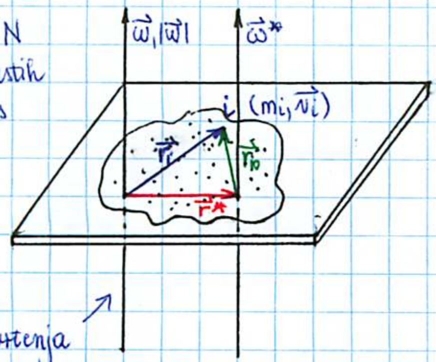
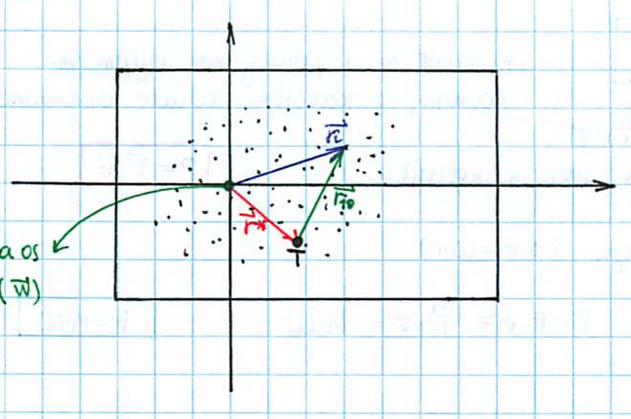
KINETIČNA ENERGIJA TOGEGA TELESA

(a) Sistem N točkastih teles



os vrtenja
(=pomembno je, da je normala ena sama)



dejanska os vrtenja ($\vec{\omega}$)

$$W_K = \frac{1}{2}\sum_{i=1}^{N} m_i v_i^2 = \frac{1}{2}\sum_{i=1}^{N} m_i(\vec{v}^* + \vec{v}_{i0})^2 =$$
$$= \frac{1}{2}\sum_{i=1}^{N} m_i (v^{*2} + 2\vec{v}^*\cdot\vec{v}_{i0} + v_{i0}^2) =$$
$$= \frac{1}{2}v^{*2}\sum_{i=1}^{N} m_i + \frac{1}{2}\sum_{i=1}^{N} m_i v_{i0}^2 + \vec{v}^*\cdot\sum m_i\vec{v}_{i0} =$$
$$= \frac{1}{2}mv^{*2} + \underbrace{\frac{1}{2}\sum_{i=1}^{N} m_i v_{i0}^2}$$

→ = v zvezi z vrtenjem glede na težišče (tega delami če se telo ne vrti in drsi, npr. led)

$$\dot{\vec{r}}_i = \vec{v}_i$$
$$\vec{v}_{i0} = \vec{v}_i - \vec{v}^* = \dot{\vec{r}}_{i0}^*$$

$$\sum_{i=1}^{N} m_i\vec{v}_{i0} = \sum_{i=1}^{N} m_i(\vec{v}_i - \vec{v}^*) =$$
$$= m\vec{v}^* - m\vec{v}^* = 0$$

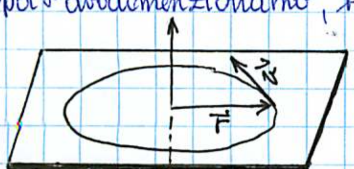
$\boxed{W_{RK} = \frac{1}{2}J\omega^2}$ rotacijska kin. energija

$$\boxed{W_K = \frac{1}{2}mv^{*2} + \frac{1}{2}\sum_{i=1}^{N} m_i v_{i0}^2 = W_P = mgh^*}$$

↑ hitrost bo manjša, če se bo telo kotalilo, sicer pa drselo bo višje (glede na W_P).

(b) 2 dimenziji + togo telo (= zanima nas hitrost $\dot{\vec{r}}^*$ glede na težišče $\overset{T}{\cdot}\xrightarrow{\vec{r}_{i0}} i$)
$\vec{v} = \vec{\omega}\times\vec{r}$ ↙ kot 90°

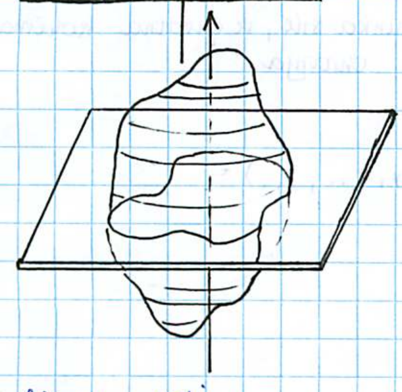
Ker je telo togo, bodo vse kotne hitrosti enako dolge $v_{i0} = v = \omega\cdot r$, ker pa je dvodimenzionalno, bodo imele še isto smer.



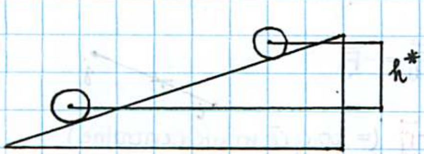
$\boxed{W_K = \frac{1}{2}mv^{*2} + \frac{1}{2}J_0\omega^2}$ $\boxed{J_0 = \sum_{i=1}^{N} m_i r_{i0}^2}$

↑ nepremična os + togo telo ↓ J_0 ostane enak

(c)



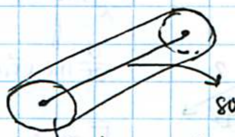
Vaja: klanec s sodi



$$v = \omega\cdot r$$
$$\frac{1}{2}mv^{*2} + \frac{1}{2}J\omega^2 = mgh^*$$
$$\frac{1}{2}mv^2 + \frac{1}{2}J\omega^2 = mgh$$
$$v^2\left(\frac{1}{2}m + \frac{1}{2}J/r^2\right) = mgh$$
$$v^2 = \frac{2mgh}{m + J/r^2}$$
$$v = \sqrt{\frac{2gh}{1 + J/mr^2}}$$

zač. W_P, končna W_K

Kako priti do $J=0$?



(1) $J_{drsenje} = 0$ — $v_1 = \sqrt{2gh}$
(2) $J = \frac{1}{2}mr^2$ — $v = \sqrt{\frac{2}{3}}v_1$
(3) $J = mr^2$ — $v = \sqrt{\frac{1}{2}}v_1$
(4) $J = \frac{2}{5}mr^2$ — $v = \sqrt{\frac{5}{7}}v_1$

MOČ

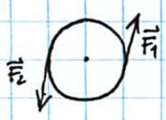
$$\boxed{P = \frac{A}{t} \;\ldots\; \frac{dA}{dt}}$$

kvocient med opravljenim delom in časom, v katerem to delo opravimo

(a) $P = \frac{d}{dt} \vec{F} \cdot \vec{s} = \vec{F} \cdot \vec{v}$

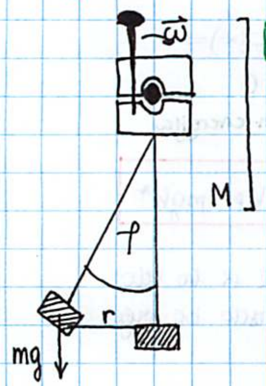
↳ či je sila konstanta $\boxed{P = \vec{F} \cdot \vec{v}}$

(b) moč pri vrtenju $(\Rightarrow v = \omega \cdot r)$



$P = F \cdot v = \frac{M}{r} \cdot v = M \cdot \omega$ $\boxed{P = M \cdot \omega}$

Pronyjeva zavora

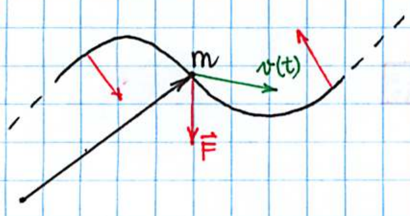


$\boxed{P_{mehanska} = M \cdot \omega}$

(= kakšno težo lahko motor dvigne)

IZREK O VRTILNI KOLIČINI

(a)



Telo se giblje z neko hitrostjo $\vec{v} = \dot{\vec{r}}$

že vemo:

$m \cdot \vec{a} = \vec{F}$

$m \frac{dv}{dt} = \frac{d}{dt} m\vec{v} = \frac{d}{dt} \vec{G} = \vec{F}$

$\vec{F} = \dot{\vec{G}}$

$\Delta \vec{G} = \int \vec{F}(t)\, dt$ ← či ni sunka sile, se gibalna količina ohranja

$\Delta \frac{1}{2} m v^2 = \int \vec{F}(\vec{r})\, d\vec{s}$

$m \cdot \vec{a} = \vec{F} \quad / \vec{r} \times$

$m \vec{r} \times \vec{a} = \vec{r} \times \vec{F} = \vec{M}$

$\frac{d}{dt}(\vec{r} \times \vec{v}) = (y v_z - z v_y, z v_x - x v_z, x v_y - y v_x)^{\cdot} = (\dot{y} v_z + y \dot{v}_z - \dot{z} v_y - z \dot{v}_y, \ldots, \ldots) =$

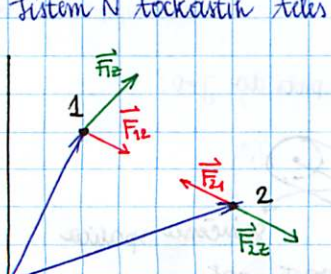
$= \dot{\vec{r}} \times \vec{v} + \vec{r} \times \dot{\vec{v}} = \vec{v} \times \vec{v} + \vec{r} \times \vec{a} = \vec{r} \times \vec{a}$

$m \frac{d}{dt}(\vec{r} \times \vec{v}) = \vec{M}$

$\boxed{\vec{\Gamma} = m \vec{r} \times \vec{v}}$ $\quad \vec{\Gamma} \ldots$ vrtilna količina

$\boxed{\frac{d}{dt} \vec{\Gamma} = \vec{M}}$

(b) Sistem N točkastih teles



$m_i \vec{a_i} = \vec{F_{iz}} + \sum_j \vec{F_{ij}} \quad i = 1, \ldots, N \quad \backslash \vec{r_i} \times, \oplus$

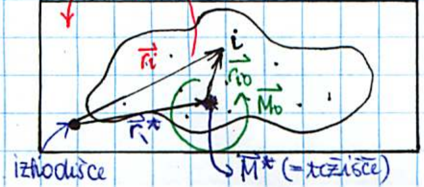
$\sum_i m_i (\vec{r_i} \times \vec{a_i}) = \sum_i (\vec{r_i} \times \vec{F_{iz}}) + \underbrace{\sum_{ij} (\vec{r_i} - \vec{r_j}) \times \vec{F_{ij}}}_{= 0} \; ; \quad \vec{F_{ij}} = -\vec{F_{ji}}$

$\frac{d}{dt} m_i (\vec{r_i} \times \vec{v_i})$

0, či $\vec{F_{ij}} \parallel \vec{r_i} - \vec{r_j}$ (= torej če so sile centralne)

$\vec{\Gamma} = \sum_i m_i (\vec{r_i} \times \vec{v_i}) = \sum_i \vec{\Gamma_i}$ $\quad \boxed{\frac{d\vec{\Gamma}}{dt} = \vec{M_z}}$

(c) nepremični osi (= vsaj vzporedne) za N teles



Definicije:

$\vec{\Gamma} = \sum_i m_i \vec{r_i} \times \vec{v_i}$

$\vec{\Gamma}^* = m \vec{r}^* \times \vec{v}^*$ (= kot da bi bila masa zbrana v težišču)

$\vec{\Gamma_0} = \sum_i m_i \vec{r_{i0}} \times \vec{v_{i0}}$ (= glede na težišče)

$\vec{M} = \sum_i \vec{r}_i \times \vec{F}_{iz}$

$\vec{M}^* = \vec{r}^* \times \sum_i \vec{F}_{iz} = \vec{r}^* \times \vec{F}$ ← navor teže

$\vec{M}_0 = \sum_i \vec{r}_{i0} \times \vec{F}_{iz}$ ← navor za vrtenje okoli težišča

Izrek: $\frac{d}{dt}\vec{\Gamma} = \vec{M}$ (= že dokazali)

$\frac{d}{dt}\vec{\Gamma}^* = \vec{M}^*$ Dokaz: $\frac{d}{dt} m\vec{r}^* \times \vec{v}^* = m(\vec{v}^* \times \vec{v}^* + \vec{r}^* \times \vec{a}^*) =$

$= \underbrace{m \cdot \vec{r}^* \times \vec{a}^*}_{\vec{F}_z} = \vec{r}^* \times \vec{F}_z$

$\frac{d}{dt}\vec{\Gamma}_0 = \vec{M}_0$ Dokaz: $\vec{\Gamma} = \vec{\Gamma}^* + \vec{\Gamma}_0$

$\vec{\Gamma} = \sum_i m_i(\vec{r}_i \times \vec{v}_i) = \sum_i m_i((\vec{r}^* + \vec{r}_{i0}) \times (\vec{v}^* + \vec{v}_{i0})) =$

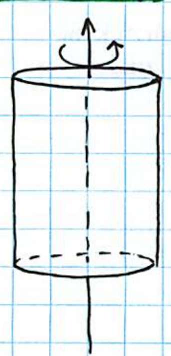
$= \sum_i m_i(\vec{r}^* \times \vec{v}^* + \vec{r}_{i0} \times \vec{v}^* + \vec{r}^* \times \vec{v}_{i0} + \vec{r}_{i0} \times \vec{v}_{i0})$

$= \vec{\Gamma}^* + \vec{\Gamma}_0$ $\quad \vec{r}_{i0} \times \vec{v}^* = 0 \quad$ $(m\vec{v}^* - m\vec{v}^*) \times \vec{r}^* = 0$

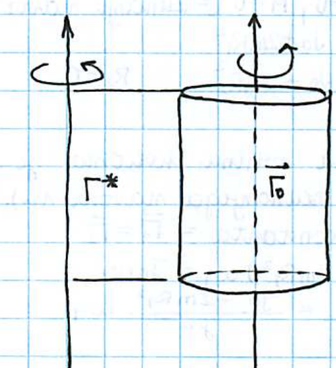
v^* ... konstanta

$\sum_i m_i \vec{r}_{i0} \times \vec{v}^* =$
$= \sum_i m_i(\vec{r}_i - \vec{r}^*) \times \vec{v}^* =$
$= (m\vec{r}^* - m\vec{r}^*) \times \vec{v}^* = 0$

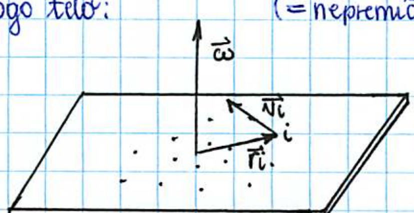
$\boxed{\vec{\Gamma} = \vec{\Gamma}^* + \vec{\Gamma}_0}$



$\vec{\Gamma} = \vec{\Gamma}_0, \ \vec{\Gamma}^* = 0$



(d) Togo telo: (= nepremična os, fiksna os)



$\frac{d}{dt}\vec{\Gamma} = \vec{M}$

$\vec{\Gamma} = \sum_i m_i \vec{r}_i \times \vec{v}_i \xrightarrow{togo} \sum_i m_i [\vec{r}_i \times (\vec{\omega} \times \vec{r}_i)] =$

$= \sum_i m_i [r_i^2 \vec{\omega} - (\vec{r}_i \cdot \vec{\omega}) \vec{r}_i] = J \vec{\omega}$

J = matrika

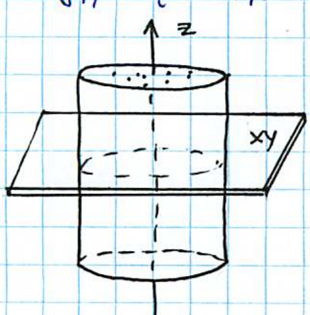
$J = \sum_i m_i (r^2 I - \vec{r} \otimes \vec{r}) = \sum_i m_i \begin{pmatrix} y^2+z^2, & xy, & xz \\ y x, & x^2+z^2, & yz \\ zx, & zy, & x^2+y^2 \end{pmatrix}_i$

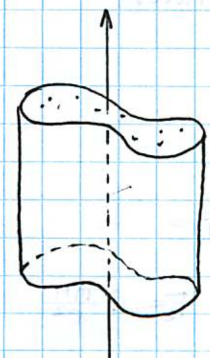
$\vec{a} \otimes \vec{b} = \begin{pmatrix} a_1 b_1 & \cdots & a_1 b_3 \\ \vdots & \ddots & \vdots \\ a_3 b_1 & \cdots & a_3 b_3 \end{pmatrix}$

$\vec{r}_i = (x_i, y_i, z_i)$

$\boxed{\vec{\Gamma} = J \cdot \vec{\omega}}$

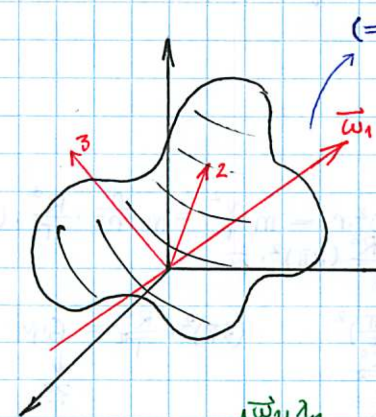
$J_{xy} = \sum_i m_i x_i y_i = 0$





(v prvem primeru je enako število neg. in poz. x, v drugem pa to ne velja)

(= če bi se vrteli v eni od teh smeri, bi se vrteli brez tresenja)

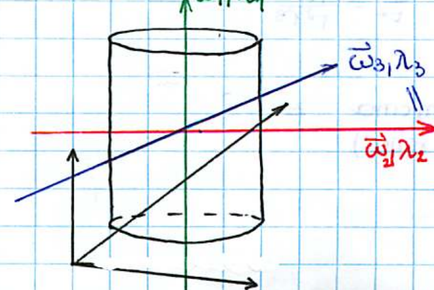


$\vec{\Gamma} = \lambda \vec{\omega} = J \vec{\omega}$

$(J - \lambda I)\vec{\omega} = 0$

$\Rightarrow \lambda_1, \vec{\omega}_1$
$\lambda_2, \vec{\omega}_2$
$\lambda_3, \vec{\omega}_3$

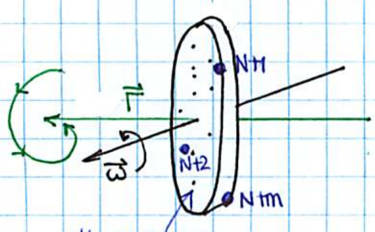
$\lambda_i \neq \lambda_j \Rightarrow \vec{\omega}_i \cdot \vec{\omega}_j = 0$



= smer vrtenja bo fiksna (= vrtilna količina je fiksna)

Primeri:

(a) kolo (= centriranje gum)



če N teles, dodamo še m teles

$\vec{\Gamma} = J \cdot \vec{\omega} = \vec{\Gamma}(t)$

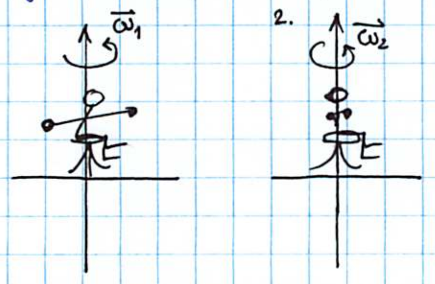
$J(t) \neq$ konstanta (ni fiksen, nekolinearen z osjo)

$\frac{d}{dt}\vec{\Gamma}(t) = \vec{M}(t) \neq 0$

$\vec{\Gamma} = J_\omega \vec{\omega}$ $\quad J\vec{\omega} = J\omega \cdot \vec{\omega} = \vec{\Gamma}, \; \dot{\vec{\Gamma}} = 0 = \vec{M}$

$\uparrow$ neka konstanta

(b) vrtljiv stol (= $\vec{\omega}_1, \vec{\omega}_2$ nepremična os)



$\vec{M}^* = 0, \; \vec{M} = 0$ (= zunanji navor)

$J_1 = J_0 + 2mR_1^2$

$J_2 = J_0 + 2mR_2^2 \qquad R_2 = 0$

J_0 ... vse razen rok in uteži

Celotna vrtilna količina je konstantna (= ker zunanjega navora ni).

$\vec{\Gamma} = \text{konstanta} = \vec{\Gamma}_1 = \vec{\Gamma}_2$

$(J_0 + 2mR_1^2)\,\omega_1 = J_0 \omega_2$

$\omega_2 = \frac{J_0 + 2mR_1^2}{J_0}\,\omega_1$

Koliko dela opravijo roke?

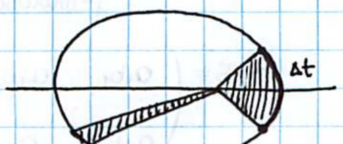
$W_{k_2} - W_{k_1} = A_{roke} = A_n$

$\frac{1}{2} J_0 \omega_2^2 - (\frac{1}{2} J_0 + 2mR_1^2)\,\omega_1^2 = A_n$

delo rok je pozitivno

GRAVITACIJSKA SILA (= Newtonovi zakoni 1687)

- Keplerjevi zakoni (= prva dva 1609, tretji 1618)
  1. Tiri planetov okoli sonca so ELIPSE.



  2. V različnih časovnih intervalih zveznica opiše enake ploščine (ponekod se giblje hitreje, ponekod počasneje).

$\Delta t = \text{konstanten} \Rightarrow \Delta S = \text{konstanten}$ } ohranitev vrtilne količine (sile so centralne).

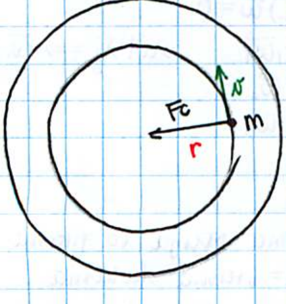
  3.

r_i, t_i

velikost elipse

$\frac{a_i^3}{t_i^2} = \text{konstanta} = \frac{(150\,000\,000\text{ km})^3}{(1\text{ leto})^2}$

- •



$F_c = m \cdot \omega^2 \cdot r = m\frac{v^2}{r} = m\frac{r}{r^3}\cdot\frac{R^3}{T^2}\cdot(2\pi)^2 =$

$= \frac{mR^3}{T^2}(2\pi)^2 \cdot \frac{1}{r^2}$

$\omega^2 = (\frac{2\pi}{t})^2$

$\frac{r^3}{t^2} = \frac{R^3}{T^2}$

$\frac{1}{t^2} = \frac{R^3}{T^2 r^3}$

$(2\pi)^2 \cdot \frac{R^3}{T^2} = G \cdot M$

$\downarrow$

??

$F_c = G\frac{mM}{r^2}$

F_c mora biti premo sorazmerna z $\propto \frac{1}{r^2}$ m M (= masa sonca)