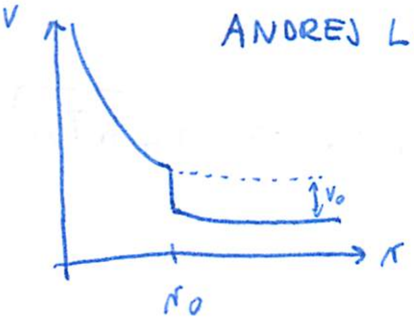
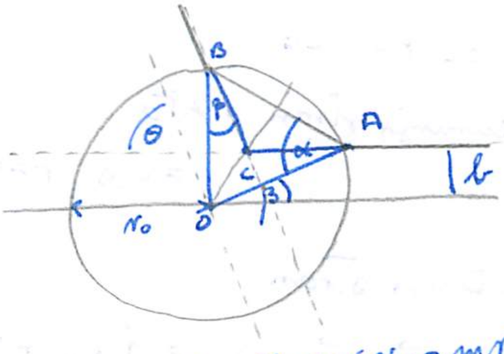
ANDREJ LOZAR

$$V(r) = \begin{cases} V_0 & ; r \leq r_0 \\ 0 & ; r > r_0 \end{cases}$$





POLNA ENERGIJA:

$$W = T + V = T_\infty$$

$$T = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} m r^2 \dot{\varphi}^2 = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} \frac{\Gamma^2}{m r^2}$$

$$\dot{\varphi} = \frac{\Gamma}{m r^2} \quad \Gamma = m r v_\varphi = m r^2 \dot{\varphi}$$

VRTILNA KOLIČINA:

$$\Gamma = m v_\infty b$$

$$W = T + V \Rightarrow \frac{1}{2} m \dot{r}^2 = W - V - \frac{1}{2} \frac{\Gamma^2}{m r^2}$$

$$\dot{r} = \frac{dr}{dt} = \frac{dr}{d\varphi} \frac{d\varphi}{dt} = r' \dot{\varphi}$$

$$\dot{r}^2 = \frac{2}{m} \left(W - V - \frac{1}{2} \frac{\Gamma^2}{m r^2} \right) \rightarrow \dot{r} = \pm \sqrt{\frac{2}{m} \left(W - V - \frac{\Gamma^2}{2 m r^2} \right)}$$

$$\left(\frac{dr}{d\varphi} \right)^2 \dot{\varphi}^2 = \frac{2}{m} \left(W - V - \frac{1}{2} \frac{\Gamma^2}{m r^2} \right)$$

$$\left(\frac{dr}{d\varphi} \right)^2 \frac{\Gamma^2}{m^2 r^4} = \frac{2}{m} \left(W - V - \frac{1}{2} \frac{\Gamma^2}{m r^2} \right) \quad / \cdot \frac{m^2}{\Gamma^2}$$

$$\frac{1}{r^4} \left(\frac{dr}{d\varphi} \right)^2 = \frac{2m}{\Gamma^2} (W - V) - \frac{1}{r^2}$$

$$\frac{1}{r^2} \frac{dr}{d\varphi} = \pm \sqrt{\frac{2m}{\Gamma^2} (W - V) - \frac{1}{r^2}}$$

Nastavek $u = \frac{1}{r}$

$$du = -\frac{dr}{r^2}$$

$$\frac{du}{d\varphi} = \mp \sqrt{\frac{2m}{\Gamma^2} (W - V) - u^2}$$

KDAJ DELEC PRODRE:

$$\Gamma^2 = m^2 v_\infty^2 b^2 = 2 T_\infty b^2 m$$

$$T_\infty = W > V_{eff}(r_0) = \frac{\Gamma^2}{2 m r_0^2} + V_0$$

$$T_\infty > \frac{T_\infty b^2}{r_0^2} + V_0$$

$$T_\infty \left(1 - \frac{b^2}{r_0^2} \right) > V_0$$

$$T_\infty > \frac{V_0}{1 - \frac{b^2}{r_0^2}} \quad \text{ali} \quad b < \sqrt{\left(1 - \frac{V_0}{T_\infty} \right)} r_0$$

A) DELEC PRODRE

zunanja stran $r \geq r_0$

$$v = v_\infty, \; r = r_0 \qquad \Gamma = m v_\infty b = b\sqrt{2 m T_\infty}$$

notranja stran

$$\vec{v} = \vec{v}_r + \vec{v}_\varphi = \dot{r}\,\hat{e}_r + \dot{\varphi}\,\hat{e}_\varphi$$

$$v_r = \dot{r} = -\sqrt{\frac{2}{m}\left(W - V - \frac{\Gamma^2}{2 m r_0^2}\right)} = -\sqrt{\frac{2}{m}\left(T_\infty - V_0 - \frac{2 T_\infty b^2 m}{2 m r_0^2}\right)}$$

(proti izhodišču)

$$v_\varphi = \dot{\varphi} r_0 = \frac{\Gamma}{m r_0} = \frac{b\sqrt{2 m T_\infty}}{m r_0} = \sqrt{\frac{2 T_\infty}{m}}\,\frac{b}{r_0}$$

ΔABC je enakokrak $\Rightarrow$ $\pi - \theta$ ... vrh

$2\times$ $\alpha - \beta$ ... stranska kota

$$(\pi - \theta) + 2(\alpha - \beta) = \pi$$

$$\boxed{\theta = 2(\alpha - \beta)}$$

$$\beta = \arcsin \frac{b}{r_0}$$

$$\operatorname{tg}\alpha = \left|\frac{v_\varphi}{v_r}\right| = \frac{\sqrt{\frac{2T_\infty}{m}}\,\frac{b}{r_0}}{\sqrt{\frac{2}{m}\left(W - V - \frac{\Gamma^2}{2 m r_0^2}\right)}} = \frac{\sqrt{\frac{2T_\infty}{m}}\,\frac{b}{r_0}}{\sqrt{\frac{2T_\infty}{m}}\sqrt{\left(1 - \frac{V_0}{T_\infty} - \frac{b^2}{r_0^2}\right)}}$$

$$\operatorname{tg}\alpha = \frac{1}{\sqrt{\frac{r_0^2}{b^2} - \frac{r_0^2}{b^2}\frac{V_0}{T_\infty} - 1}}$$

B) DELEC ODBIJE

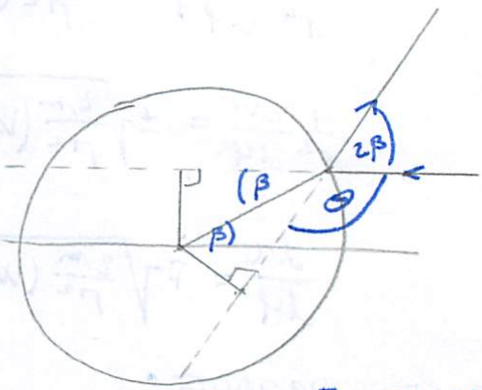
$$2\beta + \theta = 180^\circ$$

$$\theta = \pi - 2\beta = \pi - 2\arcsin\left(\frac{b}{r_0}\right)$$

$$\beta = \frac{\pi}{2} - \frac{\theta}{2}$$

$$\frac{b}{r_0} = \sin\beta = \sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = \cos\frac{\theta}{2}$$

$$\boxed{\theta = 2\arccos\frac{b}{r_0}}$$



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