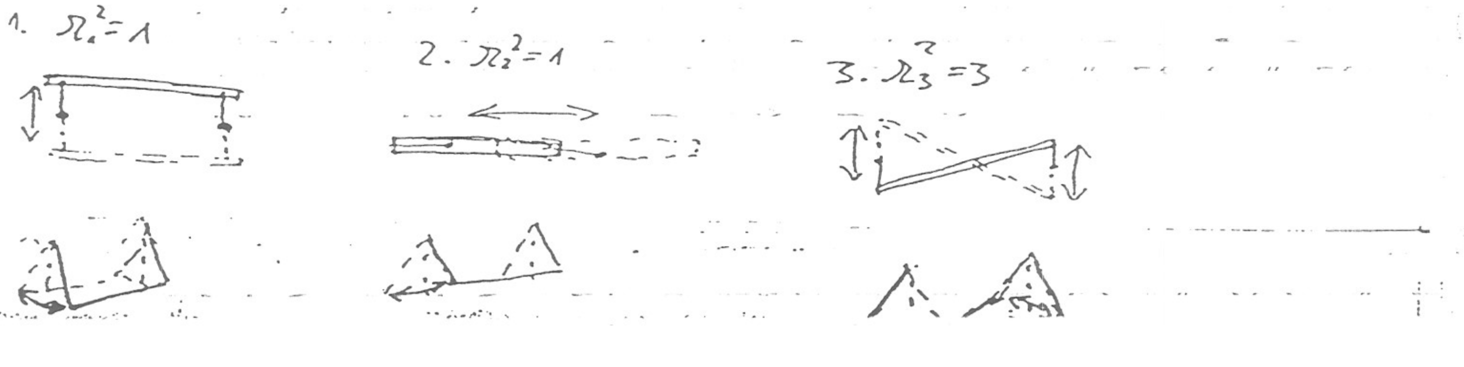
$$(\Omega^2 - 1)(\Omega^2 - 3)$$

ista l. vrednost $\Omega^2 = 1$

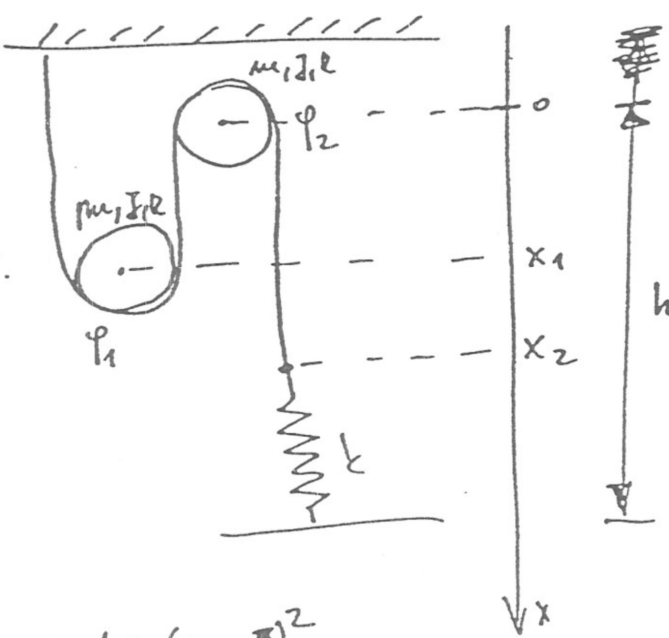
2. $\Omega_2^2 = 1 \Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad \eta_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $\quad (\Omega_2^2 = 1)$

3. $\Omega_3^2 = 3 \Rightarrow \begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & -2 \end{bmatrix} \Rightarrow \eta_3 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ $\quad (\Omega_3^2 = 3,$

SKICE (TLORIS):



24) sistem škripcev



$$l\varphi_2 = x_2$$

$$2x_1 + x_2 = l - \text{const}$$

$$l\dot{\varphi}_2 = \dot{x}_2$$

$$2x_1 = l - x_2$$

$$2\dot{x}_1 = -\dot{x}_2$$

$$2\varphi_1 = -\varphi_2$$

$$2\dot{\varphi}_1 = -\dot{\varphi}_2$$

$$V = -mgx_1 + \frac{1}{2}k(h - x_2)^2$$

$$T = \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}J\dot{\varphi}_1^2 + \frac{1}{2}J\dot{\varphi}_2^2$$

$$T = \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}J\left(\frac{\dot{x}_1}{l}\right)^2 + \frac{1}{2}J\left(\frac{\dot{x}_2}{l}\right)^2$$

$$= \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}\frac{m\cancel{l^2}}{2\cancel{l^2}}\dot{x}_1^2 + \frac{1}{2}\frac{m\cancel{l^2}}{2\cancel{l^2}}\cdot(2\dot{x}_1)^2$$

$$= m\dot{x}_1^2\left(\frac{1}{2} + \frac{1}{4} + 1\right) = \frac{7m\dot{x}_1^2}{4}$$

tako postavim merilo

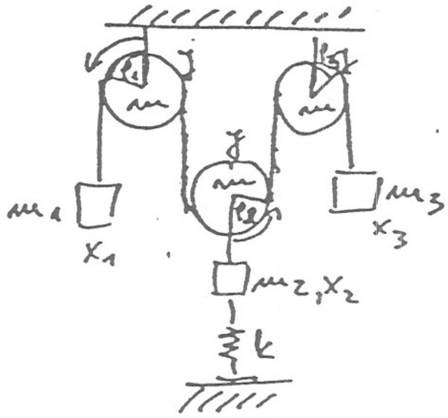
$$L = T - V = \frac{7m\dot{x}_1^2}{4} + mgx_1 - \frac{1}{2}k(\overbrace{h - l}^{\alpha=0} + 2x_1)^2$$

$$\text{E.L.:}\quad \frac{7}{2}m\ddot{x}_1 - mg + 4kx_1 = 0$$

$$\frac{7}{2}m\ddot{x}_1 + 4kx_1 = mg$$

$$\ddot{x}_1 + \underbrace{\left(\frac{8k}{7m}\right)}_{\omega^2}x_1 = mg$$

) ... mase



$$\varphi_2 \overset{!}{=} \frac{\varphi_3-\varphi_1}{2} \Rightarrow 2\dot\varphi_2 = \dot\varphi_3 - \dot\varphi_1$$

$$\ell\varphi_1 = x_1$$

$$x_1 + x_3 + 2x_2 = \ell$$

$$x_1 = \ell - x_3 - 2x_2$$

$$\dot x_1 = -(\dot x_3 + 2\dot x_2)$$

$V_1 = -m_1 g x_1, \quad V_2 = -(m_2+m) g x_2, \quad V_3 = -m_3 g x_3$

$$V = k\frac{(x_2-x_0)^2}{2}$$

$$T_1 = m_1\frac{\dot x_1^2}{2} + \frac{J\dot\varphi_1^2}{2} = \frac{1}{2}\dot x_1^2\left(m_1 + \frac{m}{2}\right)$$

$$T_3 = m_3\frac{\dot x_3^2}{2} + \frac{J\dot\varphi_3^2}{2} = \frac{1}{2}\dot x_3^2\left(m_3 + \frac{m}{2}\right)$$

$$T_2 = (m_2+m)\frac{\dot x_2^2}{2} + \frac{J\dot\varphi_2^2}{2} = \frac{1}{2}(m_2+m)\dot x_2^2 + \frac{mR^2}{4}\cdot\frac{\dot x_2^2}{R^2} = \frac{1}{2}\left(m_2 + \frac{3m}{2}\right)\dot x_2^2$$

$L = T - V$

$$\frac{1}{2}(\dot x_3 + 2\dot x_2)^2\left(m_1 + \underline{\frac{m}{2}}\right) + \frac{1}{2}\dot x_3^2\left(m_3 + \underline{\frac{m}{2}}\right) + \frac{1}{2}\dot x_2^2\left(m_2 + \underline{\frac{3m}{2}}\right)$$

$$m_1 g(\ell - x_3 - 2x_2) + (m_2 + \underline{m}) g x_2 + m_3 g x_3 - \frac{k}{2}(x_2-x_0)^2$$

podčrtani členi predstavljajo doprinos zaradi škripcev

vstavimo $m_1 = 2m, \; m_2 = 6m, \; m_3 = 3m$

$$\Rightarrow L = \frac{1}{2}\dot x_3^2\cdot 6m + \frac{1}{2}\dot x_2^2\cdot\frac{35m}{2} + \frac{1}{2}\dot x_3\dot x_2\cdot 10m$$
$$+ mgx_3 + 3mgx_2 - \frac{k}{2}(x_2-x_0)^2$$

Euler - Lagrange:

x_3: $6m\ddot x_3 + 5m\ddot x_2 - mg = 0$

x_2: $\frac{35}{2}m\ddot x_2 + 5m\ddot x_3 - 3mg + \frac{k}{m}(x_2-x_0) = 0$

V ravnovesju $x_2=0, \dot{x}_2=0, \ddot{x}_2=0$

$$\Rightarrow \ddot{x}_3 = \frac{g}{6} \qquad \left(\ddot{x}_1 = -\frac{g}{6}\right)$$

Nihanje: $\ddot{x}_3 = \frac{g-5\ddot{x}_2}{6}$

$$\Rightarrow \frac{35}{2}\ddot{x}_2 + \frac{5g}{6} - \frac{25\ddot{x}_2}{6} - 3g + \frac{k}{m}(x_2-x_0) = 0$$

$$\frac{105-25}{6}\ddot{x}_2 + \frac{k}{m}x_2 = \frac{k}{m}x_0 + 3g - \frac{5}{6}g$$

$$\Rightarrow \ddot{x}_2 + \frac{3k}{40m}x_2 = \frac{3}{40}\left(\frac{k}{m}x_0 + \frac{13}{6}g\right)$$

$$\omega^2 = \frac{3k}{40m} \longleftarrow \text{frekvenca nihanja sistema.}$$

Gibanje sistema je torej (po pričakovanju) sestavljeno iz enakomerno pospešenega gibanja uteži 1 in 3, ter iz nihanja vseh treh uteži s frekvenco ω:

$$\underline{x_2 = x_{20}\cdot\cos\omega t}$$

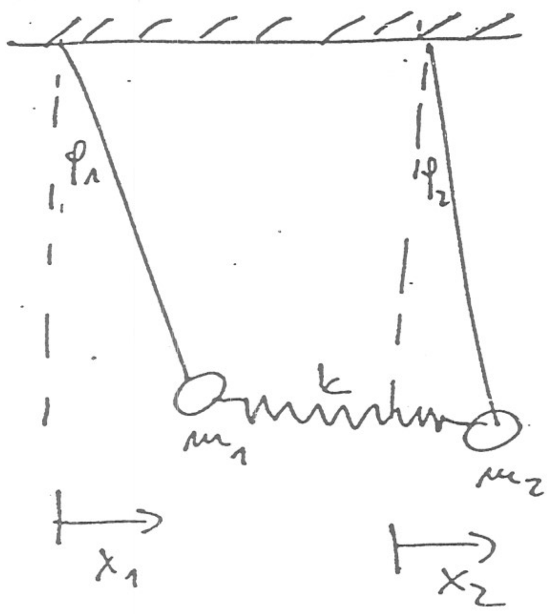
(začetna hitrost x_3: $\dot{x}_{30} = v_3$)

$$\ddot{x}_3 = \frac{g}{6} + \frac{5}{6}\omega^2 x_{20}\cos\omega t \Rightarrow \underline{x_3 = \frac{1}{2}\frac{g}{6}t^2 + \dot{x}_{30}\cdot t - \frac{5}{6}x_{20}\cos\omega t}$$

$$\underline{\ddot{x}_1 = -(\ddot{x}_3 + 2\ddot{x}_2)}$$

To je rezultat če upoštevam tudi maso škripcev. Če so škripci zanemarljivo lahki, potem pomečem podčrtane člene iz L (na prejšnji strani) ven)

6)



$x_1 = l\varphi_1 \qquad \dot{x}_1 = l\dot{\varphi}_1$

$x_2 = l\varphi_2 \qquad \dot{x}_2 = l\dot{\varphi}_2$

$$T = \frac{m_1}{2}\dot{x}_1^2 + \frac{m_2}{2}\dot{x}_2^2 = \frac{m_1}{2}l^2\dot{\varphi}_1^2 + \frac{m_2}{2}l^2\dot{\varphi}_2^2$$

$$V = -lgm_1\cos\varphi_1 - lgm_2\cos\varphi_2 + \frac{k}{2}(x_1-x_2)^2$$

$$V = -lgm_1\left(1-\frac{\varphi_1^2}{2}\right) - lgm_2\left(1-\frac{\varphi_2^2}{2}\right) + \frac{k}{2}l^2(\varphi_1-\varphi_2)^2$$

$$V' = lg\frac{m_1}{2}\varphi_1^2 + lg\frac{m_2}{2}\varphi_2^2 + \frac{k}{2}l^2(\varphi_1-\varphi_2)^2$$

$$L = T - V$$

Euler - Lagrange:

$\varphi_1: \quad m_1 l^2\ddot{\varphi}_1 + lgm_1\varphi_1 + kl^2(\varphi_1-\varphi_2) = 0$

$\varphi_2: \quad m_2 l^2\ddot{\varphi}_2 + lgm_2\varphi_2 - kl^2(\varphi_1-\varphi_2) = 0$

Označimo $\frac{g}{l} = \omega_0^2$, $\frac{k}{m_1} = \omega_1^2$, $\frac{k}{m_2} = \omega_2^2$

$$\ddot{\varphi}_1 + \omega_0^2\varphi_1 + \omega_1^2(\varphi_1-\varphi_2) = 0 \qquad (1)$$

$$\ddot{\varphi}_2 + \omega_0^2\varphi_2 - \omega_2^2(\varphi_1-\varphi_2) = 0 \qquad (2)$$

Označimo $\varphi_1 - \varphi_2 = \eta \qquad \varphi_1 = \frac{\eta+\xi}{2}$

$\varphi_1 + \varphi_2 = \xi \qquad \varphi_2 = \frac{\xi-\eta}{2}$

I. (1) − (2) :

$$\ddot{\eta} + \omega_0^2\eta + (\omega_1^2 + \omega_2^2)\eta = 0$$

$$\ddot{\eta} + \eta(\omega_0^2 + \omega_1^2 + \omega_2^2) = 0$$

$$\underline{\Omega_1^2 = \omega_0^2 + \omega_1^2 + \omega_2^2}.$$

$$\Rightarrow \underline{\underline{\eta(t) = A\cos\Omega_1 t + B\sin\Omega_1 t}}$$

A, B dobimo iz začetnih pogojev

II. (1) + (2)

$$\ddot{\xi} + \omega_0^2\xi + \omega_1^2\eta - \omega_2^2\eta$$

$$\Rightarrow \ddot{\xi} + \omega_0^2\xi = (\omega_2^2 - \omega_1^2)\eta$$

$$\underline{\Omega_2^2 = \omega_0^2}$$

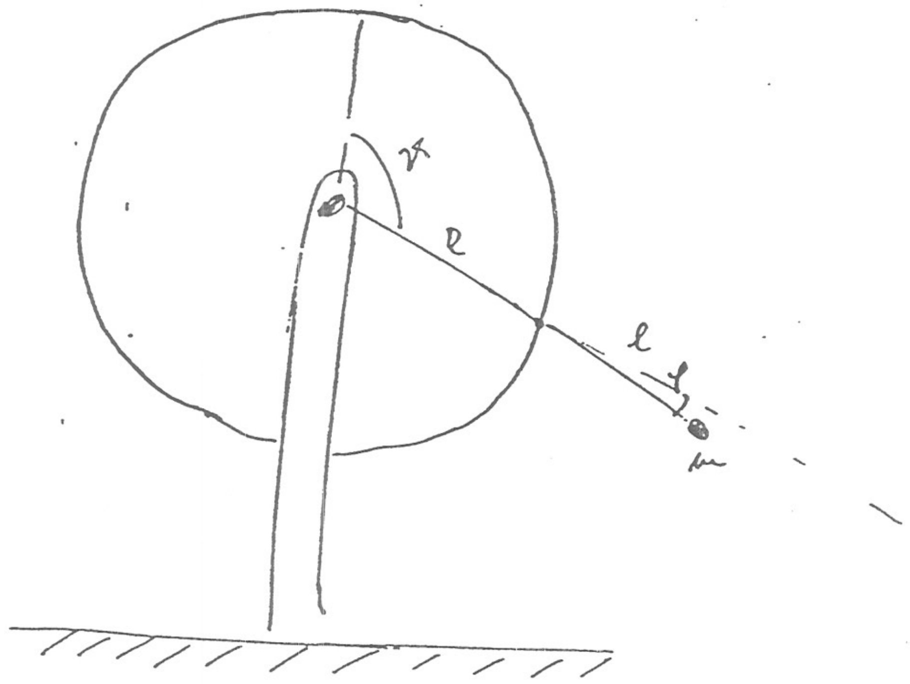
$$\Rightarrow \underline{\underline{\xi = C\cos\Omega_2 t + D\sin\Omega_2 t + \xi'}}$$

rešitev partikularnega dela

$$\underline{\varphi_1(t) = \frac{\xi(t) + \eta(t)}{2}}$$

$$\underline{\varphi_2(t) = \frac{\xi(t) - \eta(t)}{2}}$$

7)

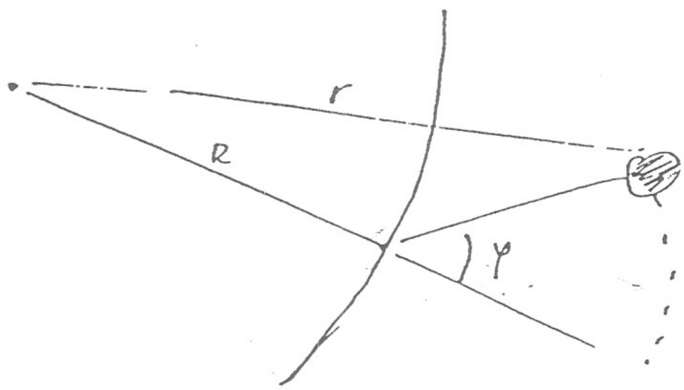


$\omega^2 \gg \frac{g}{R} \rightarrow R\omega^2 \gg g \rightarrow (R+l)\omega^2 \gg g$

...lni pospešek je bistveno večji od g zato potencialno ...ergijo težnega polja zanemarim (ali pa upoštevam le ...ajhen popravek pri različnih ϑ)

...blem je torej tak, kot če imamo g, ki linearno ...ašča z r: $\vec{F} = m\vec{a} = m\vec{r}\omega^2 = m\vec{g}' \Rightarrow \underline{\vec{g}' = \omega^2\vec{r}}$

$-\frac{m r^2 \omega^2}{2}$



$R^2 + l^2 - 2Rl\cos(\pi - \varphi) = R^2 + l^2 + 2Rl\cos\varphi$

$\frac{m\omega^2}{2}(R^2 + l^2 + 2Rl\cos\varphi)$

$\frac{m}{2}\left((R+l)\omega + l\dot{\varphi}\right)^2 = \frac{m}{2}\left((R+l)^2\omega^2 + l(R+l)\omega\dot{\varphi} + l^2\dot{\varphi}^2\right)$

Euler - Lagrange : $L = T - V$

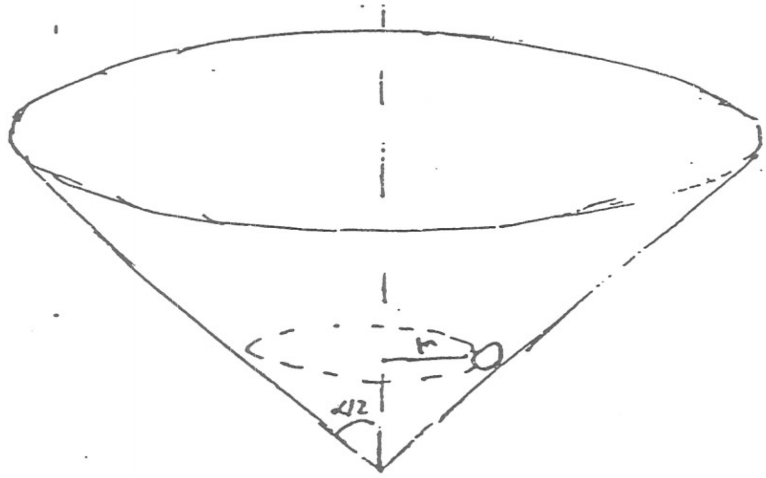
$$\cancel{m} l^{\cancel{2}} \ddot{\varphi} + \cancel{m} \omega^2 \frac{R \cancel{l}}{\cancel{l}} \sin\varphi = 0$$

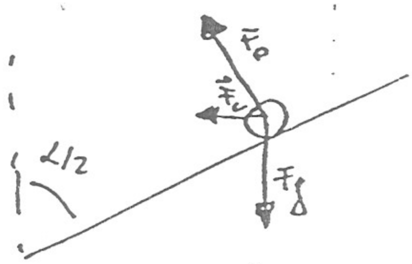
$$\Rightarrow \ddot{\varphi} + \omega^2 \frac{R}{l} \sin\varphi = 0$$

za male kote : $\Omega^2 = \omega^2 \cdot \frac{R}{l}$ (brez upoštevanja g)

$\varphi = \varphi_0 (\sin \Omega t + \phi_0)$; φ_0, ϕ_0 - začetna pogoja

8)





$$\vec F_p + \vec F_g = \vec F_c$$

$$V = -mgr\,\mathrm{ctg}\frac{\alpha}{2} = -mgr\,C$$ = ima li

$$T = \frac{1}{2} m r^2\omega^2 + \frac{1}{2} m\dot r^2 + \frac{1}{2} m \dot r^2 \mathrm{ctg}^2\frac{\alpha}{2}$$

$$= \frac{1}{2} m r^2\omega^2 + \frac{1}{2} m \dot r^2 \underbrace{\left(1+\mathrm{ctg}^2\frac{\alpha}{2}\right)}_{D}$$

$$F_r = m a_r = m\omega^2 r$$

$$\Rightarrow r = \frac{mg\cdot \mathrm{ctg}\frac{\alpha}{2}}{m\omega^2}$$

$$L = T - V = \frac{1}{2} m r^2 \omega^2 + \frac{1}{2} m \dot r^2 D + mgrC$$

φ ciklična $\Rightarrow$ $p_\varphi = \Gamma = m r^2 \omega$ $\quad (\omega = \dot\varphi)$

$$L = \frac{1}{2}\frac{\Gamma^2}{m^2 r^2}\cancel{m r^2} + \frac{D}{2} m \dot r^2 + mgrC$$

Euler - Lagrange

$$D m \ddot r - \frac{p_\varphi^2}{m r^3} - mgC = 0$$

Ravnovesje: $\ddot r = 0 \Rightarrow \frac{p_\varphi^2}{m r^3} = mgC \Rightarrow \frac{m^2 r^4 \omega^2}{m r^3} = mgC \Rightarrow r = \frac{g}{\omega^2}\cdot C$

Dobim isto.
To je dobro.

Mala motnja: $r = r_0 + x$, $x \ll r_0$

$$D m \ddot x - \frac{\Gamma^2}{m(r_0^3 + 3r_0^2 x)} = mgC$$

$$D m \ddot x - \frac{\Gamma^2}{m r_0^3 \left(1 + 3\frac{x}{r_0}\right)} = mgC$$

$$D m \ddot x + \frac{3\Gamma^2}{m r_0^4} x = mgC + \frac{\Gamma^2}{m r_0^3}$$

Upoštevam:

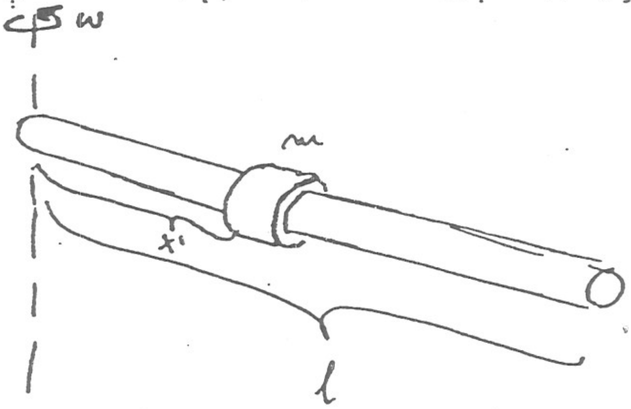
$$(r_0 + x)^3 \doteq r_0^3 + 3 r_0^2 x$$

$$\frac{1}{1+\frac{x}{r_0}} \doteq 1 - \frac{x}{r_0}$$

$$\Rightarrow \ddot x + \frac{3\Gamma^2}{D m^2 r_0^4} x = A \Rightarrow \Omega^2 = \frac{3\Gamma^2}{D m^2 r_0^4}$$

$x = x_0 \sin \Omega t$

7)



$x'\hat{i}'$

$\dot{x}'\hat{i}' + x'\dot{\hat{i}}' = \dot{x}'\hat{i}' + \omega x'\hat{j}'$

$T_1 = \frac{m}{2}(\dot{x}'^2 + \omega^2 x'^2)$

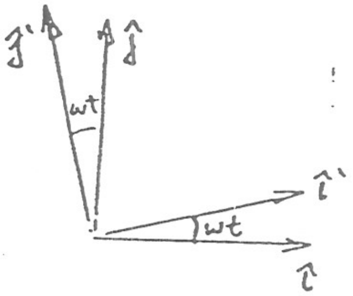
$T_2 = \frac{J}{2}\dot{\varphi}^2 = \frac{J}{2}\omega^2$

$V = 0$

$L = T - V = T = \frac{m}{2}\dot{x}'^2 + \frac{m}{2}\dot{\varphi}^2 x'^2 + \frac{J}{2}\dot{\varphi}^2$

ker $\varphi \to$ ciklična. $p_\varphi = m\dot{\varphi}x^2 + J\dot{\varphi} = \Gamma$

$$\Rightarrow \dot{\varphi} = \frac{\Gamma}{J + mx^2}$$



$\hat{i}' = \hat{i}\cos\omega t + \hat{j}\sin\omega t$

$\hat{j}' = -\hat{i}\sin\omega t + \hat{j}\cos\omega t$

$\dot{\hat{i}}' = +\omega\hat{j}'$

$\dot{\hat{j}}' = -\omega\hat{i}'$

$(x' = x)$

.: $\cancel{m}\ddot{x}' - \cancel{m}\dot{\varphi}^2 x' = 0$

$$\ddot{x} - \frac{\Gamma^2}{J^2 + 2Jmx^2 + m^2x^4}x = 0$$

1.) če je palica zelo težka oz. jo mi vrtimo s konstantno hitrostjo ω, potem

$$\ddot{x} - \frac{\Gamma^2}{J^2}x = 0 \to \ddot{x} - \omega^2 x = 0 \Rightarrow x = Ae^{\omega t} + Be^{-\omega t}$$

$$\left.\begin{array}{l} x(t=0) = x_0 = A + B \\ \dot{x}(t=0) = 0 = A - B \end{array}\right\} A = B = \frac{x_0}{2}$$

$$x(t) = x_0 \cdot \operatorname{ch}\omega t$$

a) ker pa sta J palice in mx^2 primerljiva in je palica prosto vpeta, potem:

$$L = T - V = \frac{1}{2}\left(m\dot{x}^2 + (mx^2 + J)\dot{\varphi}^2\right) = \frac{1}{2}\left(m\dot{x}^2 + \frac{p_\varphi^2}{mx^2+J}\right)$$

$$\text{E.L.} \Rightarrow \ddot{x} - p_\varphi^2 \cdot \frac{1}{(mx^2+J)^2} x = 0 \qquad \Big\} \cdot \dot{x}$$

Upoštevam

$$\dot{x}\ddot{x} = \frac{1}{2}\frac{d}{dt}(\dot{x}^2)$$

$$x\dot{x} = \frac{1}{2}\frac{d}{dt}(x^2)$$

$$\frac{1}{2}\frac{d}{dt}\left(\dot{x}^2\right) = \frac{p_\varphi^2}{(mx^2+J)^2}\, x\dot{x}$$

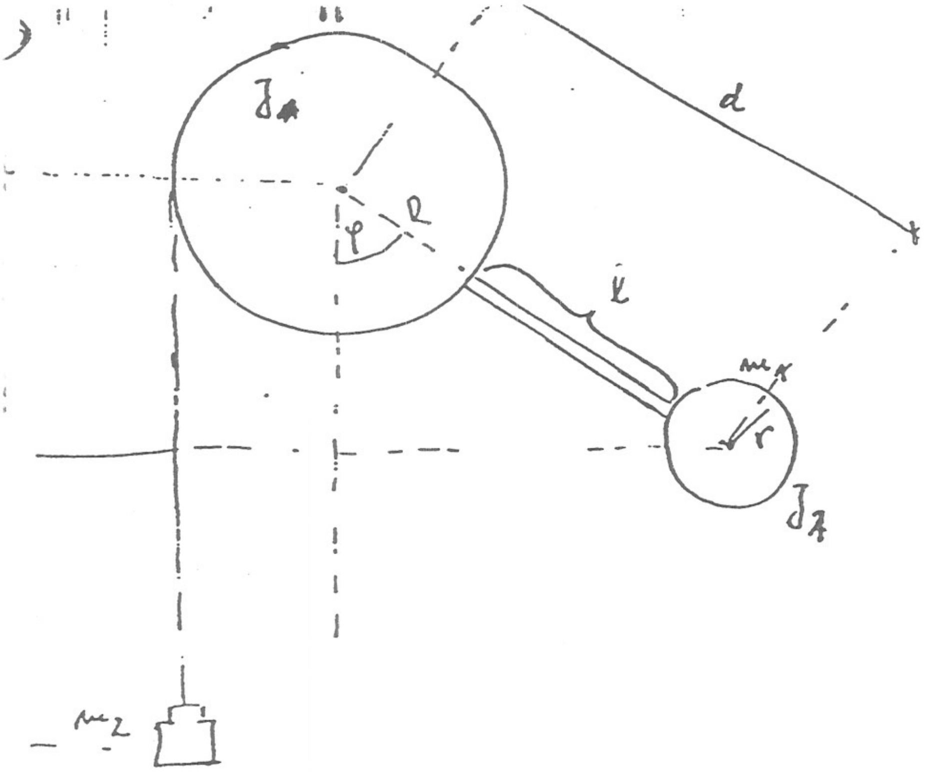
Z metodo ostrega pogleda vidim:

$$\frac{d}{dt}\left(\frac{-p_\varphi^2}{2(mx^2+J)m}\right) = \frac{p_\varphi^2}{(mx^2+J)^2}\, x\dot{x}$$

$$\Rightarrow \quad \dot{x}^2 = \frac{-p_\varphi^2}{(mx^2+J)m} + v_0^2$$

$$\begin{aligned} \dot{x}(0) &= 0 \\ x(0) &= x_0 \end{aligned} \quad \Rightarrow v_0^2 = \frac{p_\varphi^2}{(mx_0^2+J)m}$$

$$\Rightarrow v^2(x) = \frac{p_\varphi^2}{m}\cdot\left(\frac{1}{mx_0^2+J} - \frac{1}{mx^2+J}\right)$$



$(R + r + l = d)$

$$J = \frac{MR^2}{2}$$

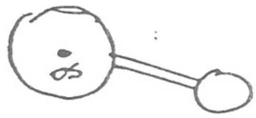
$$J_1 = \frac{2 m_1 r^2}{5}$$

$$J_s = m d^2$$ ← steine

$$J_1 + J_s = J'$$

$$J + J' = \underline{J}$$

vztr. moment celega sistema okrog osi:



$$T = \frac{m_2 \dot{x}_2^2}{2} + \frac{J \dot{\varphi}^2}{2} + \frac{J_1 \dot{\varphi}^2}{2} + \frac{J_s}{2} \dot{\varphi}^2$$

$$\Rightarrow T = \frac{m_2 \dot{x}_2^2}{2} + (J + J') \cdot \frac{\dot{\varphi}^2}{2}$$

$$\underline{T = \frac{m_2 \dot{x}_2^2}{2} + \frac{\underline{J} \dot{\varphi}^2}{2}}$$

$$x_2 = R\varphi$$
$$dx_2 = R\, d\varphi$$
$$\dot{x}_2 = R\dot{\varphi}$$

$$\underline{V = -m_2 g x_2 - m_1 g d \cos\varphi}$$

$$L = \frac{m_2 R^2 \dot{\varphi}^2}{2} + \frac{\underline{J} \dot{\varphi}^2}{2} + m_2 g R \varphi + m_1 g d \cos\varphi$$

$$L. \Rightarrow (m_2 R^2 + \underline{J})\, \ddot{\varphi} + m_1 g d \sin\varphi - m_2 g R = 0$$

Ravnovesje $\ddot{\varphi} = 0 \Rightarrow m_1 d \sin\varphi = m_2 R \Rightarrow \sin\varphi = \frac{m_2 R}{m_1 d}$

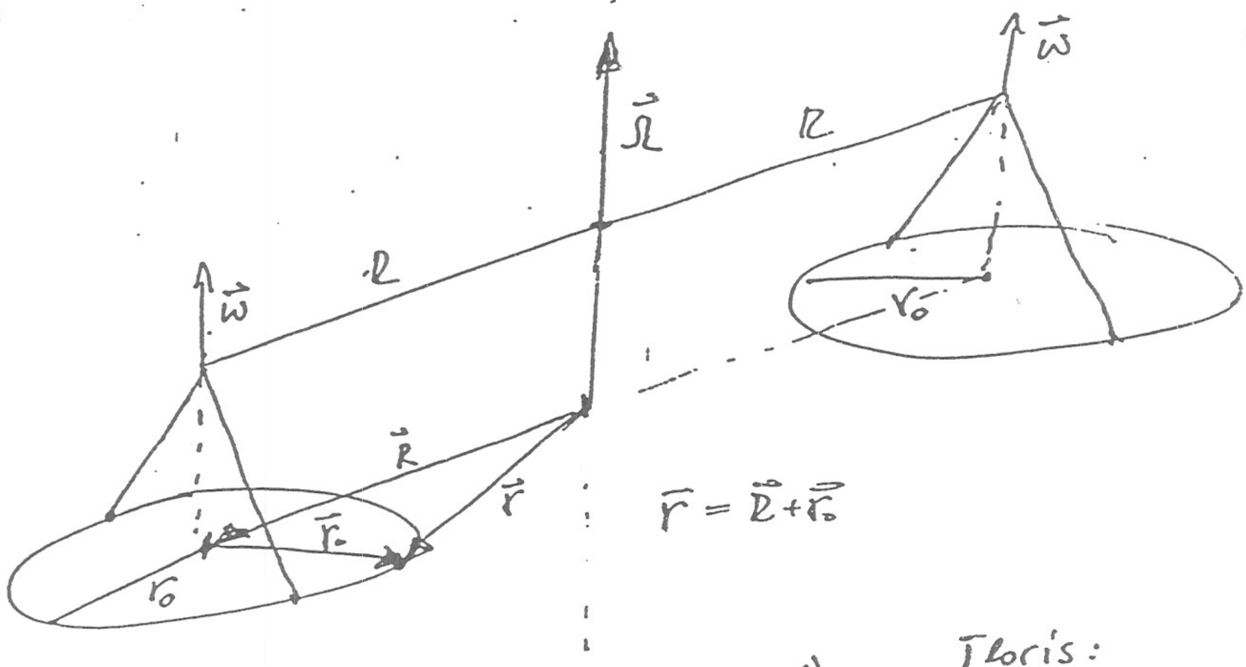
Nihanje:

$$\varphi = \varphi_0 + \alpha \quad \Rightarrow \quad \sin\varphi = \sin(\varphi_0 + \alpha) = \sin\varphi_0 + \alpha\cos\varphi_0$$

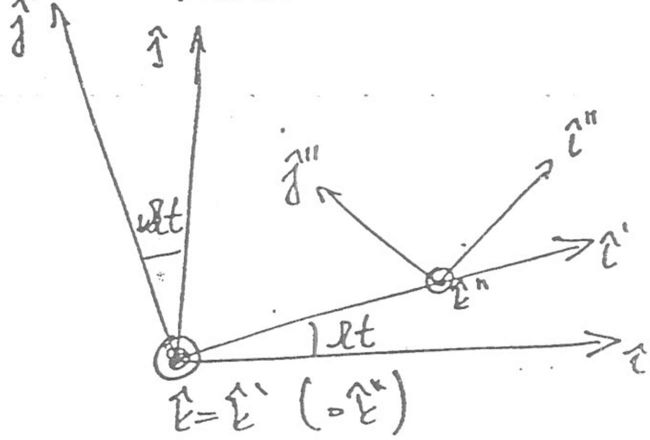
$$\Rightarrow (m_2 l^2 + \underline{J})\,\cancel{\ddot{\varphi}}\,\ddot{\alpha} + m_1 g\, d\, \alpha \cos\varphi_0 + C = 0$$

$$\ddot{\alpha} + \underbrace{\frac{m_1 g\, d \cos\varphi_0}{(m_2 l^2 + \underline{J})}}_{\Omega^2}\,\alpha = 0$$

1)



Tloris:



$= \hat{i}\cos\Omega t + \hat{j}\sin\Omega t; \quad \dot{\hat{i}}' = \Omega \hat{j}'$

$= -\hat{i}\sin\Omega t + \hat{j}\cos\Omega t; \quad \dot{\hat{j}}' = -\Omega \hat{i}'$

$= \hat{i}'\cos\omega t + \hat{j}'\sin\omega t; \quad \dot{\hat{i}}'' = \omega \hat{j}''$

$= -\hat{i}'\sin\omega t + \hat{j}'\cos\omega t; \quad \dot{\hat{j}}'' = -\omega \hat{i}''$

pravim še druge odvode:

$\Omega \dot{\hat{j}}' = -\Omega^2 \hat{i}' = +\hat{i}(-\Omega^2)\cos\Omega t + \hat{j}(-\Omega^2)\sin\Omega t = \ddot{\hat{i}}'$

$-\Omega \dot{\hat{i}}' = -\Omega^2 \hat{j}' = -\hat{i}(-\Omega^2)\sin\Omega t + \hat{j}(-\Omega^2)\cos\Omega t = \ddot{\hat{j}}'$

$= -\omega^2 \hat{i}'' = -\omega^2(\hat{i}'\cos\omega t + \hat{j}'\sin\omega t)$

$-\omega^2 \hat{j}'' = -\omega^2(-\hat{i}\sin\omega t + \hat{j}\cos\omega t)$

$\ddot{\hat{i}}'' = +\hat{i}(-\omega^2)(\cos\Omega t\cos\omega t - \sin\Omega t\sin\omega t) + \hat{j}(-\omega^2)(\sin\Omega t\cos\omega t + \cos\Omega t\sin\omega t) = +\hat{i}(-\omega^2)\cos[(\Omega+\omega)t] + \hat{j}(-\omega^2)\sin[(\Omega+\omega)t]$

$\ddot{\hat{j}}'' = -\hat{i}(-\omega^2)(\cos\Omega t\sin\omega t + \sin\Omega t\cos\omega t) + \hat{j}(-\omega^2)(\cos\Omega t\cos\omega t - \sin\Omega t\sin\omega t) = -\hat{i}(-\omega^2)\sin[(\Omega+\omega)t] + \hat{j}(-\omega^2)\cos[(\Omega+\omega)t]$

$$\vec{r} = \vec{R}\,\hat{i}' + \vec{r}_0\,\hat{i}''$$

Označim: $\omega + \Omega = \varphi$

$$\dot{\vec{r}} = R\,\dot{\hat{i}}' + r_0\,\dot{\hat{i}}''$$

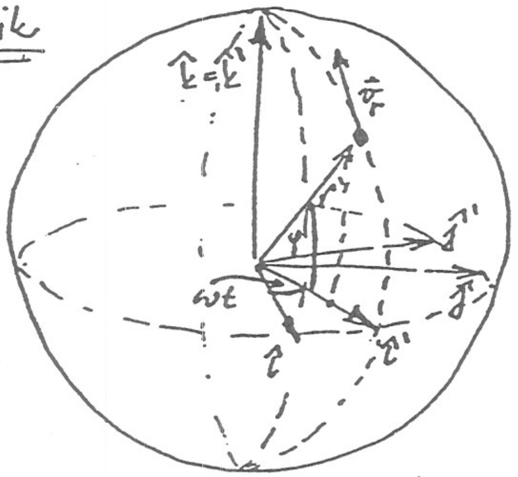
$$\ddot{\vec{r}} = R\,\ddot{\hat{i}}' + r_0\,\ddot{\hat{i}}'' = R\,\hat{i}\,(-\Omega^2)\cos\Omega t + R\,\hat{j}\,(-\Omega^2)\sin\Omega t$$
$$+ r_0\,\hat{i}\,(-\omega^2)\cos\varphi t + r_0\,\hat{j}\,(-\omega^2)\sin\varphi t$$

$$\Rightarrow \quad \boxed{\begin{aligned}\ddot{\vec{r}} &= \hat{i}\,(-R\Omega^2\cos\Omega t - r_0\omega^2\cos\varphi t) \\ &+ \hat{j}\,(-R\Omega^2\sin\Omega t - r_0\omega^2\sin\varphi t)\end{aligned}}$$

Opomba: Čas računamo šteti, ko je operovana točka - veseljak najdlje od velike osi, oz. takrat ko imata $\vec{r}_0$ in $\vec{R}$ isto smer.

Avto vozi po satelitu Europa

Poldnevnik



$\hat{i}' = \hat{i}\cos\omega t + \hat{j}\sin\omega t$

$\hat{j}' = -\hat{i}\sin\omega t + \hat{j}\cos\omega t$

$\hat{k}' = \hat{k}$

$\vec{\omega} = \omega\hat{k}$

$\vec{g} = -g\cos\varphi\,\hat{i}' - g\sin\varphi\,\hat{k}'$

$\vec{a} - m\vec{g}$

$\vec{a}_{rel} + 2\vec{\omega}\times\vec{v}_{rel} + \vec{\omega}\times(\vec{\omega}\times\vec{r}') + \cancel{\dot{\vec{\omega}}\times\vec{r}'}$ (1)

$= -v\sin\varphi\,\hat{i}' + v\cos\varphi\,\hat{k}'$, $\vec{r}' = R\cos\varphi\,\hat{i}' + R\sin\varphi\,\hat{k}'$

$-\frac{v^2}{R}\cos\varphi\,\hat{i}' - \frac{v^2}{R}\sin\varphi\,\hat{k}'$ (Upoštevam: $dx = R\,d\varphi \Rightarrow v = R\dot{\varphi}$)

so vektorji, ki jih v enačbi (1) potrebujem. Pretvorim jih v $\hat{i}, \hat{j},$

$\hat{i}(R\cos\varphi\cos\omega t) + \hat{j}(R\cos\varphi\sin\omega t) + \hat{k}(R\sin\varphi)$

$\hat{i}(-v\sin\varphi\cos\omega t) + \hat{j}(-v\sin\varphi\sin\omega t) + \hat{k}(v\cos\varphi)$

$\hat{i}(-\frac{v^2}{R}\cos\varphi\cos\omega t) + \hat{j}(-\frac{v^2}{R}\cos\varphi\sin\omega t) + \hat{k}(-\frac{v^2}{R}\sin\varphi)$ (2)

$) = R\begin{bmatrix} i & j & k \\ 0 & 0 & \omega \\ c\varphi c\omega t & c\varphi s\omega t & s\varphi \end{bmatrix} = \hat{i}(-\omega R\cos\varphi\sin\omega t) + \hat{j}(\omega R\cos\varphi\cos\omega t)\ (+\hat{k}\cdot 0)$

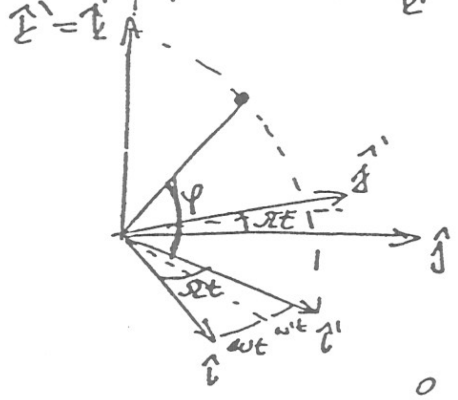
$\times\vec{r}') = \omega R\begin{bmatrix} i & j & k \\ 0 & 0 & \omega \\ -c\varphi s\omega t & c\varphi c\omega t & 0 \end{bmatrix} = \hat{i}(-\omega^2 R\cos\varphi\cos\omega t) + \hat{j}(-\omega^2 R\cos\varphi\sin\omega t)\ (+\hat{k}\cdot 0)$

$= \vec{\omega}\times\vec{\omega}\times\vec{r}'$ (3)

$= v\begin{bmatrix} i & j & k \\ 0 & 0 & \omega \\ -s\varphi c\omega t & -s\varphi s\omega t & c\varphi \end{bmatrix} = \hat{i}(\omega v\sin\varphi\sin\omega t) + \hat{j}(-v\omega\sin\varphi\cos\omega t) = \vec{\omega}\times\vec{v}_r$ (4)

$\cos\varphi\,\hat{i}' - g\sin\varphi\,\hat{k}' = \hat{i}(-g\cos\varphi\cos\omega t) + \hat{j}(-g\cos\varphi\cos\omega t) + \hat{k}(-g\sin\varphi)$ (5)

2. izpopravek: Koordinatni sistem $\hat{i}, \hat{j}, \hat{k}$ postavimo tako, da je vozilo v uglu pri mizi. Torej pišem: $\omega' = \frac{v}{R'} = \frac{v}{R\cos\varphi}$, torej: $\omega + \omega' = \Omega$



$\hat{i}' = \hat{i}\cos\Omega t + \hat{j}\sin\Omega t$

$\hat{j}' = \hat{i}(-\sin\Omega t) + \hat{j}\cos\Omega t$

$\hat{k}' = \hat{k}$

(saj $\vec{v}_{rel} = 0$) $\vec{\Omega} = \Omega\hat{k}$

Spet $F = m\vec{a} - m\vec{g}$, $\vec{a} = \cancel{\vec{A}} + \cancel{\vec{a}_{rel}} + \cancel{2\vec{\omega}\times\vec{v}_{rel}} + \vec{\omega}\times(\vec{\omega}\times\vec{r}') + \cancel{\dot{\vec{\omega}}\times\vec{r}'}$

Ostane le centrifugalna sila. Pričakovano.

$\vec{r}' = R\cos\varphi\,\hat{i}' + R\sin\varphi\,\hat{k}' = \hat{i}(R\cos\varphi\cos\Omega t) + \hat{j}(R\cos\varphi\sin\Omega t) + \hat{k}(R\sin\varphi)$

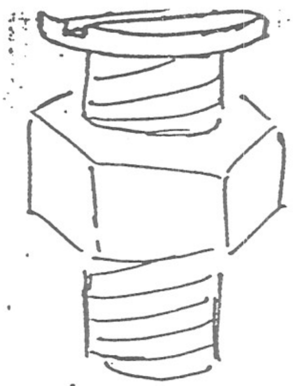
$$\vec{\Omega}\times\vec{r}' = R\begin{bmatrix} i & j & k \\ 0 & 0 & \Omega \\ c\varphi c\Omega t & c\varphi s\Omega t & s\varphi \end{bmatrix} = \begin{matrix}\hat{i}(-R\Omega\cos\varphi\sin\Omega t) \\ \hat{j}(R\Omega\cos\varphi\cos\Omega t)\end{matrix}$$

$$\vec{\Omega}\times(\vec{\Omega}\times\vec{r}) = R\Omega^2\begin{pmatrix} i & j & k \\ 0 & 0 & 1 \\ -c\varphi s\Omega t & c\varphi c\Omega t & 0 \end{pmatrix} = \begin{matrix}\hat{i}(-R\Omega^2\cos\varphi\cos\Omega t) \\ \hat{j}(-R\Omega^2\cos\varphi\sin\Omega t)\end{matrix} \quad (6)$$

$$\vec{g} = \hat{i}(-g\cos\varphi\cos\Omega t) + \hat{j}(-g\cos\varphi\sin\Omega t) + \hat{k}(-g\sin\varphi) \quad (7)$$

$\vec{F} = m(\vec{a} - \vec{g}) = m((6) - (7))$

(33)



Hod vijaka: $p\left(\frac{cm}{2\pi}\right)$

$$\Rightarrow x = p \cdot \varphi$$
$$\dot{x} = p\dot{\varphi}$$

$$= \frac{1}{2} J \dot{\varphi}^2 + \frac{1}{2} m \dot{x}^2$$

$$= \frac{1}{2} \frac{J}{p^2} \dot{x}^2 + \frac{1}{2} m \dot{x}^2 = \frac{1}{2} \underbrace{\left(\frac{J}{p^2} + m\right)}_{m'} \dot{x}^2 = \frac{1}{2} m' \dot{x}^2$$

$$= -mgx$$

$$T - V = \frac{1}{2} m' \dot{x}^2 + mgx$$

L.: $m' \ddot{x} = mg$ (ravnovesje ($\ddot{x}=0$) $\Rightarrow mg = 0$
imamo ga torej le če ali $m=0$
ali $g=0$)

$$\Rightarrow \ddot{x} = \frac{m}{m'} \cdot g$$

$\boxed{x(t) = \frac{1}{2} \ddot{x} t^2}$ – gibanje je enakomerno pospešeno

Upoštevam še silo upora: $F = -k\dot{\varphi}$

$$\Rightarrow M\ddot{\varphi} + \frac{mga}{2\pi} = F\cdot\frac{\partial F}{\partial \varphi} = -r\cdot k\dot{\varphi}$$

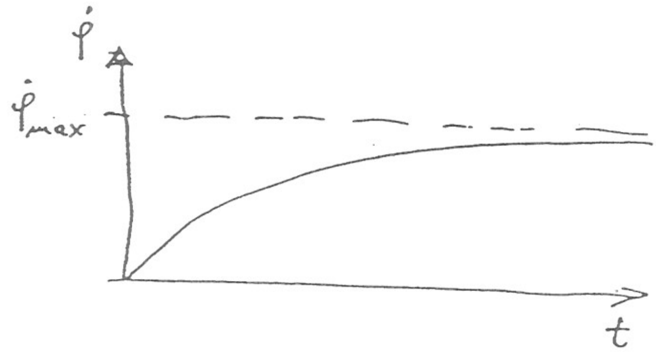
$$\ddot{\varphi} + rk\dot{\varphi} + \frac{mga}{2\pi} = 0$$

Homogeni del: $\lambda^2 + rk\lambda = 0 \quad \Rightarrow \quad \lambda_1 = 0, \; \lambda_2 = -rk$

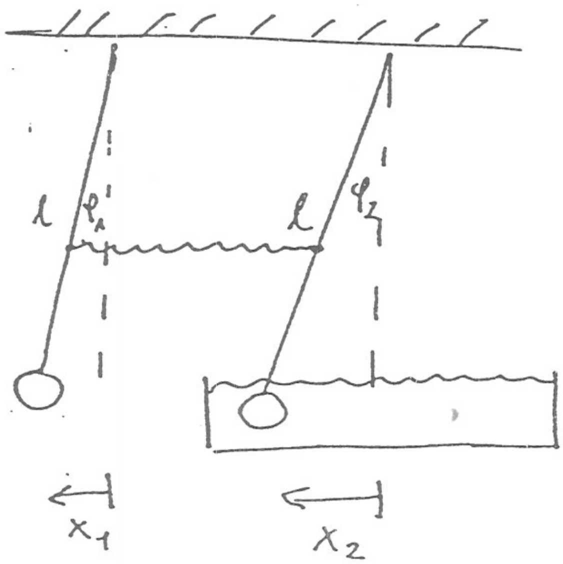
$$\Rightarrow \varphi_H = A e^{-rkt} + B$$

$$\varphi_P = -\frac{mga}{2\pi rk}\cdot t$$

Rešitev: $\varphi = \varphi_H + \varphi_P = A e^{-krt} + B - \frac{mga}{2\pi rk}\cdot t$



34) Dve nihali



za $\varphi \ll 1 \to \cos\varphi \doteq 1-\frac{\varphi^2}{2}$

$T_1 = \frac{1}{2} m_1 \dot{x}_1^2$ $\qquad l\varphi_1 = x_1$

$T_2 = \frac{1}{2} m_2 \dot{x}_2^2$ $\qquad l\varphi_2 = x_2$

$V_1 = \frac{m_1 g}{2} \varphi_1^2$

$V_2 = \frac{m_2 g}{2} \varphi_2^2$

$V_v = \frac{k}{2}\left(\frac{x_1-x_2}{2}\right)^2 = \frac{k}{8}(x_1-x_2)^2$

$$\Rightarrow L = T - V = \frac{m_1 l^2}{2}\dot{\varphi}_1^2 + \frac{m_2 l^2}{2}\dot{\varphi}_2^2 - \frac{m_1 g l}{2}\varphi_1^2 - \frac{m_2 g l}{2}\varphi_2^2 - \frac{k l^2}{8}(\varphi_1-\varphi_2)^2$$

Euler - Lagrange:

$\varphi_1: \quad \ddot{\varphi}_1 + \frac{g}{l}\varphi_1 + \frac{k}{4m_1}(\varphi_1-\varphi_2) = 0$

$\varphi_2: \quad \ddot{\varphi}_2 + \frac{g}{l}\varphi_2 + \frac{k}{4m_2}(\varphi_2-\varphi_1) = -C\cdot\dot{\varphi}_2$

značimo $\frac{g}{l} = \omega_0^2$, $\frac{k}{4m_1} = \omega_1^2$, $\frac{k}{4m_2} = \omega_2^2$

$$\to \ddot{\varphi}_1 + (\omega_0^2+\omega_1^2)\varphi_1 - \omega_1^2\varphi_2 = 0$$

$$\ddot{\varphi}_2 + (\omega_0^2+\omega_2^2)\varphi_2 - \omega_2^2\varphi_1 = -C\dot{\varphi}_2$$

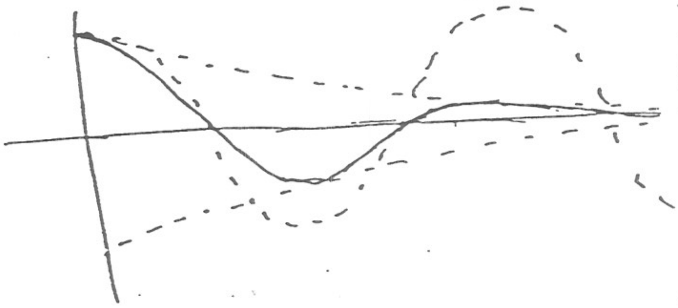
Sistem rešujem z nastavkom:

$$\varphi_1 = \varphi_0 \cdot e^{i\omega' t + \phi_1} \cdot e^{-\lambda t}$$

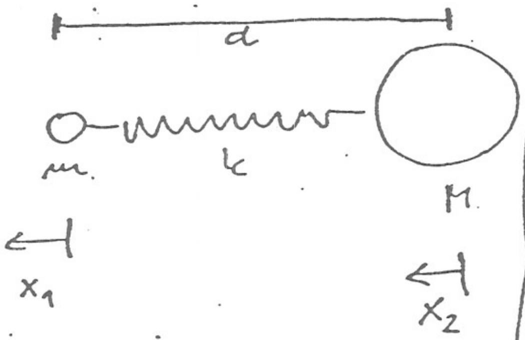
$$\varphi_2 = \varphi_0 \cdot e^{i\omega' t + \phi_2} \cdot e^{-\lambda t}$$

$\vdots$

(Analiza II)



5) Molekula m.



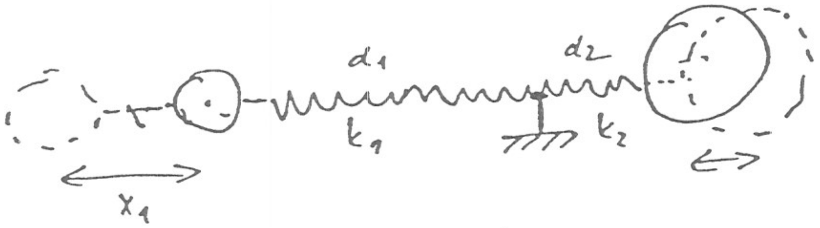
(če se molekula slučajno vrti, lahko pospravim centrifugalno silo v potencial oz. se sistem obnaša enako kot da se ne vrti in ima šibkejšo vzmet, saj je centrifug. potencial ravno tako kvadratičen kot pot. vzmeti:

$$V_{11} = \frac{k x_1'^2}{2}, \quad V_{12} = -m\omega^2 \frac{x_1'^2}{2}$$

$$\Rightarrow V_1 = \frac{1}{2} x_1'^2 \underbrace{(k - m\omega^2)}_{k'} \rightarrow \text{vrtenje})$$

Postavim se v sistem, ker težišče molekule miruje in se molekula ne vrti. (Če se →)

…em je očitno, da bo vsak delec vzmeti bližje M pri miru; uteži bosta nihali vsaka zase, vendar z isto fazo: (in frekvenco)



Iz ohranitve težišča: $d_1 m_1 = d_2 m_2$, $d_1 + d_2 = d$

$$\to d_1 = d\frac{M}{m+M}, \quad d_2 = d\frac{m}{M+m}$$

začetna vzmet: k_0

Velja $k = \frac{\alpha}{\ell}$ (dvakrat daljša vzmet ima dvakrat manjši k)

$\alpha = k_0 d$

$$\Rightarrow k_1 = \frac{k_0 d}{d_1} = k_0 \cdot \frac{m+M}{M} \Rightarrow \underline{\omega_1^2 = k_1/m_1 = k_0 \cdot \frac{m+M}{mM}}$$

$$\frac{1}{\mu} = \frac{1}{M} + \frac{1}{m}$$

$$k_2 = \quad = k_0 \cdot \frac{m+M}{m} \Rightarrow \underline{\omega_2^2 = k_2/m_2 = k_0 \cdot \frac{m+M}{mM}}$$

$\omega_1 = \omega_2$ ✓ Molekula torej niha s frekvenco $\omega_0 = k_0/\mu$

Razmerje amplitud: $\underline{\frac{x_1}{x_2} = \frac{d_1}{d_2} = \frac{M}{m}}$ $\quad \omega^2 = \frac{k_0}{\mu} = \frac{k_0}{M} + \frac{k_0}{m} = \underline{\omega_1^2 + \omega_2^2 = \omega^2}$

Še z Lagrangeom:

$$V = \frac{k}{2}(x_1 - x_2) \Rightarrow V = \frac{1}{2} x^T \underline{\underline{V}} x \Rightarrow \underline{\underline{V}} = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$T = \frac{m}{2}\dot{x}_1^2 + \frac{M}{2}\dot{x}_2^2 \Rightarrow T = \frac{1}{2}\dot{x}^T \underline{\underline{T}} \dot{x} \Rightarrow \underline{\underline{T}} = \begin{bmatrix} m & \\ & M \end{bmatrix}$$

Iščem $\det\left[-\omega^2 T + V\right] = 0$

označim: $\omega_1'^2 = \frac{k}{m}$

$\omega_2'^2 = \frac{k}{M}$

$$\Rightarrow \underline{\underline{T}}' = \underline{\underline{T}} \cdot \frac{1}{k} = \begin{bmatrix} \frac{1}{\omega_1'^2} & 0 \\ 0 & \frac{1}{\omega_2'^2} \end{bmatrix}$$

$$\underline{\underline{V}}' = \underline{\underline{V}} \cdot \frac{1}{k} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

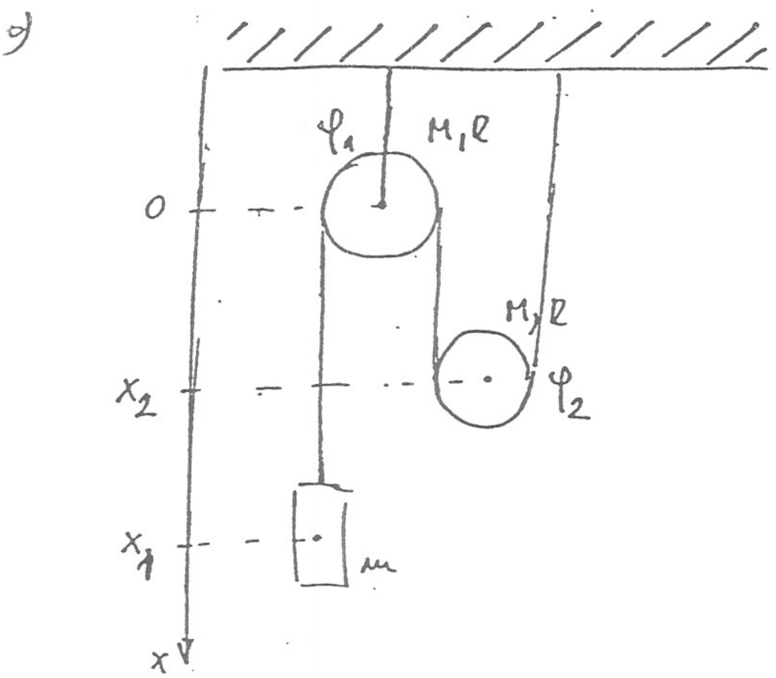
$$\det\left[-\Omega^2 \underline{\underline{T}}' + \underline{\underline{V}}'\right] = 0 \Rightarrow \det \begin{bmatrix} 1 - \frac{\Omega^2}{\omega_1'^2} & -1 \\ -1 & 1 - \frac{\Omega^2}{\omega_2'^2} \end{bmatrix} = 0$$

$$\Rightarrow -\frac{\Omega^2}{\omega_2'^2} - \frac{\Omega^2}{\omega_1'^2} + \frac{\Omega^4}{\omega_1'^2 \omega_2'^2} = 0$$

$$\Omega^2\left(-\frac{1}{\omega_2'^2} - \frac{1}{\omega_1'^2} + \frac{\Omega^2}{\omega_1'^2 \omega_2'^2}\right) = 0$$

$\Omega_0^2 = 0 \rightarrow$ trivialna rešitev

$$\underline{\Omega_1^2 = \omega_1'^2 + \omega_2'^2} = \frac{k_0}{m} + \frac{k_0}{M} = k_0/\mu \leftarrow \text{dobim isto}$$



očitno: $\dot{x}_2 = -\frac{\dot{x}_1}{2}$

$\varphi_1 = -\frac{\varphi_2}{2}$ $\left(\begin{array}{l} \varphi_1(t=0)=0 \\ \varphi_2(t=0)=0 \end{array} \right.$

$\dot{\varphi}_1 = -\frac{\dot{\varphi}_2}{2}$

$x_1 + 2x_2 = const = l$

$\frac{l}{3} = x_{10} = x_{20}$

začetna višina uteži in škripca

$= \frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} M \dot{x}_2^2 + \frac{1}{2} J \dot{\varphi}_1^2 + \frac{1}{2} J \dot{\varphi}_2^2$

$= -mgx_1 - Mgx_2$

$T = \frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} M \frac{\dot{x}_1^2}{4} + \frac{1}{2} J \frac{\dot{x}_1^2}{l^2} + \frac{1}{2} J \frac{\dot{x}_1^2}{4l^2}$ $\Big\}$ Upoštevam $J = \frac{Ml^2}{2}$

$V = -mgx_1 + \frac{M}{\cancel{2}} g \left(\frac{l - x_1}{2} \right) = x_1 \cdot g \cdot \left(\frac{M}{2} - m \right) - \frac{1}{2} Mgl$

$T = \frac{1}{2} \dot{x}_1^2 \left(m + \frac{M}{4} + \frac{M}{2} + \frac{M}{8} \right) = \frac{1}{2} \dot{x}_1^2 \underbrace{\left(m + \frac{7M}{8} \right)}_{M'}$

$= T - V = \frac{1}{2} M' \dot{x}_1^2 + x_1 g \left(m - \frac{M}{2} \right) + \frac{1}{2} Mgl$

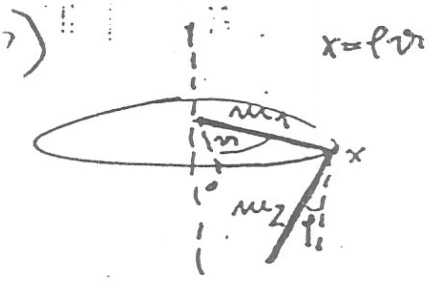
$\Rightarrow M' \ddot{x}_1 + g \left(\frac{M}{2} - m \right) = 0$

novesje: $\ddot{x}_1 = 0 \Rightarrow \frac{M}{2} = m$ $\rightarrow$ ravnovesje: ni stabilno
ni labilno
je marginalno stabilno

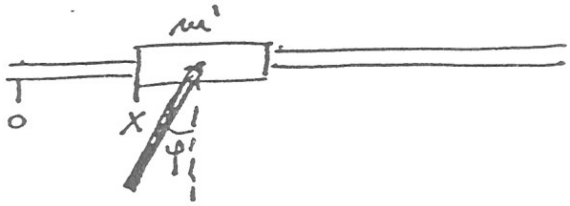
vnovesje: $\ddot{x}_1 = a = g \cdot \left(\frac{\frac{M}{2} - m}{M'} \right)$ $\rightarrow$ gibanje uteži je ~~enakomerno~~ pospešeno,

$\boxed{x_1(t) = \frac{1}{2} a t^2}$ $\left(+ \underbrace{\cancel{bt}}_{0} + c \right)$

)



toke, da ga obravnavam linearno!



Po vodilu (vodoravnem) brez trenja drsi masa m', z nje visi palica m_2. Sistem je popolnoma analogen prvemu. (ker se viseča palica ne more odklanjati „ven", v smeri prve palice)

:

$$= \frac{1}{2} J_1 \dot\vartheta^2 = \frac{1}{2}\frac{m_1 l^2}{3}\dot\vartheta^2$$

$$\vartheta = \frac{x}{l}$$

$$\Rightarrow T_1 = \frac{1}{2}\frac{m_1}{3}\dot x^2$$

$$= \frac{1}{2} m' \dot x^2 \Rightarrow m' = \frac{m_1}{3}$$

koordinate:

m': x, y $\quad (y(t) \equiv 0 \ \forall t)$

m_2 (palica): x_2, y_2 — težišče

$$x_2 = x_1 + \frac{l}{2}\sin\varphi, \quad y_2 = \frac{l}{2}\cos\varphi \qquad \left(\frac{l}{2} = d\right) \quad (x = x_1)$$

$$\dot x_2 = \dot x + \dot\varphi d\cos\varphi, \quad \dot y_2 = -d\dot\varphi\sin\varphi$$

okrog težišča

$$= \frac{1}{2} m_2 \dot x_2^2 + \frac{1}{2} m_2 \dot y_2^2 + \frac{1}{2} J_{T2}\dot\varphi^2 =$$

$$= \frac{1}{2} m_2(\dot x_1^2 + \dot\varphi^2 d^2\cos^2\varphi + 2\dot x_1\dot\varphi d\cos\varphi) + \frac{1}{2} m_2 d^2\dot\varphi^2\sin^2\varphi + \frac{1}{2}\frac{m l^2}{12}\dot\varphi^2$$

$$= \frac{1}{2} m_2 \dot x_1^2 + \frac{1}{2} m_2 d^2\dot\varphi^2 + \frac{1}{2}\frac{m_2 \cancel{4} d^2}{\cancel{12}\,3}\dot\varphi^2 + m_2 \dot x_1 \dot\varphi_1 d\cos\varphi$$

$$= \frac{1}{2} m_2 \dot x_1 + \frac{1}{2}\underbrace{\left(\frac{m_2 d^2}{3} + m_2 d^2\right)}_{\frac{4 m_2 d^2}{3} = \frac{m_2 l^2}{3} = J_{p2}}\dot\varphi^2 + m_2 \dot x_1 \dot\varphi_1 d\cos\varphi$$

okrog osi oz. pritrdišča

$$= \frac{1}{2} m_2 \dot x^2 + \frac{1}{2} J_{p2}\dot\varphi^2 + m\dot x\dot\varphi d\cos\varphi$$

Če pa se vrti zgornja palica s povprečno hitrostjo ω, upoštevam centrifugalni dodatek k $\vec g$ pri spodnji palici in ostalo ostane nespremenjeno

$$= -mg y_2 = -mgd\cos\varphi = -V_0\cos\varphi$$

$$L = T - V = \frac{1}{2}(m' + m_2)\dot x^2 + \frac{1}{2} J_{p2}\dot\varphi^2 + \frac{V_0}{g}\dot x\dot\varphi\cos\varphi + V_0\cos\varphi$$

ciklična: $p_x = (m' + m_2)\dot x + \frac{V_0}{g}\dot\varphi\cos\varphi$

$\dot\varphi = 0 \Rightarrow p_x$ = gibalna količina (oz. vrtilna v prvotnem problemu)

$\dot\varphi \neq 0$ in p_x - gibalna količina v x-smeri

Euler - Lagrange:

$$\frac{\partial}{\partial t}\left[J_{Pz}\dot\varphi + \frac{V_0}{g}\dot x\cos\varphi\right] - \left[-\frac{V_0}{g}\dot x\dot\varphi\sin\varphi - V_0\sin\varphi\right] = 0$$

$$J_{Pz}\ddot\varphi + \frac{V_0}{g}\ddot x\cos\varphi - \cancel{\frac{V_0}{g}\dot x\sin\varphi\,\dot\varphi} + \cancel{\frac{V_0}{g}\dot x\dot\varphi\sin\varphi} + V_0\sin\varphi = 0$$

$$J_{Pz}\ddot\varphi + \frac{V_0}{g}\ddot x\cos\varphi + V_0\sin\varphi = 0$$

Ravnovesna lega: $\ddot x = 0$, $\ddot\varphi = 0$

$\Rightarrow \sin\varphi = 0$, $\varphi = 0 \Rightarrow \dot\varphi = 0$

$\Rightarrow p_x = (m_1 + m_2)\dot x$

m' se enakomerno giblje $x = x_0 + \dot x_0 t$, palica po visi navpično dol, oz. v prvem primeru: zgornja palica enakomerno drsi $v = v_0 + \dot v_0 t$, spodnja navpično visi.

Mala nihanja: $\varphi \ll 1$, $\sin\varphi \doteq \varphi$, $\cos\varphi \doteq 1$ $\quad \dot x = \dot x_0 + v$

$\Rightarrow J_{Pz}\ddot\varphi + \frac{V_0}{g}\ddot x + V_0\varphi = 0$

$\rightarrow J_{Pz}\ddot\varphi + \frac{V_0}{g}\dot v + V_0\varphi = 0$ (Upoštevam J_{Pz}, V_0)

$\rightarrow \frac{\cancel{m_2}l^2}{3}\ddot\varphi + \frac{\cancel{m_2 g}l}{2\cancel{g}}\dot v + \cancel{m_2}g\frac{l}{2}\varphi = 0$

$l\frac{\ddot\varphi}{3} + \frac{\dot v}{2} + \frac{g}{2}\varphi = 0$

gib. kol. v x smeri

Postavim se v tak inercialni sistem, da $p_x = 0$. Potem:

$(m' + m_2)v + \frac{m_2 l}{2}\dot\varphi = 0 \rightarrow \dot v = -\frac{m_2 l}{2(m' + m_2)}\ddot\varphi$

$\Rightarrow \frac{l\ddot\varphi}{3} - l\frac{m_2}{4(m' + m_2)}\ddot\varphi + \frac{g}{2l}\varphi = 0$

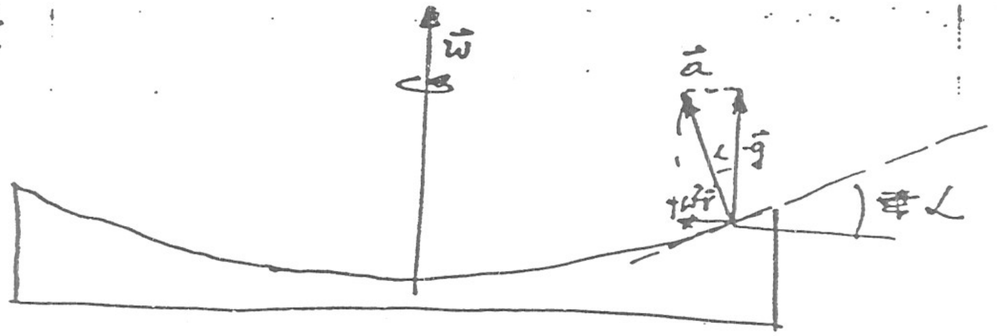
$\Rightarrow \ddot\varphi + \underbrace{\frac{6(m' + m_2)}{4m' + m_2}\cdot\frac{g}{2l}}_{\omega^2}\varphi = 0$

$$\Rightarrow \boxed{\omega^2 = \frac{6(m' + m_2)}{4m' + m_2}\cdot\frac{g}{l}}$$

1. Za $m' \gg m_2 \rightarrow$ prosto nihanje trdno vpete palice: $\omega^2 = \frac{3g}{2l}$ ✓ 

2. Za $m_2 \gg m' \rightarrow$ nihanje "drseče" vpete palice: $\omega^2 = \frac{6g}{\ldots}$ 

:8) vrsalusce



Gladina je pravokotna na vektor $\vec{a}$

$\vec{a} = a_0(-\omega^2 r, +g)$

$z' = \operatorname{tg}\alpha = \frac{r\omega^2}{g}$

$\left(z' = \frac{\partial z}{\partial r}\right)$

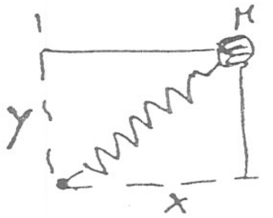
...em majhen ω,
...o da je posoda nizka
...loliko kinetično energijo
z-smeri zanemarimo.
...me:

$= \frac{m}{2}(\dot{x}^2 + \dot{y}^2)$

$V = mgz = \frac{m r^2 \omega^2}{2} = \frac{k(x^2+y^2)}{2}$ $\quad(\vec{r} = (x,y)$, $k = m\omega^2)$

$= T - V = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) - \frac{k}{2}(x^2+y^2)$

...je problem telesa na vzmeti v ravnini



$\cdot L:$

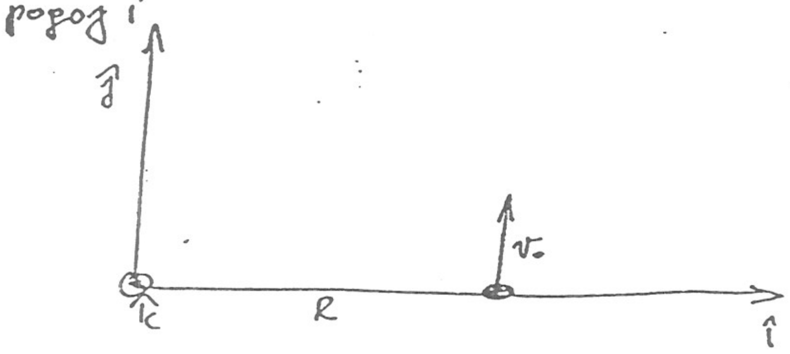
$\ddot{x} + \frac{k}{m}x = 0 \Rightarrow x = A\cos\omega' t + B\sin\omega' t$ $\qquad \omega'^2 = \frac{k}{m}$

$\ddot{y} + \frac{k}{m}y = 0 \Rightarrow y = C\cos\omega' t + D\sin\omega' t$

A, B, C, D $\to$ določimo iz začetnih pogojev

...selec se torej giblje po elipsi.

začetni pogoji



$X(0) = R \qquad \dot{x}(0) = 0$

$Y(0) = 0 \qquad \dot{y}(0) = v_0$

$$\Rightarrow \quad \underline{X = R\cos\omega' t}$$

$$y = C\cos\omega' t + D\sin\omega' t$$

$$\dot{y} = -C\omega'\sin\omega' t + D\omega'\cos\omega' t$$

$$\dot{y}(0) = D\omega' = v_0 \Rightarrow D = \frac{v_0}{\omega'}$$

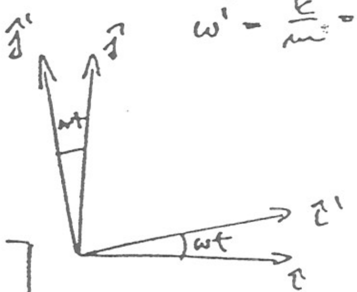
$$\Rightarrow \underline{Y = \frac{v_0}{\omega'}\sin\omega' t}$$

$$\vec{r}(t) = \begin{pmatrix} R\cos\omega' t \\ \frac{v_0}{\omega'}\sin\omega' t \end{pmatrix}$$

$$\Rightarrow \dot{\vec{r}}(t) = \begin{pmatrix} -R\omega\cos\omega t \\ v_0\cos\omega t \end{pmatrix}$$

$$\omega'^2 = \frac{k}{m} = \frac{\cancel{m}g\omega^2}{\cancel{m}g} \Rightarrow \underline{\omega'^2 = \omega^2}$$

V vrtečem sistemu:



$\vec{r}, \vec{r}'$

$$\vec{r}' = \begin{pmatrix} R_x\cos\omega t + R_y\sin\omega t \\ -R_x\sin\omega t + R_y\cos\omega t \end{pmatrix}$$

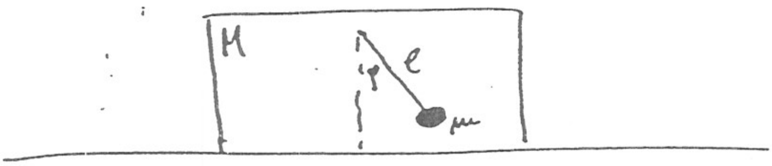
$$\vec{r}' = \begin{bmatrix} R\cos^2\omega t + \frac{v_0}{\omega}\sin^2\omega t \\ -R\cos\omega t\sin\omega t + \frac{v}{\omega}\sin\omega t\cos\omega t \end{bmatrix}$$

$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) \qquad z = \frac{\omega^2}{2g}(x^2 + y^2)$

$V = mgz = \frac{m\omega^2}{2}(x^2 + y^2)$

$L = T - V$

3) Utež na kladi:



Klada: x_1, y_1 ; $y_1(t) \equiv 0 \ \forall t$

Utež: x_2, y_2 ; $x_2 = x_1 + l\sin\varphi$, $y_2 = l\cos\varphi$

$$\dot{x}_2 = \dot{x}_1 + \dot{\varphi} l\cos\varphi \,,\quad \dot{y}_2 = -\dot{\varphi}\, l\sin\varphi$$

Klada: T

Utež: T, V

$$= \frac{1}{2}M\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2 + \frac{1}{2}m\dot{y}_2^2$$

$$= \frac{1}{2}M\dot{x}_1^2 + \frac{1}{2}m\dot{x}_1^2 + \underline{\frac{1}{2}m\dot{\varphi}^2 l^2\cos^2\varphi} + \frac{1}{2}m\cdot 2\dot{x}_1\dot{\varphi} l\cos\varphi + \underline{\frac{1}{2}m\dot{\varphi}^2 l^2\sin^2\varphi}$$

$$= \underline{\frac{1}{2}M\dot{x}_1^2 + \frac{1}{2}m\dot{x}_1^2 + m\dot{x}_1\dot{\varphi} l\cos\varphi + \frac{1}{2}m\dot{\varphi}^2 l^2 = T}$$

$$\underline{V = -mgl\cos\varphi}$$

$$T - V = \frac{1}{2}(M+m)\dot{x}_1^2 + m\dot{x}_1\dot{\varphi} l\cos\varphi + \frac{1}{2}m\dot{\varphi}^2 l^2 + mgl\cos\varphi$$

~~[illegible]~~ Vidimo, da je x_1 ciklična. $(x = x_1)$

$\underline{p_x = (M+m)\dot{x} + m\dot{\varphi} l\cos\varphi}$ – (gibalna količina v x smeri)

Euler-Lagrange: $\frac{\partial}{\partial t}\frac{\partial}{\partial\dot{\varphi}}L - \frac{\partial}{\partial\varphi}L = 0$

$$(m\underline{\dot{x}} l\underline{\cos\varphi} + m l^2\dot{\varphi}) - (-m\dot{x}_1\dot{\varphi} l\sin\varphi - mgl\sin\varphi) = 0$$

$$\ddot{x} l\cos\varphi - \cancel{m\dot{x} l\sin\varphi\dot{\varphi}} + \cancel{m} l^2\ddot{\varphi} + \cancel{m\dot{x}\dot{\varphi} l\sin\varphi} + \cancel{m}g\cancel{l}\sin\varphi = 0$$

$$\underline{\underline{\ddot{x}\cos\varphi + \ddot{\varphi} l + g\sin\varphi = 0}}$$

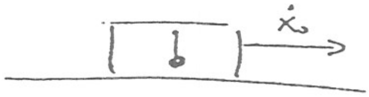
$\ddot{x}\cos\varphi + \ddot{\varphi} l + g\sin\varphi = 0$

1.) Ravnovesna lega:

$\ddot{x} = 0, \ \ddot{\varphi} = 0, \quad \to \sin\varphi = 0, \ \varphi = 0$

$\to p_x = (M+m)\cdot \dot{x}_0$ - gibalna količina

$\Rightarrow x = x_0 + \dot{x}_0 t$



začetna pogoja

Klada se giblje enakomerno z začet. hitrostjo in z navpično visečo utežj

2.) Mala nihanja

$\varphi \ll 1 \Rightarrow \sin\varphi \doteq \varphi$

$\cos\varphi \doteq 1$

$\dot{x} = \dot{x}_0 + v$

$\Rightarrow \dot{v} + l\ddot{\varphi} + g\varphi = 0$

Postavim se v tak sistem, da je težišče sistema pri miru (px ozi toref $p_x = 0$ oz.: $(M+m)\dot{x} + ml\dot{\varphi} = p_x = 0$)

$(M+m)v + ml\dot{\varphi} = 0 \Rightarrow \dot{v} = -\frac{ml}{M+m}\ddot{\varphi}$

$\Rightarrow \left(-\frac{ml}{M+m} + l\right)\ddot{\varphi} + g\varphi = 0, \quad \to \omega_0^2 = \frac{g}{l - \frac{ml}{M+m}} = \frac{g}{l}\cdot\frac{m+M}{M}$

$\varphi = \varphi_0 \cos\omega_0 t$

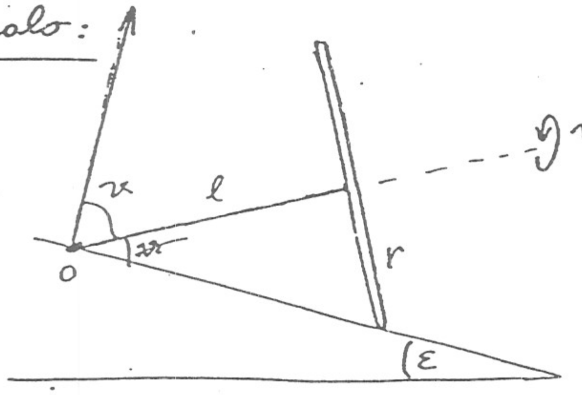
$\dot{v} = a = \omega_0^2 \frac{ml}{M+m}\varphi_0\cos\omega_0 t, \quad v = \omega_0 \frac{ml}{M+m}\varphi_0 \sin\omega_0 t \ (+\dot{x}_0)$

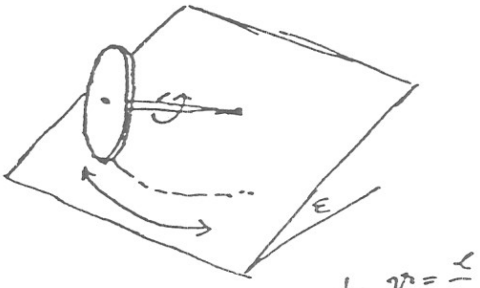
$x = -\frac{ml}{M+m}\varphi_0\cos\omega_0 t \ (+\dot{x}_0 t + x_0)$

začetni pogoji

$x = -\frac{ml}{\ }\varphi$

10) Nikolo:





$\mathrm{tg}\,\vartheta = \frac{l}{r}$

$$T = \frac{1}{2}\left(J_x' \omega_x'^2 + J_y' \omega_y'^2 + J_z' \omega_z'^2\right) = \frac{1}{2}\left(J \dot\phi \sin^2\vartheta + J'(\dot\psi + \dot\phi\cos\vartheta)^2\right)$$

ker ne zdrsuje

$$\text{Vez}: \quad \frac{l}{\sin\vartheta}\dot\phi = -r\dot\psi \quad \Rightarrow \quad \dot\psi = -\dot\phi \cdot \frac{1}{\cos\vartheta}$$

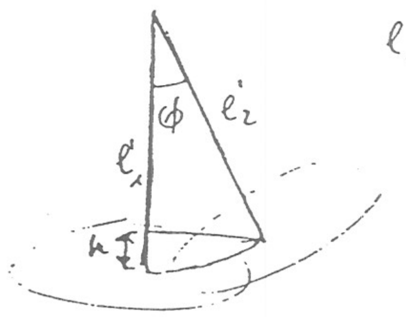
$$\Rightarrow \quad T = \frac{\dot\phi^2}{2}\left(J\sin^2\vartheta + J'\frac{\sin^4\vartheta}{\cos^2\vartheta}\right) = \frac{\dot\phi^2}{2}\sin^2\vartheta\left(J + J'\,\mathrm{tg}^2\vartheta\right)$$

Še potencial:

~~$V = mgl$~~

Za majhna nihanja okrog ravnovesne lege, in za relativno majhen kot ε smem zapisati:

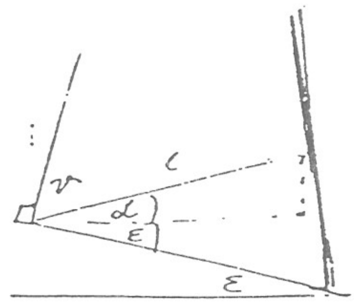
Tloris



$$l_1' = l_2' = l'$$

Potem:

$$h = l'(1 - \cos\phi)$$



$l' = l\cos\alpha$

$\alpha = \frac{\pi}{2} - \varepsilon - \vartheta$

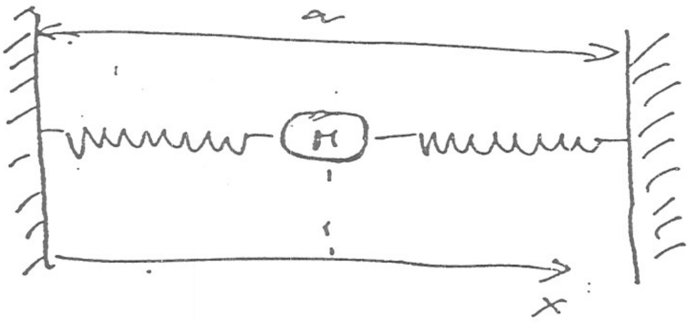
$$V = mgl'(1 - \cos\phi)\sin\varepsilon = c_1 - \overbrace{mgl'\sin\varepsilon}^{C}\cos\phi$$

$$L = T - V = \underbrace{\frac{\dot\phi^2}{2}\sin^2\vartheta\left(J + J'\,\mathrm{tg}^2\vartheta\right)}_{D} + C\cos\phi + c_1$$

Euler - Lagrange: $D\ddot\phi + C\phi = 0 \;\longrightarrow\; \ddot\phi + \frac{C}{D}\phi = 0$

$$\Rightarrow \quad \omega^2 = \frac{C}{D} = \frac{mgl'\sin\varepsilon}{\sin^2\vartheta\left(J + J'\,\mathrm{tg}^2\vartheta\right)}$$

1) Masa na vzmeti



$$V_1 = \frac{k_1 x^2}{2}$$

$$V_2 = \frac{k_2 (a-x)^2}{2}$$

$$T = \frac{m\dot{x}^2}{2}$$

$$L = T - V = \frac{1}{2} m\dot{x}^2 - \frac{k_1 x^2}{2} - \frac{k_2 (a-x)^2}{2}$$

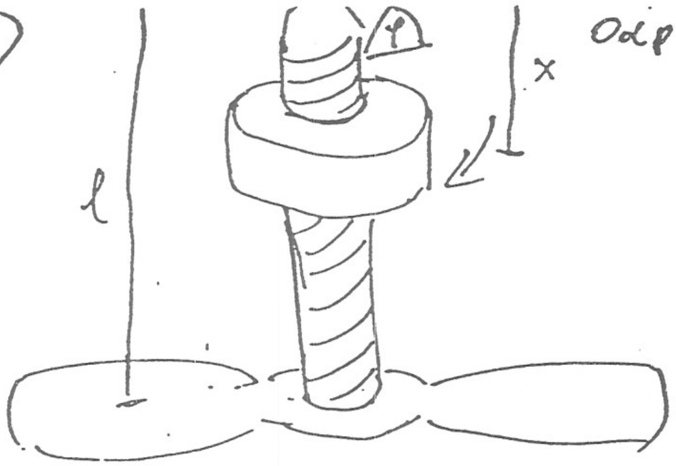
Euler – Lagrange:

$$m\ddot{x} + k_1 x - k_2 (a-x) = 0$$

$$\ddot{x} + \frac{(k_1 + k_2)}{m} x = \frac{k_2 a}{m} \qquad x = x_0 \cdot e^{i\Omega t} + \frac{k_2}{k_1 + k_2} \cdot a$$

$$\Rightarrow \boxed{\Omega^2 = \frac{k_1 + k_2}{m}}$$

2) Odpirač:



Hod $p\left(\frac{cm}{2\pi}\right)$

$$x = p\varphi$$
$$\dot{x} = p\dot{\varphi}$$

$$T = \frac{1}{2} J\dot{\varphi}^2 + \frac{1}{2} m\dot{x}^2 = \frac{1}{2}\frac{J}{p^2}\dot{x}^2 + \frac{1}{2}\dot{x}^2 m = \frac{1}{2}\underbrace{\left(\frac{J}{p^2}+m\right)}_{m'}\dot{x}^2 = \frac{1}{2}m'\dot{x}^2$$

$$V = -mgx$$

$$L = T - V = \frac{1}{2}m'\dot{x}^2 + mgx$$

Euler-Lagrange: $m'\ddot{x} = mg$

$$\Rightarrow \ddot{x} = \frac{m}{m'}g$$

$$x(t) = \frac{m}{m'}g\cdot\frac{1}{2}\cdot t^2$$

čas od vrha do tal: $l = \frac{m}{m'}g\cdot\frac{t_0^2}{2} \Rightarrow t_0 = \sqrt{\frac{2l}{g}\cdot\frac{m'}{m}}$

sunek sile:

$$F\cdot\Delta t = \Delta G = m\Delta v = m v_k$$

$$x(t) = \frac{m}{m'}g\cdot\frac{1}{2}t^2 \Rightarrow \dot{x}(t) = \frac{m}{m'}g\cdot t$$

$$v_k = \dot{x}(t_0) = \frac{m}{m'}g\sqrt{\frac{2l}{g}\cdot\frac{m'}{m}} = \sqrt{2lg\cdot\frac{m}{m'}}$$

sunek sile: $\Delta G = m v_k = m\sqrt{2lg\frac{m}{m'}}$

Sunek navora:

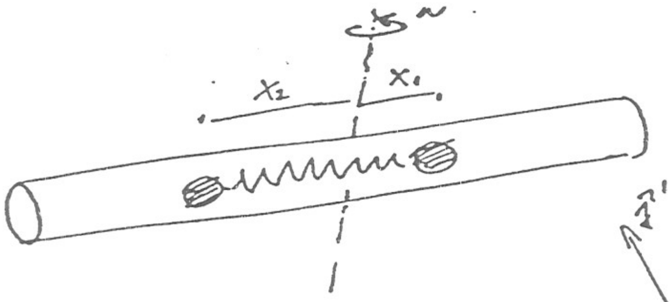
$$M \Delta t = \Delta \Gamma = J \Delta \omega = J \omega_k$$

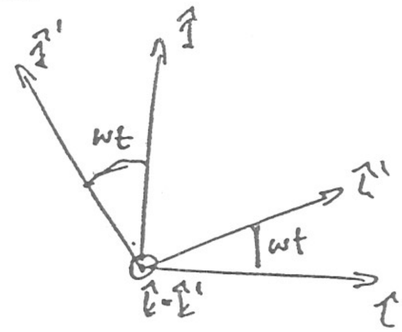
$$v_k = p \cdot \omega_k$$

$\Rightarrow$

Sunek navora: $$\underline{\underline{M \Delta t = \Delta \Gamma = J \cdot \frac{v_k}{p} = \frac{J}{p} \cdot \sqrt{2 l g \frac{m}{m'}}}}$$

43)

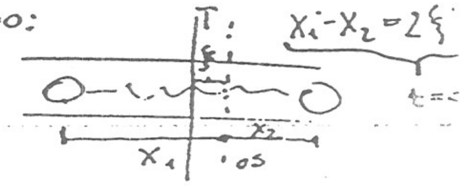




$$\vec{r}_1 = x_1\hat{i}', \quad \dot{\vec{r}}_1 = \dot{x}_1\hat{i}' + \omega x_1\hat{j}'$$
$$\vec{r}_2 = -x_2\hat{i}', \quad \dot{\vec{r}}_2 = -\dot{x}_2\hat{i}' - \omega x_2\hat{j}'$$

$$T = \frac{1}{2}m\left(\dot{x}_1^2 + \omega^2 x_1^2 + \dot{x}_2^2 + \omega^2 x_2^2\right)$$
$$V = \frac{1}{2}k\,(x_1 + x_2)^2$$

$$\hat{i}' = \hat{i}\cos\omega t + \hat{j}\sin\omega t$$
$$\hat{j}' = -\hat{i}\sin\omega t + \hat{j}\cos\omega t$$



$$L = T - V = \frac{1}{2}m\left(\dot{x}_1^2 + \omega^2 x_1^2 + \dot{x}_2^2 + \omega^2 x_2^2\right) - \frac{k}{2}\left(x_1^2 + x_2^2 + 2x_1x_2\right)$$

E.L.: $\Rightarrow$
$$\left.\begin{aligned} m\ddot{x}_1 &= m\omega^2 x_1 - kx_1 - kx_2 \\ m\ddot{x}_2 &= m\omega^2 x_2 - kx_2 - kx_1 \end{aligned}\right\} +,-$$

označimo $x_1 + x_2 = u$, $x_1 - x_2 = v$

$$\Rightarrow m\ddot{u} - m\omega^2 u + 2ku = 0 \quad \Rightarrow \quad \ddot{u} + u\left(\frac{2k}{m} - \omega^2\right) = 0 \quad (1)$$
$$m\ddot{v} - m\omega^2 v = 0 \quad (2)$$

(1): če $\frac{2k}{m} > \omega^2$: označim $\omega_1^2 = \left(\frac{2k}{m} - \omega^2\right)$

$\ddot{u} + u(\omega_1^2) = 0$

$\Rightarrow u = C\cos\omega_1 t + D\sin\omega_1 t$

$u(0) = C = 2x_0$

$x_1(t=0) = x_0 + \xi$
$x_2(t=0) = x_0 - \xi$
$u_0 = 2x_0$

$\dot{u}(0) = -\cancel{C\omega_1\sin\omega_1 t} + D\omega_1\cos\omega_1 t = 0$

$\Rightarrow D = 0$ (če $\omega_1^2 \neq 0$)

$\Rightarrow \underline{u = 2x_0\cos\omega_1 t}$ (ob predpostavki, da je nerastegnjena vzmet dolga 0 in da sta masni točki in se v središču vedno „zgrešita") ce: $\frac{2k}{m} < \omega^2$

$$\Rightarrow \omega_1^2 = \omega^2 - \frac{2k}{m}$$

$$u = Ae^{\omega_1 t} + Be^{-\omega_1 t}$$

$$u(0) = A + B = 2x_0$$

$$\dot{u}(0) = \omega(A-B) = 0 \quad \rightarrow \quad A = B = x_0$$

$$\Rightarrow \underline{u = x_0(e^{\omega_1 t} + e^{-\omega_1 t})}$$

(2) $\ddot{v} - \omega^2 v = 0$

$$\rightarrow v = Ee^{\omega t} + Fe^{-\omega t}$$

$$v(0) = x_1(0) - x_2(0) = 2\xi$$

$$\Rightarrow E + F = 2\xi$$

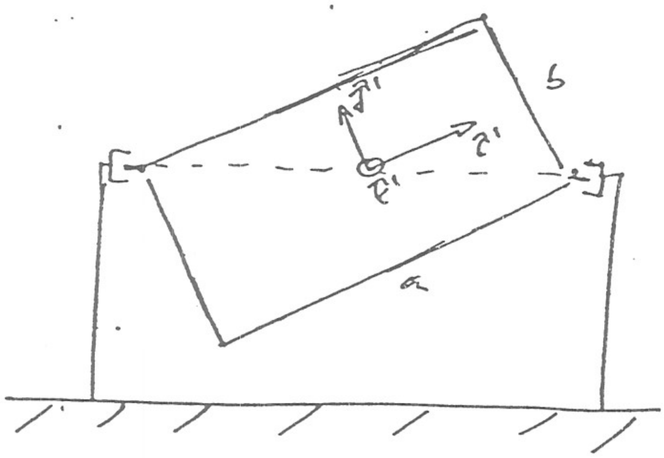
$$\dot{v}(0) = E\omega e^{\omega t} - F\omega e^{-\omega t}$$

$$\Rightarrow E = F = \xi$$

$$\rightarrow \underline{v = \xi(e^{\omega t} + e^{-\omega t})}$$

$$\Rightarrow \boxed{\begin{aligned} x_1 &= \frac{u+v}{2} \\ x_2 &= \frac{u-v}{2} \end{aligned}}$$

44



$$\vec{M} = \frac{\partial \vec{L}}{\partial t}$$

$$\vec{L} = J_x' \omega_x \hat{i}' + J_y' \omega_y \hat{j}' + J_z \omega_z \hat{k}'$$

$$\dot{\vec{L}} = J_x' (\dot{\omega}_x \hat{i}' + \omega_x \dot{\hat{i}}') + J_y (\dot{\omega}_y \hat{j}' + \omega_y \dot{\hat{j}}') + J_z (\dot{\omega}_z \hat{k}' + \omega_z \dot{\hat{k}}')$$

$$\dot{\vec{r}} = \omega \times \vec{r} \quad \Rightarrow \quad \dot{\hat{i}}' = \vec{\omega} \times \hat{i}' = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\dot{\hat{j}}' = \vec{\omega} \times \hat{j}'$$

$$\dot{\hat{k}}' = \vec{\omega} \times \hat{k}'$$

$$\begin{bmatrix} \dot{\hat{i}}' \\ \dot{\hat{j}}' \\ \dot{\hat{k}}' \end{bmatrix} = \begin{bmatrix} \vec{\omega} \times \hat{i}' \\ \vec{\omega} \times \hat{j}' \\ \vec{\omega} \times \hat{k}' \end{bmatrix} = \begin{bmatrix} (\omega_x \hat{i}' + \omega_y \hat{j}' + \omega_z \hat{k}') \times \hat{i}' \\ \cdots \\ \cdots \end{bmatrix} = \begin{bmatrix} \omega_z' \hat{j}' - \omega_y' \hat{k}' \\ \omega_x' \hat{k}' - \omega_z' \hat{i}' \\ \omega_y' \hat{i}' - \omega_x' \hat{j}' \end{bmatrix}$$

$$\Rightarrow \dot{\vec{L}} = \underline{J_x' \dot{\omega}_x' \hat{i}'} + J_x' \omega_x' \omega_z' \hat{j}' - \underline{\underline{J_x' \omega_x' \omega_y' \hat{k}'}}$$
$$+ J_y' \dot{\omega}_y' \hat{j}' + \underline{\underline{J_y' \omega_y' \omega_x' \hat{k}'}} - \underline{J_y' \omega_y' \omega_z' \hat{i}'}$$
$$+ \underline{\underline{J_z' \dot{\omega}_z' \hat{k}'}} + \underline{J_z' \omega_z' \omega_y' \hat{i}'} - J_z' \omega_z' \omega_x' \hat{j}'$$

$M_x' = \Sigma$ ____

$M_y' = \Sigma$ ____

$M_z' = \Sigma$ ____

Mi imamo $\omega_x' = \omega \cdot \frac{a}{\sqrt{a^2+b^2}}$ $\quad \dot{\omega}_x' = 0$

$\omega_y' = -\omega \cdot \frac{b}{\sqrt{a^2+b^2}}$ $\quad \dot{\omega}_y' = 0$

$\omega_z' = 0$ $\quad \dot{\omega}_z' = 0$

Torej ostane le M_z':

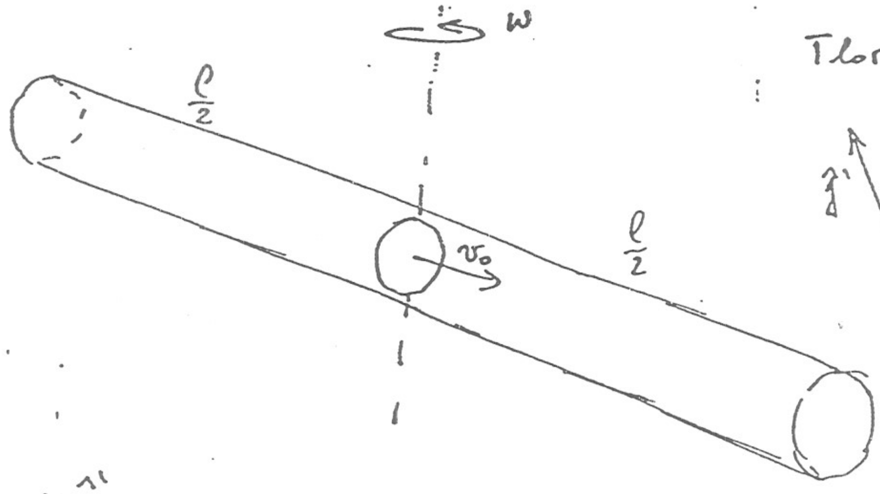
$$M_z' = -J_x' \omega_y' \omega_x' + J_y' \omega_y' \omega_x'$$

$$\underline{M_z' = \frac{\omega^2 ab}{a^2+b^2}\left(J_x' - J_y'\right)}, \quad \underline{M_x' = 0}, \quad \underline{M_y' = 0}$$

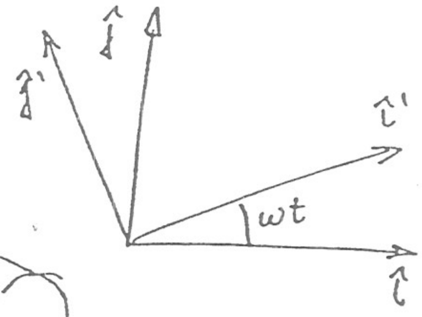
$$J_x' = \frac{mb^2}{12}, \quad J_y' = \frac{ma^2}{12}$$

$$\Rightarrow \underline{M_z' = \frac{\omega^2 ab}{a^2+b^2} \cdot \frac{m}{12} \cdot (b^2 - a^2)}$$

b)



Tloris:



$= r\hat\imath'$

$= \dot r\hat\imath' + r\omega\hat\jmath'$

$= \frac{m}{2}(\dot r^2 + \omega^2 r^2)$, $V=0$

$= T$

$\hat\imath' = \hat\imath\cos\omega t + \hat\jmath\sin\omega t$

$\hat\jmath' = -\hat\imath\sin\omega t + \hat\jmath\cos\omega t$

$\Rightarrow \dot{\hat\imath}' = \omega\hat\jmath'$

.L.: $\Rightarrow$ $\ddot r = \omega^2 r \,\}\cdot\dot r$

Upoštevam $\ddot r\dot r = \frac{1}{2}\frac{d}{dt}(\dot r^2)$

$\dot r r = \frac{1}{2}\frac{d}{dt}(r^2)$

$\Rightarrow \dot r\ddot r = \omega^2 r\dot r \Rightarrow \cancel{\frac{1}{2}}\frac{d}{dt}(\dot r^2) = \omega^2\cancel{\frac{1}{2}}\frac{d}{dt}r^2 \;\Big\}\int_0^T dt$

T -> čas ko doseže rob

$\Rightarrow v_k^2 - v_0^2 = \omega^2 l^2 \quad \left(l = \frac{\ell}{2}\right)$

$$\boxed{v_k^2 = v_0^2 + \omega^2\frac{\ell^2}{4}}$$

čas: $\ddot r = \omega^2 r$

Nastavek $r = Ae^{\omega t} + Be^{-\omega t}$, $r(0) = 0$

$\Rightarrow A + B = 0$ $\quad \dot r(0) = v_0$

$\Rightarrow \dot r = \omega A e^{\omega t} - \omega B e^{-\omega t} \Rightarrow \omega(A - B) = v_0$

$\Rightarrow A = \frac{v_0}{2\omega}$, $B = \frac{-v_0}{2\omega}$

$\Rightarrow r(t) = \frac{v_0}{2\omega}\left(e^{\omega t} - e^{-\omega t}\right) = \frac{v_0}{\omega}\cdot\underbrace{\frac{e^{\omega t} - e^{-\omega t}}{2}}_{sh(\omega t)}$

$\Rightarrow r(t) = sh(\omega t)\cdot\frac{v_0}{\omega}$

$t = \frac{1}{\omega}\,\mathrm{Arsh}\,\frac{\omega r}{v_0}$

$$\boxed{T = \frac{1}{\omega}\,\mathrm{Arsh}\,\frac{\omega l}{v_0}}$$

16) Laplace - Runge - Lenz:

$$\vec{R} = \vec{p}\times\vec{l} - mk\frac{\vec{r}}{r} \qquad H = \frac{p^2}{2m} - \frac{k}{r}$$

S pomočjo Poissonovih oklepajev:

$$[\vec{R},H] = \underbrace{[\vec{p}\times\vec{l},\, H]}_{1.} + \underbrace{[mk\frac{\vec{r}}{r},\, H]}_{2.}$$

1.) $$[\vec{p}\times\vec{l}, H]_i = \varepsilon_{ijk}\, \cancel{p_j [l_k, H]}^{\,0} + \varepsilon_{ijk}\, l_k [p_j, H]$$

$$= \varepsilon_{ijk}\, l_k r_j \left(-\frac{k}{r^3}\right) = \varepsilon_{ijk}\varepsilon_{klm} r_l p_m r_j \left(-\frac{k}{r^3}\right)$$

$$= (r_i r_j p_j - r_j r_j p_i)\left(-\frac{k}{r^3}\right)$$

$$= -\frac{k}{r^3} r_i (\vec{r}\cdot\vec{p}) + \frac{k}{r} p_i = (1)$$

2.) $$-mk\left[\frac{r_i}{r}, H\right] = -mk r_i\left[\frac{1}{r}, H\right] - mk\frac{1}{r}[r_i, \overset{p_i/m}{H}] = (*)$$

$$\left[\frac{1}{r}, H\right] = \sum_j \frac{\partial}{\partial r_j}\left(\frac{1}{r}\right)\frac{\partial H}{\partial p_j} = \sum\left(-\frac{r_j}{r^3}\right)\left(\frac{p_j}{m}\right) = -\frac{\vec{r}\cdot\vec{p}}{r^3 m}$$

$$\Rightarrow (*) = -\cancel{m} k r_i\left(-\frac{\vec{r}\vec{p}}{r^3\cancel{m}}\right) - \frac{\cancel{m}k}{r}\frac{p_i}{\cancel{m}}$$

$$= \frac{k r_i(\vec{r}\cdot\vec{p})}{r^3} - \frac{k p_i}{r} = (2)$$

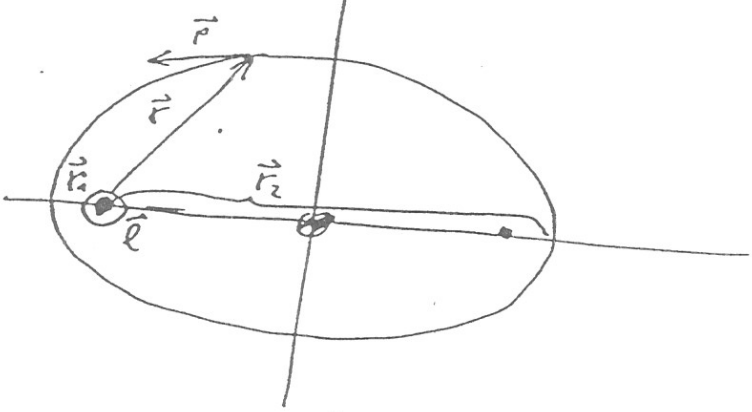
$$\Rightarrow [\vec{R}, H] = (1) + (2) = 0 \Rightarrow \dot{\vec{R}} = 0,\ \vec{R} = \text{const}$$

Geometrijski pomen LRL vektorja

$[\ell_i, \ell_j] = \varepsilon_{ijk} \ell_k$, $[L_i, L_j] = -\left(p^2 - \frac{2mk}{r}\right) \ell_3$

$\Rightarrow$ $\vec{L}$ je pravokoten na $\vec{\ell}$, torej leži v ravnini kroženja. $\vec{L}\cdot\vec{\ell} = 0$

Za orbito vzemimo elipso:



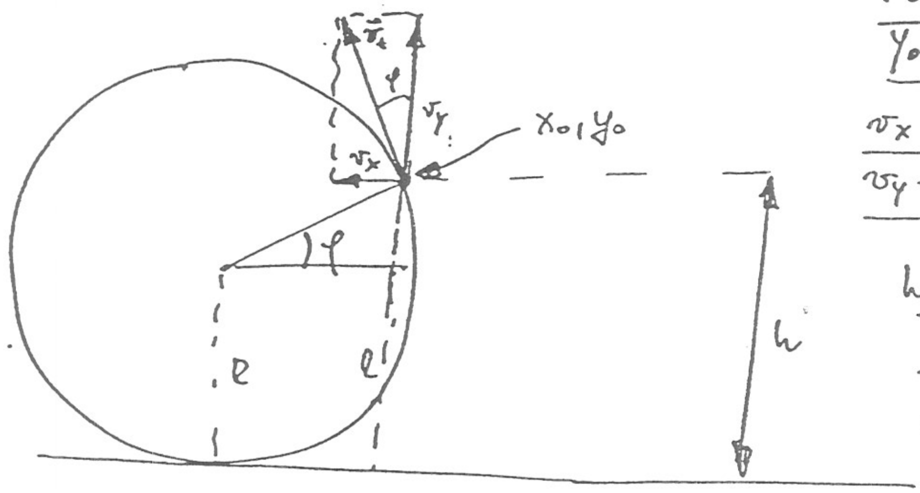
$\vec{L} = \vec{p}\times\vec{\ell} - mk\frac{\vec{r}}{r}$ $\rightarrow$ vzemimo tak $\vec{r}$, da bosta obe prispevka vzporedna – videli bomo, kam kaže $\vec{L}$. To dosežemo če $\vec{r}$ leži v smeri $\underbrace{\text{velike}}$ polosi, zato tedaj kaže tja tudi $\vec{L}$. Ker je $\dot{\vec{L}} = 0$ $\Rightarrow$ $\vec{L} = k\cdot\vec{r}_2$ $\forall t$.

Vektor vrtilne količine določa ravnino gibanja in velikost vrtilne količine, LRL pa smer velike polosi in ekscentričnost: $|\vec{L}| = mk\varepsilon$

$L^2 = m^2k^2 + 2mH\ell^2$

Z vektorjema vrtilne količine $\vec{\Gamma}$ in LRL $\vec{L}$ torej sistem popolnoma popišemo.

47) [illegible]



$x_0 = -l\cos\varphi$

$y_0 = h$

$v_x = v_t \sin\varphi$

$v_y = v_t \cos\varphi$

$\frac{h-l}{l} = \sin\varphi$

$\cos\varphi = \sqrt{1-\sin^2\varphi}$

$y_0 = h$

$v_t = v$ ← hitrost kolesa v zun. sistemu

→ tangencialna hitrost obod v sistemu kolesa

V sistemu kolesarja

$$x = x_0 + v_x t \qquad x = x(t)$$
$$y = y_0 + v_y t - \frac{gt^2}{2} \qquad y = y(t)$$

} trajektorija

V sistemu opazovalca

$$x = x_0 + v_x t + v_t t = x_0 + t(\sin\varphi + 1) v_t$$
$$y = y_0 + v_y t - \frac{gt^2}{2}$$

Kapljice letijo naprej:

$$x = x_0 + t(\sin\varphi + 1) v_t$$

$$\sin\varphi + 1 > 0 \quad \forall t$$

ali: kapljica vedno odleti tangencialno na cikloido → torej vedno naprej:

Maksimalna višina

$$\dot{y} = 0 \Rightarrow \frac{v_y}{g} = t \Rightarrow y = y_0 + \frac{v_y^2}{g} - \frac{v_y^2}{2g}$$

$$\Rightarrow y = y_0 + \frac{v_y^2}{2g}$$

Zanima nas še $v_y(y_0)$.

$$v_y^2 = v_t^2\left(1-\sin^2\varphi\right) = v_t^2\left(1-\left(\frac{h-R}{R}\right)^2\right)$$

$$\Rightarrow y = h + \frac{v_t^2}{2g}\left(1-\left(\frac{h-R}{R}\right)^2\right)$$

$$\frac{\partial y}{\partial h} = 0 \Rightarrow 1 + \frac{v_t^2}{2g}\left(-2\left(\frac{h-R}{R}\right)\cdot\frac{1}{R}\right) = 0$$

$$1 = \frac{\cancel{2}v_t^2}{\cancel{2}gR^2}h - \frac{\cancel{2}v_t^2}{2gR^2}R$$

$$\frac{\cancel{v_t^2}}{\cancel{gR^2}}h = \frac{v_t^2R + gR}{\cancel{gR^2}\,v_t^2}$$

$$\Rightarrow h = R + \frac{gR^2}{v_t^2}$$

To je višina, na kateri se kapljica, ko doseže največjo višino, odlepi.

$$\Rightarrow y_{max} = R + \frac{gR^2}{v_t^2} + \frac{v_t^2}{2g}\left(1-\left(\frac{gR^2}{v_t^2}\right)^2\right) = R + \frac{gR^2}{v_t^2} + \frac{v_t^4-g^2R^2}{2g\,v_t^2}$$

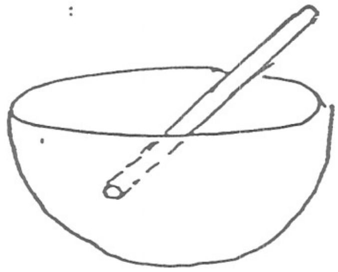
To je maksimalna višina, ki jo kapljica doseže.

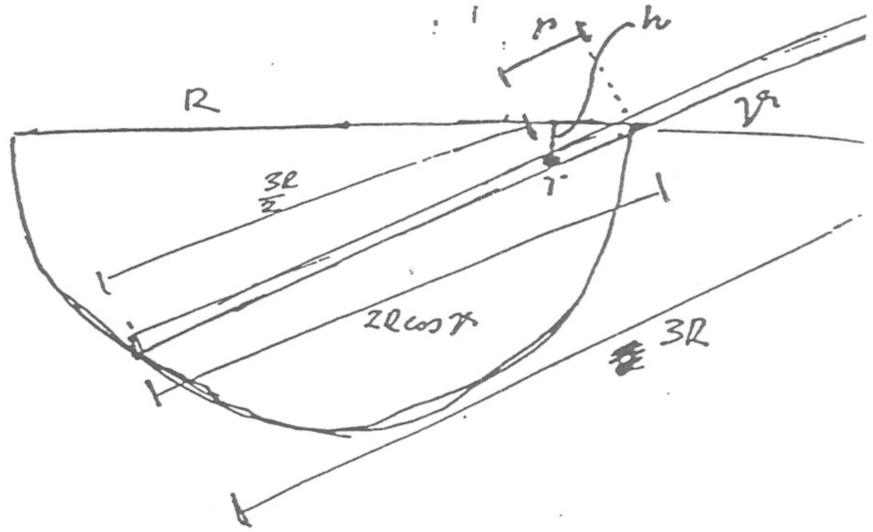
čas od trenutka odlepitve do max. višine:

$$t = \frac{v_y}{g} = \frac{v_t}{g}\cos\varphi = \frac{v_t}{g}\sqrt{(2R-h)h}\cdot\frac{1}{R} = \frac{v_t}{g}\cdot\frac{1}{R}\sqrt{\left(R-\frac{gR^2}{v_t^2}\right)\left(R+\frac{gR^2}{v_t^2}\right)} = \frac{v_t}{g}\sqrt{1-\frac{g^2R^2}{v_t^4}} =$$

Premik osi kolesa: $l = v_t\cdot t = \frac{v_t^2}{g}\sqrt{1-\frac{g^2R^2}{v_t^4}}$

(48) Palica v skledi :





$$V = -mgh$$

$$h = \left(2R\cos\vartheta - \frac{3R}{2}\right)\sin\vartheta$$

$$\frac{\partial V}{\partial \vartheta} = 0 \rightarrow \frac{\partial h}{\partial \vartheta} = 0 \Rightarrow$$

$$\Rightarrow -2\sin^2\vartheta + \left(2\cos\vartheta - \frac{3}{2}\right)\cos\vartheta = 0$$

$$4\cos^2\vartheta - \frac{3}{2}\cos\vartheta - 2 = 0$$

$$\cos\vartheta_{1/2} = \frac{3 \pm \sqrt{137}}{16}$$

Smiselna: $\cos\vartheta = \frac{3 + \sqrt{137}}{16}$ $\leftarrow$ pri tem kotu ima potencialna energija minimum - to je ravnovesna lega

Še Lagranžijan :

Translacija

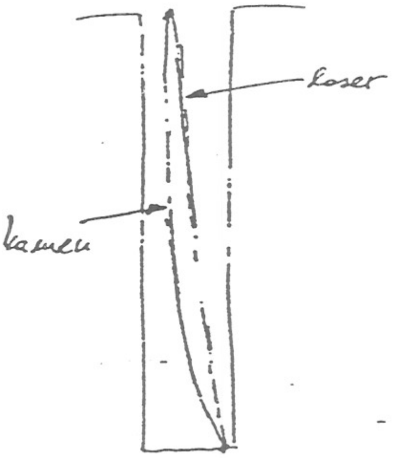
$$T_T = \frac{1}{2} m \dot{r}^2 = \frac{1}{2} m \left(2R\cos\vartheta - \frac{3R}{2}\right)^{\cdot\,2} = \frac{1}{2} m \cdot 4R^2 \sin^2\vartheta\, \dot{\vartheta}^2$$

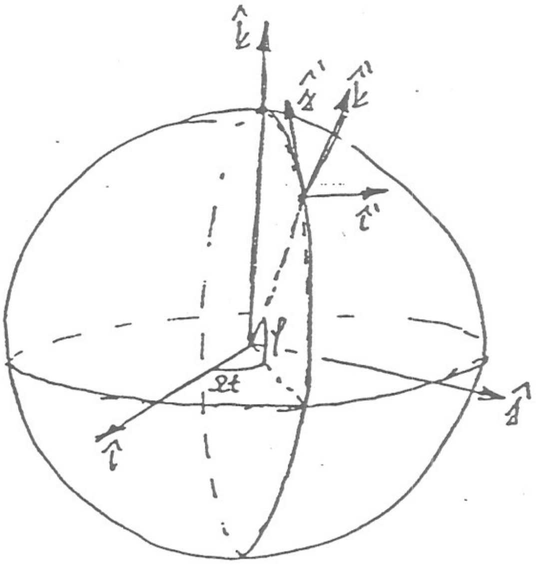
Rotacija

$$T_R = \frac{1}{2} J \dot{\vartheta}^2 = \frac{1}{2} \frac{m(3R)^2}{12} \dot{\vartheta}^2 = \frac{1}{2} \cdot \frac{3m}{4} R^2 \dot{\vartheta}^2$$

$$V = -mgh = -mgR\left(2\cos\vartheta - \frac{3}{2}\right)\sin\vartheta$$

2) Kamen pada v globok vodnjak, v točko trka posvetimo z laserjem:





$$\ddot{\vec r}' = \vec a_{rel} + 2\vec\Omega\times\vec v_{rel} + \vec\Omega\times(\vec\Omega\times\vec r')$$

$$\ddot{\vec R} = \vec\Omega\times(\vec\Omega\times\vec R)$$

(glej npr. Foucaultovo nihalo)

$$\vec R = (0, 0, R)$$

$$\vec \Omega = (0, \Omega\cos\varphi, \Omega\sin\varphi)$$

$$\vec r' = (x', y', z')$$

(*)

$$\Rightarrow \quad \ddot x' + 2\Omega(\dot z'\cos\varphi - \dot y'\sin\varphi) - x'\Omega^2 = 0$$

$$\ddot y' + 2\Omega\dot x'\sin\varphi + \Omega^2(z'\sin\varphi\cos\varphi - y'\sin^2\varphi) = -\Omega^2 R\sin\varphi\cos\varphi$$

$$\ddot z' - 2\Omega\dot x'\cos\varphi + \Omega^2(z'\cos^2\varphi + y'\sin\varphi\cos\varphi) = -g_0 + \Omega^2 R\cos^2\varphi$$

$$\eta = y'\sin\varphi - z'\cos\varphi$$

$$\Rightarrow \quad \ddot\eta + 2\Omega\dot x - \Omega^2\eta = \cos\varphi\,\overbrace{(g_0 - \Omega^2 R)}^{g}$$

$$\left.\begin{aligned} &\ddot\eta + 2\Omega\dot x' - \Omega^2\eta = 0 \\ &\ddot x - 2\Omega\dot\eta - \Omega^2 x' = 0 \end{aligned}\right\}\ \xi = x' + i\eta$$

$$\Rightarrow \quad \ddot\xi + 2i\Omega\dot\xi - \Omega^2\xi = ig\cos\varphi$$

Homogeni del: $\xi = Ae^{i\Omega t} + Be^{-i\Omega t}$

Partikularni: $\xi(0) = 0$, $\dot\xi(0) = 0$ $\Rightarrow$ $ig\cos\varphi/\Omega^2 - A = 0$

$\Rightarrow -i\Omega A = -B$

$\Rightarrow \xi = \frac{g\cos\varphi}{\Omega^2}($[illegible]$\, ie^{i\omega t} - \Omega t e^{-i\Omega t})$

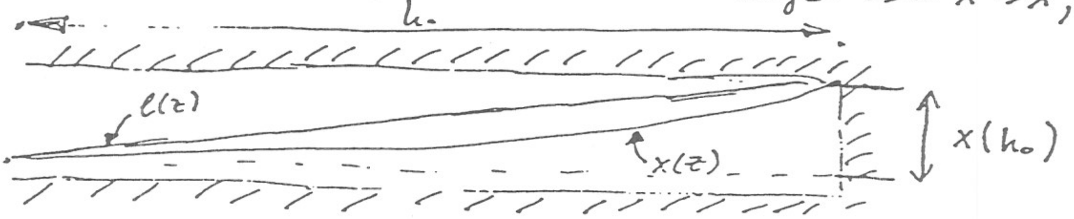
$$\Rightarrow \quad x' = \frac{g\cos\varphi}{\Omega^2}\left(\sin\Omega t - \Omega t\cos\Omega t\right) \doteq \underline{\frac{g\Omega\cos\varphi}{3}\,t^3}$$

(upoštevam $\Omega t \approx 10^{-5} \ll$, $t \approx 10\,s$)

$$y' = 0$$

$$z' = -\frac{g}{\Omega^2}\cdot\left(-1 + \cos\Omega t + \Omega t\sin\Omega t\right) \doteq \underline{-\frac{1}{2}g t^2}$$

Zanima me $x'(z')$ (Dogovor: od tu dalje velja: $x' \Rightarrow x$, $-z' \Rightarrow z$)



$$t(z) = \sqrt{\frac{2z}{g}} \quad \Rightarrow \quad x(z) = \underbrace{\frac{g\Omega\cos\varphi}{3}\cdot\sqrt{\frac{2}{g}}^{\,3}}_{A}\cdot\sqrt{z}^{\,3}$$

$l(z)$: $\quad x(h_0) = A\cdot\sqrt{h_0}^{\,3}$

$$\Rightarrow \quad l(z) = \frac{x(h_0)}{h_0}\cdot z = A\sqrt{h_0}^{\,\cancel{3}}\cdot z$$

Imam torej funkcijo $p(z) = l(z) - x(z) = A\sqrt{h_0}\cdot z - A\sqrt{z}^{\,3}$

in iščem njen ekstrem:

$$\frac{dp}{dz} = 0 \quad \Rightarrow \quad \cancel{A}\sqrt{h_0} - \cancel{A}\cdot\frac{3}{2}z^{\frac{1}{2}} = 0$$

$$\rightarrow \boxed{z = \frac{4}{9}h_0}$$