

## Spin Backflow in the Vicinity of Spin Polarons in the $t - J$ Model

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We investigate the spin backflow current in a spin-1/2 Heisenberg quantum antiferromagnet associated with the motion of a hole. The spin backflow is studied here with two different methods: (a) starting from the wave function of the hole-like quasiparticle obtained in the self-consistent Born approximation and (b) an approximation-free exact diagonalization technique. Spin currents calculated analytically are in a good agreement with small cluster diagonalization studies.

The motion of a single vacancy (hole) in a 2D antiferromagnet (AF) was extensively investigated using various analytical approaches as well as with exact diagonalization of small clusters[1]. Consensus has been achieved regarding, e.g., the ground-state energy and the quasiparticle (QP) bandwidth.

The aim of this work is to present a more detailed picture of the quasiparticle. An interesting aspect of the deformation of the spin-background is contained in the bond-spin currents

$$\mathbf{j}_{\mathbf{r}} = \langle n_0(\mathbf{S}_{\mathbf{r}_1} \times \mathbf{S}_{\mathbf{r}_2})^z \mathbf{e} \rangle \quad (1)$$

relative to the position of the hole. Here  $n_0$  is the position operator for a hole at  $\mathbf{r} = 0$  and  $\mathbf{e}$  is a unit vector  $\mathbf{e} = \mathbf{r}_2 - \mathbf{r}_1$ . This quantity follows from the equation of motion for the spin density  $\dot{\mathbf{S}}_{\mathbf{r}} = it \sum_{\mathbf{e}, s, s'} (\hat{\sigma}_{s, s'} c_{\mathbf{r}, s}^\dagger c_{\mathbf{r}+\mathbf{e}, s'} - \text{H.c.}) - 2iJ \sum_{\mathbf{e}} \mathbf{S}_{\mathbf{r}} \times \mathbf{S}_{\mathbf{r}+\mathbf{e}}$ , where  $\hat{\sigma}$  are Pauli spin matrices. Here the first term is the spin current induced by the hopping of the hole and the second term ( $\sim \mathbf{j}_{\mathbf{r}}$ ) describes the backflow in the spin system. In the following we limit ourselves to the study of the  $z$  component of the current. We note that nonvanishing cross-products are necessary prerequisites of unconventional ground states with spiral or chiral order.

The quasiparticle wave function is derived on the level of a self-consistent Born approximation (SCBA) using a reformulation of the  $t - J$  Hamiltonian in terms of spinless fermions and Schwinger bosons [2]. By means of linear spin wave theory we introduce the proper collective excitations of the Heisenberg antiferromagnet. The QP-wave

function is given by [3,4]

$$\begin{aligned} |\Psi_{\mathbf{k}}^{(n)}\rangle = & a_{\mathbf{k}} \left[ h_{\mathbf{k}}^\dagger + N^{-\frac{1}{2}} \sum_{\mathbf{q}_1} M_{\mathbf{k}\mathbf{q}_1} G_{\bar{\mathbf{k}}_1}(\bar{\omega}_1) h_{\mathbf{k}_1}^\dagger \alpha_{\mathbf{q}_1}^\dagger \right. \\ & \dots + N^{-\frac{n-1}{2}} \sum_{\mathbf{q}_1, \dots, \mathbf{q}_{n-1}} M_{\mathbf{k}\mathbf{q}_1} G_{\bar{\mathbf{k}}_1}(\bar{\omega}_1) \dots M_{\bar{\mathbf{k}}_{n-1}\mathbf{q}_{n-1}} \\ & \left. \times G_{\bar{\mathbf{k}}_n}(\bar{\omega}_n) h_{\bar{\mathbf{k}}_n}^\dagger \alpha_{\mathbf{q}_1}^\dagger \dots \alpha_{\mathbf{q}_{n-1}}^\dagger \right] |0\rangle. \end{aligned}$$

where  $h_{\mathbf{k}}^\dagger$  is the creation operator for a (spinless) hole in a Bloch state,  $\alpha_{\mathbf{q}}^\dagger$  creates an AF magnon with energy  $\omega_{\mathbf{q}}$  and the fermion-magnon coupling is given by  $M_{\mathbf{k}\mathbf{q}}$ . We calculate the Green's function for the hole  $G_{\bar{\mathbf{k}}_n}(\bar{\omega}_n)$  in SCBA, where  $\bar{\mathbf{k}}_m = \mathbf{k} - \sum_1^m \mathbf{q}_i$  and  $\bar{\omega}_m = \epsilon_{\bar{\mathbf{k}}_m} - \sum_1^m \omega_{\mathbf{q}_i}$ .

In Fig. 1 the spatial dependence of the bond spin currents is shown for  $\mathbf{k} = (5/8\pi, 5/8\pi)$  and  $J/t = 0.4$ . The calculation was performed using a  $N = 16 \times 16$  unit cell and the number of excited magnons in the wave function  $|\Psi_{\mathbf{k}}^{(n)}\rangle$  was restricted to  $n = 3$  [4].

In Fig. 2(a)  $\mathbf{j}_{\mathbf{r}}$  is presented for various bonds  $\mathbf{r}$  defined in the inset.  $\mathbf{j}_{\mathbf{r}}$  is an odd function with respect to the wave vector (at  $k = 0$ ). Because of the symmetry it vanishes also at  $\mathbf{k} = (\pi, 0)$ . Since the ground state has AF long-range order, the points  $\mathbf{k}$  and  $\mathbf{k} + (\pi, \pi)$  are equivalent, and therefore,  $\mathbf{j}_{\mathbf{r}}$  vanishes also at  $\mathbf{k} = (\pi/2, \pi/2)$ .

To test the validity of the approximations made we have performed exact diagonalization of  $N = \sqrt{18} \times \sqrt{18}$  and  $N = \sqrt{20} \times \sqrt{20}$  sites clusters. In such a small system there are only a few nonequivalent points in the Brillouin zone,

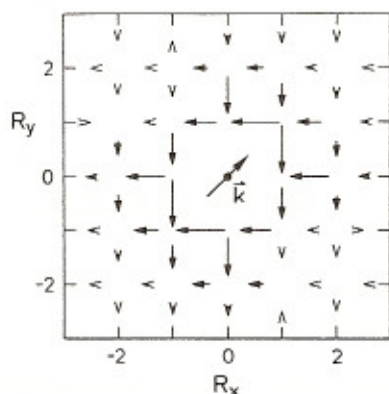


Figure 1. Bond spin currents  $\mathbf{j}_R$  for  $J/t = 0.4$  and  $\mathbf{k} = (5/8\pi, 5/8\pi)$ . The unit cell was  $N = 16 \times 16$ . The length of arrows is proportional to  $|\mathbf{j}_R|$ .

therefore we introduced in the hopping part of the  $t - J$  model the phase factors corresponding to the effect of a vector potential [5]. We considered the variation of the phase in the direction which corresponds to the momentum  $\mathbf{k} = (k, k)$ . In Fig. 2(b) we present results obtained for a unit cell with 18 sites and anisotropic Heisenberg coupling  $J_\perp/J_z = 0.5$ . The symmetry of  $\mathbf{j}_R$  around  $\mathbf{k} = (\pi/2, \pi/2)$  is the same as the result obtained within the SCBA. However, comparing the results for  $N = 18$  and  $N = 20$  sites system in the isotropic limit ( $J_\perp = J_z$ ) we found rather strong finite size effects and the analysis will be presented elsewhere.

In conclusion we have investigated spin-current correlation function around a mobile spin vacancy using the self-consistent Born approximation for the hole self-energy and the corresponding wave function. Using the same method we also found that various correlation functions decay far from the hole as a power-law. Nevertheless the QP spectral weight is finite. In the vicinity of the hole we compared the results with small cluster diagonalization studies. The agreement of the short-range behavior with the exact diagonalization results shows that approximations made in the model and by introducing the SCBA are not harmful. In particular both approaches yield a

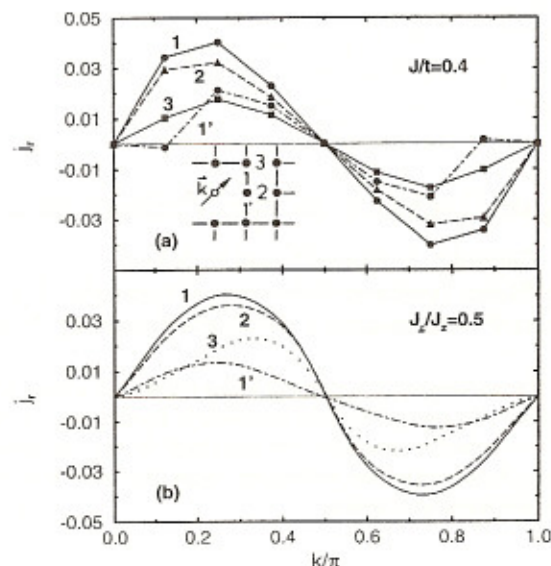


Figure 2. Momentum dependence of bond spin current along  $\mathbf{k} = (k, k)$  for  $J/t = 0.4$ . (a) Results of the SCBA. The lines connecting the symbols are a guide to the eye only. (b) Results obtained with the exact diagonalization of  $N = 18$  sites system and  $J_\perp/J_z = 0.5$ .

sign change of  $\mathbf{j}_R$  at  $\mathbf{k} = (\pi/2, \pi/2)$ .

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