Introduction

The unification of various branches of theoretical physics is a joint project of many researchers, and everyone contributes as much as he can. So far we have accumulated a great deal of knowledge and insight encoded in such marvelous theories as general relativity, quantum mechanics, quantum field theory, the standard model of electroweak interaction, and chromodynamics. In order to obtain a more unified view, various promising theories have emerged, such as those of strings and "branes", induced gravity, the embedding models of gravity, and the "brane world" models, to mention just a few. The very powerful Clifford algebra as a useful tool for geometry and physics is becoming more and more popular. Fascinating are the ever increasing successes in understanding the foundations of quantum mechanics and their experimental verification, together with actual and potential practical applications in cryptography, teleportation, and quantum computing.

In this book I intend to discuss the conceptual and technical foundations of those approaches which, in my opinion, are most relevant for unification of general relativity and quantum mechanics on the one hand, and fundamental interactions on the other hand. After many years of active research I have arrived at a certain level of insight into the possible interrelationship between those theories. Emphases will be on the exposition and understanding of concepts and basic techniques, at the expense of detailed and rigorous mathematical development. Theoretical physics is considered here as a beautiful landscape. A global view of the landscape will be taken. This will enable us to see forests and mountain ranges as a whole, at the cost of seeing trees and rocks.

Physicists interested in the foundations of physics, conceptual issues, and the unification program, as well as those working in a special field and desiring to broaden their knowledge and see their speciality from a wider perspective, are expected to profit the most from the present book. They are assumed to possess a solid knowledge at least of quantum mechanics, and special and general relativity.

As indicated in the subtitle, I will start from point particles. They move along geodesics which are the lines of minimal, or, more generally, extremal length, in spacetime. The corresponding action from which the equations of motion are derived is invariant with respect to reparametrizations of an arbitrary parameter denoting position on the worldline swept by the particle. There are several different, but equivalent, reparametrization invariant point particle actions. A common feature of such an approach is that actually there is no dynamics in spacetime, but only in space. A particle's worldline is frozen in spacetime, but from the 3-dimensional point of view we have a point particle moving in 3-space. This fact is at the roots of all the difficulties we face when trying to quantize the theory: either we have a covariant quantum theory but no evolution in spacetime, or we have evolution in 3-space at the expense of losing manifest covariance in spacetime. In the case of a point particle this problem is not considered to be fatal, since it is quite satisfactorily resolved in relativistic quantum field theory. But when we attempt to quantize extended objects such as branes of arbitrary dimension, or spacetime itself, the above problem emerges in its full power: after so many decades of intensive research we have still not yet arrived at a generally accepted consistent theory of quantum gravity.

There is an alternative to the usual relativistic point particle action proposed by Fock [**?**] and subsequently investigated by Stueckelberg [**?**], Feynman [**?**], Schwinger [**?**], Davidon [**?**], Horwitz [**?**, **?**] and many others [**?**]–[**?**]. In such a theory a particle or "event" in spacetime obeys a law of motion analogous to that of a nonrelativistic particle in 3-space. The difference is in the dimensionality and signature of the space in which the particle moves. None of the coordinates x^0, x^1, x^2, x^3 which parametrize spacetime has the role of evolution parameter. The latter is separately postulated and is Lorentz invariant. Usually it is denoted as τ and evolution goes along τ. There are no constraints in the theory, which can therefore be called *the unconstrained theory*. First and second quantizations of the unconstrained theory are straightforward, very elegant, and manifestly Lorentz covariant. Since τ can be made to be related to proper time such a theory is often called a *Fock–Schwinger proper time formalism*. The value and elegance of the latter formalism is widely recognized, and it is often used, especially when considering quantum fields in curved spaces [**?**]. There are two main interpretations of the formalism:

(i) According to the first interpretation, it is considered merely as a useful calculational tool, without any physical significance. Evolution in τ and the absence of any constraint is assumed to be fictitious and unphysical. In order to make contact with physics one has to get rid of τ in all the expressions considered by integrating them over τ. By doing so one projects unphysical expressions onto the physical ones, and in particular one projects unphysical states onto physical states.

(ii) According to the second interpretation, evolution in τ is genuine and physical. There is, indeed, dynamics in spacetime. Mass is a constant of motion and not a fixed constant in the Lagrangian.

Personally, I am inclined to the interpretation (ii). In the history of physics it has often happened that a good new formalism also contained good new physics waiting to be discovered and identified in suitable experiments. It is one of the purposes of this book to show a series of arguments in favor of the interpretation (ii). The first has roots in geometric calculus based on Clifford algebra [**?**]

Clifford numbers can be used to represent vectors, multivectors, and, in general, polyvectors (which are Clifford aggregates). They form a very useful tool for geometry. The well known equations of physics can be cast into elegant compact forms by using the geometric calculus based on Clifford algebra.

These compact forms suggest the generalization that every physical quantity is a polyvector [**?**, **?**]. For instance, the momentum polyvector in 4-dimensional spacetime has not only a vector part, but also a scalar, bivector, pseudovector and pseudoscalar part. Similarly for the velocity polyvector. Now we can straightforwardly generalize the conventional constrained action by rewriting it in terms of polyvectors. By doing so we obtain in the action also a term which corresponds to the pseudoscalar part of the velocity polyvector. A consequence of this extra term is that, when confining ourselves, for simplicity, to polyvectors with pseudoscalar and vector part only, the variables corresponding to 4-vector components can all be taken as independent. After a straightforward procedure in which we omit the extra term in the action (since it turns out to be just the total derivative), we obtain Stueckelberg's unconstrained action! This is certainly a remarkable result. The original, constrained action is equivalent to the unconstrained action. Later in the book (Sec. 4.2) I show that the analogous procedure can also be applied to extended objects such as strings, membranes, or branes in general.

When studying the problem of how to identify points in a generic curved spacetime, several authors [**?**], and, especially recently Rovelli [**?**], have recognized that one must fill spacetime with a reference fluid. Rovelli considers such a fluid as being composed of a bunch of particles, each particle carrying a clock on it. Besides the variables denoting positions of particles there is also a variable denoting the clock. This extra, clock, variable must enter the action, and the expression Rovelli obtains is formally the same as the expression we obtain from the polyvector action (in which we neglect the bivector, pseudovector, and scalar parts). We may therefore identify the pseudoscalar part of the velocity polyvector with the speed of the clock variable. Thus have a relation between the polyvector generalization of the usual constrained relativistic point particle, the Stueckleberg particle, and the DeWitt–Rovelli particle with clock.

A relativistic particle is known to posses spin, in general. We show how spin arises from the polyvector generalization of the point particle and how the quantized theory contains the Dirac spinors together with the Dirac equation as a particular case. Namely, in the quantized theory a state is naturally assumed to be represented as a polyvector wave function Φ, which, in particular, can be a spinor. That spinors are just a special kind of polyvectors (Clifford aggregates), namely the elements of the minimal left or right ideals of the Clifford algebra, is an old observation [**?**]. Now, scalars, vectors, spinors, etc., can be reshuffled by the elements of the Clifford algebra. This means that scalars, vectors, etc., can be transformed into spinors, and *vice versa.* Within Clifford algebra thus we have transformations which change bosons into fermions. In Secs. 2.5 and 2.7 I discuss the possible relation between the Clifford algebra formulation of the spinning particle and a more widely used formulation in terms of Grassmann variables.

A very interesting feature of Clifford algebra concerns the signature of the space defined by basis vectors which are generators of the Clifford algebra. In principle we are not confined to choosing just a particular set of elements as basis vectors; we may choose some other set. For instance, if e^0, e^1, e^2, e^3 are the basis vectors of a space M_e with signature $(++++)$, then we may declare the set $(e^0,\, e^0e^1,\, e^0e^2,\, e^0e^3)$ as basis vectors γ^0, γ^1, γ^2, γ^3 of some other space M_γ with signature $(+---)$. That is, by suitable choice of basis vectors we can obtain within the same Clifford algebra a space of arbitrary signature. This has far reaching implications. For instance, in the case of even-dimensional space we can always take a signature with an equal number of pluses and minuses. A harmonic oscillator in such a space has vanishing zero point energy, provided that we define the vacuum state in a very natural way as proposed in refs. [**?**]. An immediate consequence is that there are no central terms and anomalies in string theory living in spacetime with signature $(+++...---)$, even if the dimension of such a space is not critical. In other words, spacetime with such a 'symmetric' signature need not have 26 dimensions [**?**].

The principle of such a harmonic oscillator in a pseudo-Euclidean space is applied in Chapter 3 to a system of scalar fields. The metric in the space of fields is assumed to have signature $(+++...---)$ and it is shown that the vacuum energy, and consequently the cosmological constant, are then exactly zero. However, the theory contains some negative energy fields ("exotic matter") which couple to the gravitational field in a different way than the usual, positive energy, fields: the sign of coupling is reversed, which implies a repulsive gravitational field around such a source. This is the price to be paid if one wants to obtain a small cosmological constant in a straightforward way. One can consider this as a prediction of the theory to be tested by suitably designed experiments.

The problem of the cosmological constant is one of the toughest problems in theoretical physics. Its resolution would open the door to further understanding of the relation between quantum theory and general relativity. Since all more conventional approaches seem to have been more or less exploited without unambiguous success, the time is right for a more drastic novel approach. Such is the one which relies on the properties of the harmonic oscillator in a pseudo-Euclidean space.

In Part II I discuss the theory of extended objects, now known as "branes" which are membranes of any dimension and are generalizations of point particles and strings. As in the case of point particles I pay much attention to the unconstrained theory of membranes. The latter theory is a generalization of the Stueckelberg point particle theory. It turns out to be very convenient to introduce the concept of the infinite-dimensional *membrane space* $\mathcal{M}$. Every point in $\mathcal{M}$ represents an unconstrained membrane. In $\mathcal{M}$ we can define distance, metric, covariant derivative, etc., in an analogous way as in a finite-dimensional curved space. A membrane action, the corresponding equations of motion, and other relevant expressions acquire very simple forms, quite similar to those in the point particle theory. We may say that a membrane is a point particle in an infinite dimensional space!

Again we may proceed in two different interpretations of the theory:

(i) We may consider the formalism of the membrane space as a useful calculational tool (a generalization of the Fock–Schwinger proper time formalism) without any genuine physical significance. Physical quantities are obtained after performing a suitable projection.

(ii) The points in $\mathcal{M}$-space are physically distinguishable, that is, a membrane can be physically deformed in various ways and such a deformation may change with evolution in τ.

If we take the interpretation (ii) then we have a marvelous connection (discussed in Sec. 2.8) with the Clifford algebra generalization of the conventional constrained membrane on the one hand, and the concept of DeWitt–Rovelli reference fluid with clocks on the other hand.

Clifford algebra in the infinite-dimensional membrane space $\mathcal{M}$ is described in Sec. 6.1. When quantizing the theory of the unconstrained membrane one may represent states by wave functionals which are polyvectors in $\mathcal{M}$-space. A remarkable connection with quantum field theory is shown in Sec. 7.2

When studying the $\mathcal{M}$-space formulation of the membrane theory we find that in such an approach one cannot postulate the existence of a background embedding space independent from a membrane configuration. By "membrane configuration" I understand a system of (many) membranes, and the membrane configuration is identified with the embedding space. There is no embedding space without the membranes. This suggests that our spacetime is nothing but a membrane configuration. In particular, our spacetime could be just one 4-dimensional membrane (4-brane) amongst many other membranes within the configuration. Such a model is discussed in Part III. The 4-dimensional gravity is due to the induced metric on our 4-brane V_4, whilst matter comes from the self-intersections of V_4, or the intersections of V_4 with other branes. As the intersections there can occur manifolds of various dimensionalities. In particular, the intersection or the self-intersection can be a 1-dimensional worldline. It is shown that such a worldline is a geodesic on V_4. So we obtain in a natural way four-dimensional gravity with sources. The quantized version of such a model is also discussed, and it is argued that the kinetic term for the 4-dimensional metric $g_{\mu\nu}$ is induced by quantum fluctuations of the 4-brane embedding functions.

In the last part I discuss mainly the problems related to the foundations and interpretation of quantum mechanics. I show how the brane world view sheds new light on our understanding of quantum mechanics and the role of the observer.