PROSTA SIMETRIČNA VRTAVKA

Opiši gibanje proste vrtavke v lastnem sistemu $\underline{J}$ in v mirujočem sistemu. Vrtavka je osno simetrična.

NAMIGI

- SISTEM VRTAVKE:

- $\underline{J} = \begin{pmatrix} J & & \\ & J & \\ & & J' \end{pmatrix}$, v Eulerjeve enačbe vstavi $\vec{M}=0$, dobiš:

$$J'\dot{\omega}_z' = 0 \Rightarrow \omega_z' = \text{konst}$$

$$① \quad J\dot{\omega}_x' + (J'-J)\,\omega_z'\omega_y' = 0$$

$$② \quad J\dot{\omega}_y' - (J'-J)\,\omega_z'\omega_y' = 0$$

- ① $+ i$② in $\xi = \omega_x' + i\omega_y'$; $J\dot{\xi} - i\omega_z'(J'-J)\xi = 0$.

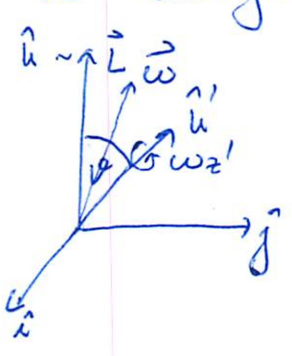
Rešitev: $\omega_x'(t) = |\xi_0|\cos(\Omega_p t + \delta)$

$(*)\ \omega_y'(t) = |\xi_0|\sin(\Omega_p t + \delta) \qquad \Omega_p = \omega_z'\dfrac{J'-J}{J}$

Precesija $\vec{\omega}$ okrog osi $\hat{k}'$.

GIBANJE V MIRUJOČEM SISTEMU:

- Mirujoči sistem postavimo tako, da $\hat{k}$ poravnan z $\vec{L} = L\hat{k}$, ki se ohranja.



1: Pod kakšnim kotom glede na $\vec{L}$ se nahaja vrtavka

$\vec{L}\cdot\hat{k}' = \text{pokaži} = J'\omega_z' = \text{konst}$

2: Kakšen kot oklepa $\vec{\omega}$ z $\vec{L}\times\hat{k}'$?

$(\vec{L}\times\hat{k}')\cdot\vec{\omega} = \text{pokaži} = 0$

$\Rightarrow \vec{L}, \vec{\omega}, \hat{k}'$ ležijo vedno v ravnini

3. Gibanje osi vrtavke:

$\dot{\hat{k}}' = \vec{\omega}\times\hat{k}' = -\omega_x'\hat{j}' + \omega_y'\hat{i}' \Rightarrow |\dot{\hat{k}}'| = |\xi_0|$

1., 2. in 3. $\Rightarrow$ os vrtavke $\hat{k}'$ in $\vec{\omega}$ precedirata okrog $\vec{L}$.

- Frekvenca precesije ~~frekvenco~~ $\Omega = ?$

$\vec{\omega} = \Omega\hat{k} + \tilde{\omega}_z'\hat{k}'$ in $\dot{\hat{k}}' = \vec{\omega}\times\hat{k}' = \ldots = \Omega\hat{k}\times\hat{k}' \Rightarrow |\dot{\hat{k}}'| = \Omega\sin\vartheta = |\xi_0|$

Ob nekem trenutku je $\omega_y' = |\xi_0| \overset{(*)}{=} \dfrac{L_y'}{J_y} = \dfrac{L\sin\vartheta}{J}$

lastni sistem

$$\Rightarrow \Omega = \frac{L}{J}$$

- $L = ?$ $\quad \omega_z' = \dfrac{L_z'}{J_z} = \dfrac{L\cos\vartheta}{J'} \Rightarrow \Omega = \dfrac{J'}{J}\omega_z'\dfrac{1}{\cos\vartheta} \underset{\vartheta\to 0}{\approx} J'/J\ \omega_z'$

Prosta vrtavka precedira z $\Omega = \frac{J'}{J}\omega_z'$ okrog vrtilne količine.