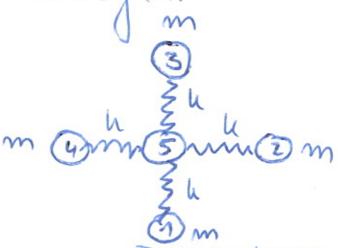
SISTEM PETIH UTEŽI V KRIŽU

Imamo sistem petih uteži z enakimi masami m, ki so povezane z vzmetmi s koeficientom k v križ. Utež 3 izmaknemo v z smeri, tako da celotni sistem zaniha le v z smeri. Kakšne so lastne frekvence in lastni nihajni načini?



NAMIGI

- Kinetična energija: $T = \dots = \frac{1}{2}\dot{\underline{z}}^T \underline{\underline{T}}\,\dot{\underline{z}}$; $\underline{\underline{T}} = m\,\mathbb{1}$ – identiteta
- Potencialna: $V = \frac{1}{2}\underline{z}^T \underline{\underline{V}}\,\underline{z}$; $\underline{\underline{V}} = k\begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ -1 & -1 & -1 & -1 & +4 \end{pmatrix}$
- Iz gibalne enačbe $\underline{\underline{V}}\,\underline{a} = \omega^2 \underline{\underline{T}}\,\underline{a}$ izračunaj ω^2! Namesto reševanja 5x5 sistema, uganemo lastne nihajne načine $\underline{a}_k^T \underline{a}_l = \delta_{kl}$ in iz njih izračunamo ω_k:

1) Tog premik: $\underline{a}_1^T = (1\ 1\ 1\ 1\ 1)$ uganemo

   Frekvenca: $\underline{\underline{V}}\,\underline{a}_1 = k\begin{pmatrix}0\\0\\0\\0\\0\end{pmatrix} = \omega_1^2 \underline{\underline{T}}\,\underline{a}_1 = \omega_1^2 m \underline{a}_1 \Rightarrow \omega_1^2 = 0$

2) 1,3 gor; 2,4 dol: $\underline{a}_2^T = (1, -1\ 1\ -1\ 0) \Rightarrow \omega_2^2 = \omega_0^2$

3) 1,2 gor; 3,4 dol: $\underline{a}_3^T = (1\ 1\ -1\ -1\ 0) \Rightarrow \omega_3^2 = \omega_0^2$

4) 1,4 gor; 2,3 dol: $\underline{a}_4^T = (1\ -1\ -1\ 1\ 0) \Rightarrow \omega_4^2 = \omega_0^2$

5) 1,2,3,4 gor, 5 dol → za koliko?

   $\underline{a}_5^T = (1\ 1\ 1\ 1\ \alpha)$; $\underline{\underline{V}}\,\underline{a}_5 = \dots = k\begin{pmatrix}1-\alpha\\1-\alpha\\1-\alpha\\1-\alpha\\-4+4\alpha\end{pmatrix} = m\omega_5^2\begin{pmatrix}1\\1\\1\\1\\\alpha\end{pmatrix}$ *)

   *) $\cancel{(1-\alpha)c = 1}$ $\cancel{(-4+4\alpha)c = \alpha}$

   $\Rightarrow \begin{pmatrix}1-\alpha\\1-\alpha\\1-\alpha\\-4+4\alpha\end{pmatrix} \cdot \underset{\text{konst}}{\underset{\downarrow}{c}} = \begin{pmatrix}1\\1\\1\\1\\\alpha\end{pmatrix}$

$\Rightarrow \dots\ c = \frac{1}{5},\ \alpha = -4$

in $\omega_5^2 = 5\frac{k}{m} = 5\omega_0^2$