1. Domača Naloga iz Nanofizike, 5.3.2024

1. *In this task, we are interested in whether a system of multiple identical scatterers in a quantum waveguide ever transitions into the ohmic regime, that is, whether its resistance increases proportionally to the number of scatterers or the length of the waveguide. Assume that for electron transport, only one transverse wave mode is significant.*

a) First consider the case of two scatterers in the coherent regime. Each scatterer should be described by a scattering matrix $(r_i, t_i), (t_i, r_i')$. An electron traveling from one to the other acquires a dynamic phase χ. Calculate the transmittance T through the system (or transcribe the result from the exercises) and sketch how it changes with χ!

b) Derive the result also for the case of classical scatterers (reflectivity R_i, transmittance T_i)!

c) Show that the result for the total transmittance can also be obtained via the relation $(1-T)/T = (1-T_1)/T_1 + (1-T_2)/T_2$. Thus, the quantity $(1-T)/T$ is additive.

d) The quantity $\rho_i = (1-T_i)/T_i$ acts as resistance normalized to the quantum of resistance $1/G_0$ (resistance $\tilde{R}_i = \rho_i/G_0$). The total resistance for a system of multiple scatterers can be obtained as the sum of individual resistances. To obtain the actual current through the system, one must also consider the serial connection of one contact resistance $1/(2G_0)$ for the left and right connection (thus in total $1/G_0$). Show that for a scatterer with transmittance T, you obtain the correct expression for current ($I = G_0TV$), if you represent the entire system as a series of scatterer and two contact resistances.

e) Show that the transmittance for N sequential scatterers with transmittance T_1

$$T = \frac{T_1}{N(1-T_1)+T_1} \tag{1}$$

Show that, if we define $N = \nu L$, where ν is the number of scatterers per unit length and L is the length, we can rewrite this result in the form $T = L_0/(L+L_0)$, where $L_0 = T_1/(\nu(1-T_1))$. In this regime (incoherent scatterers) resistance linearly increases with length and we obtain standard Ohmic behavior.

2. *In the previous task, we completely ignored interference effects. Now, however, we will show [following Datta, p. 198] that if we take into account interference effects on average, we can obtain behavior very different from Ohmic. Start with the result for two scatterers*

$$T(\chi) = \frac{T_1T_2}{1-2\sqrt{R_1R_2}\cos\chi + R_1R_2}. \tag{2}$$

From this, we can write the resistance for a pair of scatterers $\rho_{12}(\chi) = (1-T(\chi))/T(\chi)$. Assume that we have a multitude of scatterer pairs, for which we do not know the phase χ. Then the typical $\rho_{12} = (1/2\pi)\int_0^{2\pi} d\chi \rho_{12}(\chi)$.

a) Show that $\rho_{12} = \frac{1+R_1R_2-T_1T_2}{T_1T_2} = \rho_1+\rho_2+2\rho_1\rho_2$, where $\rho_i = (1-T_i)/T_i$! This result includes the incoherent sum of resistances and a correction, which on average takes into account interference effects.

b) Now we will use this expression to calculate the resistance of a conductor of length L, to which we add a piece of length ΔL and has a resistance for which we assume $\Delta\rho = \Delta L/L_0$. According to the expressions above, we therefore obtain $\rho(L+\Delta L) = \rho(L) + [1+2\rho(L)]\Delta L/L_0$. Show that the change of resistance of the conductor with length is described by the differential equation

$$d\rho(L)/dL = \frac{1+2\rho}{L_0} \tag{3}$$