THE LANDSCAPE OF THEORETICAL PHYSICS: A GLOBAL VIEW

From Point Particles to the Brane World and Beyond, in Search of a Unifying Principle

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Chapter 6

EXTENDED OBJECTS AND CLIFFORD ALGEBRA

We have seen that geometric calculus based on Clifford algebra is a very useful tool for a description of geometry and point particle physics in *flat spacetime*. In Sec. 4.2 we have encountered a generalization of the concepts of vectors and polyvectors in an infinite-dimensional $\mathcal{M}$-space, which is in general a curved space. What we need now is a more complete description, firstly, of the usual finite-dimensional vectors and polyvectors in *curved space*, and secondly, of the corresponding objects in an infinite-dimensional space (which may be $\mathcal{M}$-space, in particular).

After such mathematical preliminaries I shall discuss a membrane description which employs $\mathcal{M}$-space basis vectors. A background independent approach will be achieved by proclaiming the basis vectors themselves as dynamical objects. The corresponding action has its parallel in the one considered in the last section of previous chapter. However, now the dynamical object is not the $\mathcal{M}$-space metric, but its "square root".

Until the techniques of directly solving the functional differential equations are more fully developed we are forced to make the transition from functional to the usual partial differential equations, i.e., from the infinite-dimensional to the finite-dimensional differential equations. Such a transition will be considered in Sec. 6.2, where a description in terms of position-dependent target space vectors will be provided. We shall observe that a membrane is a sort of a fluid localized in the target space, and exploit this concept in relation to the DeWitt-Rovelli reference fluid necessary for localization of spacetime points. Finally we shall realize that we have just touched the tip of an iceberg which is the full polyvector description of the membrane together with the target space in which the membrane is embedded.