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The Extended Special Theory of Relativity[1]

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Abstract

Two systems, in which the square of the 4-dimensional vector (the quadratic form) has the opposite signs, move with a relative speed v that is greater than the speed of 1ight. The transformation matrix for such systems is derived. Between the two systems there is no physical connection, because the energy is infinite at v = c. By assuming that space is quantised, we showed that a body of given linear dimensions in the direction of the motion can move only with discrete velocities and have only discrete values of energv. The exact value v = c is not allowed, except for the special kind of particles (photons). Because of the discontinuity, the jumps are possible from the states with $v < c$ into the states with $v > c$.

The basic postulate that we start from is that natural laws must be the same in all systems of reference. Here it will be called the principle of relativity. The speed of light occurs as a physical constant in Maxwell equations and is found to have the same value in all systems of reference. From Fig. 1a it is evident that this is true for all systems with axes x and ct symmetric with respect to the line which represents the propagation of light. The line AB is the world line of a particle that is at rest in the system S', and moving uniformly in the system S.

The system, that has with respect to the system S a speed $v > c$ is illustrated in Fig. 1b (x axis is still arbitrary). The line CD represents a particle at rest with respect to the system S' , or moving with the speed $v > c$ with respect to the system S. If we choose the x axis as shown in Fig. 2, then the velocity of light is the same in both systems S and S'. The speed of light is the same also in the systems that have greater than light relative speed. Phenomena, as observed in one or another system are very different. But we must stress that the laws are the same in all systems, whereas phenomena are not.

Let us find the transformations that connect the quantities in different systems of reference. First, let us find the transfornations between the systems with their relative speed lower than the speed of light. To make the problem as simple as possible, we shall use the plot that was proposed by R.W. Brehme in 1962, (Fig. 3). From Fig. 3a it follows

[1] This is a Latex version of the original article, with typos corrected.

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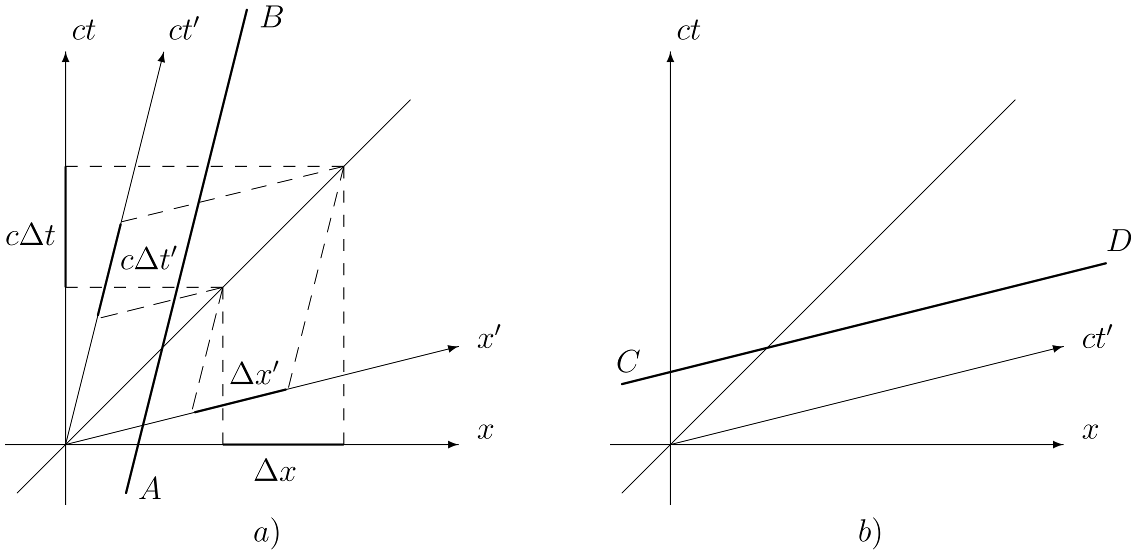

Figure 1: a) The world line of a light signal originating at the origln of the system S. b) A world line of a particle that travels faster than light with respect to the system S.

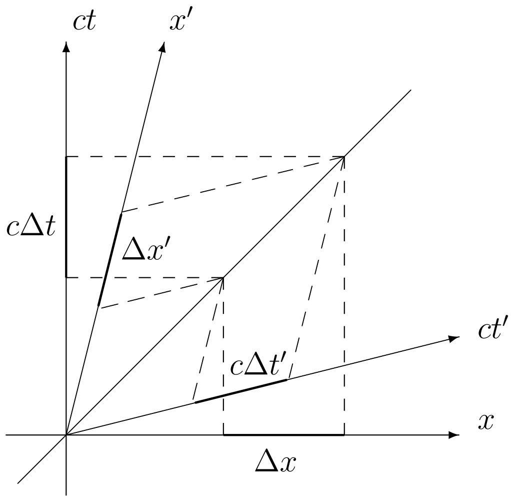

Figure 2: Graphical representation of two systems in relative motion with a velocity that is greater than the velocity of light.

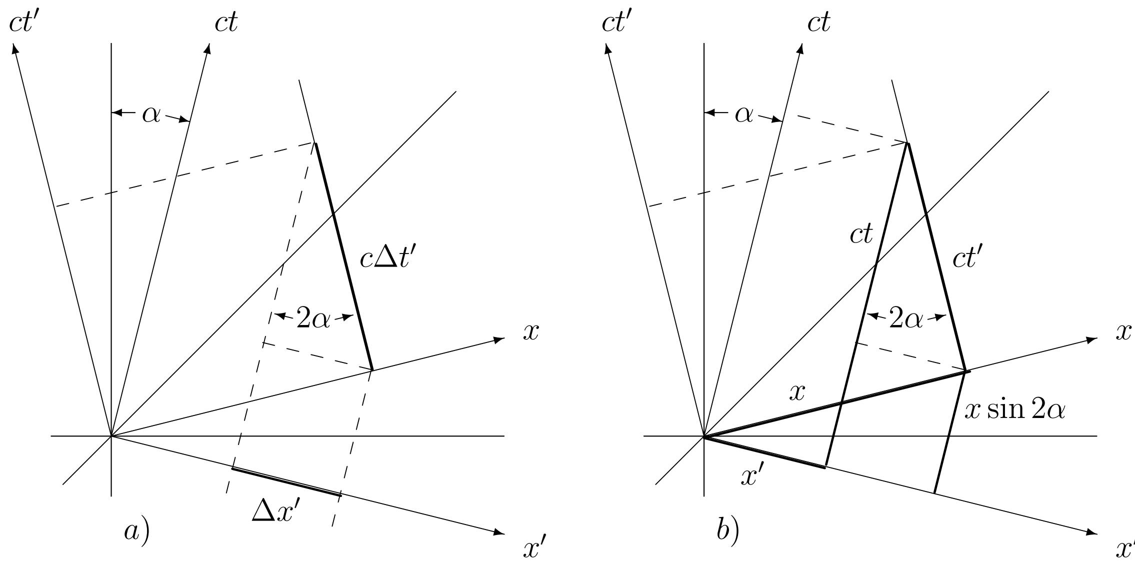

Figure 3: a) Brehme diagram for two systems in relative motion. b) Construction for obtaining the Lorentz transformation equations.

$$\begin{aligned} &\Delta x' = c\Delta t' \sin 2\alpha, \\ &\Delta x'/\Delta t' = v, \\ &v/c = \sin 2\alpha. \end{aligned} \tag{1}$$

$$\begin{aligned} &x' = (x - ct' \operatorname{tg} 2\alpha)\cos 2\alpha, \\ &ct' = (ct - x \sin 2\alpha)/\cos 2\alpha. \end{aligned} \tag{2}$$

So

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \qquad \text{and} \qquad t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}} \tag{3}$$

The systems with relative speed that is greater than the speed of light are presented in Fig. 4a and Fig. 4b. It follows

$$\begin{aligned} &c\Delta t'/\Delta x' = \sin 2\alpha, \\ &c/v = \sin 2\alpha, \end{aligned} \tag{4}$$

$$\begin{aligned} &ct' = (x - x' \operatorname{tg} 2\alpha)\cos 2\alpha, \\ &x' = (ct - x \sin 2\alpha/\cos 2\alpha. \end{aligned} \tag{5}$$

$$x' = \frac{ct - xc/v}{\sqrt{1 - c^2/v^2}},$$
$$t' = \frac{x/c - ct/v}{\sqrt{1 - c^2/v^2}}. \tag{6}$$

The inverse transformation is

$$x = \frac{ct' + x'c/v}{\sqrt{1 - c^2/v^2}} = \frac{vt' + x'}{\sqrt{\beta^2 - 1}},$$
$$t = \frac{x'/c + ct'/v}{\sqrt{1 - c^2/v^2}} = \frac{t' + vx'}{\sqrt{\beta^2 - 1}}. \tag{7}$$

where $\beta = v/c$. We see that $x'^2 - c^2t'^2 = c^2t^2 - x^2$. The square of the 4-dimensional distanee changes the sign. For the further explanation see Appendix.

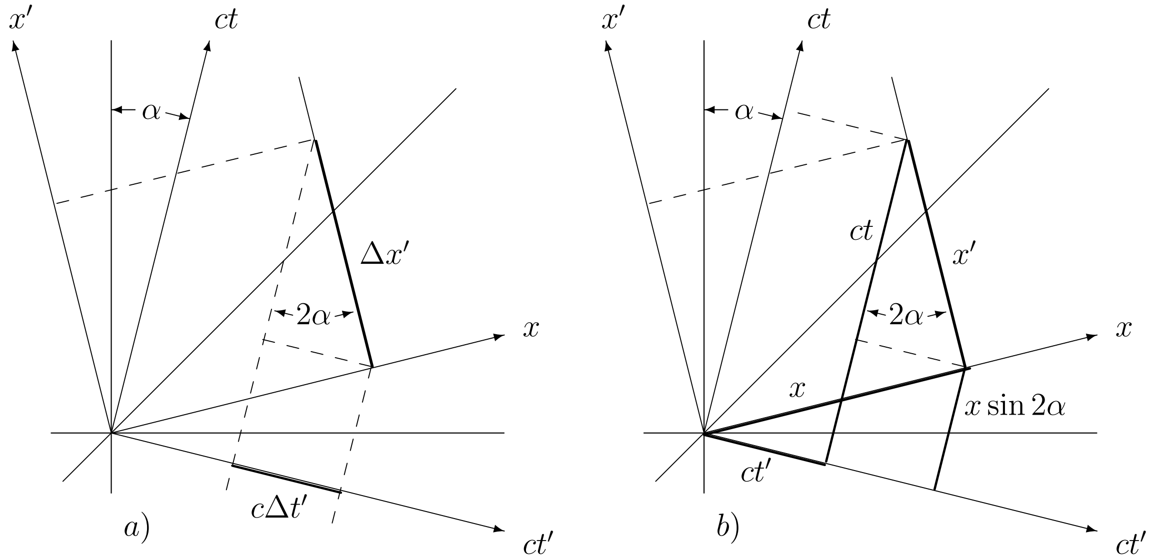

Figure 4: a) The analogous Brehme diagram for two systems in relative motion with a velocity that is greater than the volicity of light. b) Construction for obtaining the Lorentz transformation equations for systems in faster than light relative motion.

In order to find the formula for addition of velocities, we put

$$\mathrm{d}x = \frac{v\mathrm{d}t' + \mathrm{d}x'}{\sqrt{\beta^2 - 1}}\ , \qquad \mathrm{d}t = \frac{\mathrm{d}t' + v\mathrm{d}x'/c^2}{\sqrt{\beta^2 - 1}} \tag{8}$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = u = \frac{v\mathrm{d}t' + \mathrm{d}x'}{\mathrm{d}t' + v\mathrm{d}x'/c^2} = \frac{v + v'}{1 + vv'/c^2}. \tag{9}$$

This is the same formula as for the systems with relative speed $v < c$.

Examples:

$$1)\ \ v' = 0 \Rightarrow u = v.$$
$$2)\ \ v' = c,\ v = \alpha c,\ \alpha > 1 \Rightarrow u = \frac{\alpha c + c}{1 + c\alpha c/c^2} = c. \qquad (10)$$

The speed of light is constant also in the systems with relatives speeds greater than the speed of 1ight. On Fig. 5 the plot u versus v' (with v as parameter) is drawn.

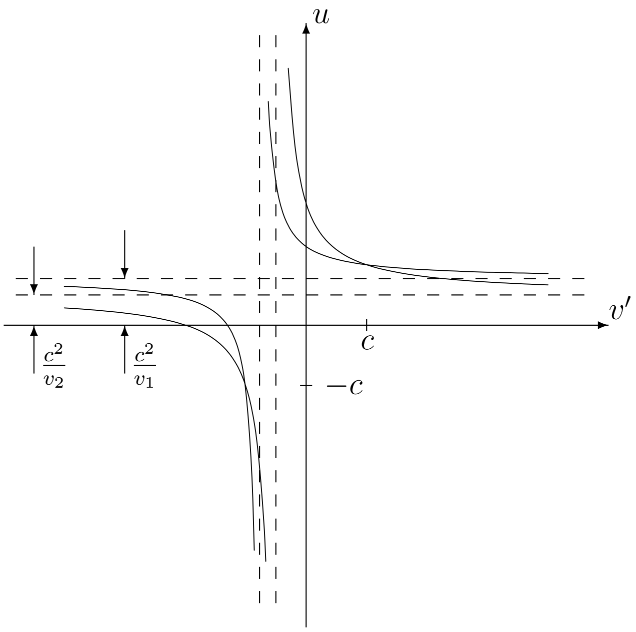

Figure 5: Graph of the velocity u as observed in the system S versus the velocity v' as observed in system S', with relative velicities v_1 and v_2 as parameters.

The time interval is

$$t_2' - t_1' = \frac{t_2 - t_1}{\sqrt{\beta^2 - 1}}(vu/c^2 - 1), \qquad (11)$$

where $u = (x_2 - x_1)/(t_2 - t_1)$. For $vu/c^2 > 1$, the time interval has the same sign, and for $vu/c^2 < 1$ the time interval has the opposite sign. All the troubles about causality remain the same as in the ordinary hypothesis of tachyons. But we could avoid the troubles by assuming that there is no need for causality in the nature, and that it is only the way of our perception of the world. If two events, as seen from a certain system of reference, seem to be in causal connection, it means in fact that some information transport is behind this. The thought experiment of R.C.Tolman (1917) is in fact the problem of possibility of information transport from the observer in one system of reference to the observer in another system. Under certain conditions (if

$u > c^2/v$, u is the speed of the tachyon, $v < c$ the relative velocity of the systems) the time interval in that experiment is negative, and no information transport is possible[3].

The inverse transformation matrix corresponding to Eq. (7) is

$$\|\alpha'\| = \begin{pmatrix} \frac{1}{\sqrt{\beta^2-1]}} & 0 & & \frac{-i\beta}{\sqrt{\beta^2-1]}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{i\beta}{\sqrt{\beta^2-1]}} & 0 & & \frac{1}{\sqrt{\beta^2-1]}} \end{pmatrix}. \tag{12}$$

The momentum fourvector in the moving system ($v > c$) is ${}^4p = (\mathbf{p}, iW/c)$, and in the system at rest is ${}^4p_0 = (0, iW_0/c)$. The transformation is ${}^4p = \|\alpha'\|\, {}^4p_0$. It follows

$$p_x = (vW_0/c^2)(\beta^2-1)^{-1/2} = mv, \tag{13}$$
$$W = W_0(\beta^2-1)^{-1/2}. \tag{14}$$

From (13) and (14) it follows that $W = mc^2$, $W_0 = m_0c^2$ and $m = m_0(\beta^2-1)^{-1/2}$. It is evident that the absolute value of the 4-dimensional vector changes the sign if the relative velocity of the systems is greater than c:

$$p^2 - W^2/c^2 = \frac{W_0^2(v^2/c^4 - 1/c^2}{v^2/c^2 - 1} = W_0/c^2. \tag{15}$$

So we have the following relations:

$$p^2 - W^2/c^2 = -W_0/c^2\ , \quad \text{if} \ \ v < c \tag{16}$$

or

$$W = (c^2p^2 + W_0^2)^{1/2} = m_0c^2(1-\beta^2)^{-1/2} \tag{17}$$

and

$$p^2 - W^2/c^2 = W_0/c^2\ , \quad \text{if} \ \ v > c \tag{18}$$

or

$$W = (c^2p^2 - W_0^2)^{1/2} = m_0c^2(\beta^2-1)^{-1/2} \tag{19}$$

So we have two distinct expressions for dependence of a particle's energy on its velocity. The first one is valid in the region $O < v < c$, and the second one in the region $c < v < \infty$ (Fig. 6). We must stress once again that in the second case we made

[3]A note added in 2011: Soon after having written this, I no longer belived that no information transfer is possible if $u > c^2/v$. I adopted an explanation that causality problems can be resolved by taking into account the Everett interpretation of quantum mechanics. Ten years later I wrote about this in *Lett. N. Cim* **30**, 111 (1981), and thirty years later in my book "The Landscape of Theoretical Physics" (Kluwer, 2001), arXiv http://arxiv.org/abs/gr-qc/0610061 .

the transformations from the system at rest (partic1e's proper system) to the moving system with velocity $v > c$. Therefore, m_0 is the particle's rest mass. It means that a particle traveling faster than light is not a special kind of particle, but an ordinary one. The question arises, however, how is it possible to accelerate the particle beyond the infinite light barrier? The present derivation would have no physical meaning if the light barier were really infinite. Mathematically the systems that are faster than light perhaps could exist, but there would be no physical connection between them.

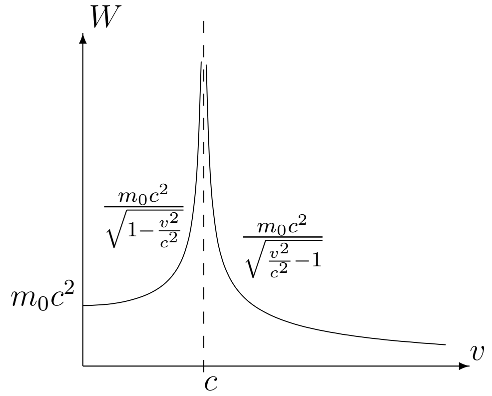

Figure 6: Dependence of a particle's energy W on its velocity v.

The first, intuitive argument against the possibility of the infinite light barrier is the appearence of a singularity. Singularities are always results of some idealisations. What could be an idealisation in our theory? We have assumed that space is continuous and that it is possible to determine a distance with infinite accuracy, and that there exist infinitely small distances. First, let us suppose that there exists a natural uncertalnty of length Δx_0. According to the theory of relativity, the length contraction occurs when a body is moving: $l = l_0(1-\beta^2)^{1/2}$, where l_0 is the body's length in the system at rest. At a certain high velocity v_c, the length would become smaller than Δx_0. If Δx_0 is the smallest observable length in the given system of reference, Lorentz transformations appoint something that physically is not true for $v_c < v < c$. The question arises, which transformations could replace the Lorentz transformations in that region? If the space is contonouos, then the Lorentz transformations are the only ones that are consistent with the principle of relativlty, and we cannot explain how to reach or pass the light speed. By supposing the existance of Δx_0, we showed only that something is wrong with the Lorentz transformations and our picture.

So the next step is to assume that space is quantised. It means that length is an integral multiple of some unit x_0:

$$\begin{aligned} x &= nx_0 \ , & n &= 0,1,2,3,... \\ \Delta x = l &= \Delta n x_0 \ , & \Delta n &= 0,1,2,3,... \end{aligned} \tag{20}$$

If l is the length of a given body or particle or part of the system, it must be finite, so the values $\Delta n = 0$ is not possible. If we insert (20) into the Lorentz transfornations, we have

$$\begin{aligned}
&n'x_0 = \frac{nx_0 - vt}{(1-\beta^2)^{1/2}}\ , \qquad \text{for} \quad v < c \\
&l = l'(1-\beta^2)^{1/2} \\
&\Delta n x_0 = \Delta n' x_0 (1-\beta^2)^{1/2} \\
&\Delta n = \Delta n'(1-\beta^2)^{1/2}\ , \qquad \Delta n = 1, 2, 3, ..., \Delta n' \\
&v/c = (1 - \Delta n^2/\Delta n'^2)^{1/2}
\end{aligned} \tag{21}$$

Special cases:

$$\begin{aligned}
&1)\ \ \Delta n' = 1, \quad \Delta n = 1 \\
&\quad v/c = 0 \\
&2)\ \ \Delta n' = 2, \quad \Delta n = 2, 1 \\
&\quad v/c = 0, \qquad \text{for}\ \ \Delta n = 2 \\
&\quad v/c = (1-(1/2)^2)^{1/2} = \sqrt{3/4}, \qquad \text{for}\ \ \Delta n = 1 \\
&3)\ \ \Delta n' \gg 1 \\
&\quad v/c = 0, \quad \text{for} \quad \Delta n = \Delta n' \\
&v/c = \left(1 - \left(\frac{\Delta n' - 1}{\Delta n'}\right)^2\right)^{1/2} \approx \sqrt{2/\Delta n'} \approx 0
\end{aligned} \tag{22}$$

The limit in which the body's length tends to infinity, is the continuous case.

These cases are graphically presented in Fig. 7, where we used the geometrical presentation of H. Minkowski. Because of our assumption of quantised coordinate, only discrete hyperbolas are drawn and only in the systems with discrete values of velocities, the length (as appears in the system S at rest) has discrete values $\Delta n\, x_0$.

If $v > c$, we have the similar situation:

$$\begin{aligned}
&n'x_0 = \frac{vt - nx_0}{(\beta^1 - 1)^{1/2}} \\
&\Delta x = -(\beta^1 - 1)^{1/2} \Delta x \qquad \text{or} \qquad \Delta n = -(\beta^1 - 1)^{1/2} \Delta n'.
\end{aligned} \tag{23}$$

Here length dilatation instead of length contraction takes pIace. Special cases:

$$\begin{aligned}
1) \qquad &\Delta n' = 1, \qquad \Delta n = 1, 2, 3... \\
&v/c = \sqrt{2}, \quad \text{for} \quad \Delta n = 1 \\
&v/c = \sqrt{3}, \quad \text{for} \quad \Delta n = 2 \\
&\text{etc.}
\end{aligned} \tag{24}$$

$$
\begin{aligned}
2) \qquad & \Delta n' = 2, \qquad \Delta n = 1, 2, 3... \\
& v/c = \sqrt{5/4} \quad \text{for} \quad \Delta n = 1 \\
& v/c = \sqrt{2} \quad \text{for} \quad \Delta n = 2 \\
& v/c = \sqrt{13/4} \quad \text{for} \quad \Delta n = 3 \\
& \text{etc.}
\end{aligned} \tag{25}
$$

For a very large $\Delta n'$ there is practically a continuous case. In Fig. 7 the cases for $v > c$ are also drawn. Under a transformation between the systems whose relative velocity is greater than c, the square of a 4-dimensional vector changes the sign and the calibration curves are $X^2 - c^2t^2 = -x_0^2$, instead of $X^2 - c^2t^2 = x_0^2$.

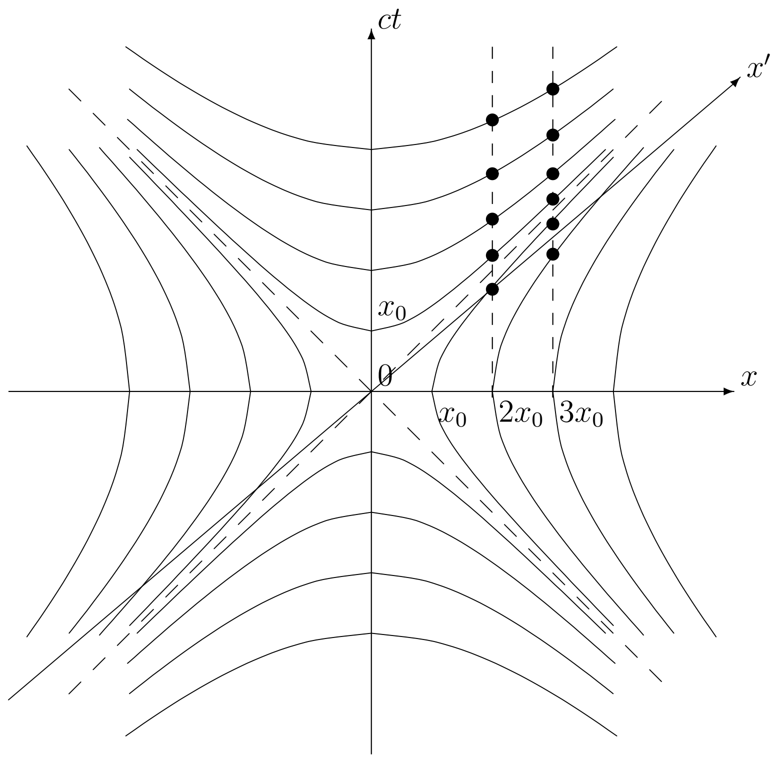

Figure 7: Discrete calibration hyperbolas. The lines $0-\bullet$ represent possible directions of the x-axes. A particle of a given linear dimensions in the direction of motion can travel only with velocities that are determined by the lines $0-\bullet$. Faster-than-light velocities are also included. Calibration curves $x^2 - c^2t^2 = x_0^2$ are valid for the transformations between the systems with $v < c$, and the curves $x^2 - c^2t^2 = -x_0^2$ for the systems withs $v > c$.

What moves in fact is not such an imaginary thing as a system of reference, but a abody (rocket) or a particle. What we observe in a reference system is the motion of a body. So if we examine the motion of the body in the whole, we must take the length l_0 of the whole body (in the direction of motion) in our equations. It may be compared to the center of mass which is introduced in order to describe the motion of the whole body, when we are not interested in details within the body. Here we also do not ask ourselves about details within the body, which we even do not see when the velocity is high enough, since the condition of quantisation is not fulfilled for these details at the same velocity as for the whole body. A "continuous" rod would be seen at this velocity as the system of discrete points. At the critical velocity, the longitudinal structure (in the direction of motion) would disappear. At the speed of light, the body as the whole would disappear (see eq..(21)), hence the speed of light is not possible [in such discretised spacetime].

What about the energy? If the length of a body is large enough ($l \gg x_0$) , the expressions

$$W = \frac{m_0 c^2}{(1-\beta^2)^{1/2}} \,, \qquad \text{if} \quad v < c \tag{26}$$

and

$$W = \frac{m_0 c^2}{(\beta^2 - 1)^{1/2}} \,, \qquad \text{if} \quad v < c \tag{27}$$

are valid quite well, with the restriction that only discrete values of velocity are allowed and that $v \neq c$. It follows that only discrete energies are possible. While a particle is being accelerated, it jumps from one energy state into another. There exist a maximum possible energ, and it is finite. A particle may also occupy the states, belonging to the velocities that are greater than the velocity of light.(Fig. 8).

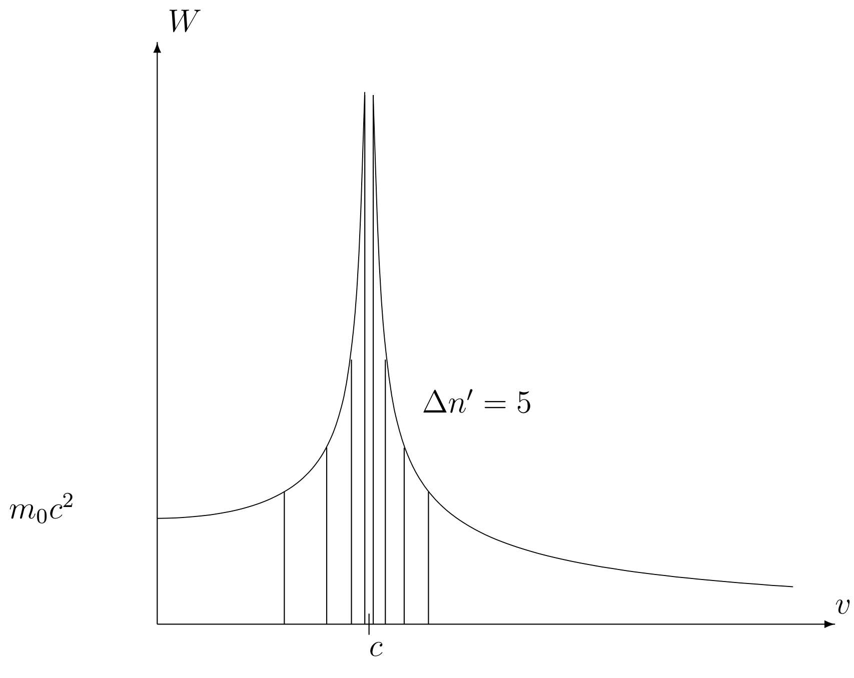

Figure 8: Discrete energies belonging to discrete velocities of a particle when its proper length $\Delta n' \, x_0$ is small.

Acceleration beyond the light barrier is thus possible. If $v < c$,

$$l = l'(1-\beta^2)^{1/2} \geq x_0 \ , \qquad \Delta n' < n < 1$$
$$(1-\beta^2)^{1/2} = l/l' = \Delta n/\Delta n'. \tag{28}$$

For $\Delta n' \gg 1$, the velocity can be practically the same as the velocity of light, but the particle's energy is finite (see eq. (30)).

If $v > c$,

$$l = l'(\beta^2-1)^{1/2} \geq x_0 \ , \qquad \Delta n' < n < 1$$
$$(\beta^2-1)^{1/2} = l/l' = \Delta n/\Delta n' \ , \quad \Delta n' > 1. \tag{29}$$

Hence

$$W = m_0c^2\Delta n'/\Delta n, \qquad \Delta n'/\Delta n > 1, \quad \text{for} \ \ v < c$$
$$\Delta n'/\Delta n < 1, \quad \text{for} \ \ v < c \tag{30}$$

For $\Delta n' \gg 1$, we have practically continuous dependenee of energy on velocity. A particle may have a velocity that is nearly the same as that of light, but, as it can be seen from eq.(30), the energy is not infinite, since the minimum value of Δn is 1 and $\Delta n'$ is never infinite (Fig. 9).

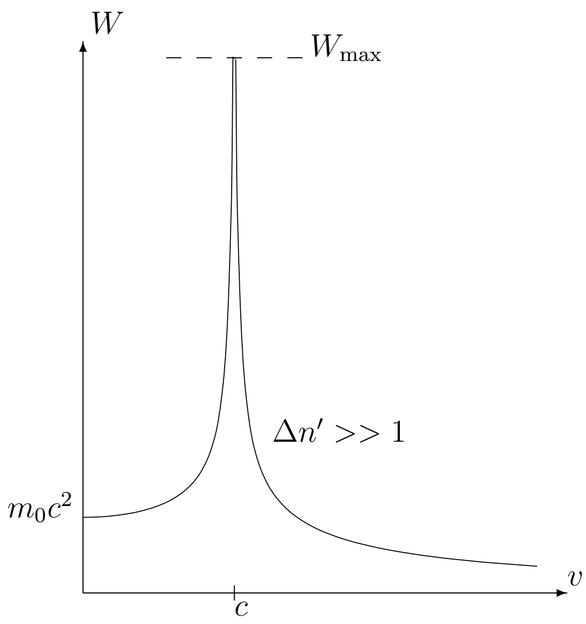

Figure 9: Practically continuous dependence of a particle's energy on its velocity when particle's proper length $\Delta n' x_0$ is very large. But there exists a maximum value of energy in agreement with Eq. (30).

Examples:

1) Estimation of $W_{\max}$ for proton: Let us suppose $x_0 = 10^{-17}$ m, the diameter of proton $l_0 = 2.4 \times 10^{-15}$ m,

$$W_{\max} = m_0 c^2 l_0 / x_0 = 931\text{MeV} \frac{2.4 \times 10^{-15}}{10^{-17}} = 220\text{GeV}$$

2) Rocket: $m_0 = 10^5$ kg, $l_0 = 100$ m,

$$W_{\max} = \frac{10^5 \times 9 \times 10^{16} \times 100}{10^{-17}} = 9 \times 10^{40}\text{J} = mc^2 \ , \quad m = 10^{24}\text{kg}.$$

Expressions (26),(27) are valid only for very big $\Delta n'$ (i.e. proper lengths of bodies), while for small values of $\Delta n'$ ($\Delta n' = 1, 2, ...$) new laws of motion should be found and new expressions for energy derived. Our hypothesis points only that such particles can exist only at few discrete velocities, and the first possible velocity is very high. Some elementary particles have perhaps such properties— they are generated at high energies, Photon is special kind of particle, since it moves with the speed of light.

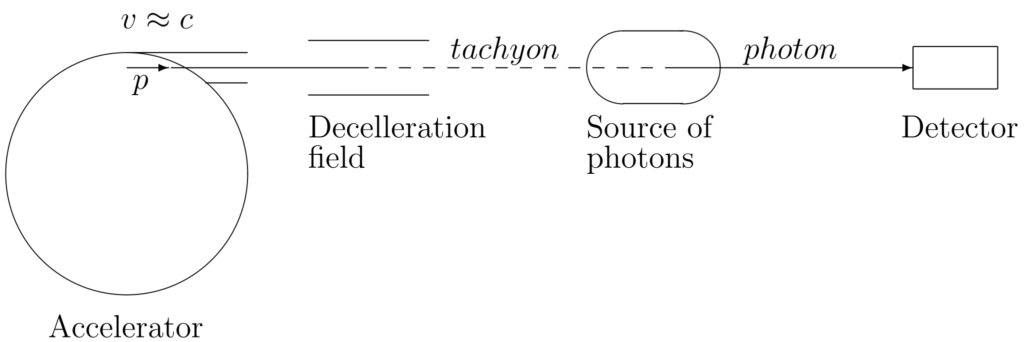

Figure 10: The scheme of experiment for detecting tachyons. Tachyons could be detected through inversely scattered photons. A tachyon that is faster than light collides with a photon that travels in the same direction as the tachyon and gives the photon a part of its momentum and energy, if the tachyon's momentum is greater than the photon's one. Hence the tachyon's velocity should not be too high.

In order to test our hypothesis, we propose the following experiment. In 300 GeV proton accelerator protons should be accelerated as much as possible ($v \approx c$), and then exposed to sudden loss of energy. A fraction of flux will become faster than light, and thus the inverse Compton effect could be observable (Fig. 10). The inverse Compton effect is possible, when a particle is faster than photon, and if the particle has greater momentum than photon. Here we said nothing about the quantisation of time. A special treatment would be neeessary if a quantum of time t_0 is of the order x_0/c. If $t_0 \ll x_0/c$, there is no need for special treatment.

Appendix

We will show here a more general treatment of transformations between systems that move relative to each other. Because the speed of light must be the same in all systems of reference, the following must be valid for two events that are connected with a light signal:

$$\begin{aligned}&(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2-c^2(t_2-t_1)^2\\&=(x_2'-x_1')^2+(y_2'-y_1')^2+(z_2'-z_1')^2-c^2(t_2'-t_1')^2=0\end{aligned} \quad (31)$$

The distance squared between two arbitrary events (that are not connected with the light signal) is

$$s_{12}^2=(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2-c^2(t_2-t_1)^2. \quad (32)$$

If two events are infinitely close together,

$$ds^2=dx^2+dy^2+dz^2-c^2dt^2. \quad (33)$$

If $ds=0$ in one system, $ds'=0$ in another system too. 0n the other hand, ds and ds' are infinitely small, hence

$$ds^2=a\,ds'^2. \quad (34)$$

The coefficient a depends only on the relatlve velocity between two systems, since the space is isotropic. Let $S,\ S_1,\ S_2$ be three systems and let $v_1,\ v_2$ be the relative velocities of S_1 and S_2 with respect to S. Then

$$\begin{aligned}&ds^2=a(v_1)ds_1^2\ ,\quad ds^2=a(v_2)ds_2^2\quad\text{and}\quad ds_1^2=a(v_{12})ds_2^2\\ \Rightarrow\quad&\frac{a(v_2)}{a(v_1)}=a(v_{12}).\end{aligned} \quad (35)$$

Because v_{12} depends on the angle between vectors $\mathbf{v}_1$ and $\mathbf{v}_2$, but the left hand side of Eq. (35) is independent of the angle, the solution is

$$\text{I.}\qquad a(v_2)=1\ ,\quad a(v_1)=1\ ,\quad a(v_{12})=1. \quad (36)$$

$$\text{II.}\qquad a(v_2)=-1\ ,\quad a(v_1)=-1\ ,\quad a(v_{12})=1. \quad (37)$$

In the first case, $ds^2=ds'^2$. If the distance is not infinitely small, then $s_{12}^2={s'}_{12}^2$. Let us introduce $x_1=x,\ x_2=y,\ x_3=z,\ x_4=ict$ and insert it into the latter equation [in which one point is the coordinate origin]:

$$x_1^2+x_2^2+x_3^2+x_4^2={x_1'}^2+{x_2'}^2+{x_3'}^2+{x_4'}^2. \quad (38)$$

This is possible only for rotations in 4-dimensional space. If the systems move along the x axis, the y and the z axis remain unchanged. So we have

$$
\begin{aligned}
x'_1 &= \alpha_{11}x_1 + \alpha_{14}x_4 \\
x'_2 &= \alpha_{22}x_2 \\
x'_3 &= \alpha_{33}x_3 \\
x'_4 &= \alpha_{41}x_1 + \alpha_{44}x_4 .
\end{aligned} \tag{39}
$$

By inserting this into (38) we obtain

$$
\begin{aligned}
&\alpha_{11}^2 + \alpha_{41}^2 = 1, \\
&\alpha_{14}^2 + \alpha_{44}^2 = 1, \\
&\alpha_{11}\alpha_{14} + \alpha_{41}\alpha_{44} = 0 \ , \qquad \frac{\alpha_{41}}{\alpha_{11}} = K, \\
&\alpha_{22}^2 = 1 \ , \qquad \alpha_{33} = 1.
\end{aligned} \tag{40}
$$

The solution is

$$
\begin{aligned}
&\alpha_{11} = \frac{\pm 1}{(1+K^2)^{1/2}} \ , \quad \alpha_{14} = \frac{\mp K}{(1+K^2)^{1/2}}, \\
&\alpha_{41} = \frac{\pm K}{(1+K^2)^{1/2}} \ , \quad \alpha_{44} = \frac{\pm 1}{(1+K^2)^{1/2}}, \\
&\alpha_{22} = \pm 1 \ , \qquad \alpha_{33} = \pm 1.
\end{aligned} \tag{41}
$$

So we have

$$
x' = \frac{\pm x \mp Kict}{(1+K^2)^{1/2}}. \tag{42}
$$

In order to determine the constant K, we set

$$
\mathrm{d}x' = \frac{\pm \mathrm{d}x \mp Kic\mathrm{d}t}{(1+K^2)^{1/2}} = 0 \ , \qquad K = \frac{v}{ic} \, . \tag{43}
$$

(The point at rest in S' moves with the velocity v with respect to S, where v is the relative velocity between the systems.) Then

$$
x' = \pm \frac{x - vt}{(1-\beta^2)^{1/2}} \ , \qquad t' = \pm \frac{t - vx/c^2}{(1-\beta^2)^{1/2}} \, . \tag{44}
$$

The transformation matrix $\|\alpha\|$ (in fact there are two transformation matrices) is:

$$
\|\alpha\| = \begin{pmatrix} \pm\gamma & 0 & 0 & \pm i\gamma v/c \\ 0 & \pm 1 & 0 & 0 \\ 0 & 0 & \pm 1 & 0 \\ \mp i\gamma v/c & 0 & 0 & \pm\gamma \end{pmatrix}, \tag{45}
$$

where $\gamma = 1/(1-\beta^2)^{1/2}$. The inverse matrix is

$$\|\alpha'\| = \begin{pmatrix} \pm\gamma & 0 & 0 & \mp i\gamma v/c \\ 0 & \pm 1 & 0 & 0 \\ 0 & 0 & \pm 1 & 0 \\ \pm i\gamma v/c & 0 & 0 & \pm\gamma \end{pmatrix}. \tag{46}$$

The transformation of the momentum 4-vector, from the system in which the particle is at rest (S') to the system in which the particle moves with velocity v (S), is

$$\begin{pmatrix} p \\ 0 \\ 0 \\ iW/c \end{pmatrix} = \|\alpha'\| \begin{pmatrix} 0 \\ 0 \\ 0 \\ iW_0/c \end{pmatrix}, \tag{47}$$

$$\begin{aligned} p &= \mp(-vW_0\gamma/c^2) = mv, \\ W &= \pm W_0\gamma = mc^2, \end{aligned} \tag{48}$$

where

$$W_0 = m_0c^2\ , \qquad m = \pm m_0\gamma. \tag{49}$$

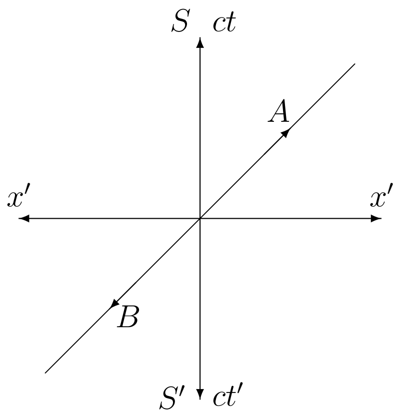

Figure 11: The reference systems that are inversed to each other. The arrow A represents a possible direction of a particle's 4-momentum, whereas the arrow B represents the opposite direction of 4-momentum.

As we see from (48), two kinds of transformations that conserve the square of 4-dimensional vector are possible: (i) Under the first kind of transformation the sign of the components of a 4-vectors remains the same; (ii) under the second kind of transformations the sign is changed. If the relative velocity of the systems is zero, the transformations, where the signs of components of 4-vectors are changed, are possible. For instance, if $v = 0$, the first kind of transformation is identity: $x' = x,\ y' = y,\ z' = z,\ t' = t$ or $p'_x = p_x,\ p'_y = p_y,\ p'_z = p_z,\ W' = W$, and the second kind of transformation is $x' = -x$, $y' = -y$, $z' = -z$, $t' = -t$, or $p'_x = -p_x$, $p'_y = -p_y$, $p'_z = -p_z$, $W' = -W$. This means that the laws of nature are invariant under rotations and under the inversion of coordinate frame in 4-dimensional space. According to this picture, transitions between the positive and the negative energy states are possible. A particle or a matter that have positive momentum and energy, when observed from the inverse reference frame, we see in our reference frame as a particle with negative momentum and energy (Fig. 11). Something similar (but not the same) was expressed by Feynman. According to him, the electrons that move backwards in time have negative energies [6].

According to the Case II (Eq. (37), the square of a 4-vector can change the sign:

$$s'^2_{12} = -s^2_{12}, \tag{50}$$

or

$$x'^2_1 + x'^2_2 + x'^2_3 + x'^2_4 = -x^2_1 - x^2_2 - x^2_3 - x^2_4. \tag{51}$$

The procedure is similar as before:

$$\begin{aligned} x'_1 &= \alpha_{11}x_1 + \alpha_{14}x_4, \\ x'_2 &= \alpha_{22}x_2, \\ x'_3 &= \alpha_{33}x_3, \\ x'_4 &= \alpha_{41}x_1 + \alpha_{44}x_4, \end{aligned} \tag{52}$$

$$\begin{aligned} &\alpha^2_{11} + \alpha^2_{41} = -1, \\ &\alpha^2_{14} + \alpha^2_{44} = -1, \\ &\alpha_{11}\alpha_{14} + \alpha_{41}\alpha_{44} = 0\ , \qquad \frac{\alpha_{41}}{\alpha_{11}} = K, \\ &\alpha^2_{22} = -1\ , \qquad \alpha_{33} = -1. \end{aligned} \tag{53}$$

The solution is

$$\begin{aligned} &\alpha_{11} = \frac{\pm i}{(1+K^2)^{1/2}}\ , \quad \alpha_{14} = \frac{\mp iK}{(1+K^2)^{1/2}}, \\ &\alpha_{41} = \frac{\pm iK}{(1+K^2)^{1/2}}\ , \quad \alpha_{44} = \frac{\pm i}{(1+K^2)^{1/2}}, \\ &\alpha_{22} = \pm i\ , \quad \alpha_{33} = \pm i, \end{aligned} \tag{54}$$

$$x' = \frac{\pm ix \mp iKict}{(1+K^2)^{1/2}}. \tag{55}$$

In order to determine the constant K, we set

$$\mathrm{d}x' = \frac{\pm i\mathrm{d}x \mp iKic\mathrm{d}t}{(1+K^2)^{1/2}} = 0\ , \qquad K = \frac{v}{ic}. \tag{56}$$

(The point at rest in S' moves with the velocity v with respect to S, where v is the relative velocity between the systems.) Then

$$x' = \frac{\pm ix \mp ivt}{(1-\beta^2)^{1/2}} = \pm\frac{x - vt}{(\beta^2-1)^{1/2}}\ , \qquad t' = \pm\frac{t - vx/c^2}{(\beta^2-1)^{1/2}}, \tag{57}$$

where $\beta = v/c$. This means that x' and t' are real only if $v > c$, while y' and z' are always imaginary [under such transformation]. The inverse transformation matrix is

$$\|\alpha'\| = \begin{pmatrix} \pm i\gamma & 0 & 0 & \pm\gamma v/c \\ 0 & \pm i & 0 & 0 \\ 0 & 0 & \pm i & 0 \\ \mp\gamma v/c & 0 & 0 & \pm i\gamma \end{pmatrix}, \tag{58}$$

$$\begin{pmatrix} p \\ 0 \\ 0 \\ iW/c \end{pmatrix} = \|\alpha'\| \begin{pmatrix} 0 \\ 0 \\ 0 \\ iW_0/c \end{pmatrix}, \tag{59}$$

$$\begin{aligned} p &= \pm iW_0\gamma v/c^2 = mv, \\ W &= \pm iW_0\gamma = mc^2, \end{aligned} \tag{60}$$

where

$$W_0 = m_0c^2\ , \qquad m = \frac{\pm im_0}{(1-\beta^2)^{1/2}}\,. \tag{61}$$

According to these transformations, energy and momentum (the component in the direction of motion) are imaginary if $v < c$ and real if $v > c$. So we have two sets of transformations: one for the systems with relative velocity $v < c$, and the other for the systems with $v > c$. The first solution to Eq. (35) corresponds to $v_1 < c,\ v_2 < c,\ v_{12} < c$, whilst the second solution corresponds to $v_1 > c,\ v_2 > c,\ v_{12} < c$ This means that, if two systems S_1 and S_2 move with respect to the third system S faster than light, then the relative velocity between S_1 and S_2 is less than the velocity of light. Under a transformation to a system with $v > c$, only the x-components are rea1, whereas the y and z components are imaginary. The spatial distance, as seen from the system "at rest", is

$$\begin{aligned} d &= (x'^2 + y'^2 + z'^2)^{1/2} = (x^2 - y^2 - z^2)^{1/2} \\ d^2 &= x_1^2 - x_2^2 - x_3^2 \end{aligned} \tag{62}$$

If $x_2^2 + x_3^2 < x_1^2$, the distance d is real. The 3-space metrics, that is Euclidean in the system moving with $v > c$, when observed from the system at rest, is not Euclidean, but one with coefficients $g_{11} = 1,\ g_{22} = -1,\ g_{33} = -1$, and $g_{ik} = 0$ for $i \neq k$. (Generaly, $d^2 = \sum_{i,k} g_{ik}x_ix_k$.)

Conclusion

Though the present hypothesis has perhaps some logical incorrectnesses, it may help to solve the dilemma as to whether tachyons do exist or not. By our reasoning we obtained the same expressions for velocity dependence of energy and momentum as in the [ordinary] theory of tachyons. If both derivations are equivalent, or if the present derivation is more general than the previous one, incorrectnesses in our hypothesis will show that tachyons do not exist (or at least that they cannot be explained by the special theory of relativity).

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Note added in 2011

From my present perspective, the description of quantised space(time) in this old paper of mine, is preliminary, and should eventually be replaced or supplemented by more sophisticated theoretical methods.