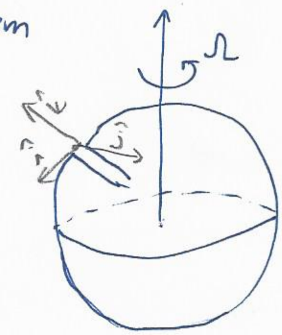
DOMAČA NALOGA ZA KLASIČNO MEHANIKO:
PADANJE KAMNA V JAŠEK

Splošno: $\frac{d^2\vec{r}'}{dt^2} = \left(\frac{d^2\vec{r}'}{dt^2}\right)_{rel} + 2\vec{\Omega}\times\left(\frac{d\vec{r}'}{dt}\right)_{rel} + \cancel{\vec{\Omega}\times(\Omega\times\vec{r})}$ → ne upoštevamo: (velika v_{rel}) ~~[illegible]~~

v resnici bi morali upoštevati na zemlji, ker ni majhen.

v neinercialnem v inercialnem



Začetni pogoji:

$x'(0) = y'(0) = z'(0) = 0$

$\dot{x}'(0) = \dot{y}'(0) = \dot{z}'(0) = 0$

$\vec{\Omega} = \Omega\cos\theta\,\hat{k}' - \Omega\sin\theta\,\hat{i}'$

v vrtečem

1.) Perturbativno: Člene prvega reda $2\vec{\Omega}\times\vec{v}$ zanemarimo: (Dobro za majhne t)

$$\frac{d^2\vec{r}'}{dt^2} = \left(\frac{d^2\vec{r}'}{dt^2}\right)_{rel}$$

$$-g\hat{k}' = \ddot{x}_1'\hat{i}' + \ddot{y}_1'\hat{j} + \ddot{z}_1'\hat{k}'$$

$$\ddot{z}_1' = -g \qquad , \ddot{x}_1' = 0 \ , \ \ddot{y}_1' = 0$$

$$\dot{z}_1' = -gt + C$$

Začetni pogoji $\Rightarrow$ $\vec{v}_{rel} = -gt\,\hat{k}'$

$\vec{v}_{rel}$ vstavimo nazaj v enačbo:

$$-g\hat{k}' = \ddot{x}_2'\hat{i} + \ddot{y}'\hat{j} + \ddot{z}'\hat{k} - 2\Omega g t\sin\theta\,\hat{j}$$

$$\vec{\Omega}\times\vec{v}_{rel} = \Omega\begin{vmatrix} i & j & k \\ -\sin\theta & 0 & \cos\theta \\ 0 & 0 & -gt \end{vmatrix} = \Omega(0, -gt\sin\theta, 0)$$

Po komponentah:

$\hat{i}'$: $\ddot{x}_2' = 0 \Rightarrow x_2'(t) = 0$

$\hat{j}'$: $2\Omega g t\sin\theta = \ddot{y}'$

$\dot{y}' = 2\Omega g\sin\theta\frac{t^2}{2}$, $y'(t) = \frac{1}{3}\Omega g\sin\theta\, t^3$

$\hat{k}'$: $-g = \ddot{z}'$ $\quad z'(t) = -g\frac{t^2}{2}$

$$\vec{r}'(t) = \left(0, \frac{1}{3}\Omega g\sin\theta\, t^3, -g\frac{t^2}{2}\right)$$

2.) Eksplicitno:

$$-g\hat{k}' = \ddot{x}'\hat{i}' + \ddot{y}'\hat{j}' + \ddot{z}'\hat{k}' + 2\Omega\left(-\sin\theta\,\hat{i}' + \cos\theta\,\hat{k}'\right) \times \left(\dot{x}'\hat{i}' + \dot{y}'\hat{j}' + \dot{z}'\hat{k}\right)$$

$$\vec{\Omega} \times \vec{v}_{rel} = 2\Omega \begin{vmatrix} i & j & k \\ -\sin\theta & 0 & \cos\theta \\ \dot{x}' & \dot{y}' & \dot{z}' \end{vmatrix} = 2\Omega\left(-\cos\theta\,\dot{y}',\ \cos\theta\,\dot{x}' + \sin\theta\,\dot{z}',\ -\sin\theta\,\dot{y}'\right)$$

Po komponentah:

$\hat{i}$: $\ddot{x}' - 2\Omega\cos\theta\,\dot{y}' = 0$ $\quad / \cdot \cos\theta$

$\hat{j}$: $2\Omega(\cos\theta\,\dot{x}' + \sin\theta\,\dot{z}') + \ddot{y}' = 0$

$\hat{k}$: $\ddot{z}' - \sin\theta\, 2\Omega\dot{y} = -g$ $\quad / \cdot \sin\theta$

Seštejemo $\hat{i}$ in $\hat{k}$: $\cos\ddot{x}' - 2\Omega\dot{y}' + \sin\theta\,\ddot{z}' = -g\sin\theta$

- Nova spremenljivka: $\alpha = x'\cos\theta + z'\sin\theta$

1.) $\ddot{\alpha} - 2\Omega\dot{y}' = -g\sin\theta$ $\qquad$ 2.) $\ddot{y}' + 2\Omega\dot{\alpha} = 0$

- Nova spremenljivka: $\beta = y + \alpha i$

$2.) + 1.)\cdot i \rightarrow \ddot{y}' + \ddot{\alpha}i + 2\Omega(\dot{\alpha} - \dot{y}'i) = -ig\sin\theta$

$\ddot{\beta} + \frac{2\Omega}{i}(\dot{y}' + \dot{\alpha}i) = -ig\sin\theta$

$\ddot{\beta} - 2\Omega i\dot{\beta} = -ig\sin\theta$ $\quad \leftarrow$ lahko bi: $\dot{\beta} = u$ ....

Nastavek: $\beta = Ae^{i\lambda t} + Bt + C$

$\rightarrow$ v enačbo: $-\lambda^2 A e^{i\lambda t} - 2\Omega i(Ai\lambda e^{i\lambda t} + B) = -ig\sin\theta$

$-\lambda^2 A e^{i\lambda t} + 2\Omega A\lambda e^{i\lambda t} - 2\Omega iB = -ig\sin\theta$

$\hookrightarrow B = \frac{g\sin\theta}{2\Omega}$

$\lambda^2 = 2\Omega\lambda$ $\quad \lambda_1 = 0$ $\leftarrow$ je še v konstanti C

$\lambda_2 = 2\Omega$

$$\beta = Ae^{i2\Omega t} + \frac{g\sin\theta}{2\Omega}t + C = y + \alpha i = y + (x'\cos\theta + z'\sin\theta)i$$

$t=0$: $A = -C$

Odvajamo: $\dot{y}' + (\dot{x}'\cos\theta + \dot{z}'\sin\theta)i = A2\Omega i\, e^{i2\Omega t} + \frac{g\sin\theta}{2\Omega}$

$t=0$: $A = -\frac{g\sin\theta}{i4\Omega^2} = i\frac{g\sin\theta}{4\Omega^2}$

$-\frac{g\sin\theta}{}$

$$\beta = y' + \alpha i = i\,\frac{g\sin\theta}{4\Omega^2}\,e^{i2\Omega t} + \frac{g\sin\theta}{2\Omega}\,t - i\,\frac{g\sin\theta}{4\Omega^2} =$$

$$= i\,\frac{g\sin\theta}{4\Omega^2}\cdot\left(\cos(2\Omega t) + i\sin(2\Omega t)\right) + \frac{g\sin\theta}{2\Omega}\,t - i\,\frac{g\sin\theta}{4\Omega^2}$$

$$\mathrm{Re}(\beta) = y'(t) = -\frac{g\sin\theta}{4\Omega^2}\sin(2\Omega t) + \frac{g\sin\theta}{2\Omega}\,t = \frac{g\sin\theta}{2\Omega}\left(t - \frac{\sin(2\Omega t)}{2\Omega}\right)$$

Prvotna enačba: $\ddot{x}' = 2\Omega\cos\theta\,\dot{y}' = \cancel{2\Omega}\cos\theta\cdot\frac{g\sin\theta}{\cancel{2\Omega}}\cdot\left(1-\cos(2\Omega t)\right)$

$$\dot{x}' = g\sin\theta\cos\theta\,t - g\sin\theta\cos\theta\,\frac{1}{2\Omega}\,\sin(2\Omega t)$$

$$x' = \frac{g}{2}\sin\theta\cos\theta\,t^2 + \frac{g\sin\theta\cos\theta}{4\Omega^2}\cdot\cos(2\Omega t) + C = g\sin\theta\cos\theta\cdot\left(\frac{t^2}{2} + \frac{\cos(2\Omega t)}{4\Omega^2} - \frac{1}{4\Omega^2}\right)$$

konst., da je $x(0)=0$

$$\mathrm{Im}(\beta) = \alpha = x'\cos\theta + z'\sin\theta = \frac{g\sin\theta}{4\Omega^2}\cos(2\Omega t) - \frac{g\sin\theta}{4\Omega^2}$$

$$z' = \frac{g}{4\Omega^2}\left(\cos(2\Omega t) - 1\right) - g\cos^2\theta\left(\frac{t^2}{2} + \frac{\cos(2\Omega t)}{4\Omega^2}\right) + g\,\frac{\cos^2\theta}{4\Omega^2}$$

konst., da je $z(0)=0$

$$= \frac{g}{4\Omega^2}\left(\cos(2\Omega t)\cdot\sin^2\theta - 1\right) - g\cos^2\theta\,\frac{t^2}{2}$$

$$r'(t) = \left(g\sin\theta\cos\theta\left(\frac{t^2}{2} - \frac{1}{4\Omega^2} + \frac{\cos(2\Omega t)}{4\Omega^2}\right),\ \frac{g\sin\theta}{2\Omega}\left(t - \frac{\sin(2\Omega t)}{2\Omega}\right),\ \frac{g}{4\Omega^2}\left(\cos(2\Omega t)\cdot\sin^2\theta - 1\right) - g\cos^2\theta\,\frac{t^2}{2} + g\,\frac{\cos^2\theta}{4\Omega^2}\right)$$

V približku ko je t majhen:

$\hat{i}$: Če razvijemo $\cos(2\Omega t)$ se ~~po~~ odštejeta 1 in t^2, pride šele t^4.

$\hat{j}$: Pri razvoju $\sin(2\Omega t)$ se člen z t odšteje, ostane t^3

$\hat{z}$: Spet pri razvoju šele od t^4.

$\Rightarrow$ Se njema z preturbacijo do t^3.