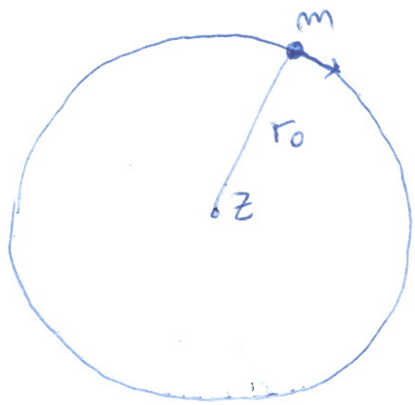
SATELIT OKROG ZEMLJE – 1. kolokvij 2015

Okrog Zemlje na oddaljenosti $r_0 = 42000\,km$ kroži satelit. V nekem trenutku vanj trči lahek meteorit, tako da satelit izgubi 2% kin. energije, smer gibanja se mu ob trku ne spremeni. Zapiši in skiciraj za pred in po trku $V_{ef}(r)$ in ga primerjaj s celotno energijo. Po kakšni orbiti se bo satelit gibal po trku? Kolikšna je najmanjša razdalja med središčem Zemlje in satelitom v tej orbiti?



Sistem pred trkom ob $t=0$:

$t<0$: Kroženje v potencialu $V = -\frac{K}{r}$ pri radiju r_0. Poiščimo T_0: začetno kinetično. Pri kroženju je $v_r = \dot{r} = 0$ in $v_\varphi = \dot{\varphi} r \neq 0$, (obodno hitrost v_φ dobimo iz centripetalnega pospeška:)

$$\vec{a}_c = \frac{1}{m}(-\nabla V) = -\frac{1}{m}\frac{\partial V}{\partial r}\hat{e}_r = -\frac{K}{m}\frac{1}{r^2}\hat{e}_r$$

$$|\vec{a}_c|\Big|_{t<0} = \frac{K}{r_0^2 m} \underset{\text{ker kroženje}}{=} \dot{\varphi}^2 r_0$$

$$\Rightarrow \dot{\varphi}^2 = \frac{K}{m r_0^3} \equiv \dot{\varphi}_0^2$$

$$p_{\varphi_0}^2 = (m\dot{\varphi}_0 r^2)^2 = K m r_0$$

Po tej delamo

Druga pot do $\dot{\varphi}_0$:

Iz efektivnega potenciala: vemo, da je delec v minimumu efektivnega potenciala, ker kroži.

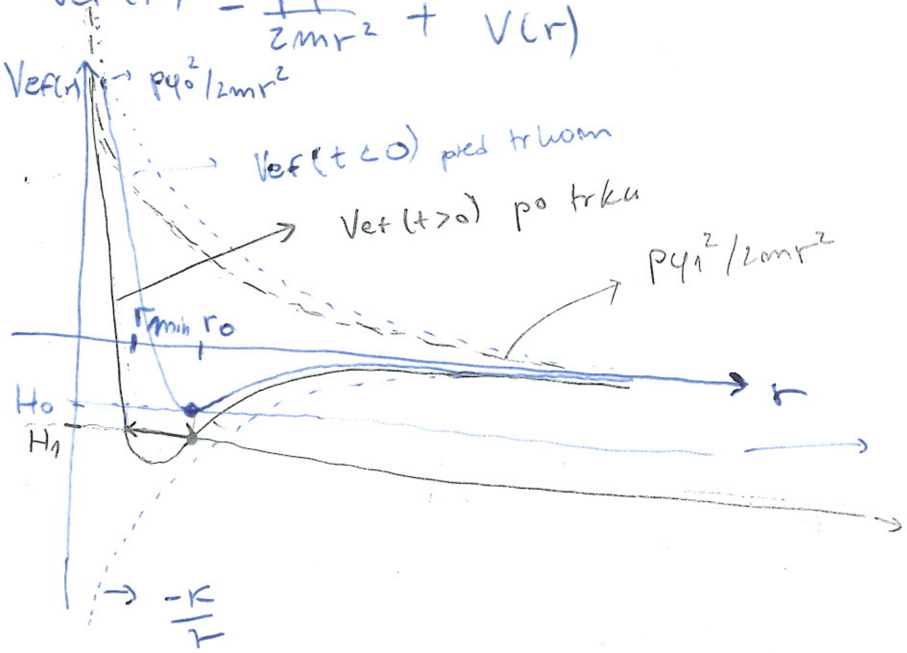
$$V_{ef}(r) = V(r) + \frac{p_\varphi^2}{2mr^2} = \frac{-K}{r} + \frac{p_\varphi^2}{2mr^2}$$

$$\frac{\partial V_{ef}}{\partial r}\Big|_{r_0} = 0 = +\frac{K}{r_0^2} - 2\frac{p_{\varphi_0}^2}{2mr_0^3} \Rightarrow p_{\varphi_0}^2 = mKr_0$$

$\underline{t<0}$:
$$r = r_0$$
$$p_{\varphi 0}^2 = k m r_0$$
$$T_0 = \frac{p_{\varphi 0}^2}{2 m r_0^2} = \frac{k}{2 r_0} \; ; \quad H_0 = T_0 + V_0 = -\frac{k}{2r_0}$$

Ob $\underline{t=0}$: Trk, satelit izgubi 2% ~~T~~ kinetične energije, torej:
$$T(t=0) = T_0 \eta \quad ; \quad \eta = 0{,}98.$$
$$r = r_0$$
$$p_{\varphi 1}^2 = 2 m r_0^2 T(t=0) = 2 m r_0^2 T_0 \eta = p_{\varphi 0}^2 \eta$$
Nova konstanta gibanja - zmanjšanje vrtilne količine
$$H_1 = -\frac{k}{r_0} + \frac{p_{\varphi 0}^2}{2 m r_0^2}\eta$$

Efektivni potencial:
$$H = T + V = \frac{1}{2} m \dot{r}^2 + \frac{p_\varphi^2}{2 m r^2} + V(r) = \frac{1}{2} m \dot{r}^2 + V_{ef}(r)$$
$$V_{ef}(r) = \frac{p_\varphi^2}{2 m r^2} + V(r)$$



energija pred trkom, ker je, saj imamo kroženje.

energija po trku, ker je, saj je satelit pri $r=r_0$ in ima $\dot{r}=0$

Gibal se bo po elipsi.

Energija:

$\underline{t<0}$: $\quad H_0 = -\frac{k}{r_0} + \frac{p_{\varphi 0}^2}{2 m r_0^2}$

$\underline{t>0}$:
$$H_1 = -\frac{k}{r} + \frac{p_{\varphi 1}^2}{2 m r^2} + \frac{1}{2} m \dot{r}^2 =$$
$$H_1 = -\frac{k}{r_0} + \frac{p_{\varphi 0}^2 \eta}{2 m r_0^2}$$

Poenostavi

②

Najmanjša razdalja, ki jo bo imel satelit do Zemlje, $r = r_{min}$, tam bo $\dot{r} = 0$. Izraz za r_{min} dobimo iz energije:

$$H_1 = \cancel{T} - \frac{K}{r_*} + \frac{p_\varphi^2}{2mr_*^2} = -\frac{K}{r_*} + \frac{\eta K m r_0}{2mr_*^2} = \frac{K}{r_0}\left(-\frac{r_0}{r_*} + \frac{r_0^2}{r_*^2}\frac{\eta}{2}\right)$$

r_* – radij kjer $\dot{r} = 0$, torej tam max ali min razdalja

$x = \frac{r_*}{r_0}$ uvedemo za lažje računanje

$$H_1 = \frac{K}{r_0}\left(-\frac{1}{x} + \frac{1}{x^2}\frac{\eta}{2}\right) \quad \Big/ \cdot x^2 \Big/ \cancel{\tfrac{K}{r_0}}\, H_1$$

Izrazimo x:

$$x^2 = -\frac{K}{r_0 H_1}x + \frac{K}{r_0 H_1}\eta/2$$

$$\frac{K}{r_0 H_1} = \frac{K}{r_0\left(-\frac{K}{r_0} + \eta\frac{Kmr_0}{2mr_0^2}\right)} = \frac{1}{(-1+\eta/2)}$$

$$\boxed{x^2 + \frac{1}{(-1+\eta/2)}x - \frac{\eta/2}{(-1+\eta/2)} = 0}$$ kvadratna enačba

Rešitvi sta:

$$x_{1,2} = \frac{-\frac{1}{-1+\eta/2} \pm \sqrt{\left(\frac{1}{-1+\eta/2}\right)^2 + 4\frac{\eta/2}{(-1+\eta/2)}}}{2} =$$

$$= \frac{1}{2(-1+\eta/2)}\left(-1 \pm \sqrt{1 + 2\eta(-1+\eta/2)}\right)$$

$$\cancel{x_{min} = \frac{r_{min}}{r_0}} = \frac{1}{2(1-\eta/2)}\left(1 \pm \sqrt{\underbrace{1 - 2\eta + \eta^2}_{(1-\eta)^2}}\right) = \frac{1}{2(1-\eta/2)}\left(1 \pm (1-\eta)\right)$$

$$\boxed{x_{min} = \frac{r_{min}}{r_0} = \frac{1-1+\eta}{2(1-\eta/2)} = \frac{\eta/2}{1-\eta/2}}$$

$$x_{max} = \frac{r_{max}}{r_0} = \frac{1+1-\eta}{2(1-\eta/2)} = \frac{2(1-\eta/2)}{2(1-\eta/2)} = 1$$ → Največja oddaljenost pri začetnem r.