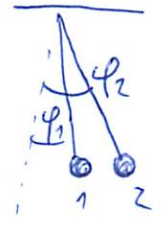
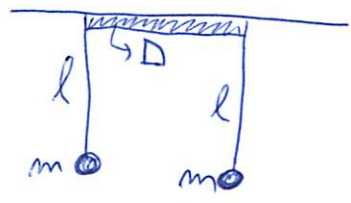


TORZIJSKO SKLOPLJENI TEŽNI MIHALI

Obravnavajmo dve težni mihali, ki se lahko prosto vrhita drugo drugo skupne osi, ki deluje hkrati kot torzijska sklopitev z velikostjo D . Poišči stacionarne stabilne lege, in lastne frekvence in lastne mihajne maxime drugo drugih!



Sistem opišemo s koordinatama zasuka $\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$

NAMIGI

• $V = -mgl(\cos\varphi_1 + \cos\varphi_2 - \frac{d}{2}(\varphi_1 - \varphi_2)^2)$; $d = \frac{D}{mgl} > 0$

• Ramovesna lega:

① $\frac{\partial V}{\partial \varphi_1} \Big|_{\varphi_0} = 0$

② $\frac{\partial V}{\partial \varphi_2} \Big|_{\varphi_0} = 0$

rešujemo

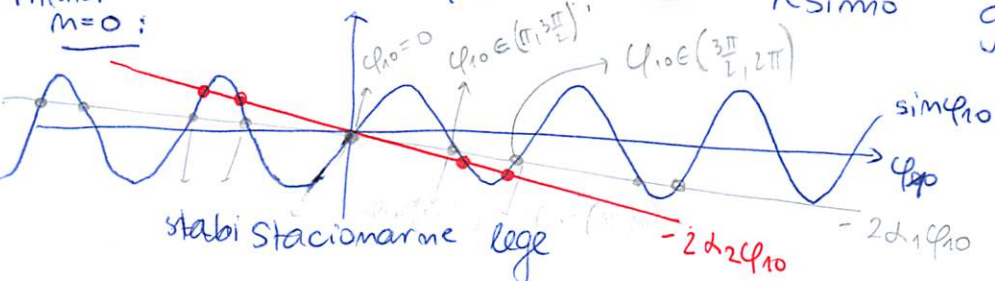
① + ② : $\sin\varphi_{10} = -\sin\varphi_{20} \Rightarrow$

① - ② : $\begin{cases} \sin\varphi_{10} + 2d\varphi_{10} - \pi m = 0 \\ \sin\varphi_{10} - 2d\varphi_{10} - \pi m = 0 \end{cases}$

$\varphi_{10} = \begin{cases} -\varphi_{20} + 2\pi m \\ \varphi_{20} + \pi + 2\pi m \end{cases}$

Enačbo $\sin\varphi_{10} + 2d\varphi_{10} - \pi m = 0$ rešimo

grafično:



$d_2 > d_1$; za d_2 manj rešitev

• Stabilna lega, če $V(\varphi_0 + \delta\varphi) > V(\varphi_0)$ v bližini stacionarne φ_0

Taylor: $V(\varphi_0 + \delta\varphi) = V(\varphi_0) + \sum_i \frac{\partial V}{\partial \varphi_i} \Big|_{\varphi_0} \delta\varphi_i + \frac{1}{2} \sum_{ij} \frac{\partial^2 V}{\partial \varphi_i \partial \varphi_j} \Big|_{\varphi_0} \delta\varphi_i \delta\varphi_j + \dots$

$\Rightarrow \varphi_0$ stabilna, če:

$\delta\varphi^T \tilde{V} \delta\varphi > 0$ za $\forall \delta\varphi \neq 0$ je \tilde{V} pozitivno definitna (vse lastne vrednosti pozitivne)

• Zapiši matriko \tilde{V} , kjer $\tilde{V}_{ij} = \frac{\partial^2 V}{\partial \varphi_i \partial \varphi_j}$ in poišči lastne vrednosti

$\tilde{V} = mgl \begin{pmatrix} \cos\varphi_{10} + d & -d \\ -d & \cos\varphi_{10} + d \end{pmatrix}$; $\lambda_1 = \cos\varphi_{10} mgl$, $\lambda_2 = (\cos\varphi_{10} + 2d) mgl$

• stabilne lege za $m=0$, prve tri:

a) $\varphi_{10} = 0$: $\lambda_1 > 0$
 $\lambda_2 > 0$

stabilna ✓

b) $\varphi_{10} \in (\pi, \frac{3\pi}{2})$: $\lambda_1 < 0$

labilna

c) $\varphi_{10} \in (\frac{3\pi}{2}, 2\pi)$: $\lambda_1 > 0$
 $\lambda_2 > 0$

stabilna

• Izračun lastnih mihanij: Zapiši \tilde{T} in \tilde{V} za majhne odmike iz φ_0

$$\tilde{T} = \frac{1}{2} \delta \dot{\varphi}^T \tilde{\underline{I}} \delta \dot{\varphi} \quad ; \quad \tilde{\underline{I}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} ml^2, \text{ v imamo}$$

• Lastna mihanija: z nastankom $\delta \varphi = \varphi_0 a e^{-i\omega t}$ iz Euler-Lag. enačb dobimo:

$$\tilde{\underline{V}} \underline{a} = \omega^2 \tilde{\underline{I}} \underline{a} \quad \Rightarrow \quad \det(\tilde{\underline{V}} - \underbrace{\omega^2 ml^2}_{\lambda} \underline{1}) = 0$$

λ_1, λ_2 že poznamo, torej

$$\lambda_1 = mgl \cos \varphi_0 = \omega_1^2 ml^2$$

$$\Rightarrow \omega_1^2 = \omega_0^2 \cos \varphi_0; \omega_0^2 = \frac{g}{l}$$

$$\omega_2^2 = (\cos \varphi_0 + 2) \omega_0^2$$

• Lastni vektorji, izračunaj!

$$\lambda_1 \text{ pripada } \underline{a}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \lambda_2 \text{ pripada } \underline{a}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Lastni mihanijni načini so torej:

$$\delta \varphi_1 = \varphi \quad \delta \varphi_2 = \varphi$$

$$\underline{\delta \varphi}_1 = (A_1 \cos \omega_1 t + B_1 \sin \omega_1 t) \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\underline{\delta \varphi}_2 = (A_2 \cos \omega_2 t + B_2 \sin \omega_2 t) \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

