Hamilton-Jacobijeva enačba

Spomnimo se akcije,

$$S = \int_{t_1}^{t_2} L(q, \dot{q}, t)\, dt,$$

kjer je začetna in končna lega določena,

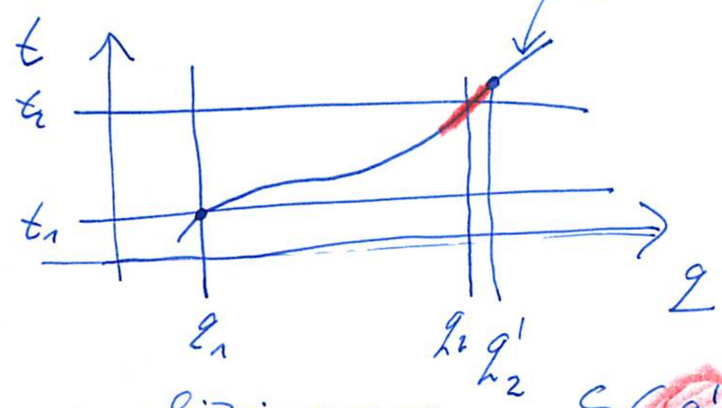
$q(t_1) = q_1$ in $q(t_2) = q_2$ in $q(t)$ zadošča enačbi gibanja ≡ „klasična pot".

Zamislimo si sedaj akcijo kot funkcijo klasične poti, kjer je q_1 fiksiran in S gledamo kot funkcijo končne lege, pri čemer je $q(t)$ že klasična rešitev,

$$S = S(q_2).$$

Sedaj gremo s q_2' malo naprej ali malo nazaj od prave končne točke, vendar vzdolž prave poti:

$q(t)$ klasična pot iz E-L enačba



in analiziramo $S(q_2')$.

Od nje že znamo variirati S, $t_2 \to t_2'$

$$\delta S = \int_{t_1}^{t_2} \Big[\underbrace{\frac{\partial L}{\partial q} - \frac{d}{dt}\Big(\frac{\partial L}{\partial \dot q}\Big)}_{0}\Big]\delta q\, dt + \frac{\partial L}{\partial \dot q}\,\delta q(t)\Big|_{t_1}^{t_2}.$$

Vzdolž klasične poti $q(t)$ je prvi člen $\equiv 0$, zato ostane

$$\delta S = \frac{\partial L}{\partial \dot q}\Big|_{t_2}\delta q(t_2) - \frac{\partial L}{\partial \dot q}\underbrace{\delta q(t_1)}_{0} =$$

$\leftarrow$ se tudi variira in na koncu vzame $t_2' = t_2$.

$$= \frac{\partial L}{\partial \dot q}(t_2)\,\underbrace{\delta q(t_2)}_{q_2' - q_2}$$

variacija vzdolž klasične poti

$$\frac{\partial S(q_2')}{\partial q_2'} = \frac{\partial L}{\partial \dot q}\Big|_{t=t_2} = p\Big|_{t=t_2} = p_2$$

Lahko zapišemo tudi

$$\boxed{\frac{\partial S(q)}{\partial q} = p},$$

če namesto q_2, p_2 pišemo q, p.

Poglejmo sedaj še

$$\frac{\partial S}{\partial t_2}.$$

$$\frac{dS}{dt_2} = \frac{\partial S}{\partial t_2} + \frac{\partial S}{\partial q}\dot q\Big|_{t=t_2} = \frac{\partial S}{\partial t_2} + p\dot q\Big|_{t=t_2}$$

Po drugi strani velja itak

$$\frac{dS}{dt_2} = L \qquad \text{(odvod po zgornji meji)}$$

Zato

$$\frac{\partial S}{\partial t_2} = -p\dot q\Big|_{t_2} + L(q_2, \dot q_2, t_2) =$$

$$= -(p\dot q - L) = -H(q_2, p_2, t_2).$$

Rezultat ($t_2 \to t$)

$$\boxed{H + \frac{\partial S}{\partial t} = 0 \quad \text{in} \quad \frac{\partial S}{\partial q} = p}$$

$$H = H(q, p, t) = H\left(q, \frac{\partial S}{\partial q}, t\right)$$

H.-J. enačba

gibalna količina odvisna od skalarnega polja tudi v 3D, $\vec p = \nabla S$.

To je poseben primer kanonične transformacije, kjer je $K = 0$ in

$$F_2(q, P) = F_2 = S$$

gibanje delca torej opiše nekakšno valovanje (polje).

$$p = \frac{\partial S}{\partial q}$$

$$K = 0 = H + \frac{\partial S}{\partial t}$$

če $H = \frac{p^2}{2m} + V(q)$

$$\Rightarrow \quad -\frac{\partial S}{\partial t} = \frac{1}{2m}\left(\frac{\partial S}{\partial q}\right)^2 + V(q)$$

podoben izraz dobimo v QM za fazo, $\psi = R e^{iS}$

če $H \neq H(t)$ in

$H\left(q, \frac{\partial S}{\partial q}\right) = E$, je S „Hamiltonova glavna funkcija". „principal"

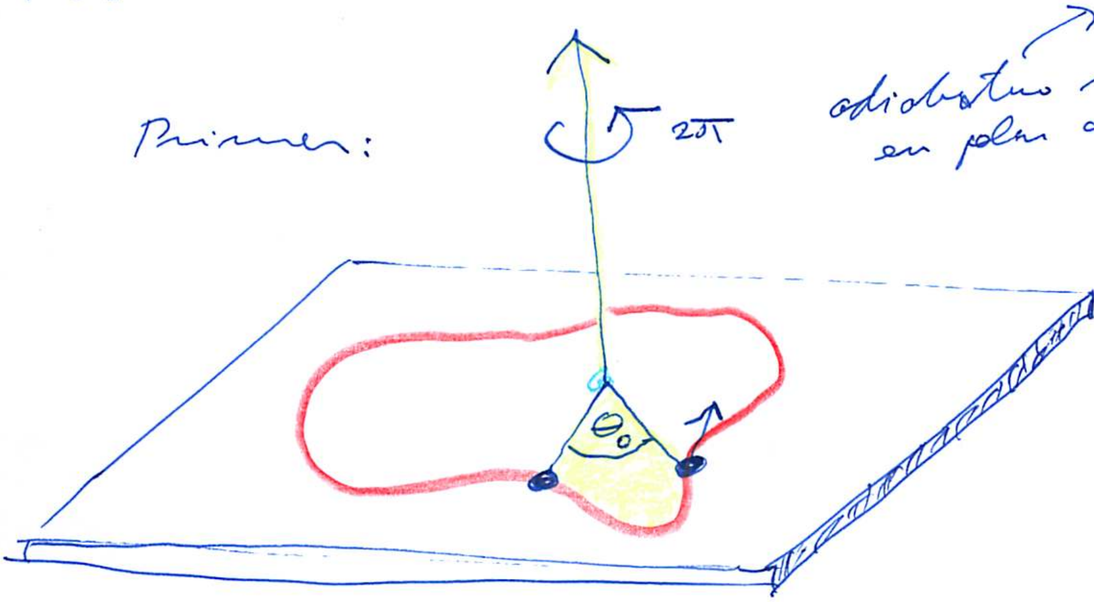
Hannayev kot (1984)

Akcija: $I = \frac{1}{2\pi}\oint p\,dq$

kot θ

$\Big\}$ (I, θ) formalizem

Primer:



adiabatno naredimo en poln obrat → zelo zelo počasi

$$\Theta_0 = -8\pi^2 \frac{S}{l^2}$$

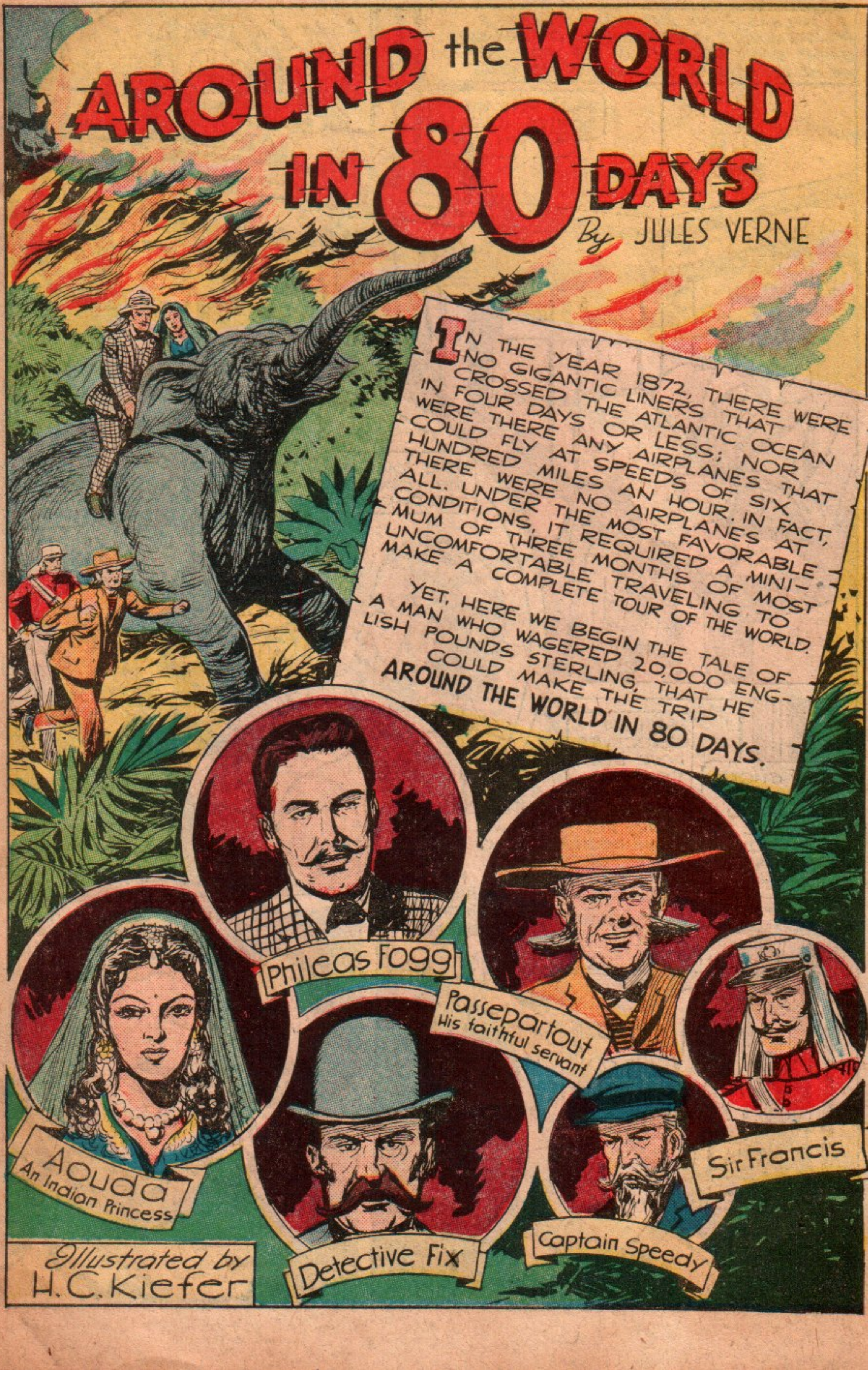
S ← ploščina zanke

l ← obseg

Za krog, $S = \pi R^2$, $l = 2\pi R \Rightarrow \Theta = -2\pi$.

Jules Verne,
V 80 dneh okoli sveta
(ni povsem isto...)

— K O N E C —



AROUND the WORLD IN 80 DAYS
By JULES VERNE
IN THE YEAR 1872, THERE WERE NO GIGANTIC LINERS THAT CROSSED THE ATLANTIC OCEAN IN FOUR DAYS OR LESS; NOR WERE THERE ANY AIRPLANES THAT COULD FLY AT SPEEDS OF SIX HUNDRED MILES AN HOUR. IN FACT, THERE WERE NO AIRPLANES AT ALL. UNDER THE MOST FAVORABLE CONDITIONS, IT REQUIRED A MINIMUM OF THREE MONTHS OF MOST UNCOMFORTABLE TRAVELING TO MAKE A COMPLETE TOUR OF THE WORLD.
YET, HERE WE BEGIN THE TALE OF A MAN WHO WAGERED 20,000 ENGLISH POUNDS STERLING, THAT HE COULD MAKE THE TRIP
AROUND THE WORLD IN 80 DAYS.
Phileas Fogg
Passepartout
His faithful servant
Sir Francis
Aouda
An Indian Princess
Detective Fix
Captain Speedy
Illustrated by
H. C. Kiefer