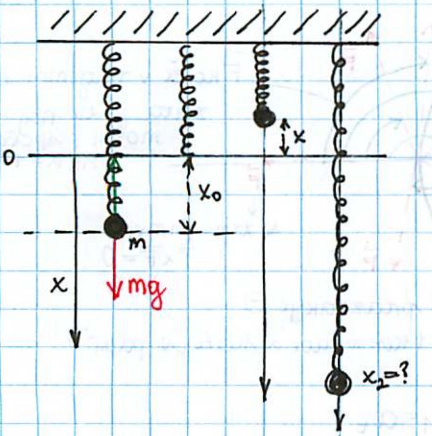
Primer:



k... hookov zakon (pove nam, kolikšna vzmet je)

$mg = kx_0$

$x_0 = \frac{mg}{k}$

Zunanje sile so sile vzmeti in teže, vendar jih ne štejemo, ker so že uporabljene v energiji.

V točki x_2 vzmet miruje.

$$\left(\underbrace{\tfrac{1}{2}mv^2}_{=0} + (-mg\,x_2) + \tfrac{1}{2}kx^2\right) - \left(\underbrace{\tfrac{1}{2}mv_1^2}_{=0} + mg\,x_1(-1) + \tfrac{1}{2}kx_1^2\right) = 0$$

x_2 je štet navzdol ; $x_0 = 0$

Izrazimo x_2:

x_2 .....

$v_2(x) = ?$

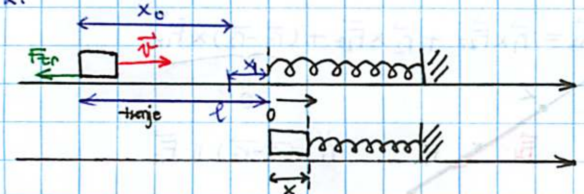
$W_p = mgz = -mgx$ $\quad x = -z$

mgz_1

mgz_2

$$\tfrac{1}{2}mv_2^2 + mgz_2 - \left(\tfrac{1}{2}mv_1^2 + mgz_1\right) = 0$$

Primer 2:



(a) $\frac{1}{2}m0^2 + \frac{1}{2}kx^2 - \frac{1}{2}mv^2 - 0 = 0$ ← Če ni drugih sil (ne trenja, nekonkol, itd.)

$x^2 = \frac{m}{k}v^2$

$x = \sqrt{\frac{m}{k}} \cdot v$

(b) Če je trenje:

(b.1) $0 - \frac{1}{2}mv^2 = -F_{tr}x_0$

↑ končna hitrost

$x_0 = \frac{1}{2}\frac{mv^2}{F_{tr}} = \frac{1}{2}\frac{mv^2}{mgk_{tr}} = \ldots$

(b.2) Telo pride do vzmeti in takrat ima še neko hitrost.

$0 + \frac{1}{2}kx^2 - \frac{1}{2}mv^2 - 0 = A_{drs} = -F_{tr}(l+x)$

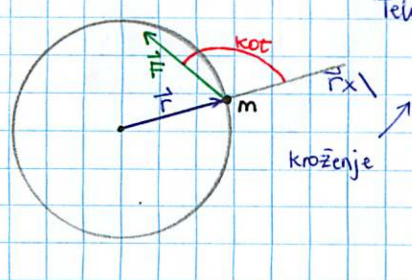
↑ začetna kin. E ↑ začetna prož. E

↳ kako daleč pride pri vzmeti

$\Rightarrow x = ?$ (= kvadratna enačba)

NAVOR

(a) v ravnini...



Telo se giblje po krožnici.

$m\vec{a} = \vec{F}$

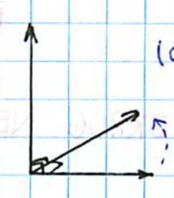
$m(\vec{a}_r + \vec{a}_t) = \vec{F}$

kroženje ↗ $m(\vec{r} \times (\vec{a}_r + \vec{a}_t)) = \vec{r} \times \vec{F}$

$m(\vec{r} \times \vec{a}_r) + m(\vec{r} \times \vec{a}_t) = \vec{r} \times \vec{F}$

$m(\vec{r} \times \vec{a}_r) = 0$, ker sta vzporedna

$0 + m \cdot r \cdot a_t = |\vec{r} \times \vec{F}|$



(desno sučno, ker kaže navzgor)

($a_t = r\alpha$ $\quad \alpha = \ddot{\varphi}$, $\alpha = \dot{\omega}$, $\omega = \dot{\varphi}$)

$\vec{r} \perp \vec{a}_t$ $\quad \sin\alpha = 1$

M,... navor

$\boxed{a_t = \alpha r}$, $\boxed{\vec{r} \times \vec{F} = \vec{M}}$

$m \cdot r \cdot a_t = |\vec{r} \times \vec{F}|$

$m \cdot r \cdot \alpha \cdot r = M$

$\boxed{\underbrace{mr^2}_{\text{vztrajnostni moment } J} \cdot \alpha = M}$

$\boxed{J \cdot \alpha = M}$ ← Newtonov zakon za tovrsten primer

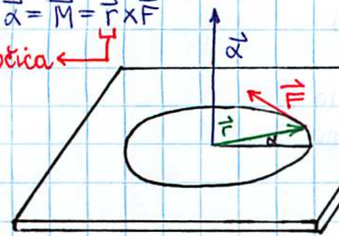
↑ vloga mase ↘ sile (vloga)

$J \cdot \alpha = M = r \cdot F \sin\varphi$

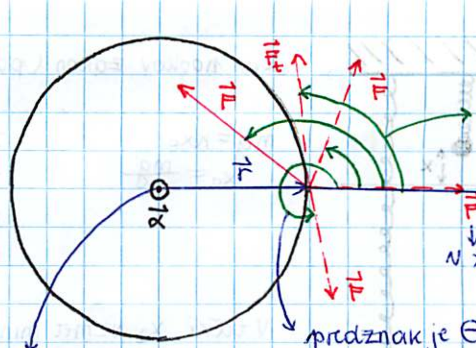
$J\vec{\alpha} = \vec{M} = \vec{r} \times \vec{F}$

ročica

$\vec{\alpha} = \alpha \vec{e}_z$



($\vec{r}$ zavrtiš v $\vec{F}$ in dobiš vektor, ki kaže navzgor)



$\vec{F}$ kaže v tagentni smeri, zato je tu največji možni pospešek, $\sin\alpha = 1$

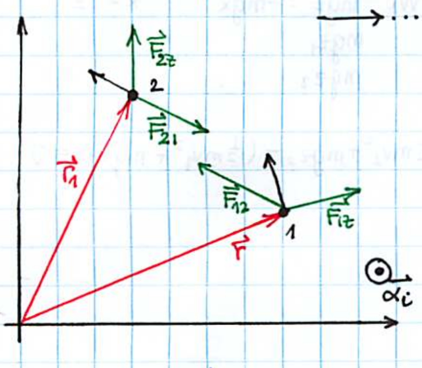
v tem primeru $\vec{r} \times \vec{F} = 0$

vektor, ki kaže pravokotno iz lista

predznak je ⊖ (ker vektor α kaže „v papir")

$r_1 \cdot \alpha_1 = a_t$

(b) $N = 2$



⟶ … tagentni pospešek

$m_1 \vec{a}_1 = \vec{F}_{1z} + \vec{F}_{12} \quad \vec{r}_1 \times \backslash$

$m_2 \vec{a}_2 = \vec{F}_{2z} + \vec{F}_{21} \quad \vec{r}_2 \times \backslash$

samo tagentni pospešek

$$\left.\begin{aligned} m_1(\vec{r}_1 \times \vec{a}_1) &= \vec{r}_1 \times \vec{F}_{1z} + \vec{r}_2 \times \vec{F}_{12} \\ m_2(\vec{r}_2 \times \vec{a}_2) &= \vec{r}_2 \times \vec{F}_{2z} + \vec{r}_2 \times \vec{F}_{21} \end{aligned}\right\} \oplus \qquad \vec{F}_{12} = -\vec{F}_{21}$$

$$m_1 r_1^2 \vec{\alpha}_1 + m_2 r_2^2 \vec{\alpha}_2 = \vec{r}_1 \times \vec{F}_{1z} + \vec{r}_2 \times \vec{F}_{2z} + \underbrace{(\vec{r}_1 - \vec{r}_2) \times \vec{F}_{12}}_{\overset{?}{=} 0}$$

Centralna sila (povezuje center teh točk; edini primer, kjer ne velja: to ni res pri magnetih)

če $(\vec{r}_1 - \vec{r}_2) \parallel \vec{F}_{12}$

Za $N > 2$, očitno!

Vzitem N točkastih teles.

Naj bo togo telo.

$\vec{\alpha}_i = \vec{\alpha}$ ← vsi so enaki

$J \cdot \vec{\alpha} = \vec{M}$

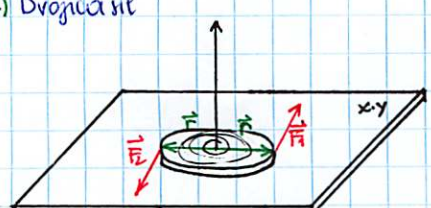
$J = \sum_{i=1}^{N} m_i r_i^2$

$\vec{M} = \sum_{i=1}^{N} \vec{r}_i \times \vec{F}_{iz}$

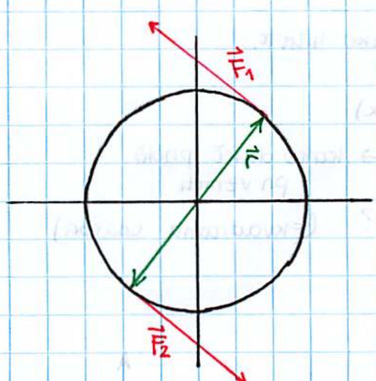
↑ ročice (vektor)

Primeri:

(a) Dvojica sil



Valj, ki se vrti okoli neke osi.



Nasprotni sili

$m\vec{a}^* = \vec{F}_1 + \vec{F}_2 = 0$

(giblje se s konstantno hitrostjo, če pa je na začetku mirovalo, bo ves čas mirovalo)

$J\vec{\alpha} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 =$

$= \vec{r}_1 \times \vec{F}_1 - \vec{r}_2 \times \vec{F}_1 =$

$= \vec{r}_1 \times \vec{F}_1 + \vec{r}_1 \times \vec{F}_1 =$

$= 2\vec{r}_1 \times \vec{F}_1$

$J\alpha = 2rF_1$

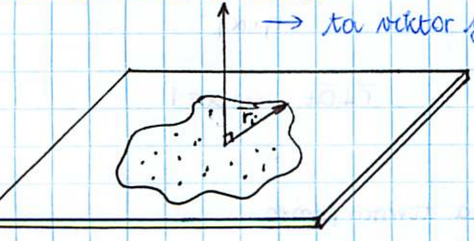
dvojica sil, ala dvojno …?

Težišče miruje.

VRTENJE TOGEGA TELESA OKROG NEPREMIČNE (FIKSNE) OSI

(a) Množica N teles v ravnini

⟶ ta vektor je lahko $\vec{\alpha}, \vec{\omega}, \vec{\varphi}$



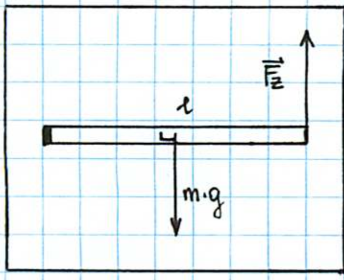
Vsakemu telesu lahko pripišemo lego $\vec{r}_i$.

$$J \cdot \vec{\alpha} = \vec{M} = \sum_{i=1}^{N} \vec{r}_i \times \vec{F}_{iz}$$

$$\sum_{i=1}^{N} m r_i^2 \vec{\alpha}_i = \sum_{i=1}^{N} \vec{r}_i \times \vec{F}_{iz} \qquad J = \sum_{i=1}^{N} m_i r_i^2$$

↑ vsi deli se vrtijo sinhrono, torej z enakim kotnim pospeškom.

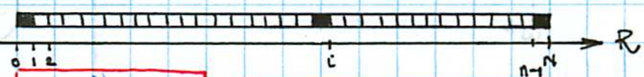
(b)



(=ravnina)

$$J\cdot\alpha = \frac{l}{2} m\cdot g$$
$$\alpha = \frac{l\cdot m g}{2\cdot J}$$
$$\varphi = \frac{1}{2}\alpha t^2$$

Palica:



$$\boxed{J = \sum_{i=1}^{N} m_i r_i^2}$$

$$\boxed{\Delta m_i = \frac{m}{l}\Delta r_i = \frac{m}{l}\cdot\frac{l}{N} \quad r_i = i\cdot\frac{l}{N}}$$

$$J = \sum_{i=1}^{N}\frac{m}{l}\cdot\frac{l}{N}\cdot\frac{l^2}{N^2} i^2 = \frac{m l^2}{N^3}\sum_{i=1}^{N} i^2$$

$$\sum_{i=1}^{N} i^2 = \frac{N(N+1)(2N+1)}{6}$$

Problem je, ker ne vemo izračunati vsote.

$$\frac{N(N+1)(2N+1)}{6} = \frac{N}{6}(2N^2+3N+1)$$

$$J = \frac{ml^2}{3}\left(1+\frac{3}{2}N^{-1}+\frac{1}{2}N^{-2}\right) \xrightarrow{N\to\infty} \frac{ml^2}{3}$$

$$dm = \frac{m}{l}\cdot dr$$

$$\boxed{J = \int r^2 dm = \int r^2\cdot\frac{m}{l}\,dr = \frac{m}{l}\cdot\frac{l^3}{3} = \frac{ml^2}{3}}$$

enak rezultat

po celem telesu

$$\boxed{\alpha = \frac{l m g\cdot 3}{2 m l^2} = \frac{3}{2} g/l}$$

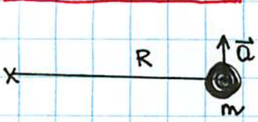
$$a = l\cdot\alpha = l\cdot\frac{3}{2} g/l = \frac{3}{2} g$$

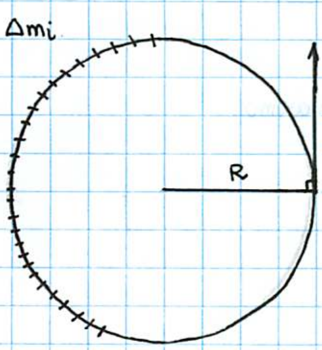
Očitno je, da je vztrajnostni moment odvisen od „lege osi." ← Ali je to sploh prav?

(c) **Obroč:**

(r je vedno enak)

$$\boxed{J = \sum_{i=1}^{N}\Delta m_i r_i^2 = m R^2}$$





(d) **okrogla plošča:**

R, m, d (= debelina nas ne zanima)

$$\boxed{J = \sum_{i=1}^{N} r_i^2 \Delta m_i} \xrightarrow{\Delta m_i \to 0} \int r^2 dm$$

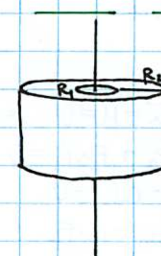
$$\Delta m = \frac{m}{\pi R^2 d}\, 2\pi r\,\Delta r\, d = 2\frac{m}{R^2} r\,\Delta r$$ ... koliko kilogramov na kubični meter

zelo majhno

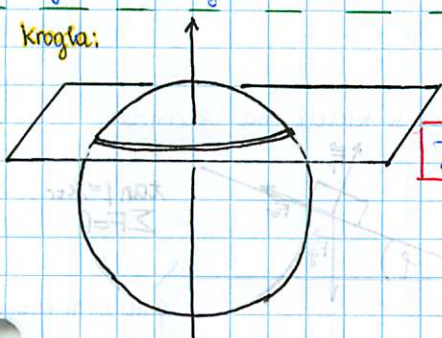
$$\int r^2 dm = \frac{2m}{R^2}\int_0^R r^3 dr = \boxed{\frac{1}{2} m R^2}$$

(e) **Valj:**

$$\boxed{J = \frac{1}{2} m R^2}$$



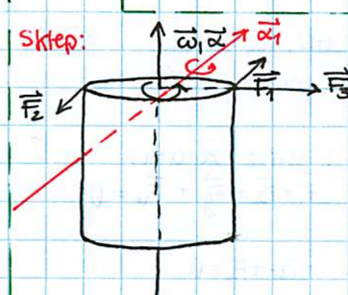
(f) **Krogla:**



$\frac{1}{2} m R^2$

$$\boxed{J = \frac{2}{5} m R^2}$$

Sklep:



$\vec{F_3}$ ne prispeva k pospešku. Težišče miruje.

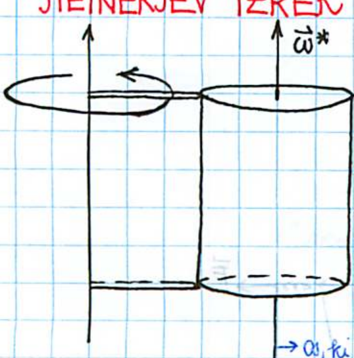
$$J\vec{\alpha} = \vec{M} = \sum_i \vec{r_i}\times\vec{F_{iz}}$$

$J = \frac{1}{2} m R^2$ ← ni odvisen (J) od višine (ker smo ga definirali na ravninskih likih)

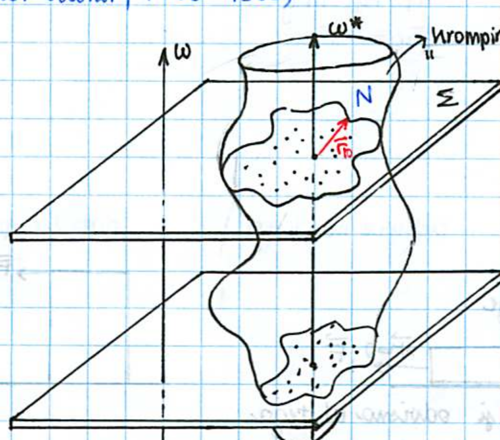
radij valja

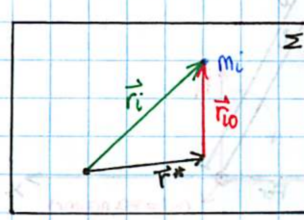
STEINERJEV IZREK

(Jakob Steiner, 1796 - 1863)



→ ω, ki gre skozi izhodišče





Sistem točkastih teles (=N) obravnavamo kot togo telo.

$$J = \sum_{i=1}^{N} m_i r_i^2 = \sum_{i=1}^{N} m_i\vec{r_i}\cdot\vec{r_i} =$$
$$= \sum_{i=1}^{N} m_i(\vec{r_{i0}}+\vec{r^*})\cdot(\vec{r_{i0}}+\vec{r^*}) =$$
$$= \sum_{i=1}^{N} m_i\left(r_{i0}^2 + r^{*2} + 2\vec{r_{i0}}\cdot\vec{r^*}\right) =$$
$$= J_0 + m r^{*2}$$

$$\sum_{i=1}^{N} m_i r_{i0}^2 = J_0$$

$$\sum_{i=1}^{N} m_i r^{*2} = m\cdot r^{*2}$$

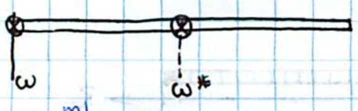
$$2\vec{r^*}\sum m_i\vec{r_{i0}} = 2\vec{r^*}\sum m_i(\vec{r_i}-\vec{r^*}) = 2\vec{r^*}\sum m_i\vec{r_i} - 2\vec{r^*}\vec{r^*}\sum m_i = 0$$

Steinerjev izrek:

$$\boxed{J_\omega = m r^{*2} + J_0;\quad J_0 = \sum_{i=1}^{N} m_i r_{i0}^2}$$

Če je telo zelo majhno, je J_0 prav tako zelo majhen - ga zanemarimo.

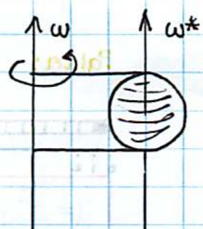
Primer:



$J = \frac{ml}{3} = J_\omega$

$J_0 = 2\cdot(\frac{l}{2})^2 \cdot \frac{m}{2} \cdot \frac{1}{3} = \frac{1}{12} ml^2$

$J_\omega = m(\frac{l}{2})^2 + \frac{1}{12} ml^2 = \frac{1}{3} ml^2$



$J_\omega = ml^2 + \frac{2}{5} mR^2$

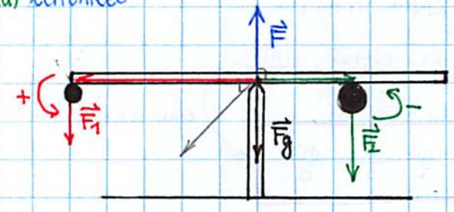
STATIKA

Imamo N teles, hitrost teh teles je $v_i = 0$.

(a) tehtnica

$\vec{F_g}$... teža palice (=jo zanemarimo)

$\vec{F}$... sila tehtnice



1. pogoj:

$\vec{v}^* = 0$; $\vec{\alpha}^* = 0$ (= če je $\vec{\omega} = 0$)

$\sum \vec{F_{iz}} = 0$ ← vsota vseh zunanjih sil na palico je enaka 0, če je tehtnica v ravnovesju

$\vec{F} + \vec{F_1} + \vec{F_2} = 0 = m \cdot \vec{a}^*$

$\vec{F} = -(\vec{F_1} + \vec{F_2})$; $\vec{r_1}, \vec{r_2}$

2. pogoj:

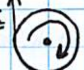
$\vec{\omega} = 0$, $\vec{\alpha} = 0$; $J \cdot \vec{\alpha} = \vec{M} = 0$ ← navor je 0, na katerokoli os, ker je $\vec{\alpha} = 0$.

$0 \cdot \vec{F} + 0 \cdot \vec{F_g} + \vec{r_1} \times \vec{F_1} - \vec{F_2} \times \vec{r_1} = 0$

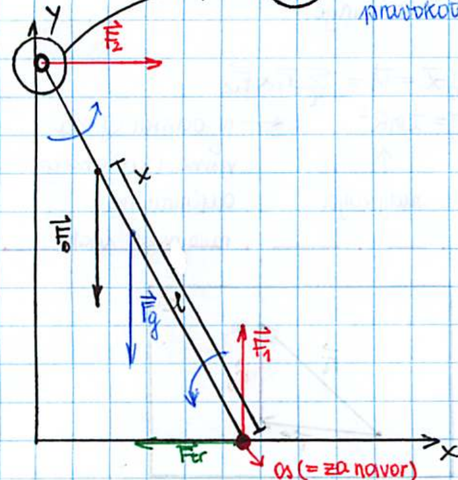
$r_1 \cdot F_1 - r_2 F_2 = 0$

$r_1 F_1 = r_2 \cdot F_2 \Longrightarrow m_1 \cdot r_1 = m_2 \cdot r_2$

(b) lestev:



kolešček se ne vrti, saj je sila pravokotna in navor je 0!



1. lestev že drsi:

$\vec{F_1} + \vec{F_2} + \vec{F_g} + \vec{F_{tr}} = 0$

x: $-F_{tr} + F_2 = 0$

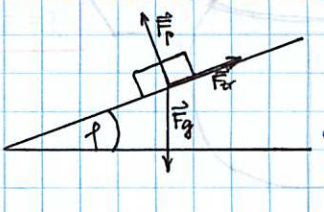
$F_2 = F_{tr} = k \cdot F_1 = k m g$

y: $F_1 - F_g = 0$

$F_1 = F_g = m \cdot g$

$F_{tr} = k \cdot F_1$

Spomnimo se klade na klancu:



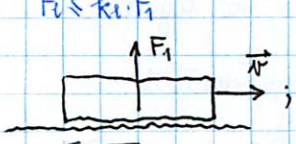
$\tan\varphi = k_{tr}$

$\sum F = 0$

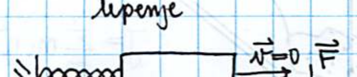
drsenje, ni vrtenja

2. lestev še ne drsi: (=lepenje) ; $Mg x \cos\varphi$ ... delavec na lestvi

$F_t \leq k_l \cdot F_1$

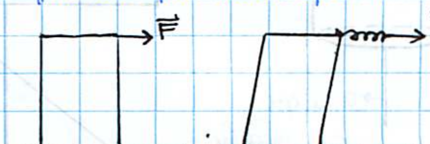


;



↳ sila je odvisna od tega raztezka

„Kateder pod mikroskopom":



$F_2 \leq k_l \cdot mg$

$\sum \vec{F} = 0$

$\sum \vec{M} = 0$ ← glede na katerokoli os

$0 + 0 + \vec{M_g} + \vec{M_2} = 0$

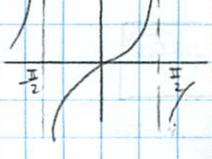
$\frac{1}{2} mg\cos\varphi = l \cdot F_2 \cdot \sin\varphi$; $F_2 \leq mg \cdot k$

$(Max\cos\varphi) + \frac{1}{2} m \cdot l \cdot \cos\varphi = l \cdot mg \cdot k \sin\varphi$

$\frac{1}{2}\cos\varphi = k \sin\varphi$

$\frac{1}{2}\frac{\cos\varphi}{\sin\varphi} = k$

$\frac{1}{k} = 2 \cdot \tan\varphi$



Čim večji je k, tem večje je trenje