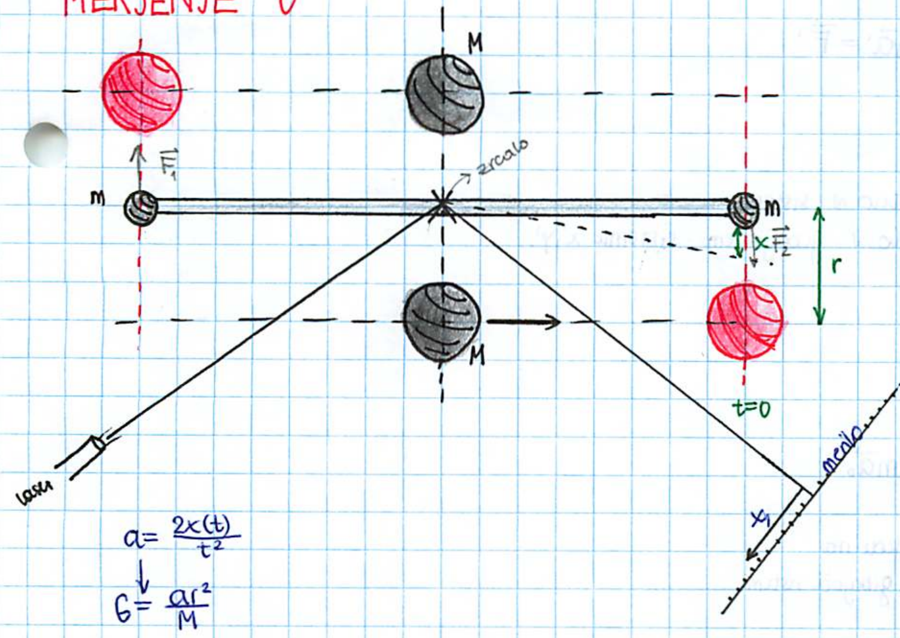
MERJENJE G



$$a = \frac{2x(t)}{t^2}$$

$$\downarrow$$

$$G = \frac{ar^2}{M}$$

$$G = \frac{2x(t)r^2}{M\cdot t^2}$$

Cavendish 1798

$$F = G\frac{mM}{r^2} = m\cdot a$$

$$G\frac{M}{r^2} = a$$

$$G = \frac{ar^2}{M}$$

$r = 3{,}8$ cm

$M = 3$kg

$$\frac{M\cdot m}{r^2}G = F \; ; \quad x \ll r$$

$$\dots \frac{1}{(r-x)^2}$$

$$F = m\cdot a = m\ddot{x}$$

$$x(t) = \frac{1}{2}at^2$$

$$a = \frac{2x(t)}{t^2}$$

$N = kg\,m/s^2$

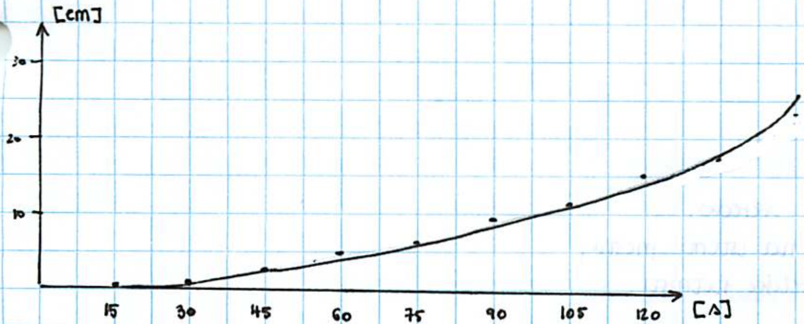
$$x_1 = \frac{x}{2{,}6\cdot 10^{-3}} \qquad x_1 = 10\text{ cm} \sim 100\text{ s}$$

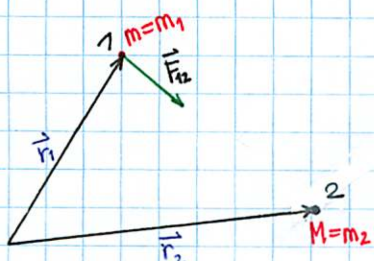
$$G = \frac{2\cdot 2{,}6\cdot 10^{-3}\cdot 10\text{cm}\cdot 3{,}8^2\cdot 10^{-6}\text{m}^2}{3\text{kg}\cdot 10^4\text{ s}^2} =$$

$$\doteq 6{,}67428\cdot 10^{-11}\ \text{Nm}^2/\text{kg}^2$$

Cavendisheva naprava za merjenje G:
- laser
- tanka nitka z majhnim prožnostnim koeficientom
- zrcalo
- svinčeni krogli, ki se gibljeta po medianosti cevi

v ravnovesju



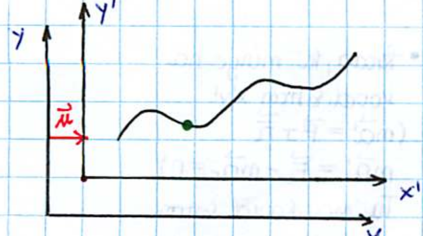


$$\vec{F}_{12} = G\frac{m_1m_2}{|\vec{r}_1-\vec{r}_2|^2}\cdot\frac{(\vec{r}_2-\vec{r}_1)}{|\vec{r}_2-\vec{r}_1|}$$

→ enotski vektor, ki kaže od telesa 1

$$\vec{F}_{12} \parallel \vec{r}_2-\vec{r}_1$$

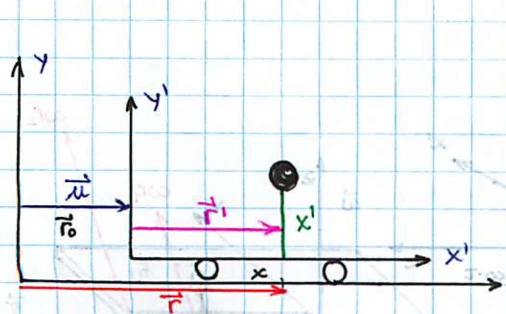
NEINERCIALNI (=POSPEŠENI) KOORDINATNI SISTEMI



$$\vec{r} = (x(t), y(t), z(t))$$

$\vec{u}(t) \Rightarrow$ ni nujno, da je konstanten

Primer:



$$\int_{t_1}^{t_2} u(t)\,dt$$

ni odvod, le oznaka

$$\vec{r}'(t) = (x'(t), y'(t), z'(t))$$

$$\dot{\vec{u}} = \vec{a}$$

$$\vec{r}' = \vec{r} - \vec{r}_0$$

$$\dot{\vec{r}}' = \vec{v}' = \dot{\vec{r}} - \dot{\vec{r}}_0 = \vec{v} - \vec{u} \; ;$$

hitrost gibajočega telesa

$$\boxed{\text{Galilejeva transformacija: } \vec{v} = \vec{u} + \vec{v}'}$$

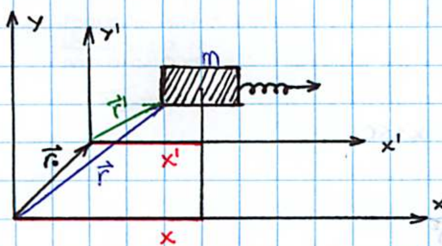
vlak se giblje 50 km/h, človek 5 km/h → skupaj 55 km/h

$$\ddot{\vec{r}}' = \vec{a}' = \dot{\vec{v}}' = \vec{a} - \vec{a}_0 \qquad \vec{a} = \vec{a}_0 + \vec{a}'$$

1. npr. $a_0 = 0$: $\vec{u} = \text{konstanta} = \vec{u}_0$

$$\vec{r}' = \vec{r} - \vec{u}_0 \cdot t$$
$$\vec{v}' = \vec{v} - \vec{u}_0 \quad \Longleftrightarrow \quad \vec{F} = m\vec{a} = m\vec{a}' = \vec{F}'$$
$$\vec{a}' = a \qquad\qquad \vec{F} = \vec{F}'$$

SISTEMSKE SILE



Mirujoče telo v sistemu xy se bo gibalo v gibajočem sistemu x', y'.

$\vec{r} = \vec{r}' + \vec{r}_0(t)$

$\vec{v} = \dot{\vec{r}} = \vec{v}' + \vec{u}_0$

$\vec{a} = \vec{a}' + \vec{a}_0$

$\vec{a}'$: gibajoč; $\vec{a}_0$: glede na mirujoč

$\vec{F} = m \cdot \vec{a} = m\vec{a}' + m\vec{a}_0$

$\vec{F}$: telo v mirujočem sistemu; $m\vec{a}'$: glede na gibajoči sistem

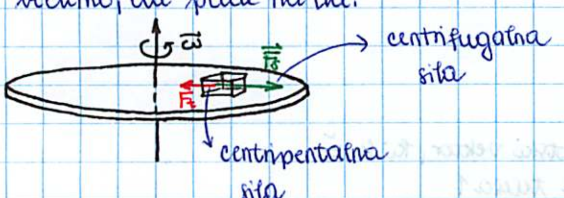
$m\vec{a}' = \vec{F} \underbrace{- m\vec{a}_0}_{\text{sistemska sila}} = \vec{F} + \vec{F}_s$ $\qquad \boxed{\vec{F}_s = -m\vec{a}_0}$

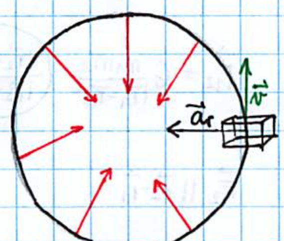
$m\vec{a}'$: glede na gibajoči sistem

sistemska sila: glede na sistem, ki ga gledaš

1. $\vec{a}_0 = \text{konstanten}$
   - dvigalo se dviga $10\,m/s^2$ navzgor. Spustimo kredo. Kreda pade na tla, a tistu ostane na istem mestu, le da se mi, ki jo opazujemo, zaradi velike hitrosti, vidimo, da pade na tla.

2.





$|\vec{F}_t| = k_{tr} \cdot m \cdot g$

$\vec{F}_{cp} = m \cdot \vec{a}$

$\vec{F}_{cp} = m \cdot \omega^2 \cdot \vec{r}\,(-1) = -m\omega^2\vec{r}$

$\vec{a}_r = -\omega^2 \cdot \vec{r}$

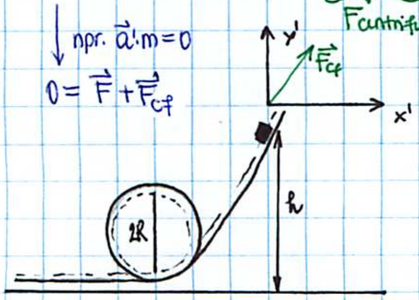
$\omega = \frac{v}{r} \quad \Longrightarrow \quad v = \omega \cdot r = \left(\frac{2\pi}{24h}\right) \cdot 6400\,km = 2000\,km/h$

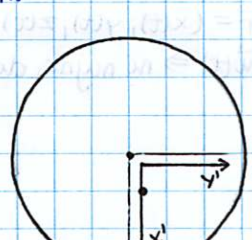
$m\vec{a}' = \vec{F} + \vec{F}_s$

$m\vec{a}' = \vec{F} - (m\omega^2\vec{r}) = \vec{F} \underbrace{+ m\omega^2\vec{r}}_{\text{Fcentrifugalna}}$

$\downarrow$ npr. $\vec{a}' \cdot m = 0$

$0 = \vec{F} + \vec{F}_{cf}$

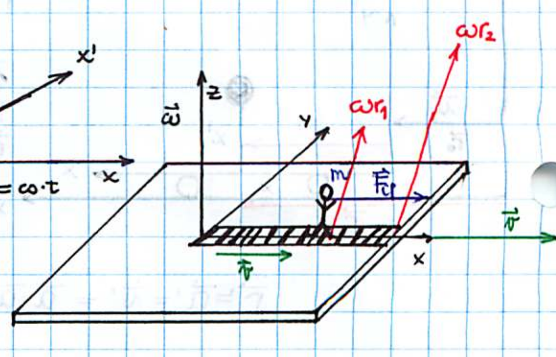




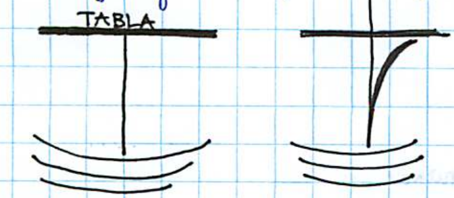
- Kreda, ki miruje na koord. sistem x'y'
  ($m\vec{a}' = \vec{F} + \vec{F}_c$
  $m\vec{a}' = \vec{F}_t - m\vec{a}_r = 0$)
  Ta nov koord. sistem ima lasten pospešek.

$\varphi = \omega \cdot t$

3. Coriolisova sila
   - radialna hitrost je vedno enaka
   - ↗ = tagentna hitrost, ki z bližanjem ekvatorju povečuje
   - v = konstantna

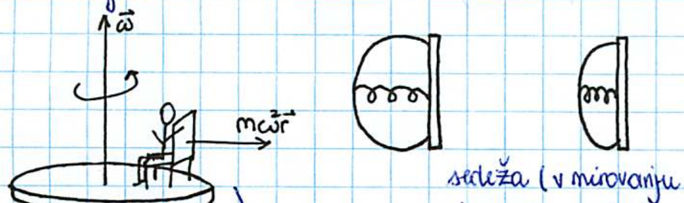


Primer: Merjeno glede na predavalnico



Poskusimo hoditi naravnost, a nas Coriolisova sila ($=\vec{F}_{cf}$) vleče desno.

Primer: Vrtiljak



sedeža (v mirovanju in med vrtenjem)

stol nas zadrži, da ne zletimo z vrtiljaka

$\vec{r} = v\cdot t\,(\cos\omega t, \sin\omega t, 0)$ $\qquad v=$ konstanta

$\dot{\vec{r}} = \vec{v} = v\,(\cos\omega t, \sin\omega t, 0) + v\cdot t\,(-\omega\sin\omega t, \omega\cos\omega t, 0)$ $\qquad \omega=$ konstanta

$\ddot{\vec{r}} = \vec{a} = v\,(-\omega\sin\omega t, \omega\cos\omega t, 0) + v\,(-\omega\sin\omega t, \omega\cos\omega t, 0) + v\cdot t\,(-\omega^2\cos\omega t, -\omega^2\sin\omega t, 0)$

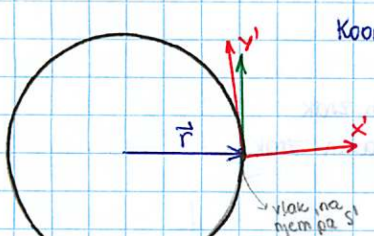
$= \underbrace{2v(-\omega\sin\omega t, \omega\cos\omega t, 0)}_{2\vec{\omega}\times\vec{r}\,;\ \vec{\omega}\perp\vec{v},\ \vec{v}\perp\vec{r}} + \underbrace{v\cdot t(-\omega^2\cos\omega t, -\omega^2\sin\omega t, 0)}_{-\omega^2\vec{r}}$

$\vec{e}_\perp = (-\sin\omega t, \cos\omega t, 0)$

↳ pravokoten na $\vec{r}$

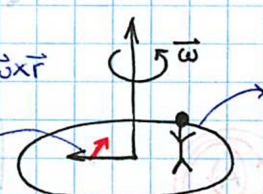
centripentalna (vleče navznoter)

$$\boxed{\vec{a} = -\omega^2\vec{r} + 2\vec{\omega}\times\vec{r}}$$



Koordinatni sistem S' glede na vlak

$m\vec{a}' = \vec{F} - m\vec{a}_0 = \vec{F} + m\omega^2\vec{r} - 2m\vec{\omega}\times\vec{r}$



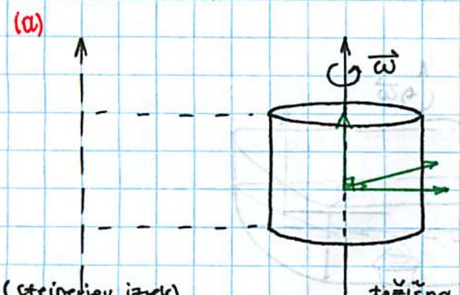
pri vrtenju nas "ven" vleče centrifugalna sila ($\vec{F} = m\omega^2\vec{r}$). Bolj proti sredini stojimo, manj nas vleče "ven".

Če med vrtenjem hodimo, nas v desno vleče Coriolisova sila.

POSKUSI

1. Vrtilna količina

(a)



(steinerjev izrek)

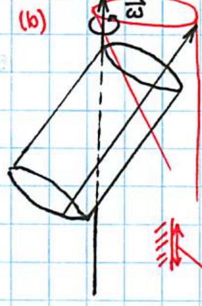
matrika

$J_\omega\vec{\omega} = \vec{\Gamma}$

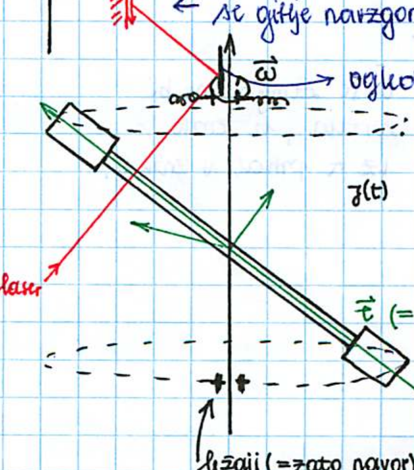
$\vec{\Gamma} = \sum_i m_i\vec{r}_i\times\vec{v}_i$

$\vec{v}_i = \vec{\omega}\times\vec{r}_i$

(b)



$$\vec{\Gamma} = \begin{pmatrix} J_{xx} & J_{xy} & J_{xz} \\ \vdots & \vdots & \vdots \end{pmatrix}_{3\times3}\vec{\omega} = J\vec{\omega}$$



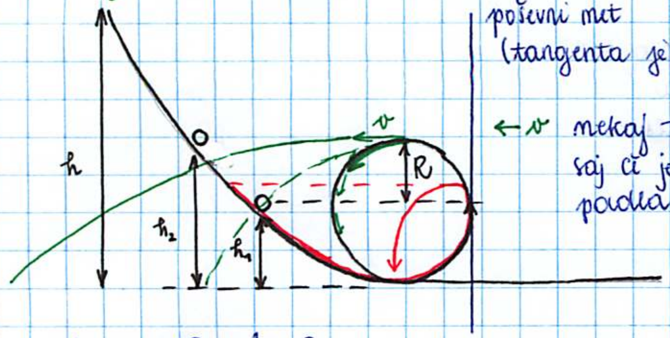
← se giblje navzgor in navzdol (zaradi navora)

$J(t)\,\vec{\omega} = \vec{\Gamma}(t)$

$\frac{d\vec{\Gamma}(t)}{dt} = \vec{M}(t)$

$\vec{\Gamma}$ (= vztrajnostni moment ni vzporeden z $\vec{\omega}$)

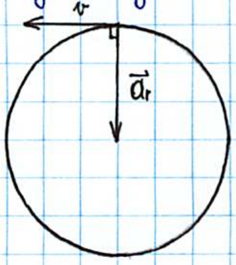
2. Looping



poševni met
(tangenta je poševna)

$\leftarrow v$ nekaj hitrosti mora biti, saj či je ne bi bilo, bi kroglica padla navzdol (višina je enaka višini, s katere je kroglica spuščena)

$$mgh_2 = mg2R + \frac{1}{2}mv^2$$



$$\vec{a}_r = \frac{v^2}{R} = g$$

$h_2 = 2R + \frac{R}{2} = \frac{5}{2}R$ ... drsenje kroglice

$mgh_2 = mg2R + \frac{1}{2}mv^2 + \frac{1}{2}J\omega^2$ .... kotaljenje kroglice

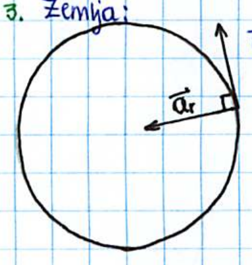
$h_2 = \frac{5}{2}R$

$J = \frac{2}{5}mr^2$

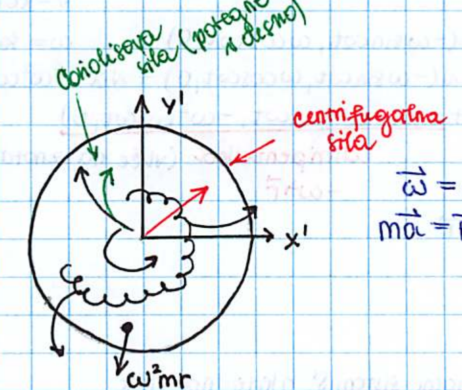
$\omega = \frac{v}{r}$

$$h_2 = 2R + \frac{1}{2}R + \frac{1}{2}\left(\frac{2}{5}\right)R = \frac{27}{10}R$$

3. Zemlja:



$$\vec{a}_r = \frac{v^2}{r} = \omega^2 r$$



$$\vec{\omega} = \omega \vec{R}_z$$

$$m\vec{a} = \vec{F} + m\omega^2\vec{r} - 2m\vec{\omega} \times \vec{v}$$

4. Vreme

$1m^3$ .... 1,013 kg suh zrak

$1m^3$ ..... 0,923 kg vlažen zrak

O_2 ... 32 g/mol

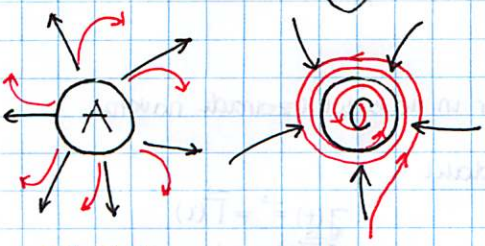
N_2 ... 28 g/mol

CO_2 .... 48 g/mol

18 g/mol .... $+H_2O$

29 kg/mol

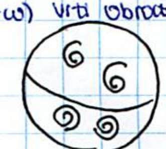
1 kmol = $6{,}0221417930 \cdot 10^{26}$ delcev/mol $\in \mathbb{N}$



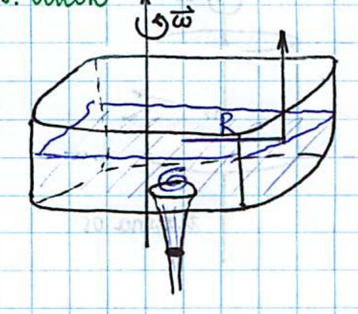
suh zrak
(hladen zrak je težji)

Če se Zemlja ne bi vrtela, bi zrak iz A pihal v zrak C.

Na južni polobli se zrak zaradi negativnega predznaka ω (−ω) vrti obratni smeri.



5. Odtok



Zaradi višinske razlike (ko nekaj vode odteče) in da bi se ohranila vrtilna količina, se mora voda, ki je na nižji višini vrtet hitreje.