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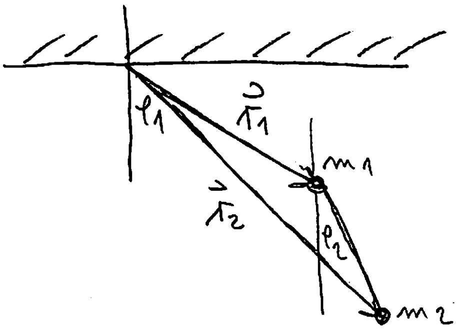

$$\vec{r}_1 = (l_1 \sin\varphi_1, l_1 \cos\varphi_2) = (x_1, y_1)$$
$$\vec{r}_2 = (l_1 \sin\varphi_1 + l_2 \sin\varphi_2, l_1\cos\varphi_1 + l_2 \cos\varphi_2) = (x_1+x_2, y_1+y_2)$$

$$x_i = l_i \sin\varphi_i \ ; \ \dot{x}_i = l_i \cos\varphi_i \dot{\varphi}_i$$
$$y_i = l_i \cos\varphi_i \ ; \ \dot{y}_i = -l_i \sin\varphi_i \dot{\varphi}_i$$
$$\dot{x}_i^2 + \dot{y}_i^2 = l_i^2 \dot{\varphi}_i^2$$

$$\dot{\vec{r}}_1 = (\dot{x}_1, \dot{y}_1)$$
$$\dot{\vec{r}}_2 = (\dot{x}_1 + \dot{x}_2, \dot{y}_1 + \dot{y}_2)$$
$$\dot{\vec{r}}_1^{\,2} = \dot{x}_1^2 + \dot{y}_1^2 = l_1^2 \dot{\varphi}_1^2$$
$$\dot{\vec{r}}_2^{\,2} = \dot{x}_1^2 + \dot{x}_2^2 + 2\dot{x}_1\dot{x}_2 + \dot{y}_1^2 + \dot{y}_2^2 + 2\dot{y}_1\dot{y}_2$$
$$= l_1^2 \dot{\varphi}_1^2 + l_2 \dot{\varphi}_2^2 + 2 l_1 l_2 \dot{\varphi}_1 \dot{\varphi}_2 (\cos\varphi_1 \cos\varphi_2 + \sin\varphi_1 \sin\varphi_2)$$
$$= l_1^2 \dot{\varphi}_1^2 + l_2 \dot{\varphi}_2^2 + 2 l_1 l_2 \dot{\varphi}_1 \dot{\varphi}_2 \Big(1 - \underbrace{\Big(\frac{\varphi_1^2}{2} - \frac{\varphi_2^2}{2} + \frac{\varphi_1^2\varphi_2^2}{4} + \varphi_1\varphi_2\Big)}_{\substack{\Downarrow \\ \text{zanemarimo,} \\ \text{ker so višji red}}}\Big)$$

$$\boxed{\begin{aligned}\cos\varphi &= 1 - \frac{\varphi^2}{2} \\ \sin\varphi &= \varphi\end{aligned}}$$

$$T = \frac{1}{2} m_1 \dot{\vec{r}}_1^{\,2} + \frac{1}{2} m_2 \dot{\vec{r}}_2^{\,2}$$
$$= \frac{1}{2} m_1 l_1^2 \dot{\varphi}_1^2 + \frac{1}{2} m_2 l_1^2 \dot{\varphi}_1^2 + \frac{1}{2} m_2 l_2^2 \dot{\varphi}_2^2 + m_2 l_1 l_2 \dot{\varphi}_1 \dot{\varphi}_2$$

$$V = -m_1 g l_1 \cos\varphi_1 - m_2 g (l_2 \cos\varphi_2 + l_1 \cos\varphi_1)$$
$$= -m_1 g l_1 - m_2 g l_2 - m_2 g l_1 + \frac{1}{2}(m_1 + m_2) g l_1 \varphi_1^2 + \frac{1}{2} m_2 g l_2 \varphi_2^2$$

$$L = T - V = \frac{1}{2}(m_1 + m_2) l_1^2 \dot{\varphi}_1^2 + \frac{1}{2} m_2 l_2^2 \dot{\varphi}_2^2 + m_2 l_1 l_2 \dot{\varphi}_1 \dot{\varphi}_2$$
$$+ \frac{1}{2}(m_1 + m_2) g l_1 \varphi_1^2 + \frac{1}{2} m_2 g l_2 \varphi_2^2$$

$$\frac{\partial}{\partial t}\Big(\frac{\partial L}{\partial \dot{q}_i}\Big) - \frac{\partial L}{\partial q_i} = 0$$

i) $$\frac{\partial}{\partial t}\Big(\frac{\partial L}{\partial \dot{\varphi}_1}\Big) - \frac{\partial L}{\partial \varphi_1} = 0$$
$$(m_1 + m_2) l_1^2 \ddot{\varphi}_1 + m_2 l_1 l_2 \ddot{\varphi}_2 + (m_1 + m_2) g l_1 \varphi_1 = 0$$

ii) $$\frac{\partial}{\partial t}\Big(\frac{\partial L}{\partial \dot{\varphi}_2}\Big) - \frac{\partial L}{\partial \varphi_2} = 0$$
$$m_2 l_2^2 \ddot{\varphi}_2 + m_2 l_1 l_2 \ddot{\varphi}_1 + m_2 g l_2 \varphi_2 = 0$$

i) $\varphi_1 = -\frac{1}{(m_1+m_2) g l_1}\left[(m_1+m_2) l_1^2 \ddot{\varphi}_1 + m_2 l_1 l_2 \ddot{\varphi}_2\right]$

$$\varphi_1 = -\frac{l_1}{g}\ddot{\varphi}_1 - \frac{m_2}{m_1+m_2}\frac{l_2}{g}\ddot{\varphi}_2$$

ii) $\varphi_2 = -\frac{1}{m_2 g l_2}\left[m_2 l_2^2 \ddot{\varphi}_2 + m_2 l_1 l_2 \ddot{\varphi}_1\right]$

$$\varphi_2 = -\frac{l_2}{g}\ddot{\varphi}_2 - \frac{l_1}{g}\ddot{\varphi}_1$$

$\omega_i^2 = \frac{g}{l}$

$$A = \begin{bmatrix} -\frac{l_1}{g} & -\frac{m_2}{m_1+m_2}\frac{l_2}{g} \\ -\frac{l_1}{g} & -\frac{l_2}{g} \end{bmatrix} = \begin{bmatrix} -\omega_1^{-2} & -\frac{m_2}{m_1+m_2}\omega_2^{-2} \\ -\omega_1^{-2} & -\omega_2^{-2} \end{bmatrix}$$

$$\vec{\varphi} = A\ddot{\vec{\varphi}} \quad ; \quad \vec{\varphi} = a e^{i\omega t}\vec{v}$$

$$\Rightarrow a e^{i\omega t}\vec{v} = -A\omega^2\vec{v}\, a e^{i\omega t}$$

$$-\frac{1}{\omega^2}\vec{v} = A\vec{v}$$

$$\det\left(A + \frac{1}{\omega^2} I\right) = 0$$

$m_2 = m_1$
$l_1 = l_2$

$$0 = \begin{vmatrix} -\omega_1^{-2} + \omega^{-2} & -\frac{m_2}{m_1+m_2}\omega_2^{-2} \\ -\omega_1^{-2} & -\omega_2^{-2} + \omega^{-2} \end{vmatrix} = \begin{vmatrix} -\omega_1^{-2} + \omega^{-2} & -\frac{1}{2}\omega_2^{-2} \\ -\omega_1^{-2} & -\omega_2^{-2} + \omega^{-2} \end{vmatrix}$$

$$(\omega^{-2} - \omega_1^{-2})(\omega^{-2} - \omega_2^{-2}) - \frac{1}{2}\omega_1^{-2}\omega_2^{-2} = 0 \quad / \cdot \omega^4 \omega_1^2 \omega_2^2$$

$$(\omega_1^2 - \omega^2)(\omega_2^2 - \omega^2) - \frac{1}{2}\omega^4 = 0$$

$$\omega_1^2\omega_2^2 - \omega_1^2\omega^2 - \omega_2^2\omega^2 + \frac{1}{2}\omega^4 = 0$$

$$\omega_{I,II}^2 = \frac{\omega_1^2 + \omega_2^2 \pm \sqrt{(\omega_1^2+\omega_2^2)^2 - 2\omega_1^2\omega_2^2}}{\frac{1}{2}\cdot 2} = \omega_1^2 + \omega_2^2 \pm \sqrt{\omega_1^4 + \omega_2^4}$$

$$\underline{\underline{\omega_{I,II}^2 = \omega_1^2 + \omega_2^2 \pm \sqrt{\omega_1^4 + \omega_2^4}}} = 2\omega_1^2 \pm \sqrt{2}\,\omega_1^2$$

$l_1 = l_2$

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