THE LANDSCAPE OF THEORETICAL PHYSICS: A GLOBAL VIEW

From Point Particles to the Brane World and Beyond, in Search of a Unifying Principle

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Chapter 8

SPACETIME AS A MEMBRANE IN A HIGHER-DIMENSIONAL SPACE

When studying dynamics of a system of membranes, as seen from the $\mathcal{M}$-space point of view, we have arrived in Chapter 5 at a fascinating conclusion that all that exists in such a world model is a membrane configuration. *The membrane configuration itself is a 'spacetime'.* Without membranes there is no spacetime. According to our basic assumption, at the fundamental level we have an $\mathcal{M}$-space — the space of all possible membrane configurations — and nothing else. If the membrane configuration consists of the membranes of various dimensions n, lower and higher than the dimension of our observed word ($n = 4$), then we are left with a model in which our 4-dimensional spacetime is one of those (4-dimensional) membranes (which I call *worldsheets*).

What is the space our worldsheet is embedded in? It is just the space formed by the other n-dimensional ($n = 0, 1, 2, ...$) extended objects (say membranes) entering the membrane configuration. If all those other membranes are sufficiently densely packed together, then as an approximation a concept of a continuous embedding space can be used. Our spacetime can then be considered as a 4-dimensional worldsheet embedded into a higher-dimensional space.

8.1. THE BRANE IN A CURVED EMBEDDING SPACE

We are now going to explore a brane moving in a curved background embedding space V_N. Such a brane sweeps an n-dimensional surface which

I call *worldsheet*[1] The dynamical principle governing motion of the brane requires that its worldsheet is a minimal surface. Hence the action is

$$I[\eta^a] = \int \sqrt{|\tilde{f}|}\, \mathrm{d}^n x, \tag{8.1}$$

where

$$\tilde{f} = \det \tilde{f}_{\mu\nu} \ , \qquad \tilde{f}_{\mu\nu} = \partial_\mu \eta^a \partial_\nu \eta^b \gamma_{ab}. \tag{8.2}$$

Here x^μ, $\mu = 0, 1, 2, ..., n-1$, are coordinates on the worldsheet V_n, whilst $\eta^a(x)$ are the embedding functions. The metric of the embedding space (from now on also called *bulk*) is γ_{ab}, and the induced metric on the worldsheet is $\tilde{f}_{\mu\nu}$.

In this part of the book I shall use the notation which is adapted to the idea that our world is a *brane*. Position coordinates in our world are commonly denoted as x^μ, $\mu = 0, 1, 2, ..., n-1$, and usually it is assumed that $n = 4$ (for good reasons, of course, unless one considers Kaluza–Klein theories). The notation in (8.1) (8.2) is "the reverse video" of the notation used so far. The correspondence between the two notations is the following

worldsheet coordinates	$\xi^a, \ \xi^A, \phi^A$	x^μ
embedding space coordinates	x^μ	η^a
embedding functions	$X^\mu(\xi^a)\, , X^\mu(\phi^A)$	$\eta^a(x^\mu)$
worldsheet metric	$\gamma_{ab}\, , \gamma_{AB}$	$g_{\mu\nu}$
embedding space metric	$g_{\mu\nu}$	γ_{ab}

Such a reverse notation reflects the change of role given to spacetime. So far 'spacetime' has been associated with the embedding space, whilst the brane has been an object in spacetime. Now spacetime is associated with a brane, so spacetime itself is an object in the embedding space[2].

For the extended object described by the minimal surface action (8.1) I use the common name *brane*. For a more general extended object described by a Clifford algebra generalization of the action (8.1) I use the name *membrane* (and occasionally also *worldsheet*, when I wish to stress that the object of investigation is a direct generalization of the object V_n described by (8.1) which is now understood as a special kind of (generalized) worldsheet).

[1] Usually, when $n > 2$ such a surface is called a *world volume*. Here I prefer to retain the name *worldsheet*, by which we can vividly imagine a surface in an embedding space.

[2] Such a distinction is only manifest in the picture in which we already have an effective embedding space. In a more fundamental picture the embedding space is inseparable from the membrane configuration, and in general is not a manifold at all.

Suppose now that the metric of V_N is conformally flat (with η_{ab} being the Minkowski metric tensor in N-dimensions):

$$\gamma_{ab} = \phi\, \eta_{ab}. \tag{8.3}$$

Then from (8.2) we have

$$\tilde{f}_{\mu\nu} = \phi\, \partial_\mu \eta^a \partial_\nu \eta^b \eta_{ab} \equiv \phi f_{\mu\nu}\ , \tag{8.4}$$

$$\tilde{f} \equiv \det \tilde{f}_{\mu\nu} = \phi^n \det f_{\mu\nu} \equiv \phi^n f \tag{8.5}$$

$$\sqrt{|\tilde{f}|} = \omega |f|\ , \quad \omega \equiv \phi^{n/2}. \tag{8.6}$$

Hence the action (8.1) reads

$$I[\eta^a] = \int \omega(\eta)\, \sqrt{|f|} \mathrm{d}^n x, \tag{8.7}$$

which looks like an action for a brane in a flat embedding space, except for a function $\omega(\eta)$ which depends on the position[3] η^a in the embedding space V_N.

Function $\omega(\eta)$ is related to the fixed background metric which is arbitrary in principle. Let us now assume [88] that $\omega(\eta)$ consists of a constant part ω_0 and a singular part with support on another brane's worldsheet $\widehat{V}_m$:

$$\omega(\eta) = \omega_0 + \kappa \int \mathrm{d}^m \hat{x}\, \sqrt{|\hat{f}|}\, \frac{\delta^N(\eta - \hat{\eta})}{\sqrt{|\gamma|}}. \tag{8.8}$$

Here $\hat{\eta}^a(\hat{x})$ are the embedding functions of the m-dimensional worldsheet $\widehat{V}_m$, $\hat{f}$ is the determinant of the induced metric on $\widehat{V}_m$, and $\sqrt{|\gamma|}$ allows for taking curved coordinates in otherwise flat V_N.

The action for the brane which sweeps a worldsheet V_n is then given by (8.7) in which we replace $\omega(\eta)$ with the specific expression (8.8):

$$I[\eta] = \int \omega_0\, \mathrm{d}^n x\, \sqrt{|f|} + \kappa \int \mathrm{d}^n x\, \mathrm{d}^m \hat{x}\, \sqrt{|f|}\, \sqrt{|\hat{f}|}\, \frac{\delta^N(\eta - \hat{\eta})}{\sqrt{|\gamma|}}. \tag{8.9}$$

If we take the second brane as dynamical too, then the kinetic term for $\hat{\eta}^a$ should be added to (8.9). Hence the total action for both branes is

$$I[\eta, \hat{\eta}] = \int \omega_0\, \mathrm{d}^n x\, \sqrt{|f|} + \int \omega_0\, \mathrm{d}^m \hat{x} \sqrt{|\hat{f}|} + \kappa \int \mathrm{d}^n x\, \mathrm{d}^m \hat{x}\, \sqrt{|f|}\, \sqrt{|\hat{f}|}\, \frac{\delta^N(\eta - \hat{\eta})}{\sqrt{|\gamma|}}. \tag{8.10}$$

[3]We use here the same symbol η^a either for position coordinates in V_N or for the embedding functions $\eta^a(x)$.

The first two terms are the actions for free branes, whilst the last term represents the interaction between the two branes. The interaction occurs when the branes intersect. If we take $m = N - n + 1$ then the *intersection* of V_n and $\widehat{V}_m$ can be a (one-dimensional) line, i.e., a *worldline* V_1. In general, when $m = N - n + (p+1)$, the intersection can be a $(p+1)$-dimensional worldsheet representing the motion of a p-brane.

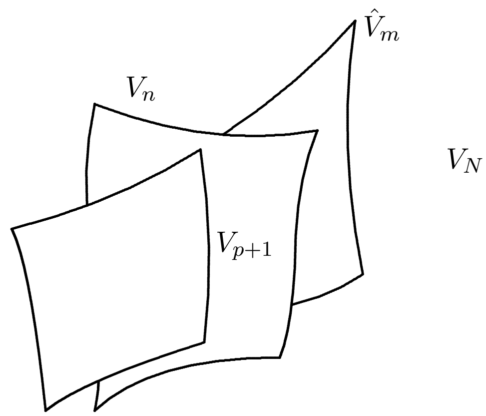

Figure 8.1. The intersection between two different branes V_n and $\hat{V}_m$ can be a p-brane V_{p+1}.

In eq. (8.10) we assume contact interaction between the branes (i.e., the interaction at the intersection). This could be understood by imagining that gravity decreases so quickly in the transverse direction from the brane that it can be approximated by a δ-function. More about this will be said in Section 4.

The equations of motion derived from the (8.10) by varying respectively η^a and $\hat{\eta}$ are:

$$\partial_\mu \left[\sqrt{|f|} \partial^\mu \eta_a \left(\omega_0 + \kappa \int \mathrm{d}^m \hat{x} \sqrt{|\hat{f}|} \, \frac{\delta^N(\eta - \hat{\eta})}{\sqrt{|\gamma|}} \right) \right] = 0 \tag{8.11}$$

$$\hat{\partial}_{\hat{\mu}} \left[\sqrt{|\hat{f}|} \hat{\partial}^{\hat{\mu}} \hat{\eta}_a \left(\omega_0 + \kappa \int \mathrm{d}^n x \sqrt{|f|} \, \frac{\delta^N(\eta - \hat{\eta})}{\sqrt{|\gamma|}} \right) \right] = 0 \tag{8.12}$$

where $\partial_\mu \equiv \partial / \partial x^\mu$ and $\hat{\partial}_{\hat{\mu}} \equiv \partial / \partial \hat{x}^{\hat{\mu}}$. When deriving eq. (8.11) we have taken into account that

$$\frac{\partial}{\partial \eta^a} \int \kappa \, \mathrm{d}^m \hat{x} \, \sqrt{|\hat{f}|} \, \delta^N(\eta - \hat{\eta}) = - \int \kappa \, \mathrm{d}^m \hat{x} \, \sqrt{|\hat{f}|} \frac{\partial}{\partial \hat{\eta}^a} \, \delta^N(\eta - \hat{\eta}) \tag{8.13}$$
$$= \kappa \int \mathrm{d}^m \hat{x} \, \frac{\partial \sqrt{|\hat{f}|}}{\partial \hat{\eta}^a} \, \delta^N(\eta - \hat{\eta}) = 0,$$

since

$$\frac{\partial \sqrt{|\hat{f}|}}{\partial \hat{\eta}^a} = \frac{\partial \sqrt{|\hat{f}|}}{\partial \hat{f}} \, \frac{\partial \hat{f}_{\mu\nu}}{\partial \hat{\eta}^a} = 0, \tag{8.14}$$

because

$$\hat{f}_{\mu\nu} = \hat{\partial}_{\hat{\mu}} \hat{\eta}^a \, \hat{\partial}_{\hat{\nu}} \hat{\eta}_a \, , \quad \frac{\partial \hat{f}_{\mu\nu}}{\partial \hat{\eta}^a} = 0. \tag{8.15}$$

Analogous holds for eq. (8.12).

Assuming that the intersection $V_{p+1} = V_n \bigcap \widehat{V}_m$ does exist, and, in particular, that it is a worldline (i.e., $p = 0$), then we can write

$$\int \mathrm{d}^m \hat{x} \sqrt{|\hat{f}|} \, \frac{\delta^N(\eta - \hat{\eta})}{\sqrt{|\gamma|}} = \int \mathrm{d}\tau \frac{\delta^n(x - X(\tau))}{\sqrt{|f|}} \, (\dot{X}^\mu \dot{X}_\mu)^{1/2}. \tag{8.16}$$

The result above was obtained by writing

$$\mathrm{d}^m \hat{x} = \mathrm{d}^{m-1} \hat{x} \, \mathrm{d}\tau, \qquad \sqrt{|\hat{f}|} = \sqrt{|\hat{f}^{(m-1)}|} (\dot{X}^\mu \dot{X}_\mu)^{1/2}$$

and taking the coordinates η^a such that $\eta^a = (x^\mu, \eta^n, \eta^{n+1}, ..., \eta^{N-1})$, where x^μ are (curved) coordinates on V_n. The determinant of the metric of the embedding space V_N in such a curvilinear coordinates is then $\gamma = \det \partial_\mu \eta^a \partial_\nu \eta_a = f$.

In general, for arbitrary intersection we have

$$\int \mathrm{d}^m \hat{x} \sqrt{|\hat{f}|} \, \frac{\delta^N(\eta - \hat{\eta})}{\sqrt{|\gamma|}} = \int \mathrm{d}^{p+1} \xi \, \frac{\delta^n(x - X\xi)}{\sqrt{|f|}} \, (\det \partial_A X^\mu \partial_B X_\mu)^{1/2}, \tag{8.17}$$

where $X^\mu(\xi^A)$, $\mu = 0, 1, 2, ..., n-1$, $A = 1, 2, ..., p$, are the embedding functions of the p-brane's worlsdsheet V_{p+1} in V_n.

Using (8.16) the equations of motion become

$$\partial_\mu \left[\sqrt{|f|} (\omega_0 f^{\mu\nu} + T^{\mu\nu}) \partial_\nu \eta_a \right] = 0, \tag{8.18}$$

$$\hat{\partial}_{\hat{\mu}} \left[\sqrt{|\hat{f}|} (\omega_0 \hat{f}^{\hat{\mu}\hat{\nu}} + \widehat{T}^{\hat{\mu}\hat{\nu}}) \hat{\partial}_{\hat{\nu}} \hat{\eta}_a \right] = 0, \tag{8.19}$$

where

$$T^{\mu\nu} = \int \frac{\kappa}{\sqrt{|f|}} \, \delta^n(x - X(\tau)) \, \frac{\dot{X}^\mu \dot{X}^\nu}{(\dot{X}^\alpha \dot{X}_\alpha)^{1/2}} \, \mathrm{d}\tau \tag{8.20}$$

and

$$\hat{T}^{\hat{\mu}\hat{\nu}} = \int \frac{\kappa}{\sqrt{|\hat{f}|}} \, \delta^n(\hat{x} - \hat{X}(\tau)) \, \frac{\dot{\hat{X}}^{\hat{\mu}} \dot{\hat{X}}^{\hat{\nu}}}{(\dot{\hat{X}}^{\hat{\alpha}} \dot{\hat{X}}_{\hat{\alpha}})^{1/2}} \, \mathrm{d}\tau \tag{8.21}$$

are the stress–energy tensors of the point particle on V_n and $\hat{V}_m$, respectively.

If dimensions m and n are such that the intersection V_{p+1} is a worldsheet with a dimension $p \geq 1$, then using (8.17) we obtain the equations of motion of the same form (8.18),(8.19), but with the stress–energy tensor

$$T^{\mu\nu} = \int \frac{\kappa}{\sqrt{|f|}} \, \delta^n(x - X(\xi)) \, \partial_A X^\mu \partial^A X^\nu \, (\det \partial_C X^\alpha \partial_D X_\alpha)^{1/2} \, \mathrm{d}^{p+1}\xi, \tag{8.22}$$

$$\hat{T}^{\hat{\mu}\hat{\nu}} = \int \frac{\kappa}{\sqrt{|\hat{f}|}} \, \delta^n(\hat{x} - \hat{X}(\xi)) \, \partial_A \hat{X}^{\hat{\mu}} \partial^A \hat{X}^{\hat{\nu}} \, (\det \partial_C \hat{X}^{\hat{\alpha}} \partial_D \hat{X}_{\hat{\alpha}})^{1/2} \, \mathrm{d}^{p+1}\xi. \tag{8.23}$$

This can also be seen directly from the action (8.9) in which we substitute eq. (8.16)

$$I[\eta^a, X^\mu] = \omega_0 \int \mathrm{d}^n x \, \sqrt{|f|} + \kappa \int \mathrm{d}^n x \, \mathrm{d}\tau \, \delta^n(x - X(\tau))(f_{\mu\nu} \dot{X}^\mu \dot{X}^\nu)^{1/2} \tag{8.24}$$

or if we substitute (8.17)

$$\begin{aligned} I[\eta^a, X^\mu] \;&=\; \omega_0 \int \mathrm{d}^n x \, \sqrt{|f|} \\ &\quad + \kappa \int \mathrm{d}^n x \, \mathrm{d}^{p+1}\xi \, \delta^n(x - X(\xi)) \, (\det \partial_A X^\mu \partial_B X^\nu f_{\mu\nu})^{1/2}. \end{aligned} \tag{8.25}$$

Remembering that

$$f_{\mu\nu} = \partial_\mu \eta^a \partial_\nu \eta^b \eta_{ab} \tag{8.26}$$

we can vary (8.24) or (8.25) with respect to $\eta^a(x)$ and we obtain (8.18).

Eq.(8.18) can be written as

$$\omega_0 \mathrm{D}_\mu \mathrm{D}^\mu \eta_a + \mathrm{D}_\mu (T^{\mu\nu} \partial_\nu \eta_a) = 0. \tag{8.27}$$

where D_μ denotes covariant derivative in V_n. If we multiply the latter equation by $\partial^\alpha \eta^a$, sum over a, and take into account the identity

$$\partial^\alpha \eta^a \mathrm{D}_\mu \mathrm{D}_\nu \eta_a = 0, \tag{8.28}$$

which follows from $\mathrm{D}_\mu(\partial_\rho\eta^a\partial_\sigma\eta_a) = \mathrm{D}_\mu f_{\rho\sigma} = 0$, we obtain

$$\mathrm{D}_\mu T^{\mu\nu} = 0. \tag{8.29}$$

Equation (8.29) implies that $X^\mu(\tau)$ is a geodesic equation in a space with metric $f_{\mu\nu}$, i.e., $X^\mu(\tau)$ is a geodesic on V_n. This can be easily shown by using the relation

$$\mathrm{D}T^{\mu\nu} = \frac{1}{\sqrt{|f|}}\,\partial_\mu(\sqrt{|f|}\,T^{\mu\nu}) + \Gamma^\nu_{\rho\mu}T^{\rho\mu} = 0. \tag{8.30}$$

Taking (8.20) we have

$$\int \mathrm{d}\tau \frac{\partial}{\partial x^\mu}\delta^n(x - X(\tau))\frac{\dot{X}^\mu\dot{X}^\nu}{(\dot{X}^\alpha\dot{X}_\alpha)^{1/2}}\mathrm{d}^n x$$

$$+\,\Gamma^\nu_{\rho\mu}\int \mathrm{d}\tau\delta^n(x - X(\tau))\frac{\dot{X}^\mu\dot{X}^\nu}{(\dot{X}^\alpha\dot{X}_\alpha)^{1/2}}\mathrm{d}^n x = 0. \tag{8.31}$$

The first term in the latter equation gives

$$-\int \mathrm{d}\tau\,\frac{\partial}{\partial X^\mu(\tau)}\,\delta^n(x - X(\tau))\,\frac{\dot{X}^\mu\dot{X}^\nu}{(\dot{X}^\alpha\dot{X}_\alpha)^{1/2}}\,\mathrm{d}^n x$$

$$= -\int \mathrm{d}\tau\,\frac{\mathrm{d}}{\mathrm{d}\tau}\delta^n(x - X(\tau))\,\frac{\dot{X}^\nu}{(\dot{X}^\alpha\dot{X}_\alpha)^{1/2}}\mathrm{d}^n x$$

$$= \int \mathrm{d}\tau\,\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{\dot{X}^\nu}{(\dot{X}^\alpha\dot{X}_\alpha)^{1/2}}\right). \tag{8.32}$$

Differentiating eq. (8.31) with respect to τ we indeed obtain the geodesic equation.

In a similar way we find for $T^{\mu\nu}$, as given in eq. (8.22), that (8.30) implies

$$\frac{1}{\sqrt{|\mathrm{det}\partial_C X^\alpha\partial_D X_\alpha|}}\partial_A(\sqrt{|\mathrm{det}\partial_C X^\alpha\partial_D X_\alpha|}\,\partial^A X^\nu) + \Gamma^\nu_{\rho\mu}\partial_A X^\rho\partial^A X^\mu = 0, \tag{8.33}$$

which is the equation of motion for a p-brane in a background metric $f_{\mu\nu} = \partial_\mu\eta^a\partial_\nu\eta_a$. Do not forget that the latter p-brane is the intersection between two branes:

$$V_{p+1} = V_n \cap \widehat{V}_m\ . \tag{8.34}$$

It is instructive to integrate (8.27) over $\mathrm{d}^n x$. We find

$$\omega_0 \oint \sqrt{|f|}\, \mathrm{d}\Sigma_\mu\, \partial^\mu \eta_a \tag{8.35}$$
$$= -\kappa \int \mathrm{d}^{p+1}\xi (|\det \partial_C X^\alpha \partial_D X_\alpha|)^{1/2}\, \partial_A X^\mu \partial^A X^\nu\, \mathrm{D}_\mu \mathrm{D}_\nu \eta_a \Big|_{x=X(\xi)}$$

where $\mathrm{d}\Sigma_\mu$ is an element of an $(n-1)$-dimensional hypersurface Σ on V_n. Assuming that the integral over the time-like part of Σ vanishes (either because $\partial^\mu \eta_a \to 0$ at the infinity or because V_n is closed) we have

$$\omega_0 \int \sqrt{|f|}\, \mathrm{d}\Sigma_\mu\, \partial^\mu \eta_a \Big|_{\tau_2} - \omega_0 \int \sqrt{|f|}\, \mathrm{d}\Sigma_\mu\, \partial^\mu \eta_a \Big|_{\tau_1} \tag{8.36}$$
$$= -\kappa \int \mathrm{d}\tau\, \mathrm{d}^p\xi (|\det \partial_C X^\alpha \partial_D X_\alpha|)^{1/2}\, \partial_A X^\mu \partial^A X^\nu\, \mathrm{D}_\mu \mathrm{D}_\nu \eta_a \Big|_{x=X(\xi)}$$

or

$$\frac{\mathrm{d}P_a}{\mathrm{d}\tau} = -\kappa \int \mathrm{d}^p\xi (|\det \partial_C X^\alpha \partial_D X_\alpha|)^{1/2}\, \partial_A X^\mu \partial^A X^\nu\, \mathrm{D}_\mu \mathrm{D}_\nu \eta_a \Big|_{x=X(\xi)} \tag{8.37}$$

where

$$P_a \equiv \omega_0 \int \sqrt{|f|}\, \mathrm{d}\Sigma_\mu\, \partial^\mu \eta_a\ . \tag{8.38}$$

When $p = 0$, i.e., when the intersection is a worldline, eq. (8.37) reads

$$\frac{\mathrm{d}P_a}{\mathrm{d}\tau} = -\kappa\, \frac{\dot X^\mu \dot X^\nu}{(\dot X^\alpha \dot X_\alpha)^{1/2}}\, \mathrm{D}_\mu \mathrm{D}_\nu \eta_a \Big|_{x=X(\xi)}\ . \tag{8.39}$$

8.2. A SYSTEM OF MANY INTERSECTING BRANES

Suppose we have a sytem of branes of various dimensionalities. They may intersect and their intersections are the branes of lower dimensionality. The action governing the dynamics of such a system is a generalization of (8.10) and consists of the free part plus the interactive part $(i, j = 1, 2, ...)$:

$$I[\eta_i] = \sum_i \int \omega_0 \sqrt{|f_i|}\, \mathrm{d}x_i + \frac{1}{2} \sum_{i \neq j} \int \omega_{ij}\, \frac{\delta^N(\eta_i - \eta_j)}{\sqrt{|\gamma|}} \sqrt{|f_i|} \sqrt{|f_j|}\, \mathrm{d}x_i \mathrm{d}x_j. \tag{8.40}$$

The equations of motion for the i-th brane are

$$\partial_\mu \left[\sqrt{|f_i|} \partial^\mu \eta_i^a \left(\omega_0 + \sum_{i \neq j} \int \omega_{ij} \, \frac{\delta^N(\eta_i - \eta_j)}{\sqrt{|\gamma|}} \, \sqrt{|f_j|} \mathrm{d}x_j \right) \right] = 0. \qquad (8.41)$$

Neglecting the kinetic term for all other branes the action leading to (8.41) is (for a fixed i)

$$I[\eta_i] = \int \omega_0 \sqrt{|f_i|} \, \mathrm{d}x_i + \sum_{i \neq j} \int \omega_{ij} \, \frac{\delta^N(\eta_i - \eta_j)}{\sqrt{|\gamma|}} \sqrt{|f_i|} \sqrt{|f_j|} \, \mathrm{d}x_i \mathrm{d}x_j \qquad (8.42)$$

or

$$I[\eta_i] = \int \omega_i(\eta) \sqrt{|f_i|} \, \mathrm{d}x_i \; , \qquad (8.43)$$

with

$$\omega_i(\eta) = \omega_0 + \sum_j \kappa_j \frac{\delta^N(\eta - \eta_j)}{\sqrt{|\gamma|}} \, \sqrt{|f_j|} \mathrm{d}x_j \; , \qquad (8.44)$$

where $\kappa_j \equiv \omega_{ij}$.

Returning now to eqs. (8.3)–(8.6) we see that $\omega_i(\eta)$ is related to the conformally flat background metric as experienced by the i-th brane. The action (8.43) is thus the action for a brane in a background metric γ_{ab}, which is conformally flat:

$$I[\eta_i] = \int \sqrt{|\tilde{f}|} \, \mathrm{d}x_i. \qquad (8.45)$$

Hence the interactive term in (8.40) can be interpreted as a contribution to the background metric in which the i-th brane moves. Without the interactive term the metric is simply a flat metric (multiplied by ω); with the interactive term the background metric is singular on all the branes within our system.

The total action (8.40), which contains the kinetic terms for all the other branes, renders the metric of the embedding space V_N dynamical. The way in which other branes move depends on the dynamics of the whole system. It may happen that for a system of many branes, densely packed together, the effective (average) metric could no longer be conformally flat. We have already seen in Sec 6.2 that the effective metric for a system of generalized branes (which I call *membranes*) indeed satisfies the Einstein equations.

Returning now to the action (8.42) as experienced by one of the branes whose worldsheet V_n is represented by $\eta_i^a(x_i) \equiv \eta^a(x)$ we find, after integrating out x_j, $j \neq i$, that

$$\begin{aligned} I[\eta^a, X^\mu] &= \omega_0 \int \mathrm{d}^n x \sqrt{|f|} \\ &+ \sum_j \kappa_j \int \mathrm{d}^n x \, \mathrm{d}^{p_j+1}\xi \, (\mathrm{det}\partial_A X_j^\mu \partial_B X_j^\nu f_{\mu\nu})^{1/2} \delta^n(x - X_j(\xi)). \end{aligned} \tag{8.46}$$

For various p_j the latter expression is an action for a system of point particles ($p_j = 0$), strings ($p_j = 1$), and higher-dimensional branes ($p_j = 2, 3, ...$) moving in the background metric $f_{\mu\nu}$, which is the induced metric on our brane V_n represented by $\eta^a(x)$. Variation of (8.46) with respect to X_k^μ gives the equations of motion (8.33) for a p-brane with $p = p_k$. Variation of (8.46) with respect to $\eta^a(x)$ gives the equations of motion (8.18 for the $(n-1)$-brane. If we vary (8.46) with respect to $\eta^a(x)$ then we obtain the equation of motion (III1.18) for an $(n-1)$-brane. The action (8.46) thus describes the dynamics of the $(n-1)$-brane (world sheet V_n) and the dynamics of the p-branes living on V_n.

We see that the interactive term in (8.40) manifests itself in various ways, depending on how we look at it. It is a manifestation of the metric of the embedding space being curved (in particular, the metric is singular on the system of branes). From the point of view of a chosen brane V_n the interactive term becomes the action for a system of p-branes (including point particles) moving on V_n. If we now adopt the *brane world view*, where V_n is our spacetime, we see that *matter on V_n comes from other branes' worldsheets which happen to intersect our worldsheet V_n.* Those other branes are responsible for the non trivial metric of the embedding space, also called the *bulk*.

THE BRANE INTERACTING WITH ITSELF

In (8.42) or (8.46) we have a description of a brane interacting with other branes. What about self-interaction? In the second term of the action (8.40) (8.42) we have excluded self-interaction. In principle we should not exclude self-interaction, since there is no reason why a brane could not interact with itself.

Let us return to the action (8.9) and let us calculate $\omega(\eta)$, this time assuming that $\widehat{V}_m$ coincides with our brane V_n. Hence the intersection is the brane V_n itself, and according to (8.17) we have

$$\begin{aligned} \omega(\eta) &= \omega_o + \kappa \int \mathrm{d}^n \hat{x} \sqrt{|\hat{f}|} \, \frac{\delta^N(\eta - \hat{\eta}(\hat{x})}{\sqrt{|\gamma|}} \\ &= \omega_0 + \kappa \int \mathrm{d}^n \xi \, \frac{\delta^n(x - X(\xi))}{\sqrt{|f|}} \sqrt{|\hat{f}|} \\ &= \omega_0 + \kappa \int \mathrm{d}^n x \, \delta^n(x - X(x)) = \omega_0 + \kappa. \end{aligned} \tag{8.47}$$

Here the coordinates ξ^A, $A = 0, 1, 2, ..., n-1$, cover the manifold V_n, and $\hat{f}_{AB}$ is the metric of V_n in coordinates ξ^A. The other coordinates are x^μ, $\mu = 0, 1, 2, ..., n-1$. In the last step in (8.47) we have used the property that the measure is invariant, $\mathrm{d}^n\xi\sqrt{|\hat{f}|} = \mathrm{d}^n x\sqrt{|f|}$.

The result (8.47) demonstrates that we do not need to separate a constant term ω_0 from the function $\omega(\eta)$. For a brane moving in a background of many branes we can replace (8.44) with

$$\omega(\eta) = \sum_j \kappa_j \frac{\delta^N(\eta - \eta_j)}{\sqrt{|\gamma|}} \sqrt{|f_j|}\mathrm{d}x_j, \tag{8.48}$$

where j runs over *all* the branes within the system. Any brane feels the same background, and its action for a fixed i is

$$I[\eta_i] = \int \omega(\eta_i)\sqrt{|f|}\mathrm{d}x = \sum_j \int \kappa_j \frac{\delta^N(\eta_i - \eta_j)}{\sqrt{|\gamma|}} \sqrt{|f_i|}\sqrt{|f_j|}\mathrm{d}x_i\,\mathrm{d}x_j. \tag{8.49}$$

However the background is self-consistent: it is a solution to the variational principle given by the action

$$I[\eta_i] = \sum_{i \geq j} \omega_{ij}\,\delta^N(\eta_i - \eta_j)\,\sqrt{|f_i|}\sqrt{|f_j|}\mathrm{d}x_i\,\mathrm{d}x_j, \tag{8.50}$$

where now also i runs over *all* the branes within the system; the case $i = j$ is also allowed.

In (8.50) the self-interaction or self coupling occurs whenever $i = j$. The self coupling term of the action is

$$\begin{aligned}
I_{\mathrm{self}}[\eta_i] &= \sum_i \kappa_i \int \delta^N(\eta_i(x_i) - \eta_i(x_i'))\sqrt{|f_i(x_i)|}\sqrt{|f_i(x_i')|}\mathrm{d}x_i\mathrm{d}x_i' \\
&= \sum_i \kappa_i \int \delta^N(\eta - \eta_i(x_i))\delta^N(\eta - \eta_i(x_i')) \\
&\qquad \times\sqrt{|f_i(x_i)|}\sqrt{|f_i(x_i')|}\mathrm{d}x_i\mathrm{d}x_i'\mathrm{d}^N\eta \\
&= \sum_i \kappa_i \int \delta^N(\eta - \eta_i(x_i))\delta^{n_i}(x_i' - x_i'')\sqrt{|f_i(x_i)|}\mathrm{d}x_i\mathrm{d}x_i'\mathrm{d}^N\eta \\
&= \sum_i \kappa_i \sqrt{|f_i(x_i)|}\mathrm{d}^{n_i}x_i\ ,
\end{aligned} \tag{8.51}$$

where we have used the same procedure which led us to eq. (8.17) or (8.47). We see that the interactive action (8.50) automatically contains the minimal surface terms as well, so they do not need to be postulated separately.

A SYSTEM OF MANY BRANES CREATES THE BULK AND ITS METRIC

We have seen several times in this book (Chapters 5, 6) that a system of membranes (a membrane configuration) can be identified with the embedding space in which a single membrane moves. Here we have a concrete realization of that idea. We have a system of branes which intersect. The only interaction between the branes is owed to intersection ('contact' interaction). The interaction at the intersection influences the motion of a (test) brane: it feels a potential because of the presence of other branes. If there are many branes and a test brane moves in the midst of them, then on average it feels a metric field which is approximately continuous. Our test brane moves in the effective metric of the embedding space.

A single brane or several branes give the singular conformal metric. Many branes give, on average, an arbitrary metric.

There is a close inter-relationship between the presence of branes and the bulk metric. In the model we discuss here the bulk metric is singular on the branes, and zero elsewhere. Without the branes there is no metric and no bulk. Actually the bulk consists of the branes which determine its metric.

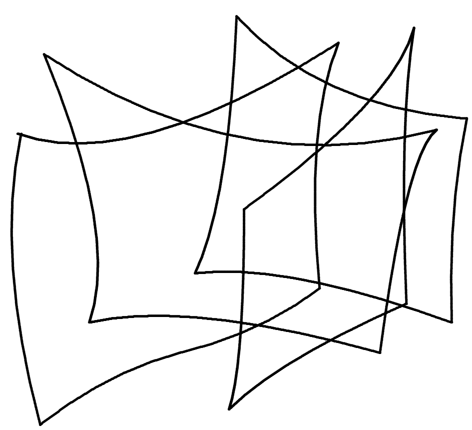

Figure 8.2. A system of many intersecting branes creates the bulk metric. In the absence of the branes there is no bulk (no embedding space).

8.3. THE ORIGIN OF MATTER IN THE BRANE WORLD

Our principal idea is that we have a system of branes (a brane configuration). With all the branes in the system we associate the embedding space (bulk). One of the branes (more precisely, its worldsheet) represents our spacetime. Interactions between the branes (occurring at the intersections) represent *matter* in spacetime.

MATTER FROM THE INTERSECTION OF OUR BRANE WITH OTHER BRANES

We have seen that matter in V_n naturally occurs as a result of the intersection of our worldsheet V_n with other worldsheets. We obtain exactly the stress–energy tensor for a dust of point particles, or p-branes in general. Namely, varying the action (8.46) with respect to $\eta^a(x)$ we obtain

$$\omega_0 \mathrm{D}_\mu \mathrm{D}^\mu \eta_a + \mathrm{D}_\mu (T^{\mu\nu} \partial_\nu \eta_a) = 0, \tag{8.52}$$

with

$$T^{\mu\nu} = \sum_j \kappa_j \int \mathrm{d}^{p_j+1}\xi \, (\det \partial_A X_j^\mu \partial_B X_j^\nu \, f_{\mu\nu})^{1/2} \, \frac{\delta^n(x - X_j(\xi))}{\sqrt{|f|}} \tag{8.53}$$

being the stress–energy tensor for a system of p-branes (which are the intersections of V_n with the other worldsheets). The above expression for $T^{\mu\nu}$ holds if the extended objects have any dimensions p_j. In particular, when *all* objects have $p_j = 0$ (point particles) eq. (8.53) becomes

$$T^{\mu\nu} = \sum_j \kappa_j \int \mathrm{d}\tau \, \frac{\dot{X}^\mu \dot{X}^\nu}{\sqrt{\dot{X}^2}} \, \frac{\delta(x - X(\tau))}{\sqrt{|f|}}. \tag{8.54}$$

From the equations of motion (8.53) we obtain (see eqs. (8.27)–(8.29))

$$\mathrm{D}_\mu T^{\mu\nu} = 0, \tag{8.55}$$

which implies (see (8.30)–(8.33)) that any of the objects follows a geodesic in V_n.

MATTER FROM THE INTERSECTION OF OUR BRANE WITH ITSELF

Our model of intersecting branes allows for the possibility that a brane intersects with itself, as is schematically illustrated in Fig. 8.3. The analysis used so far is also valid for situations like that in Fig. 8.3, if we divide the worldsheet V_n into two pieces which are glued together at a submanifold C (see Fig. 8.4).

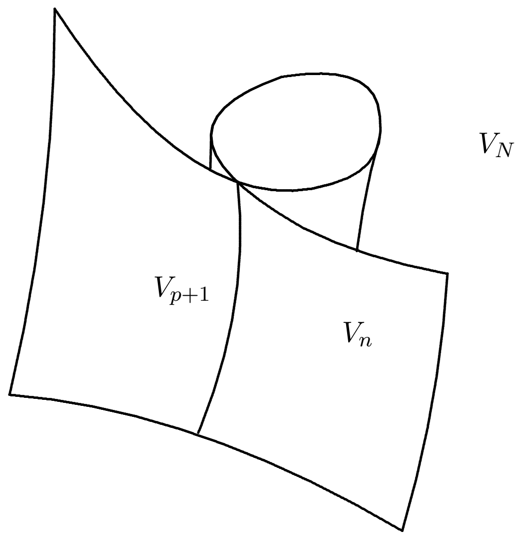

Figure 8.3. Illustration of a self-intersecting brane. At the intersection V_{p+1}, because of the contact interaction the stress–energy tensor on the brane V_n is singular and it manifests itself as matter on V_n. The manifold V_{p+1} is a worldsheet swept by a p-brane and it is is minimal surface (e.g., a geodesic, when $= 0$) in V_n.

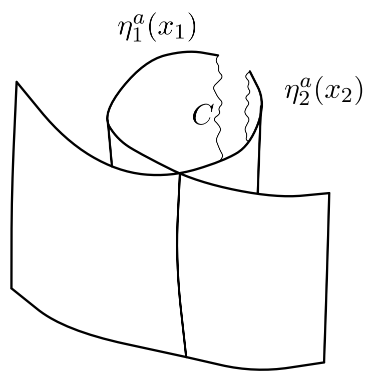

Figure 8.4. A self-intersecting worldsheet is cut into two pieces, described by $\eta_1^a(x_1)$ and $\eta_2^a(x_2)$, which are glued together at a submanifold C where the boundary condition $\eta_1^a(x_1)|_C = \eta_2^a(x_2)|_C$ is imposed.

There is a variety of ways a worldsheet can self-intersect. Some of them are sketched in Fig. 8.5.

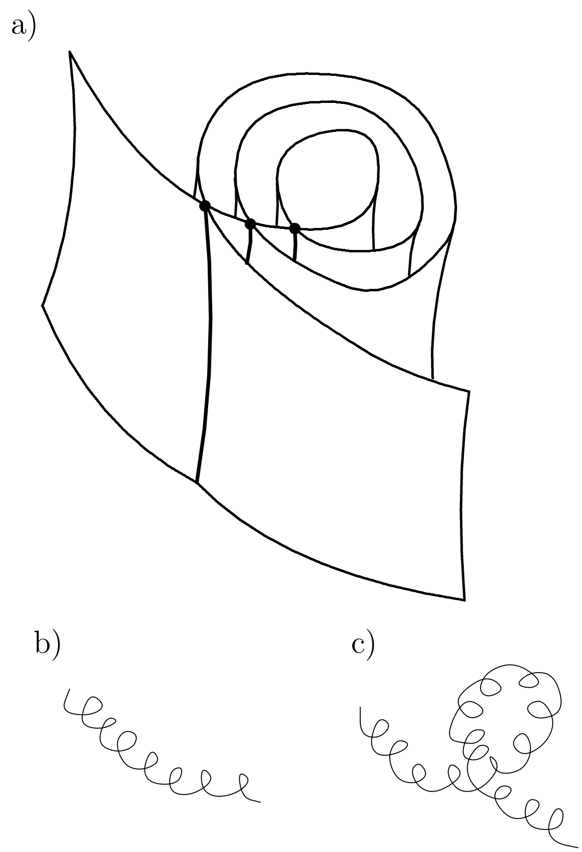

Figure 8.5. Some possible self-intersecting branes.

In this respect some interesting new possibilities occur, waiting to be explored in detail. For instance, it is difficult to imagine how the three particles entangled in the topology of the situation (a) in Fig. 8.5 could be separated to become asymptotically free. Hence this might be a possible classical model for hadrons composed of quarks; the extra dimensions of V_n would bring, via the Kaluza–Klein mechanism, the chromodynamic force into the action.

To sum up, it is obvious that a self-intersecting brane can provide a variety of matter configurations on the brane. This is a fascinating and intuitively clear mechanism for the origin of matter in a brane world.

8.4. COMPARISON WITH THE RANDALL–SUNDRUM MODEL

In our brane world model, which starts from the $\mathcal{M}$-space Einstein equations, we have assumed that gravity is localized on the brane. This was formally represented by the δ-function. In a more conventional approach the starting point is Einstein's equations in the ordinary space, not in $\mathcal{M}$-space. Let us therefore explore a little what such an approach has to say about gravity around a brane embedded in a "bulk".

Randall and Sundrum [95] have considered a model in which a 3-brane with tension κ is coupled to gravity, the cosmological constant Λ being different from zero. After solving the Einstein equations they found that the metric tensor decreases exponentially with the distance from the brane. Hence gravity is localized on the brane.

More precisely, the starting point is the action

$$I = \kappa \int \mathrm{d}^n x \sqrt{|f|}\, \delta^N(\eta - \eta(x))\, \mathrm{d}^N \eta + \frac{1}{16\pi G^{(N)}} \int \mathrm{d}^N \eta \sqrt{|f|}(2\Lambda + R), \quad (8.56)$$

which gives the Einstein equations

$$G_{ab} \equiv R_{ab} - \tfrac{1}{2} R\, \gamma_{ab} = -\Lambda \gamma_{ab} - 8\pi G^{(N)} T_{ab}\ , \quad (8.57)$$

$$T_{ab} = \int \kappa\, \mathrm{d}^n x \sqrt{|f|}\, f^{\mu\nu}\, \partial_\mu \eta_a \partial_\nu \eta_b\, \delta^N(\eta - \eta(x)). \quad (8.58)$$

Let us consider a 3-brane ($n = 4$) embedded in a 5-dimensional bulk ($N = 5$). In a particular gauge the worldsheet embedding functions are $\eta^\mu = x^\mu$, $\eta^5 = \eta^5(x^\mu)$. For a flat worldsheet $\eta^5(x^\mu) = y_0$, where y_0 is independent of x^μ, it is convenient to take $y_0 = 0$. For such a brane located at $\eta^5 \equiv y = 0$ the appropriate Ansatz for the bulk metric respecting the symmetry of the brane configuration is

$$\mathrm{d}s^2 = a^2(y) \eta_{\mu\nu}\, \mathrm{d}x^\mu\, \mathrm{d}x^\nu - \mathrm{d}y^2. \quad (8.59)$$

The Einstein equations read

$$\begin{aligned} {G^0}_0 &= {G^1}_1 = {G^2}_2 = {G^3}_3 \\ &= \frac{3a''}{a} + \frac{3a'^2}{a^2} = -\Lambda - 8\pi G^{(N)} {T^0}_0\ , \end{aligned} \quad (8.60)$$

$${G^5}_5 = \frac{6a'^2}{a^2} = -\Lambda\ , \quad (8.61)$$

where

$$T_{\alpha\beta} = \int \kappa\, \mathrm{d}^4 x \sqrt{|f|} f_{\alpha\beta}\, \delta^4(\eta^\mu - x^\mu) \delta(y) = \kappa \sqrt{|f|} f_{\alpha\beta} \delta(y), \quad (8.62)$$

whilst $T_{\alpha 5} = 0$, $T_{55} = 0$. The induced metric is

$$f_{\alpha\beta} = \partial_\alpha \eta^a \partial_\beta \eta_a = \eta_{\alpha\beta}\, a^2(y).$$

Hence

$$T^0{}_0 = T^1{}_1 = T^2{}_2 = T^3{}_3 = \kappa\, a^4 \delta(y). \tag{8.63}$$

From eq. (8.61), which can be easily integrated, we obtain

$$a = a_0\, \mathrm{e}^{-|y|\sqrt{-\Lambda/6}}. \tag{8.64}$$

Such a solution makes sense if $\Lambda < 0$ and it respects the symmetry $a(y) = a(-y)$, so that the bulk metric is the same on both sides of the brane.

Introducing $\alpha' = a'/a$ (where $a' \equiv \mathrm{d}a/\mathrm{d}y$) eq. (8.60) can be written as

$$3\alpha'' = -8\pi G^{(N)} \kappa\, a^4 \delta(y). \tag{8.65}$$

Integrating both sides of the latter equation over y we find

$$3(\alpha'(0^+) - \alpha'(0^-)) = -8\pi G^{(N)}\, \kappa a^4(0). \tag{8.66}$$

Using (8.64) we have

$$\frac{a'(0^+)}{a} = \alpha'(0^+) = -\sqrt{\frac{-\Lambda}{6}}\,,$$

$$\frac{a'(0^-)}{a} = \alpha'(0^-) = \sqrt{\frac{-\Lambda}{6}}\,, \tag{8.67}$$

$$a(0) = a_0 = 1.$$

Hence (8.66) gives

$$6\sqrt{\frac{-\Lambda}{6}} = 8\pi G^{(N)} \kappa \tag{8.68}$$

which is a relation between the cosmological constant Λ and the brane tension κ.

From (8.59) and (8.64) it is clear that the metric tensor is localized on the brane's worldsheet and falls quickly when the transverse coordinates y goes off the brane.

An alternative Ansatz. We shall now consider an alternative Ansatz in which the metric is conformally flat:

$$\mathrm{d}s^2 = b^2(z)(\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu - \mathrm{d}z^2). \tag{8.69}$$

The Einstein equations read

$$G^0{}_0 = \frac{3b''}{b^3} = -\Lambda - 8\pi G^{(N)}\kappa b^4(z) \tag{8.70}$$

$$G^5{}_5 = \frac{6b'^2}{b^4} = -\Lambda \tag{8.71}$$

The solution of (8.71) is

$$b = -\frac{1}{C + \sqrt{\frac{-\Lambda}{6}}|z|}. \tag{8.72}$$

From (8.70) (8.71) we have

$$3\left(\frac{b''}{b^2} - \frac{b'^2}{b^3}\right) = -8\pi G^{(N)}\kappa b^5\delta(z). \tag{8.73}$$

Introducing $\beta' = b'/b^2$ the latter equation becomes

$$3\beta'' = -8\pi G^{(N)}\kappa b^5\delta(z). \tag{8.74}$$

After integrating over z we have

$$3(\beta'(0^+) - \beta'(0^-)) = -8\pi G^{(N)}\kappa b^5(0), \tag{8.75}$$

where

$$\beta'(0^+) = C^{-4}\sqrt{\frac{-\Lambda}{6}}\,, \quad \beta'(0^-) = -C^{-4}\sqrt{\frac{-\Lambda}{6}}\,, \quad b(0) = -C^{-1}. \tag{8.76}$$

Hence

$$6\sqrt{\frac{-\Lambda}{6}} = 8\pi G^{(N)}\kappa C^{-1}. \tag{8.77}$$

If we take $C = 1$ then the last relation coincides with (8.68). The metric in the Ansatz (8.69) is of course obtained from that in (8.59) by a coordinate transformation.

THE METRIC AROUND A BRANE IN A HIGHER-DIMENSIONAL BULK

It would be very interesting to explore what happens to the gravitational field around a brane embedded in more than five dimensions. One could set an appropriate Ansatz for the metric, rewrite the Einstein equations and attempt to solve them. My aim is to find out whether in a space of sufficiently high dimension the metric — which is a solution to the Einstein equations — can be approximated with the metric (8.3), the conformal factor being localized on the brane.

Let us therefore take the Ansatz

$$\gamma_{ab} = \Omega^2 \bar{\gamma}_{ab} \; . \tag{8.78}$$

We then find

$$R_a{}^b = \Omega^{-2} \bar{R}_a^b + (N-2)\Omega^{-3}\Omega_{;a}{}^{;b} - 2(N-2)\Omega^{-4}\Omega_{,a}\Omega^{,b} + \Omega^{-3}\delta_a{}^b \Omega_{;c}{}^{;c} + (N-3)\Omega^{-4}\delta_a{}^b\Omega_{,c}\Omega^{,c} \; , \tag{8.79}$$

$$R = \Omega^{-2}\bar{R} + 2(N-1)\Omega^{-3}\Omega_{;c}{}^{;c} + (N-1)(N-4)\Omega^{-4}\Omega_{,c}\Omega^{,c}. \tag{8.80}$$

Splitting the coordinates according to

$$\eta^a = (x^\mu, y^{\bar{\mu}}), \tag{8.81}$$

where $y^{\bar{\mu}}$ are the transverse coordinates and assuming that Ω depends on $y^{\bar{\mu}}$ only, the Einstein equations become

$$\begin{aligned} G_\mu{}^\nu &= \Omega^{-2}\bar{G}_\mu^\nu + \Omega^{-3}\delta_\mu{}^\nu \Big[(2-N)\Omega;\bar{\mu}^{;\bar{\mu}} + (N-3)\Omega^{-1}\Omega,\bar{\mu}\Omega^{,\bar{\mu}}\Big] \\ &= -8\pi G^{(N)} T_\mu{}^\nu - \Lambda \delta_\mu{}^\nu \; , \end{aligned} \tag{8.82}$$

$$\begin{aligned} G_{\bar{\mu}}{}^{\bar{\nu}} &= \Omega^{-2}\bar{G}_{\bar{\mu}}^{\bar{\nu}} + \Omega^{-3}\Big[(N-2)\Omega_{;\bar{\mu}}{}^{;\bar{\nu}} - 2(N-2)\Omega^{-1}\Omega_{,\bar{\mu}}\Omega^{,\bar{\nu}} \\ &\quad + \delta_{\bar{\mu}}{}^{\bar{\nu}}\Big((2-N)\Omega_{;\bar{\alpha}}{}^{;\bar{\alpha}} + \Omega^{-1}\Omega_{,\bar{\alpha}}\Omega^{,\bar{\alpha}}\left((N-3)+(N-1)(N-4)\right)\Big)\Big] \\ &= -8\pi G^{(N)} T_{\bar{\mu}}{}^{\bar{\nu}} - \Lambda\delta_{\bar{\mu}}{}^{\bar{\nu}} \, . \end{aligned} \tag{8.83}$$

Let $T_a{}^b$ be the stress–energy tensor of the brane itself. Then $T_{\bar{\mu}}{}^{\bar{\nu}} = 0$ (see eq. (8.62). Using (8.83) we can express $\Omega^{-1}\Omega_{,\bar{\alpha}}\Omega^{,\bar{\alpha}}$ in terms of $\Omega_{;\bar{\alpha}}{}^{;\bar{\alpha}}$ and insert it into (8.82). Taking[4] $N > 5$, $\Lambda = 0$ and assuming that close to the

[4] If dimension $N = 5$ then Λ must be different from zero, otherwise eq. (8.83) gives $\Omega_{,5}\Omega^{,5} = 0$, which is inconsistent with eq. (8.82).

brane the term $\Omega^{-2}\bar{G}_a^b$ can be neglected we obtain the Laplace equation for Ω

$$\Omega_{;\bar{\mu}}{}^{;\bar{\mu}} = 16\pi G^{(N)} T\, \Omega^3 A \tag{8.84}$$

where

$$\begin{aligned} A &= (N-2)(N-1) \\ &\times \left[2 - (N-4) \frac{(N-2) + \bar{n}(2-N)}{-2(N-2) + \bar{n}(N-3) - \frac{\bar{n}}{2}(N-1)(N-4)} \right], \end{aligned} \tag{8.85}$$

$\bar{n}$ being the dimension of the transverse space, $\bar{n} = \delta_{\bar{\mu}}{}^{\bar{\mu}}$, and $T \equiv T_a{}^a = T_\mu{}^\mu$.

The above procedure has to be taken with reserve. Neglect of the term $\Omega^{-2}\bar{G}_a^b$ in general is not expected to be consistent with the Bianchi identities. Therefore equation (8.85) is merely an approximation to the exact equation. Nevertheless it gives an idea about the behavior of the function $\Omega(y^{\bar{\mu}})$.

The solution of eq. (8.84) has the form

$$\Omega = -\frac{k}{r^{\bar{n}-2}}, \tag{8.86}$$

where r is the radial coordinate in the transverse space. For a large transverse dimension $\bar{n}$ the function Ω falls very quickly with r. The gravitational field around the brane is very strong close to the brane, and negligible anywhere else. The interaction is practically a *contact interaction* and can be approximated by the δ-function. Taking a cutoff r_c determined by the thickness of the brane we can normalize Ω according to

$$\int_{r_c}^{\infty} \Omega(r)\, \mathrm{d}r = \frac{k}{(\bar{n}-1) r_c^{\bar{n}-1}} = 1 \tag{8.87}$$

and take $\Omega(r) \approx \delta(r)$.

The analysis above is approximate and requires a more rigorous study. But intuitively it is clear that in higher dimensions gravitational interaction falls very quickly. For a *point particle* the gravitational potential has the asymptotic behavior $\gamma_{00} - 1 \propto r^{-(N-3)}$, and for a sufficiently high spacetime dimension N the interaction is practically contact (like the Van der Waals force). Particles then either do not feel each other, or they form bound states upon contact. Network-like configurations are expected to be formed, as shown in Fig.8.6. Such configurations mimic very well the intersecting branes considered in Secs. 8.1–8.3.

In this section we have started from the conventional theory of gravitation and found strong arguments that in a space of very high dimension the

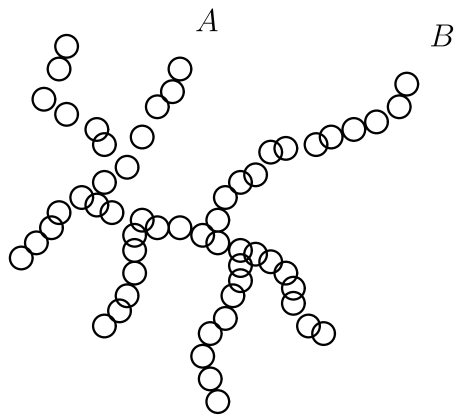

Figure 8.6. In a space of very large dimension separated point particles do not feel gravitational interaction, since it is negligible. When two particles meet they form a bound system which grows when it encounters other particles. There is (practically) no force between the 'tails' (e.g., between the points A and B). However, there is tension within the tail. (The tail, of course, need not be 1-dimensional; it could be a 2, 3 or higher-dimensional brane.)

gravitational force is a contact force. Various *network-like configurations* are then possible and they are stable. Effectively there is no gravity outside such a network configuration. Such a picture matches very well the one we postulated in the previous three sections of this chapter, and also the picture we considered when studying the $\mathcal{M}$-space formulation of the membrane theory.