KLM zveza med T in V

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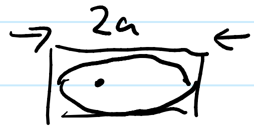
Naloga:

pokaži, da za $V(r) = \frac{-\alpha}{r}$, $\alpha > 0$

velja $2\bar{T} = -\bar{V}$; $\bar{V} = \frac{1}{T_0}\int_0^{T_0} dt\, V$

predpostaviš lahko še



$$E = -\frac{\alpha}{2a} \quad (1) \qquad \& \qquad \int \frac{d\varphi}{1+\varepsilon\cos\varphi} = \frac{2\pi}{\sqrt{1-\varepsilon^2}} \quad (2)$$

Rešitev

spomnimo se še (velja za $V(r) = -\alpha/r$)

$$r = \frac{r_0}{1+\varepsilon\cos\varphi} \quad (3); \qquad r_0 = \frac{l^2}{m\alpha} \quad (4);$$

čas obhoda $T_0 = 2\pi a^{3/2}\sqrt{\frac{m}{\alpha}}$ (5)

Iz elipse vidimo

$$\frac{r_0}{1+\varepsilon} + \frac{r_0}{1-\varepsilon} = \frac{2r_0}{1-\varepsilon^2} = 2a$$

$$r_0 = a(1-\varepsilon^2) \quad (6)$$

Poenostavimo najprej nalogo z uporabo $E = T + V$ ($E = \text{konst}$, $\bar{E} = E$)

$$2\bar{T} + \bar{V} = 2E - \bar{V} = 0$$

torej, pokazati moramo $\bar{V} = 2E$,

se pravi

$$\bar{V} = \frac{1}{T_0}\int^{T_0} V\, dt \qquad 2E = -\alpha \ldots$$

$$\bar V = \frac{1}{T_0}\int_0^{T_0} \underbrace{V}_{-\alpha/r}\, dt = 2E = -\frac{\alpha}{a}$$

torej

$$\frac{1}{T_0}\int \frac{1}{r}\, dt \overset{?}{=} \frac{1}{a}$$

Naprej:

$$dt = d\varphi\, \frac{dt}{d\varphi} = d\varphi\, \frac{m r^2}{p_\varphi}$$

$$\underbrace{\frac{1}{2\pi a^{3/2}}\frac{\sqrt{\alpha}}{\sqrt{m}}}_{(5)}\frac{m}{p_\varphi}\int_0^{2\pi} r\, d\varphi \overset{?}{=} \frac{1}{a}$$

$$\frac{1}{2\pi a^{3/2}}\underbrace{\sqrt{\frac{m\alpha}{p_\varphi^2}}\, r_0}_{(4)}\int_0^{2\pi} \underbrace{\frac{1}{1+\varepsilon\cos\varphi}}_{(2)}\, d\varphi =$$

$$= \underbrace{\frac{1}{a^{3/2}}\sqrt{r_0}}_{(6)}\ \frac{1}{\sqrt{1-\varepsilon^2}} = \frac{1}{a}$$

QED.