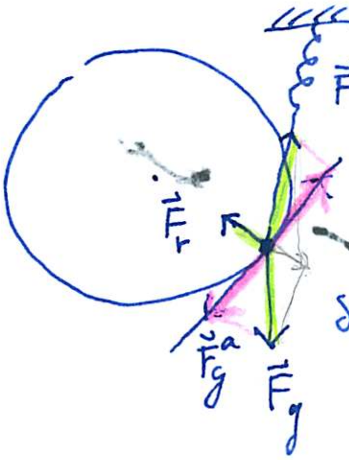
primeri za virtualno delo in D'A princip

II. Statika



$\sum_i \vec{F}_i = 0$

$\delta A = \sum_i \vec{F}_i^{a} \, \delta \vec{r}_i = 0$

$\vec{F}_g^{a} = \vec{F}_{g\,\text{tangenta}}^{a}$

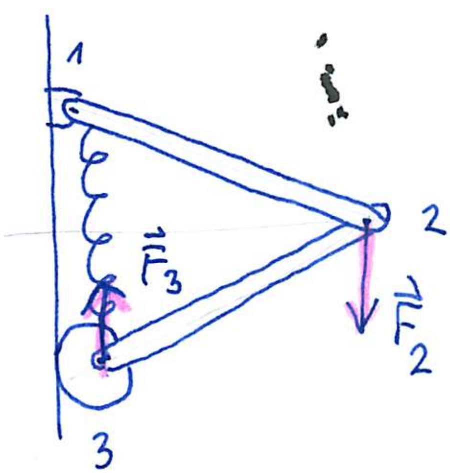
$\delta A = F_{gt} R \delta\varphi + F_{vt} R \delta\varphi = 0$

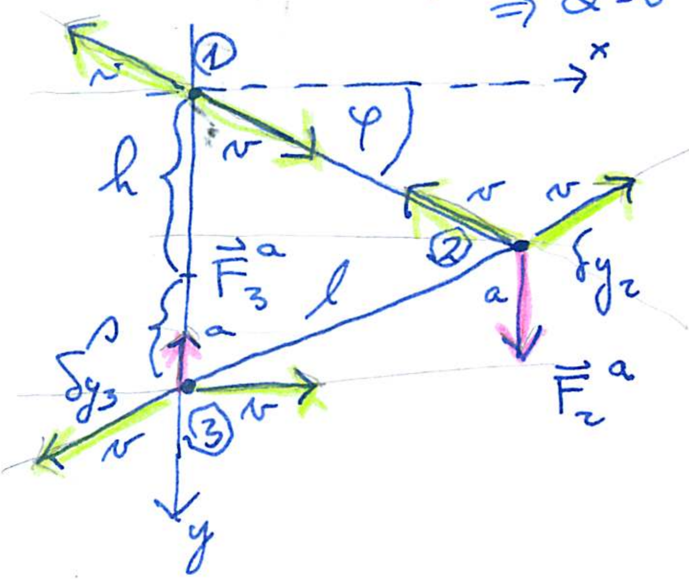
$Q \, \delta\varphi = 0$

$\leftarrow$ tangentna sila $\times R$

$\Rightarrow Q = 0$

III.





δA = virtualno delo $= 0$

za $\forall$ točko 1, 2, 3 je $\sum \vec{F} = 0$

$$\vec{F}_i = \vec{F}_i^{a} + \vec{F}_i^{v}$$

določimo ravnovesni φ in F_3, če poznamo F_2

nedaztegnjena vzmet – $h = y_3$ (nerazt.)

koeficient vzmeti – k

$y_2 = l \sin\varphi \qquad \delta y_2 = l \cos\varphi \, \delta\varphi \qquad s = y_3 - h =$

$y_3 = 2l \sin\varphi \qquad \delta y_3 = 2l \cos\varphi \, \delta\varphi \qquad = 2l\sin\varphi - h$

$F_3 = ks$

$$\delta A = \sum_i \vec{F}_i^{a} \cdot \delta \vec{r}_i = F_2 \delta y_2 - F_3 \delta y_3 = 0$$

$$F_2 l \cos\varphi \, \delta\varphi - k(2l\sin\varphi - h) 2l\cos\varphi \, \delta\varphi = 0$$

$Q \, \delta\varphi = 0$ $\leftarrow$ posplošena koordinata

$\uparrow$ posplošena sila $\Rightarrow Q = 0$ (ena sama)

$\varphi = q_1$: en sam neodvisni parameter

$$\Rightarrow \sin\varphi = \frac{F_2 + 2kh}{4kl} \quad \text{in} \quad F_3 = k\left(2l\,\frac{F_2 + 2kh}{4kl} - h\right) = \frac{F_2}{2}.$$

V ravnovesju je virtualno delo $\delta A = 0$. To je nujen pogoj za statiko. Kaj, če nismo povsem v ravnovesju (t.j. sila F_2 se poveča in spravi vse v gibanje)?

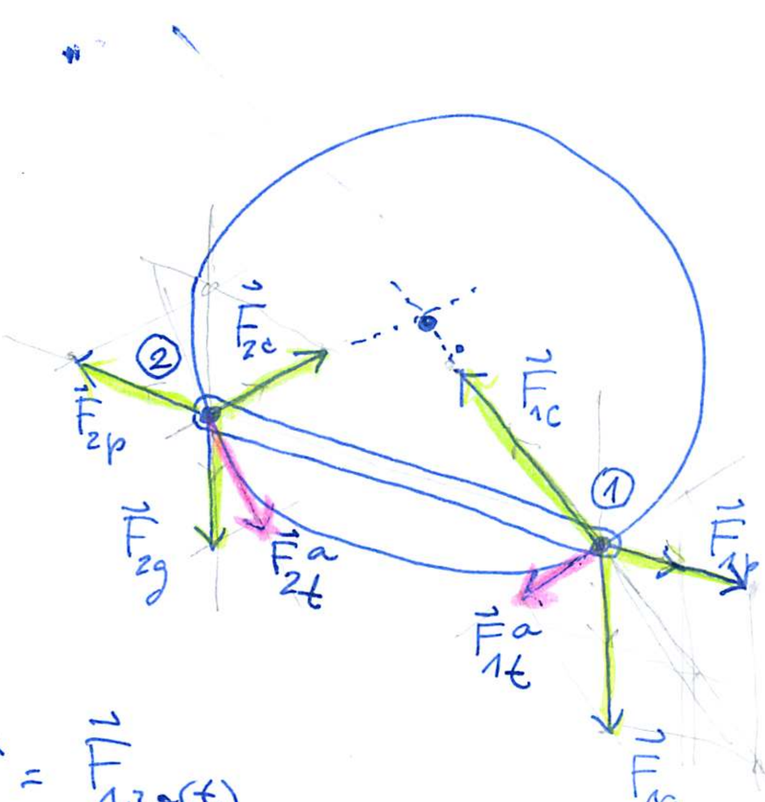
$$0 = \sum_i \left(\vec F_i^{\,a} - m_i \vec a_i\right)\cdot \delta \vec r_i = \sum_i \vec F_i^{\,a}\cdot\delta\vec r_i - \sum_i m_i \dot{\vec v}_i\cdot\vec v_i\,\delta t$$

$$\frac{d}{dt}\left(\frac{1}{2}v_i^2\right)$$

D'Alambert

$$\delta A - \delta T = 0.$$

IV. podobno kot II. (statika)



$$\vec F_{1,2}^{\,a} = \vec F_{1,2g(t)}$$

v $\forall$ točki $i = 1,2$ je $\sum$ sil $= 0$; zelene = vezi
rdeče = aktivne (lahko premaknejo)

$$\delta A = F_{1t} R\,\delta\varphi - F_{2t} R\,\delta\varphi = 0$$

$$Q\,\delta\varphi = 0.$$

torej, če enkrat zapišimo D'A princip,

$$\sum_i (\vec F_i^a - m_i \vec a_i)\cdot \delta \vec r_i = 0$$

$\delta \vec r_i$ so med sabo povezani z vezmi in torej niso neodvisni (gl. III npr.).
$\exists$ manjše število koordinat, ki so neodvisne in so $\delta \vec r_i$ od njih odvisni (npr. $\delta\varphi$ v III.).

Kako najdemo δq_j iz q_j? Kakor veš in znaš!

torej

$$\vec r_i = \vec r_i(q_1, q_2, \dots, q_n, t)$$

$$d\vec r_i = \sum_j \frac{\partial \vec r_i}{\partial q_j} dq_j + \frac{\partial \vec r_i}{\partial t} dt; \quad \vec v_i = \sum_j \frac{\partial \vec r_i}{\partial q_j}\dot q_j + \frac{\partial \vec r_i}{\partial t}.$$

gledamo v nekem trenutku ($dt=0$) virtualni premik (= tak, da je dovoljen glede na vezi; računamo se več t_i-jev premakne)

$$\delta \vec r_i = \sum_j \frac{\partial \vec r_i}{\partial q_j}\delta q_j$$

(gl. primere od prej!)

Pogoj statike:

$\delta A = \sum_i \vec F_i^a \cdot \delta \vec r_i = 0$ zapišemo z q_j-ji,

$$\sum_i \vec F_i^a \cdot \sum_j \frac{\partial \vec r_i}{\partial q_j}\delta q_j = \sum_j \left(\sum_i \vec F_i^a \cdot \frac{\partial \vec r_i}{\partial q_j}\right)\delta q_j =$$

$$= \sum_j Q_j \delta q_j = 0 \quad \text{za } \forall\, \delta q_j$$

$\Rightarrow Q_j = 0$, ker q_j neodvisni

$$\boxed{Q_j = \sum_i \vec F_i^a \cdot \frac{\partial \vec r_i}{\partial q_j}}$$

posplošena (generalizirana) sila

nima enote sile; odvisno od izbire q_j

gl. primere! (npr.: navor (tangentna sila)).

Nadaljujemo z D'Alembertovo formulo, in sicer s členi $-m_i \vec{a}_i$, ki jih izrazimo s q_j

→ to spremenimo na 1. odvode $(\dot{\vec{r}}_i)$

$$\sum_i m_i \vec{a}_i \cdot \delta \vec{r}_i = \sum_i \left[m_i \vec{a}_i \cdot \sum_j \frac{\partial \vec{r}_i}{\partial q_j} \right] \delta q_j =$$

$$= \sum_{ij} \left[\frac{d}{dt} m_i \dot{\vec{r}}_i \cdot \boxed{\frac{\partial \vec{r}_i}{\partial q_j}} - m_i \dot{\vec{r}}_i \cdot \boxed{\frac{d}{dt} \frac{\partial \vec{r}_i}{\partial q_j}} \right] \delta q_j$$

• poglejmo najprej $\frac{\partial \vec{r}_i}{\partial q_j}$:

$$\frac{d\vec{r}_i}{dt} = \vec{v}_i = \sum_j \frac{\partial \vec{r}_i}{\partial q_j} \dot{q}_j + \frac{\partial \vec{r}_i}{\partial t}$$

$$\boxed{\frac{\partial \vec{v}_i}{\partial \dot{q}_j} = \frac{\partial \vec{r}_i}{\partial q_j}}$$

$\left(\frac{\partial \vec{r}_i}{\partial \dot{q}_j} = 0, \text{ ni odvisno od } \dot{q}_j \right)$

•• in še

$$\boxed{\frac{d}{dt} \frac{\partial \vec{r}_i}{\partial q_j}} = \sum_k \frac{\partial^2 \vec{r}_i}{\partial q_j \partial q_k} \dot{q}_k + \frac{\partial^2 \vec{r}_i}{\partial q_j \partial t} =$$

$$= \frac{\partial}{\partial q_j} \left(\sum_k \frac{\partial \vec{r}_i}{\partial q_k} \dot{q}_k + \frac{\partial \vec{r}_i}{\partial t} \right) = \boxed{\frac{\partial}{\partial q_j} \frac{d\vec{r}_i}{dt}}$$

torej,

$$\sum_i m_i \ddot{\vec{r}}_i \cdot \frac{\partial \vec{r}_i}{\partial q_j} = \sum_i \left(\frac{d}{dt} \underbrace{\left(m_i \vec{v}_i \cdot \frac{\partial \vec{v}_i}{\partial \dot{q}_j} \right)}_{\frac{\partial}{\partial \dot{q}_j} T_i} - \underbrace{m_i \vec{v}_i \cdot \frac{\partial \vec{v}_i}{\partial q_j}}_{\frac{\partial}{\partial q_j} T_i} \right) = \frac{d}{dt} \frac{\partial T}{\partial \dot{q}_j} - \frac{\partial T}{\partial q_j}$$

To vstavimo v D'Alembertovo enačbo,

$$\sum_j \left(Q_j - \frac{d}{dt} \frac{\partial T}{\partial \dot{q}_j} + \frac{\partial T}{\partial q_j} \right) \delta q_j = 0 \quad \text{za } \forall\ j = 1, 2, \ldots n$$

in poljuben q_j

torej, ker so q_j neodvisni,

$$\boxed{\frac{d}{dt}\frac{\partial T}{\partial \dot q_j} - \frac{\partial T}{\partial q_j} = Q_j .}$$

To velja tudi, če sile niso konzervativne (trenje itd.)

primer od prej:

$$\frac{d}{dt}\frac{\partial\left(\frac{1}{2}mR^2\dot\varphi^2\right)}{\partial\dot\varphi} - \frac{\partial T}{\partial\varphi} = (R\,F_{gt} - R\,F_{vt})_\varphi$$

Naj sile so konzervativne

→ ne pišemo a, je pa mišljeno

$$\vec F_i = -\nabla_i V$$

$$Q_j = \sum_i \vec F_i \cdot \frac{\partial \vec r_i}{\partial q_j} = -\sum_i \nabla_i V \cdot \frac{\partial \vec r_i}{\partial q_j} = -\frac{\partial V}{\partial q_j}$$

$$\underbrace{\sum_i \left(\frac{\partial V}{\partial x_i}\frac{\partial x_i}{\partial q_j} + \frac{\partial V}{\partial y_i}\frac{\partial y_i}{\partial q_j} + \frac{\partial V}{\partial z_i}\frac{\partial z_i}{\partial q_j}\right)}_{\frac{\partial V}{\partial q_j}}$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot q_j} - \frac{\partial T}{\partial q_j} = -\frac{\partial V}{\partial q_j} \qquad *$$

če V ni funkcija $\dot q_j$ (hitrosti), je $\frac{\partial V}{\partial \dot q} = 0$ in

$$\boxed{\frac{d}{dt}\frac{\partial L}{\partial \dot q_j} - \frac{\partial L}{\partial q_j} = 0 \; ; \quad L = T - V.}$$

Lagrangeva funkcija

za $j = 1, 2, \dots n$ neodvisnih q_j

to so Euler-Lagrangeve enačbe

Če so potenciali odvisni od hitrosti, $U(q_j, \dot q_j)$, lahko definiramo sile,

primer: $\vec F = e\vec v \times \vec B$

$$Q_j = -\frac{\partial U}{\partial q_j} + \frac{d}{dt}\frac{\partial U}{\partial \dot q_j} \quad \text{in} \quad L = T - U \quad \text{in}$$

očitno vse o.k. → *