Po izreku: $\Delta \vec{G} = \int \vec{F_z}(t)\,dt$

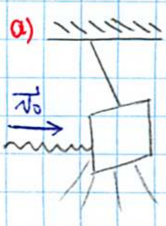
x: $-x \cdot v_0 + 0 = F\Delta t$ (Δt je ravno tako „dolg", da Δm pride od „t_1 do t_2")

$-G_x(t_1) + G_x(t_2)$

$\frac{-\Delta m}{\Delta t} v_0 = F = -F_c$ ← sila curka

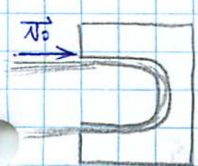
$$\boxed{\begin{aligned} &F_c = v_0 \frac{\Delta m}{\Delta t} = v_0 \cdot \Phi_m \quad ; \quad \Phi_m = \frac{dm}{dt} \\ &\rho = \frac{m}{V} \quad \text{gostota} \\ &\Phi_V = \frac{1}{\rho}\Phi_m \quad \text{volumski tok} \\ &\Phi_m = \rho \Phi_V \quad \text{masni tok} \end{aligned}}$$

a)



(tvoj realni primer). Voda se razprši.

b)



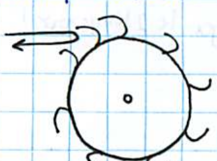
$G(t_1) = \Delta m v_0$

$G(t_2) = -\Delta m v_0$

$-\Delta m v_0 - \Delta m v_0 = F\Delta t$ ← sila stene posode na vodo

$F_c = 2v_0\Phi_m$

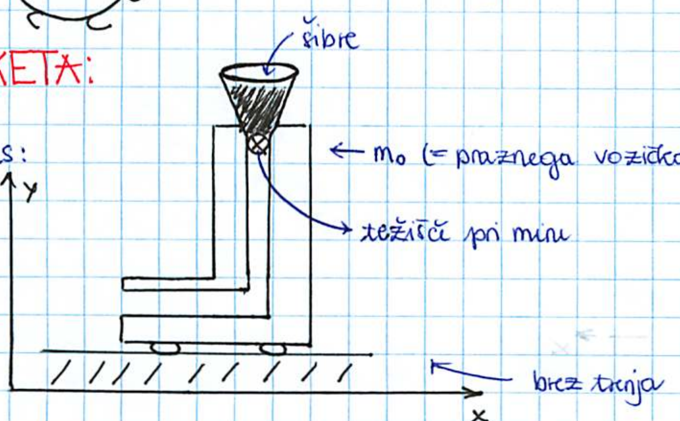
Princip turbin

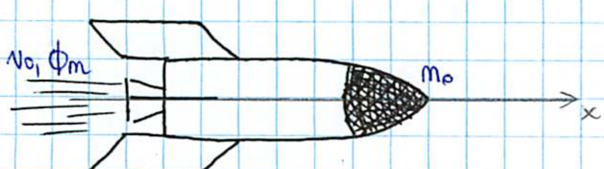


popolno je, če se odbije "vodoravno nazaj"

RAKETA:

Poskus:





m.... masa rakete

m_1.... masa v trenutku 1 (polna raketa)

$F_c = v_0 \Phi_m$

$\Delta G = F\Delta t = v_0 \Phi_m \Delta t$

$m(t)\Delta v = v_0 \Phi_m \Delta t$ ← odvisno od časa, saj izgoreva

$m = m_1 - \Phi_m(t)$

$(m_1 - \Phi_m t)\Delta v = v_0 \Phi_m \Delta t \quad | : v_0$ ← hitrost „kroglic" glede na raketo

$$\sum \frac{\Delta v}{v_0} = \sum \frac{\Phi_m \Delta t}{m_1 - \Phi_m t} \xrightarrow{\Delta t \to 0} \int_0^{v_1} \frac{dv}{v_0} = \int_0^{t_1} \frac{\Phi_m\, dt}{m_1 - \Phi_m t}$$

$$\frac{v_1}{v_0} = -\ln(\Phi_m t - m_1)\Big|_0^{t_1} = -\ln \frac{\Phi_m t_1 - m_1}{-m_1} = \ln \frac{m_1}{m_1 - \Phi_m t_1}$$

$$v_1 = v_0 \cdot \ln \frac{m_1}{m_1 - \Phi_m t_1}$$

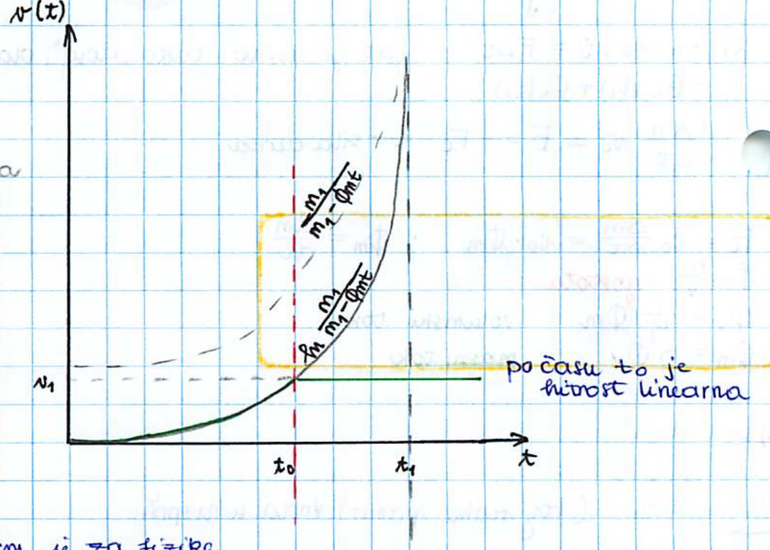
$v(t) = v_0 \ln \frac{m_1}{m_1 - \Phi_m t}$

$t_1 = \frac{m_1}{\Phi_m}$

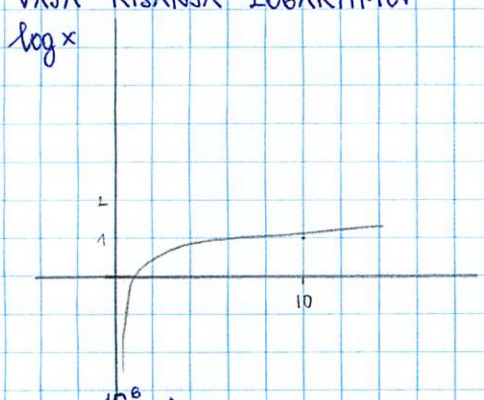
$t_0 = \frac{m_1 - m_0}{\Phi_m}$ ← izraz za čas, ko zmanjka goriva

$v_1 = v_0 \ln \frac{m_1}{m_0}$ končna hitrost

v_0 ~ hitrost izhajajočih plinov
$v_0 = 1000\ m/s$



VAJA RISANJA LOGARITMOV



Funkcija logaritem je za fizike konstantna (pač zanemarjo).

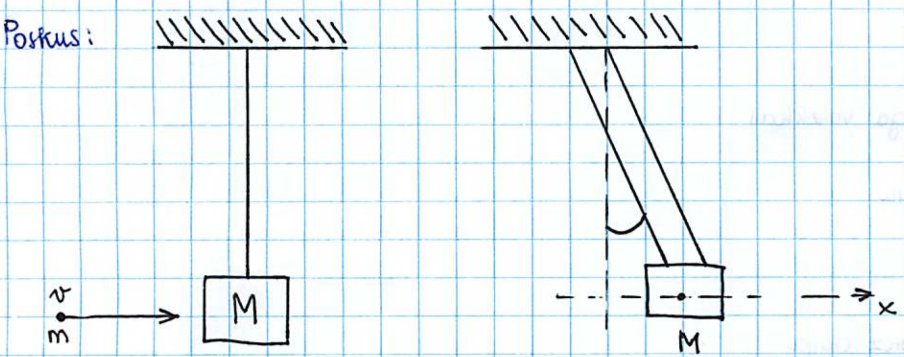
10^6 kg – masa celotne rakete
10^3 kg – koristni tovor

$\ln \frac{10^6}{10^3} \doteq 7$
$\ln 10^4 \doteq 9$ $\Rightarrow v_1 = 1\ km/s \cdot 9 = 9\ km/s$

Torej ne da se narediti hitrih raket. Či masa goriva 1000 krat težja od koristnega tovora, komaj spravimo raketo iz orbite, če 10 000 krat težja le 2 km/s več!

BALISTIČNO NIHALO:

Poskus:



$\omega = \sqrt{g/l}$
$x = \frac{v_0}{\omega} \sin(\omega t)$

$v = ?$

→ hitrost je 0 ob času 0 (klade)

$mv + M \cdot 0 = (m+M)\, v_0$

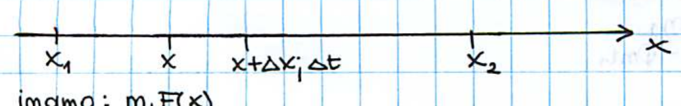
$v = \frac{M+m}{m} \sqrt{g/l}\, x_0$

$M = 310\,g$
$m = 0{,}5\,g$
$l = 590\,cm$
$x_0 = (20 \pm 1)\,cm$

x_0 [m]	v [m/s]
0,19	154
0,20	162
0,21	170

IZREK O KINETIČNI ENERGIJI; DELO:

1) 1 dimenzija in 1 točkasto telo:



imamo: m, F(x)

$F = m \cdot a$

$F \cdot \Delta x = m \cdot a\, \Delta x$ → če je Δt zadosti majhen

$m \frac{\Delta v}{\Delta t} \Delta x = F \Delta x$

$m\, \Delta v \frac{\Delta x}{\Delta t} = F \Delta x$ → približno v limiti $\Delta t \to 0$

$m v\, \Delta v = F \Delta x$

$\int_{v_1}^{v_2} m v\, dv = \int_{x_1}^{x_2} F(x)\, dx = A$ delo

Recimo $F = F_0$, konstantna sila

delo $\rightarrow$ $F_0(x_2 - x_1) = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2F_0 s}{m}}$

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \int_{x_1}^{x_2} F(x)\,dx$$

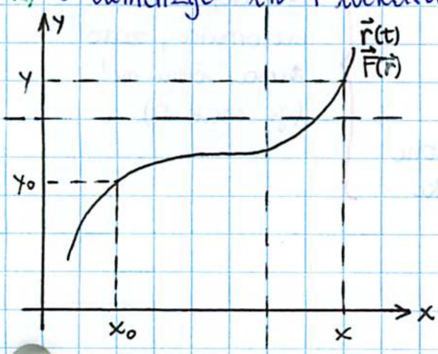
$\boxed{W_k = \frac{1}{2}mv^2}$ kinetična energija

prosti pad $F_0 = m \cdot g$

$v = \sqrt{\frac{2mgs}{m}} = \sqrt{2gh}$; $h = s$

Želimo vedeti, kolika je hitrost po določeni razdalji

2) 3 dimenzije in 1 točkasto telo



$\vec{r}(t) = (x(t), y(t), z(t))$

$\vec{F}(\vec{r}) = (F_x(\vec{r}), F_y(\vec{r}), F_z(\vec{r}))$

$\vec{r}_0$ ... začetna lega

$\vec{r}$ ... končna lega

$$x:\ \frac{1}{2}mv_x^2 - \frac{1}{2}mv_{x_0}^2 = \int_{x_0}^{x} F_x(x, y(x), z(x))\,dx$$

$$y:\ \frac{1}{2}mv_y^2 - \frac{1}{2}mv_{y_0}^2 = \int_{y_0}^{y} F_y(x(y), y, z(y))\,dy$$

$$z:\ \frac{1}{2}mv_z^2 - \frac{1}{2}mv_{z_0}^2 = \int_{z_0}^{z} F_z(x(z), y(z), z)\,dz$$

$\oplus$

F_z: sila v smeri z; dz: premik v smeri z

krivulja se lahko gleda kot krivulja časa, ali pa npr. odvisna od "x"..

$x, y(x), z(x)$ ali

$x(y), y, z(y)$ ali

$x(z), y(z), z$

$\frac{1}{2}m|\vec{v}|^2 - \frac{1}{2}m|\vec{v}_0|^2 = A$; če $\vec{F} \neq \vec{F}_0$:

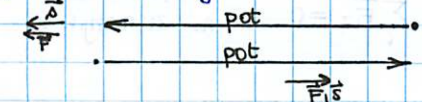
$F_x(x - x_0) + F_y(y - y_0) + F_z(z - z_0) = \vec{F} \cdot \vec{s}$; $\vec{s} = \vec{r} - \vec{r}_0$

Kdaj je delo $A = 0$?

1. $|\vec{F}| = 0$
2. $|\vec{s}| = 0$
3. $\vec{F} \perp \vec{s}$

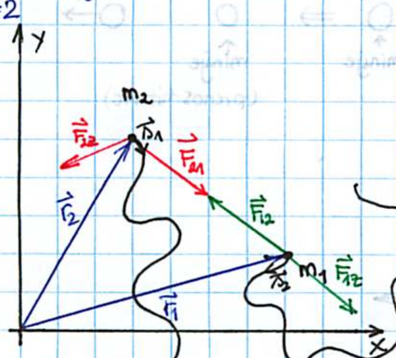
$$\boxed{A = \int_{\vec{r}_0}^{\vec{r}} \vec{F}\,d\vec{s}}$$ integral po poti $\vec{r}(t)$

Če je $s = 0$, ko je začetna točka hkrati končna, je delo vseeno opravljeno, ker je pomembno le, kakšna pot je bila opravljena. Delo se sešteva.



3.) 3 dimenzije in N teles (= točkastih)

N=2



$m_1\vec{a}_1 = \vec{F}_{1z} + \vec{F}_{12}$ $\quad \int ds_1$

$m_2\vec{a}_2 = \vec{F}_{2z} + \vec{F}_{21}$ $\quad \int ds_2$

$$(1)\ \frac{1}{2}m_1v_1^2 - \frac{1}{2}m_1v_{10}^2 = \int \vec{F}_{1z}\,d\vec{s}_1 + \int \vec{F}_{12}\,d\vec{s}_1$$

hitrost v nekem času; začetna hitrost

$$(2)\ \frac{1}{2}m_2v_2^2 - \frac{1}{2}m_2v_{20}^2 = \int \vec{F}_{2z}\,d\vec{s}_2 + \int \vec{F}_{21}\,d\vec{s}_2$$

integrala po poti

$$\int \vec{F}\,d\vec{s} = \int F_x\,dx + \int F_y\,dy + \int F_z\,dz$$

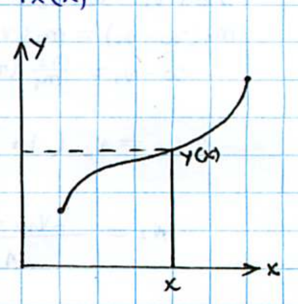
Pri gibalni količini smo integrali po t, pri kinetični energiji pa po poti oz. kraju.

$v_2^2 = |\vec{v}_2|^2$

$v_1^2 = |\vec{v}_1|^2$

Sila je odvisna od poti $\vec{F}(x,y,z) = \vec{F}(x, y(x), z(x)) = (F_x, F_y, F_z)$

$F_x(x)$



pot je krivulja, tir.

Seštejemo (1) in (2):

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \left(\frac{1}{2}m_1v_{10}^2 + \frac{1}{2}m_2v_{20}^2\right) = W_k - W_{k0} = A_Z + A_N$$

delo zunanjih sil; delo notranjih sil

skupna kin. energija; skupna kin. energija na začetku

Sili $\vec{F}_{12} = -\vec{F}_{21}$ sta nasprotni, a se integrala ne izničita, saj nimata iste poti.

$\boxed{W_k - W_{k0} = A_Z + A_N}$

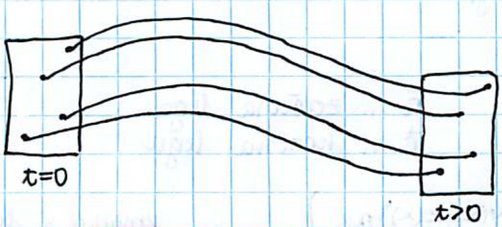
$W_k = \sum_{i=1}^{N} \frac{1}{2}m_i v_i^2$ $\qquad A_Z = \sum_{i=1}^{N} \int \vec{F}_{iz}\,d\vec{s}_i$

pot i-te točke

$$A_N = \underbrace{\int \vec{F}_{12}\, d\vec{s}_1 - \int \vec{F}_{12}\, d\vec{s}_2}_{} + \underbrace{\int \vec{F}_{13}\, d\vec{s}_1 - \int \vec{F}_{13}\, d\vec{s}_3}_{=0} + \dots$$

a) v splošnem ni enako 0 (druga pot, sili obratno nasprotni)

b) Vsi tiri so enaki, torej so dva po dva integrala enaka 0.



To (•) so toga telesa, poteka le translacija (ni rotacije) $\Longrightarrow A_n = 0$

$\overset{*}{\Longrightarrow} A_n = 0$

* tudi če se telo vrti (=rotacija) ↗ sile so pravokotne na premike

togo telo (množica N točk) se ne deformira, zato tega člena ni! (Je enak 0).

togo telo: $\boxed{W_k - W_{k0} = A_z}$

Primeri:

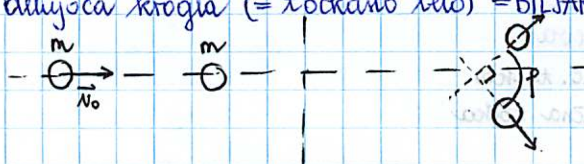
1. $\Delta W_k = A_z + \underbrace{A_n}_{0,} = \sum_i \int \vec{F}_{iz}\, d\vec{s}_i$

če je togo telo

$\Delta \vec{G} = \int \vec{F}_z\, dt$

↳ rezultanta zunanjih sil (kot, da je le eno točkasto telo)

- delujoča krogla (= točkasto telo) = BILJARD

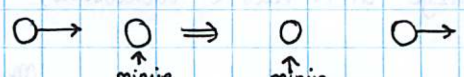


$\sum_i \vec{F}_{iz} = 0$ Predpostavka: ni trenja, to ni zunanjih sil

$m\vec{v}_0 = m\vec{v}_1 + m\vec{v}_2$
$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$

$$\left.\begin{array}{l} \vec{v}_0 = \vec{v}_1 + \vec{v}_2 \\ v_0^2 = v_1^2 + v_2^2 \end{array}\right\} \Longrightarrow \begin{array}{l} \vec{v}_1 \cdot \vec{v}_2 = 0 \\ |\vec{v}_0|^2 = v_1^2 + v_2^2 + \underbrace{2\vec{v}_1 \cdot \vec{v}_2}_{0} \end{array}$$

Če ustrelimo OSNO:



2. prožen trk v 1 dimenziji: $A_N = 0$
($A_z = 0$; ni nujno) ← ni teže

$m_1 v_{10} + m_2 v_{20} = m_1 v_1 + m_2 v_2 \iff m_1(v_{10} - v_1) = m_2(v_2 - v_{20})$ (1)

$\cancel{\frac{1}{2}} m_1 v_{10}^2 + \cancel{\frac{1}{2}} m v_{20}^2 = \cancel{\frac{1}{2}} m_1 v_1^2 + \cancel{\frac{1}{2}} m_2 v_2 \iff m_1(v_{10}^2 - v_1^2) = m_2(v_2^2 - v_{20}^2)$

$m_1(v_{10} - v_1)(v_{10} + v_1) = m_2(v_2 - v_{20})(v_2 + v_{20}) \quad / : (1)$

$v_{10} + v_1 = v_2 + v_{20}$

$$\left.\begin{array}{l} v_{10} + v_1 = v_2 + v_{20} \\ m_1(v_0 - v_1) = m_2(v_2 - v_{20}) \quad / : m_1 \neq 0 \\ v_0 - v_1 = \frac{m_2}{m_1}(v_2 - v_{20}) \end{array}\right\} \oplus$$

$$2v_{10} = v_2\left(1 + \frac{m_2}{m_1}\right) + v_{20}\left(1 - \frac{m_2}{m_1}\right)$$

$$v_2 = \frac{2v_{10} - v_{20}\left(1 - \frac{m_2}{m_1}\right)}{1 + \frac{m_2}{m_1}}$$

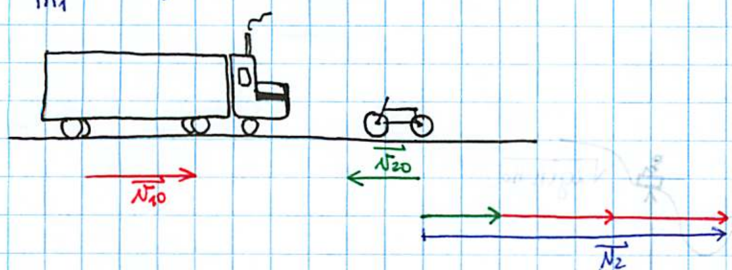
$$v_1 = v_2 + v_{20} - v_{10}$$

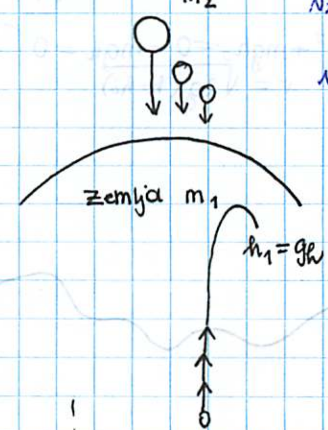
Posebni primeri:

1. $m_1 = m_2$

$v_2 = v_{10}$
$v_1 = v_{20}$

2. $\frac{m_2}{m_1} \to 0$ $\quad A_N = 0$

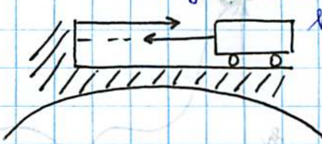




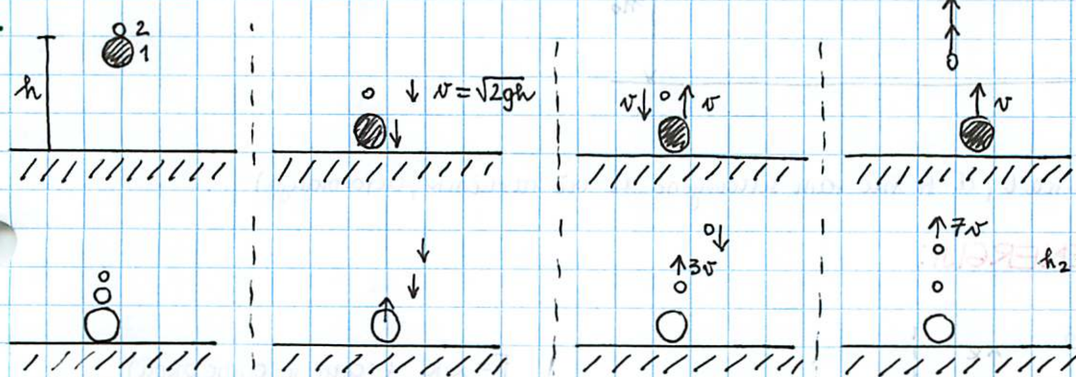
$v_2 = \frac{2v_{10} - v_{20}}{1}$ ← limitni rezultat

$v_1 = v_{10}$

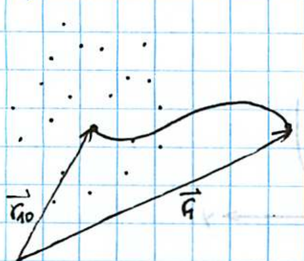
Če se telo zaleti v mirujoče telo, se odbije z enako hitrostjo.



3.



Sklep:



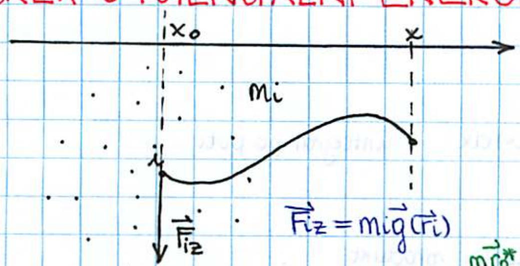
$$W_K = \sum_{i=1}^{N} \frac{1}{2} m_i v_i^2 \qquad v_i = |\vec{v}_i|$$

$$A_Z = \sum_{i=1}^{N} \int_{\vec{r}_{i0}}^{\vec{r}_i} \vec{F}_{iz} \, d\vec{s}_i$$

Togo telo: a) vse poti so enake
b) sila je pravokotna na premik (=rotacije) $\Big\}$ $A \approx 0$

Delo zunanjih sil je enako nič:
+ npr. na ledu $\vec{F}_p$, $\vec{s}$, $\vec{F}_g$ (trenja ni)
+ v breztežnem prostoru

IZREK O POTENCIALNI ENERGIJI:



$\vec{F}_{iz} = m_i \vec{g}(\vec{r}_i)$

I. $g = 9{,}81\ m/s^2$
g je konstanta.
$\vec{g}(\vec{r}_i) = \vec{g}_0 = $ konstanta

II. planeti
("oddaljevanje od Zemlje je sorazmerno z zmanjševanjem g")

Če h, h_0 (os z)

Def.: $\int\limits_{\text{po poti}} \vec{g} \, d\vec{s} = \int g_x dx + \int g_y dy + \int g_z dz =$

$= g_{x_0} \int_{x_0}^{x} dx + g_{y_0} \int_{y_0}^{y} dy + g_{z_0} \int_{z_0}^{z} =$

$= g_{x_0}(x - x_0) + g_{y_0}(y - y_0) + g_{z_0}(z - z_0)$

$$A_Z = \sum_{i=1}^{N} \int_{\vec{r}_{i0}}^{\vec{r}_i} m_i \vec{g}_0 \, d\vec{s}_i + \underset{\text{druge zunanje sile}}{A} = \sum_{i=1}^{N} \overbrace{\underbrace{m_i \vec{g}_0 \cdot (\vec{r}_i - \vec{r}_{i0})}_{m\vec{r}^*}}^{m\vec{r}_0^*} + A_{dzs} = m \cdot \vec{g}_0 (\vec{r}^* - \vec{r}_0^*) + A_{dzs} =$$

$$= m(0, 0, -g) \cdot (\Delta x, \Delta y, \Delta z) + A_{dzs} = -mg\Delta h + A_{dzs}$$

$\Delta h = h - h_0$

$\boxed{A_Z = -mg\Delta h + A_{dzs}}$ ($\Leftarrow$ to delo je neodvisno od poti!, le višinska razlika je pomembna)

$\boxed{W_p = m \cdot gh}$ potencialna energija

$\boxed{W_p = m U_p}$ $\quad U_p$... gravitacijski potencial

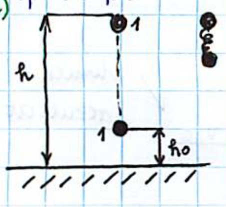
$\boxed{U_p = g \cdot h}$

$\boxed{W_K + W_p - W_{K0} - W_{p0} = \underset{\text{vse sile razen teže}}{A_{dzs}} + A_N}$ $\quad$ Izrek o potencialni energiji

Če je $W_K + W_p - W_{K0} + W_{p0} = 0$, potem se W_K in W_p ohranjata „skupaj".

Primeri:

a) prosti pad



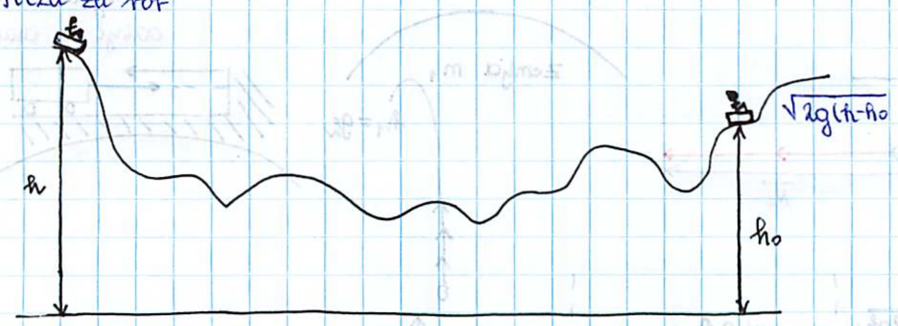
najprej spustimo spodnjo, nato še zgornjo. Vidimo, da hitrost ni enaka.

$$\sum_{i=1}^{N} \frac{1}{2} m_i v_i^2 = W_k = \frac{1}{2} v^{*2} \sum_{i=1}^{N} m_i = \frac{1}{2} m v^{*2} = \frac{1}{2} m v^2$$

$$\frac{1}{2} m v^2 + mgh_0 - 0 - mgh = 0 \rightarrow \text{ni dela}$$

$$v = \sqrt{2g(h-h_0)}$$

b) steza za rolke



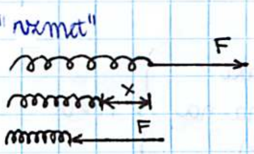
$A_Z = 0$ (= težo odmislimo)

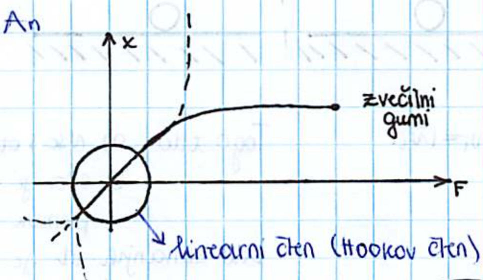
$A_N = 0$ (= če ne bi bila enaka 0, če bi bile sani sestavljene iz več različnih; deformacija)

IZREK O PROŽNOSTNI ENERGIJI:

$$W_k + W_p - W_{k0} - W_{p0} = A_z + A_n$$

"vzmet"

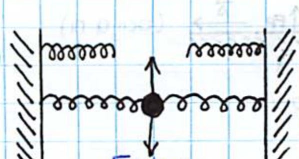




$F = x \cdot k$ (= graf se obrne okoli)

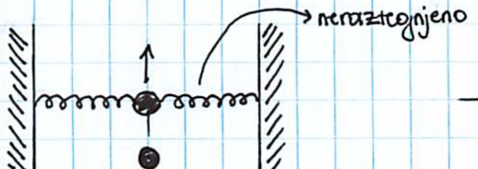
Hookov zakon

(a)

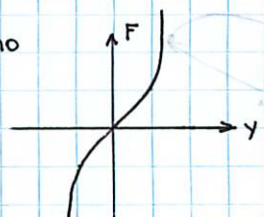


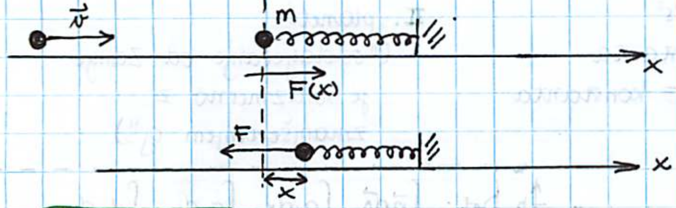
sin ωt

(b)



$F = gx^3$; eliptične funkcije (t)





$$A = \ominus \int_{x_1}^{x_2} F(x)\,dx$$ integral po poti

$\vec{F} \cdot \vec{s}$ → skalarni produkt je negativen

vektor v levo (ker zavira kroglica, ko ta prileti na vzmet).

vektor desno

$$\boxed{A = -\int_{x_1}^{x_2} F(x)\,dx} =$$

$$= + \int^{x_0} (-kx)\,dx = -\frac{1}{2} k x_0^2$$

$$\boxed{A = -\frac{1}{2} k x_2^2 + \frac{1}{2} k x_1^2}$$

$$\boxed{W_{pr} = \frac{1}{2} k x^2 = \frac{1}{2} k (x - x_{neraztegnjeno})^2}$$ x merjeno od mirovne lege

$$W_k + W_p + W_{pr} - W_{k0} - W_{p0} = A_{drs} + A_N$$

$$\left(\frac{1}{2} m v^2 + 0 + \frac{1}{2} k x^2\right) - \left(\frac{1}{2} m v_0^2 + 0\right) = A_{drs} + A_N$$

$\overset{če}{=} 0$ ← togo telo, ni drugih sil (= preprostejše)