Vprašanje je : smo v EMP → v neki umeritvi, imamo ψ
spremenimo umeritev, kako se spremeni ψ?
Fizikalna realnost naj se ne bi spremenila.

Izkazalo se bo, da se spremeni. So podelili Nobelovo za to, ker so merili pretok. V bistvu nista le $\vec{E}$ in $\vec{B}$ carja, tudi pretok je... svašta.

Umeritvena transformacija in Ψ

1) $\vec{E}, \vec{B}$

$\vec{B} = \nabla \times \vec{A}$

$\vec{E} = -\nabla \phi$

Ψ

vpeljemo $\vec{A}' = A + \nabla \Lambda$ (veliki lambda; lahko dodamo, $\vec{B}$ se ne spremeni) $\Lambda \in \mathbb{R}$ → realno polje

$$\phi' = \phi - \frac{\partial \Lambda}{\partial t} \qquad \Psi' = ?$$

Damo v Schröd. enačbo :

$$\left[\frac{1}{2m}(-i\hbar\nabla - e\vec{A})^2 + e\phi\right]\psi(\vec{r},t) = i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t)$$

ψ recimo, da poznamo.

Trdimo, da je $\psi'(\vec{r},t) = \underbrace{e^{i\frac{e}{\hbar}\Lambda(\vec{r},t)}}_{\text{faza}}\psi(\vec{r},t)$. $|\psi'| = |\psi|$ to mi merimo. To ostane enako.

Pokažimo, da je res :

$$\left[\frac{1}{2m}(-i\hbar\nabla - e\vec{A}')^2 + e\phi'\right]\psi'(\vec{r},t) = i\hbar\frac{\partial}{\partial t}\psi'(\vec{r},t)$$

$$A_x' = A_x + \frac{\partial}{\partial x}\Lambda$$
$$\phi' = \phi - \frac{\partial}{\partial t}\Lambda$$

pomožni račun: če imamo $f(x)$ in $g(x)$ in odvajamo : $\frac{\partial}{\partial x}e^f g - \left(\frac{\partial f}{\partial x}\right)e^f g =$

$$= e^f\left(\frac{\partial f}{\partial x}\right)g + e^f\frac{\partial g}{\partial x} - \left(\frac{\partial f}{\partial x}\right)e^f g$$

$$= \underbrace{e^f\frac{\partial g}{\partial x}}_{\text{stara Schrö. en.}} = \underbrace{\left(\frac{\partial}{\partial x} - \frac{\partial f}{\partial x}\right)e^f g}_{\text{nova schrö. en.}}$$

Damo to v enačbo zdaj →

$$\left\{\frac{1}{2m}\left[\Big[-i\hbar\frac{\partial}{\partial x} - eA_x - \underbrace{\left(-i\hbar\frac{\partial}{\partial x}\left\{\frac{ie}{\hbar}\Lambda\right\}\right)}_{\frac{\partial}{\partial x}(e\Lambda)}\Big]^2 + (y)^2 + (z)^2 + e\phi\right]\right\} e^{\overset{\text{moj } f}{\frac{ie\Lambda}{\hbar}}}\underset{\text{moj } g}{\psi} =$$

$$= i\hbar\left(\frac{\partial}{\partial t} - \frac{ie}{\hbar}\frac{\partial\Lambda}{\partial t}\right)e^{\frac{ie\Lambda}{\hbar} = f}\,\psi = g$$

$$= e^{\frac{ie\Lambda}{\hbar}}\cdot i\hbar\cdot\frac{\partial\psi}{\partial t}$$

* tudi uporabimo $\left(\frac{\partial}{\partial x} - \frac{\partial f}{\partial x}\right)e^f g = e^f\frac{\partial g}{\partial x}$

In vidimo, da dobimo Schrödingerjevo enačbo za Ψ

2. Kvantizacija magnetnega pretoka

$$\vec{B} = \nabla\times\vec{A}(\vec{r},t) = 0$$
$$\vec{A} = \nabla\Lambda(\vec{r},t)$$

Rešimo problem brez polja in brez A, drugič pa z $\vec{B}=0$ in $A\neq 0$

$$\Lambda(\vec{r}_2,t) = \Lambda(\vec{r}_1,t) + \int_{\vec{r}_1}^{\vec{r}_2}\vec{A}\cdot d\vec{s}$$

$$\vec{E} = -\nabla U \;;\; \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} = 0$$

$$-\int_1^2\vec{E}\cdot d\vec{s} = U(2) - U(1)$$

$$\oint\vec{A}\cdot d\vec{s} = \int(\underbrace{\text{rot}\,\vec{A}}_{B})\,d\vec{S} = \int\vec{B}\,d\vec{S} = \phi_m$$

magnetni pretok

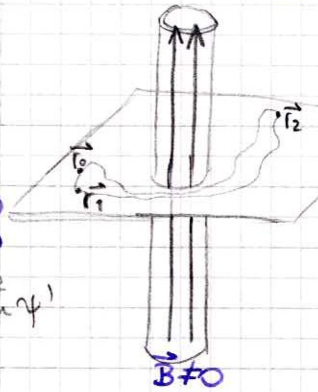
Zdaj pa poiščimo ψ, ki ustreza. Dan je Λ, dobimo A in recimo, da poznamo ψ: $i\hbar\frac{\partial\psi}{\partial t} = \frac{1}{2m}(-i\hbar\nabla - e\vec{A})^2\psi + V\psi$

naredimo novo umeritev: $\vec{A}' = \vec{A} + \nabla(-\Lambda) = 0$ in $\vec{B}' = 0$

Poiščemo $\psi'(\vec{r},t) = \psi(\vec{r},t)e^{\frac{ie}{\hbar}(-\Lambda)}$ → to je rešitev

oziroma → $\psi(\vec{r},t) = e^{\frac{ie}{\hbar}\Lambda}\psi'(\vec{r},t)$

← Le znotraj valja $\vec B \neq 0$.



$\vec B \equiv 0$
$\vec A = 0$
↓ paše zraven ψ'

$$\psi(\vec r_1, t) = \psi'(\vec r_1, t)\, e^{\frac{ie}{\hbar}\int_{\vec r_0}^{\vec r_1} \vec A\, d\vec s}$$

ker delec ne more vedet ali je v valju $\vec B$ ali ne.

naredimo zanko okrog valja: $\psi(\vec r_1, t) = \psi'(\vec r_1, t)\, \underbrace{e^{\frac{ie}{\hbar}\oint \vec A\, d\vec s}}_{\equiv 1} = \psi'(\vec r_1, t)$

gremo po področju z $\vec B = 0$.

→ ψ' je funkcija potem ko damo v valj $\vec B \neq 0$

torej: $e^{\frac{ie}{\hbar}\phi_m} = 1 = e^{i2\pi n}$

↳ $\phi_m = n\frac{2\pi\hbar}{e} = n\frac{h}{2e}$

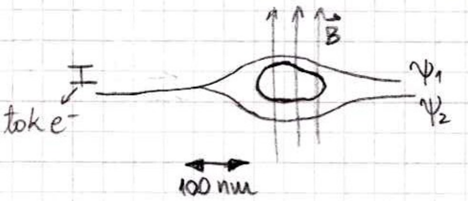
Kvant prevodnosti

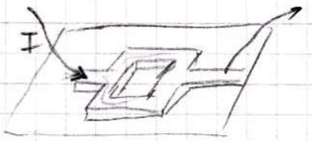
→ to dvojko je treba dodat. Kuiperjevi pari ko, in je $2e_0$.

z eksperimentom se pokaže, da to je res.
Tako se meri šibka polja.

SQUID → merijo lahko $\vec B$ zaradi pretakanja ionov / krvi po žilah

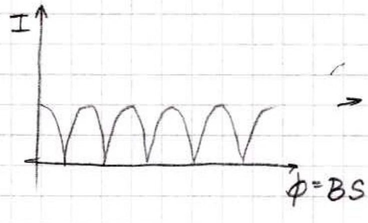
Aharonov - Bohmov pojav





$$|\psi_1 + \psi_2|^2 = |\psi_1|^2 + |\psi_2|^2 + 2\,\mathrm{Re}\,\psi_1^* \psi_2$$

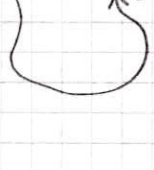
↓ $\sim e^{i\frac{e}{\hbar}\phi_m}$



→ ko spreminjaš magnetno polje, tok oscilira.

→ lahko meriš B → zelo male polja, in noro natančno lahko meriš

Spin



zanka, po njej teče tok. klasično: $\vec p_m = I\vec S$

zadnjič smo pokazali: $\vec\mu_L = g_L\frac{e}{2m}\vec L$ sklopitev: $-\vec\mu_L \cdot \vec B$

Landéjev faktor

↳ klasično je giromagn. razmerje 1, delci s spinom 1/2 pa imajo $g_L \neq 1$.

Splošno: $\vec\mu = \frac{e}{2m}(\vec L + g\vec S)$