

DELEC V POTENCIALU $V(r) = -k/r^2$

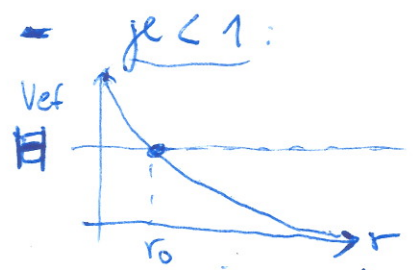
Poišči vse možne orbite delca v $V(r) = -\frac{k}{r^2}$ potencialu. Najprej preko efektivnega potenciala poišči tipe orbit, potem pa iz celotne energije $r(\varphi)$!

NAMIGI:

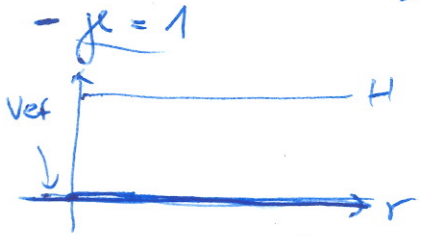
- centralni potencial \Rightarrow ravninsko gibanje $\Rightarrow (r, \varphi)$ polarne koordinate.
- $P_\varphi = \frac{\partial L}{\partial \dot{\varphi}} = m r \dot{\varphi}^2 = \text{konst saj } L \neq L(\varphi)$

① celotna energija: $H = \frac{m}{2} \dot{r}^2 + V_{\text{ef}}(\varphi)$; $V_{\text{ef}}(r) = V(r) + \frac{P_\varphi^2}{2mr^2}$

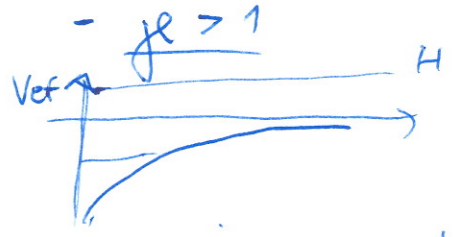
• Tipi orbit: $V_{\text{ef}}(r) = \frac{1}{r^2} \left(\frac{P_\varphi^2}{2m} - k \right) = \frac{P_\varphi^2}{2m} (1 - \gamma) \frac{1}{r^2}$; $\gamma = \frac{2mk}{P_\varphi^2}$



Neomejeno gibanje delca, min. razdalja pri r_0



$H > 0$: padanje
 $H = 0$: $\frac{m \dot{r}^2}{2} = 0$
 kroženje



padanje v center

• Poišči $r(\varphi)$: \dot{r}^2 izrazi iz ①. Spremeni Uredi novo spremenljivo $u = \frac{1}{r}$ in $\frac{d}{dt} \rightarrow \frac{d}{d\varphi} \dot{\varphi}$. Izrazi vse $\dot{\varphi}$ s P_φ , dobi se enačbo:

② $(u')^2 = \frac{2m}{P_\varphi^2} H - (1 - \gamma) u^2$; $u' = \frac{du}{d\varphi}$

• ② $\frac{d}{d\varphi} \Rightarrow$ ③: $u'(u'' + (1 - \gamma)u) = 0$

• Rešitve enačbe ③:

- $u' = 0 \Rightarrow u(\varphi) = u_0$ in $r(\varphi) = \frac{1}{u_0}$

- $u'' + (1 - \gamma)u = 0$; $|1 - \gamma| = d^2$ nova konstanta

- $\gamma < 1$: $u(\varphi) = A \cos[2(\varphi - \varphi_0)]$, $\varphi \in (-\frac{\pi}{2d}, \frac{\pi}{2d})$
- $\gamma = 1$: $u'' = 0 \Rightarrow u(\varphi) = A\varphi + B$
- $\gamma > 1$: $u(\varphi) = A e^{-2\varphi} + B e^{2\varphi}$