SIPALNI PRESEK ZA ZDRUŽITEV DELCEV

Imejmo privlačni potencial $V(r) = -\frac{K}{r^n}$; $n \geq 2$.

Poišči sipalni presek za združitev delcev!

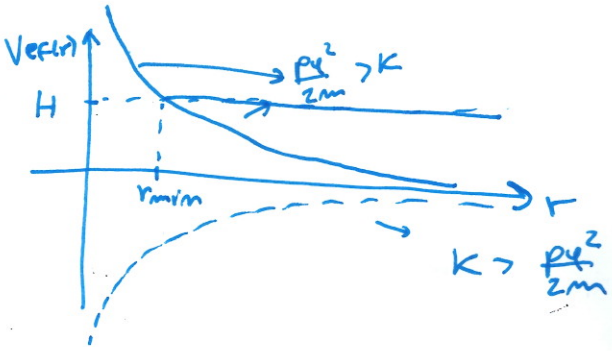
~~Delca se združita~~

Posebej obravnavamo primere $n=2$ in $n>2$, saj so kvalitativno različni.

$$H = \frac{1}{2}mv^2 + \underbrace{\frac{p_\varphi^2}{2mr^2} + V(r)}_{V_{ef}(r)}$$

$$V_{ef}(r) = \frac{p_\varphi^2}{2mr^2} - \frac{K}{r^n}$$ efektivni potencial

① $n=2$



$$V_{ef}(r) = \frac{p_\varphi^2}{2mr^2} - \frac{K}{r^2} = \frac{1}{r^2}\left(\frac{p_\varphi^2}{2m} - K\right)$$

Če je $\frac{p_\varphi^2}{2m} > K$, je $V_{ef} > 0$, če

če $\frac{p_\varphi^2}{2m} < K$, je $V_{ef} < 0$.

Pogoj, da se delca združita je, da pade delec v center, $r_{min} = 0$.

Torej mora biti $V_{ef} < 0$, torej $\frac{p_\varphi^2}{2m} < K$.

$p_\varphi = m v_0 b$

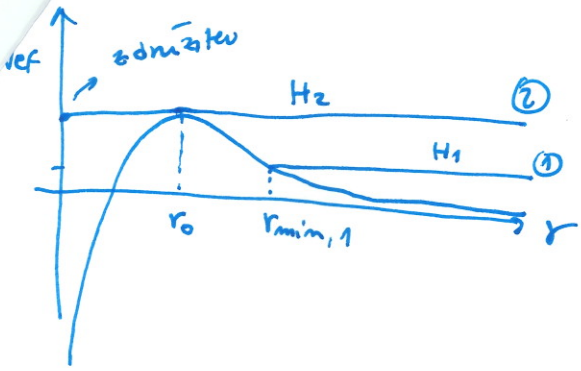
Mejni b, ko se še združita:

$$\frac{b_{max}^2 m^2 v_0^2}{2m} = K \Rightarrow b_{max}^2 = \frac{2K^{n-1}}{m v_0^2}$$

Sipalni presek za združitev je torej:

$$\sigma_{združitev} = \pi b_{max}^2 = \frac{2\pi K}{m v_0^2}$$

$n = 3$, ostali primeri kvalitativno enaki



$$V_{ef} = \frac{p_\varphi^2}{2mr^2} - \frac{K}{r^3}$$

Pogoj za združitev delcev je, da projektil premaga potencialno bariero z maksimumom v r_0:

$$H > V_{ef}(r_0)$$

① Delcu z energijo H_1 ne uspe priti čez bariero. Delec se sipље.

② Delcu z H_2 pa ravno uspe.

Poiščimo r_0:

$$\left.\frac{\partial V_{ef}}{\partial r}\right|_{r=r_0} = -\frac{2p_\varphi^2}{2mr_0^3} + \frac{3K}{r_0^4} = 0$$

$$\Rightarrow \quad r_0 = \frac{3Km}{p_\varphi^2}$$

Potencialna bariera:

$$V_{ef}(r_0) = \left(\frac{p_\varphi^2}{2m} - \frac{K}{r_0}\right)\frac{1}{r_0^2} = \frac{1}{r_0^2}\left(\frac{p_\varphi^2}{2m} - \frac{p_\varphi^2}{3m}\right) = \frac{1}{r_0^2}\frac{p_\varphi^2}{m}\frac{1}{6}$$

Kriterij: $H > V_{ef}(r_0)$

$$\frac{1}{2}mv_0^2 > \frac{1}{r_0^2}\frac{p_\varphi^2}{m}\frac{1}{6} \qquad ; \quad \frac{1}{2}mv_0^2 = H(\infty) = H(r_0)$$

$p_\varphi = mv_0 b$

Rejmi b: $\frac{1}{2}mv_0^2 = \frac{1}{r_0^2}\frac{p_\varphi^2}{m}\frac{1}{6}$

$$\frac{1}{2}mv_0^2 = \frac{1}{6}\frac{1}{r_0^2}\,mv_0^2 b_{max}^2 = \left(\frac{p_\varphi^2}{3Km}\right)^2\frac{1}{6}mv_0^2 b_{max}^2 = \frac{1}{18K}v_0^2 b_{max}^2 \; m^2 v_0^2 b_{max}^2$$

$$= \frac{m^4 v_0^4 b_{max}^4}{3^3\cdot 2\,K^2 m^2}\cdot mv_0^2 b_{max}^2 = \frac{1}{2\cdot 3^3 K^2}\,m^3 v_0^6 b_{max}^6$$

$$\Rightarrow \quad b_{max}^2 = \left(\frac{3^3K^2}{m^2v_0^4}\right)^{1/3} = 3\left(\frac{K}{mv_0^2}\right)^{2/3}$$

Celotni sipalni presek za združitev:

$$\sigma_{združitev} = \pi b_{max}^2 = 3\pi\left(\frac{K}{mv_0^2}\right)^{2/3}$$