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Seminar Assignment

Quantum Teleportation

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Abstract

The basics of quantum entanglement, its features and the Einstein-Podolsky-Rosen paradox are explained. Then, the focus is on quantum dense coding, Bell states and the algebra of entanglement in practice. Quantum teleportation is debated in a simple, yet thorough manner, moving to the seminar's motivation, the explanation of the 2004 Vienna experiment, a successful teleportation of a photon across an 800-meter-distance which was carried out by a team of Austrian scientists under the leadership of dr. Anton Zeilinger of University of Vienna's Institute of Experimental Physics.

1 Introduction

In the year 2004, a remarkable experiment has been carried out by a team of Austrian scientists. They have succeeded in teleporting a quantum state over a distance of 600 meters across the Danube river in Vienna [1]. Since all previous experiments involving teleportation were conducted over laboratory-scale distances, this experiment is therefore regarded as a milestone in the difficult and murky path towards practical quantum computing, quantum communication and teleportation.

This is of course bound to fascinate a physicist's mind since almost every one of us has definitely been reading science fiction novels or watching similar movies and has sooner or later been confronted with a convenient device - the teleporter. Imagine how such a device would transform travel as we know it today. But what is the physics behind this phenomenon? When will we be able to simply teleport ourselves to work instead of going through the ordeal of the everyday commute?

2 Quantum entanglement and the EPR paradox

Let's begin with some theory and return to the actual experiment later. What makes teleportation theoretically possible is a phenomenon called *Quantum entanglement* which is a relationship between physical quantities that has no analogue in classical physics.

We will start our explanation by reviewing some basic quantum mechanics. The state of each quantum system is described by a wave function ψ or, equivalently, using the Dirac bra-ket notation, by a state vector in Hilbert space $|\psi\rangle$. This state vector is a linear superposition of orthonormal base vectors. The choice of basis is not unique. It can be the eigenstates of the system's hamiltonian or the eigenstates of some linear operator which commutes with the hamiltonian. For a spin-$\frac{1}{2}$ particle these are the well known spin-up ($|\uparrow\rangle$) and spin-down ($|\downarrow\rangle$) states (eigenstates of the z-axis spin operator). So, the state of one single spin-$\frac{1}{2}$ particle can be written as

$$|\psi\rangle = a\,|\uparrow\rangle + b\,|\downarrow\rangle \tag{1}$$

where a and b are complex numbers. The square of each of them represents the probability of finding the particle in the corresponding state. They must obey the relation, $|a|^2 + |b|^2 = 1$, since the total probability of finding a particle in each of the possible states must be unity.

Consider a case of two spin-$\frac{1}{2}$ particles. The state of such a system must now be written in a basis which is a tensor product of each particle's base vectors.

$$|\psi\rangle = \sum_{i,j} c_{i,j} |i\rangle \otimes |j\rangle \equiv \sum_{i,j} c_{i,j} |ij\rangle \tag{2}$$

The sum is carried out over all of the first particle's Hilbert space base vectors ($|i\rangle$) and the second particle's Hilbert space base vectors ($|j\rangle$). According to (2), the general decomposition of the system state vector $|\psi\rangle$ is

$$|\psi\rangle = a|\uparrow\uparrow\rangle + b|\uparrow\downarrow\rangle + c|\downarrow\uparrow\rangle + d|\downarrow\downarrow\rangle \tag{3}$$

where a, b, c, and d are again complex numbers which, when squared, give the possibility of finding the system in a corresponding state. They must obey the normalization relation $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$, for analogue reasons as in the one particle case. The first symbol in each of the ket's denotes the state of the first particle and the second symbol the state of the second particle. If the state $|\psi\rangle$ is equivalent to one of the base vectors, then $|\psi\rangle$ is called a *separable* or *product* state.

For a counterexample, consider this *entangled* state:

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle \right) \tag{4}$$

This state is entangled, because it's impossible to decompose this state into either particle's product states. The particles are linearly superposed with one another. Again, we underline that this is a phenomenon without any analogue explanation in classical physics. But, there is more. Suppose we have two human beings, one called Alice and the other Bob. They are each given one particle of the system described in (4). For the sake of argument, let us suppose that both Alice and Bob are capable of storing each one's particle in such a way that both particles are left undisturbed after they have been prepared in the state (4). Alice and Bob can be separated by an arbitrary long distance. If Alice performs a measurement on her particle, the outcome will be either a $|\uparrow\rangle$ state or a $|\downarrow\rangle$ state, both with 50% probability. Now, let's recall one of the postulates of quantum mechanics - upon measuring, the system will remain in its measured state. This means that if Alice performs her measurement on (4) and determines that her particle is in a $|\uparrow\rangle$

state, Bob's particle will instantly collapse in a $|\downarrow\rangle$ state with 100% probability, although no interaction whatsoever had been made between Alice and Bob's particles over an arbitrary long distance. This is the core argument of EPR or Einsten-Podolsky-Rosen paradox.

The EPR paradox was Einstein's attack on quantum physics [3]. Upon discovering the entanglement, he argued that this "spooky action at a distance", e.g. the means of communication between both particles, is unphysical and that it can only be explained by introducing some hidden parameters (that we are still not aware of), which determine the results of individual measurements. Hence quantum mechanics is an incomplete physical theory[1]. Although Niels Bohr responded swiftly [4], it was not until much later that Bell presented a mathematical proof that the conclusions made in [3] are false. Bell's conclusion is, that if we believe quantum mechanical statistical predictions are valid and then, in order to determine the results of individual measurements, add some parameters to the theory, there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. That signal must propagate instantaneously, so such a theory isn't Lorentz invariant and therefore defies special relativity [5].

We must point out that quantum entanglement does *not* defy the theory of relativity. While it is true that, in case of (4), Bob's particle will instantly collapse in an opposite spin state as Alice's, no information can be transferred that way. The outcome of Alice's measurement cannot be known in advance so causality is preserved.

It may seem that entanglement is some exotic and rarely occurring phenomenon. Quite the contrary is true. It turns out that the $|\psi\rangle$ decomposed as in (3) describes a product state only if $ad - bc = 0$. So, entanglement is a much more common state for a pair of interacting quantum particles to be in that a product state. For instance, the two electrons shared by the molecule H_2 are in an entangled state. There have been successful experiments carried out, which produced separated, but entangled, photons from successive emissions of photons from a single atom. Even a pair of separated entangled atoms was produced by using a microwave cavity as a catalyst [2]. Note that the mathematics of describing entanglement is essentially the same for photons or spin-$\frac{1}{2}$ particles, because they are all binary systems - each has only two orthogonal (i.e. distinctly measurable) states. For photons, these

[1]this is a more modern version of the argument, but its essence is the same

are the horizontal and vertical polarization.

Entanglement is a very important resource. Apart from being useful in dense coding and teleportation it is widely used in quantum computation and quantum information theory. There exist several ways of quantifying this resource, depending on the field of use. One of the definitions is the *coucurrence*, C, defined as

$$C = 4\left|ad - bc\right|^2 \tag{5}$$

where a, b, c and d are the familiar coefficients of decomposition, defined in (3). The concurrence of two binary particles or *qubits* is a number between zero and one. To further clarify this concept, imagine having a hundred quantum dots which have no means of interacting with each other. In every quantum dot there is a pair of qubits. Averaging the concurrence of each pair over the whole system tells us how many qubit pairs are maximally entangled[2] and therefore useful in those processes. Concurrence is not the only way to quantify the entanglement. Apart from it, there exist the *entanglement of formation*, *relative entropy of entanglement* and *distillable entanglement*[2] but explaining those would steer us off this seminar's topic.

Entanglement is not limited to two qubits only. We can add as many as we like to the system, following the same algebra and somewhat modifying the quantification definitions.

Before we explain the basics of quantum teleportation, we focus on *dense coding*. It is, like teleportation, an application of entanglement, but a little bit easier to explain and understand.

3 Quantum dense coding

It is an intuitive fact that a spin-$\frac{1}{2}$ particle, or any other qubit for that matter, can only carry one bit of information. It is either in an $|\uparrow\rangle$ ("1") or $|\downarrow\rangle$ ("0") state. What is not intuitive, but mathematically proven, is that we cannot do better. Kholevo's theorem guarantees that there doesn't exist a coding scheme which can be used to transmit more than $\log_2 n$ bits in a quantum particle that has exactly n orthogonal states [7]. But this theorem applies to unentangled particles only. A particle, which is a part of an entangled pair, can be used to send a full two bit of information.

[2] sadly, we don't know exactly which ones

Let's say that Alice wants to send two bits of information (one of four possible choices) to Bob using only one spin-1/2 particle she directly manipulates with. First, both Alice and Bob are each given one particle of an entangled pair of particles

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \tag{6}$$

The first particle in each ket is given to Alice and second to Bob. We suppose that Bob's particle remains unperturbed (perhaps in some quantum dot or an infinite potential well). Alice can now apply four different transformations. Recall that the rotation of a z-axis spin-$\frac{1}{2}$ operator eigenstate ($|\uparrow\rangle$ or $|\downarrow\rangle$) is described by

$$\exp\left(-i\frac{\varphi}{2}\vec{\sigma}\vec{n}\right)|\uparrow\rangle = \left(\mathcal{I}\cos\frac{\varphi}{2} - i\vec{\sigma}\vec{n}\sin\frac{\varphi}{2}\right)|\uparrow\rangle \tag{7}$$

where φ is the angle of rotation around the axis described by the normal $\vec{n}$, $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is a vector whose components are the well known Pauli matrices and $\mathcal{I}$ is a 2x2 identity matrix. Again, the latter equation is equivalent for a $|\downarrow\rangle$ state. Alice had agreed with Bob that she will make one of the four operations on her qubit, depending on the message she wishes to send:

message	rotation	$\lvert\psi\rangle$ after rotation
A	none	$\lvert\Psi^-\rangle$
B	180^0 around x axis ($\varphi = \pi$, $\vec{n} = (1,0,0)$)	$i\lvert\Phi^-\rangle$
C	180^0 around y axis ($\varphi = \pi$, $\vec{n} = (0,1,0)$)	$\lvert\Phi^+\rangle$
D	180^0 around z axis ($\varphi = \pi$, $\vec{n} = (0,0,1)$)	$-i\lvert\Psi^+\rangle$

The calculations can be easily verified by applying the necessary algebra. After Alice's operation[3] on her particle, the system is (except for an irrelevant phase factor) in one of the four *Bell* or maximally entangled ($C = 1$) states:

$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle) \tag{8}$$

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle) \tag{9}$$

These states are orthogonal and hence form a convenient basis for describing dense coding and, as we shall see, quantum teleportation. After Alice has done her job, she sends her particle to Bob. Of course, the particle must not be disturbed during transportation. Bob then applies a *Bell measurement* to

[3]or, equally, measurement

both his and Alice's particles. Bell measurement is any measurement that can clearly distinguish between each of the four possible Bell states. In principle, this can be achieved, because the Bell states are orthogonal. In practice this can be done only with ion qubits in ion traps. A less ambitious measurement can be performed with entangled photons. This measurement can distinguish between $|\Phi^+\rangle$ and $|\Phi^-\rangle$'s photonic counterparts, but not between the other two states. But, as we shall see in the next section, this is enough for successful quantum teleportation.

One can argue, quite legitimately, that this process is far too complicated to have a practical use. It is very difficult to imagine a situation in which, using current technology, it would be more economical to use dense coding rather than just plain ordinary coding with twice as many particles. The fact that entanglement can do classical coding using half the resources is nevertheless a fascinating theoretical discovery. Local actions on a single quantum particle can express more information that can be possibly stored in the particle itself. The first steps into an era of quantum computing are definitely being made, on a theoretical level at least.

4 Quantum teleportation

Finally we can discuss quantum teleportation. The aim of quantum teleportation is to convey quantum states themselves. Suppose Alice had been given a single spin-$\frac{1}{2}$ particle in an unknown state $|\psi\rangle$ and she would like to convey this state to Bob (one of the reasons for this might be that Bob has a better measurement apparatus than Alice and could manipulate $|\psi\rangle$ in a way that Alice isn't able to). Of course, the easiest way for Alice would be to simply send the state to Bob (let's say via a highly delicate transport mechanism). But, what if Bob is too far away for this transportation to be efficient (on Mars, for example)? Or, what if Alice simply doesn't know where Bob is? Classically, Alice could make copies of the particle and, for instance, transmit them to all places Bob might have been (if these places would be close). This is not possible, since a no-cloning theorem forbids copying of quantum states [6]. Quantum information is therefore nothing like its classical counterpart.

Again, entanglement comes to the rescue. There is a way in which Alice can divide the information about $|\psi\rangle$'s state into two parts, one purely classical and the other a quantum one. These parts are transmitted via two separate communication channels. Although entanglement is used in the process, special relativity is not violated, since Bob still requires information from both channels to accurately replicate $|\psi\rangle$ on his location. The speed of relaying classical information is, of course, limited with the speed of light. Alice's original is, as we shall see, destroyed in the process, obeying the no-cloning theorem. The net result of teleportation therefore is: removing the $|\psi\rangle$ from Alice's location and appearing at Bob's at a suitable time later.

First, two entangled particles are produced in a Bell $|\Psi^-_{2,3}\rangle$ state (8). The subscripts label the particles in the pair, first subscript denotes the first particle in each of the ket's. The particle 2 is given to Alice and 3 to Bob, prior to any teleportation being done. Again, both particles are kept in some vessel which does not perturb each one's states after the entanglement has been carried out successfully.

The whole system (the unknown particle and the entangled pair) are in a product state, i.e. $|\psi_1\rangle\,|\Psi^-_{2,3}\rangle$, having neither experienced quantum entanglement nor an interaction of any type (Alice had taken precautions). If she were to measure her part of the entangled pair (or, for that matter, the whole pair), no information would be gained about the $|\psi_1\rangle$. She therefore entangles the three particles together.

The unknown state $|\psi_1\rangle$ can be conveniently written as

$$|\psi_1\rangle = a\,|\uparrow_1\rangle + b\,|\downarrow_1\rangle \tag{10}$$

so, the state of all three particles is (recalling (8))

$$|\phi_{1,2,3}\rangle \equiv |\psi_1\rangle\,|\Psi^-_{2,3}\rangle = \frac{a}{\sqrt{2}}\left(|\uparrow_1\uparrow_2\downarrow_3\rangle - |\uparrow_1\downarrow_2\uparrow_3\rangle\right) + \frac{b}{\sqrt{2}}\left(|\downarrow_1\uparrow_2\downarrow_3\rangle - |\downarrow_1\downarrow_2\uparrow_3\rangle\right) \tag{11}$$

which can be expressed in the Bell basis as

$$\begin{aligned} |\phi\rangle &= \frac{1}{2}\big\{|\Psi^-_{1,2}\rangle\left(-a\,|\uparrow_3\rangle - b\,|\downarrow_3\rangle\right) + |\Psi^+_{1,2}\rangle\left(-a\,|\uparrow_3\rangle + b\,|\downarrow_3\rangle\right) + \\ &+ |\Phi^-_{1,2}\rangle\left(a\,|\downarrow_3\rangle + b\,|\uparrow_3\rangle\right) + |\Phi^+_{1,2}\rangle\left(a\,|\downarrow_3\rangle - b\,|\uparrow_3\rangle\right)\big\} \end{aligned} \tag{12}$$

Now, Alice makes a Bell measurement and so, entangles the three particles. It is not hard to see that the four measurement outcomes are equally likely, each with $\frac{1}{4}$ probability, regardless of the unknown state $|\psi_1\rangle$. Furthermore, Bob's particle, the number 3, will be instantaneously projected into a state which will depend upon Alice's measurement. But, since Alice knows the outcome of her measurement, she can contact Bob via a classical communication channel and tell him the outcome of her measurement. Depending on Alice's findings, he must transform his particle according to the following table

Alice has measured	Bob must do
$\|\Psi^-_{1,2}\rangle$	nothing
$\|\Psi^+_{1,2}\rangle$	a 180^0 rotation around the z axis
$\|\Phi^-_{1,2}\rangle$	a 180^0 rotation around the x axis
$\|\Phi^+_{1,2}\rangle$	a 180^0 rotation around the y axis

in order to make an exact replica of Alice's unknown state (the irrelevant phase factor has again been omited). Alice is left with an entangled pair of particles in one of the four Bell states so the no-cloning theorem is not violated. The teleportation of $|\psi_1\rangle$ thus has the side effect of producing two random bits of information.

Omitting further explanation, let's nevertheless briefly mention some facts [6]. Since teleportation is a *linear* operation, it can also be used to teleport an entangled state. Let the unknown state $|\psi_1\rangle$ be a part of an entangled pair with a particle labeled 0. The outcome of the teleportation will be that the particles 0 and 3 will be left in a singlet state, although they were each a part of a different entangled pair. Also all of what was said applies to particles with more than 2 orthogonal states. Furthermore, two bits of classical information (four possible outcomes) are needed to avoid sending superluminal messages and if Bob tries to guess Alice's measurement and performs his rotations *before* knowing the outcome of Alice's measurement, this results in Bob reconstructing his $|\psi_1\rangle$ as a maximally mixed state of all three particles. Again, a correlation between an input an guessed signal would result in sending informations with superluminal velocity.

Figure 1 gives us a schematic comparison between quantum teleportation (left) and dense coding (right). We can observe that the rotation of a spin-$\frac{1}{2}$ particle and Bell measurement are not always performed by the same persons. Both processes require sharing of an entangled pair of particles prior to any operation.

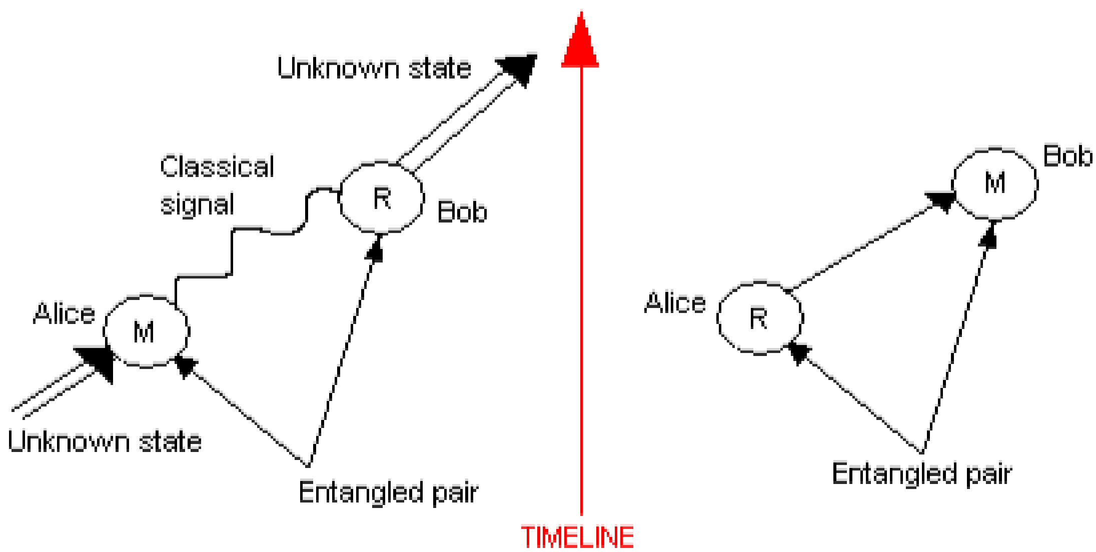

Figure 1: A comparison between dense coding and quantum teleportation

5 The Vienna experiment

Now that we have explained the basics of quantum entanglement and quantum communication, we can see that the theory is not so hard to understand. Actually doing an experiment is not quite so easy, as one can only begin to imagine all the difficulties of performing a Bell measurement, creating stable pairs of entangled qubit pairs and transporting them unperturbed during even laboratory distances. Actually teleporting a quantum state across a distance of 800 meters was with present technology definitely not simple.

We will briefly mention the setup and outcome of the experiment which was the motivation for this seminar assignment. Photons were used instead of spin-$\frac{1}{2}$ particles, as their manipulation is much easier. Spin-$\frac{1}{2}$ rotations that must be performed on Bob (the receiver's) part can be achieved with a suitable setup of various wave plates as is known from optics. The quantum channel was an 800-metre-long optical fibre installed in a public sewer system underneath the Danube river in Vienna. The site was deliberately chosen as it does not undergo ideal laboratory conditions, but on the contrary experiences temperature fluctuations and other environmental factors. A pulsed laser was used to pump a β-barium borate crystal (BBO) that generated the entangled photon pair. Singlet state photons, also generated by this technique, were used as the teleportation input ('Alice's' unknown state) and as the trigger for electronic logic, respectively. The unknown state was guided into a single-mode optical-fibre beam splitter (BS), connected to polarizing beam splitters (PBS), which performed the Bell measurement. This scheme allowed the Alice logic to identify two of the four Bell states ($|\Psi^-_{b,c}\rangle$ or $|\Psi^+_{b,c}\rangle$). The result was conveyed through a classical channel (Radio-Frequency units) to the receiver's end ('Bob' logic) which used an electro-optic modulator (EOM) to transform the photon received via the quantum channel into the unknown initial state $|\psi_1\rangle$ (see Figure 2). If Alice logic observed the $|\Psi^+_{b,c}\rangle$ state, Bob logic applied a π phase shift between the horizontal and vertical components of the received photon by applying a voltage pulse on the EOM. The classical signal arrived about $1.5ms$ before the quantum photon, because of the reduced velocity of light in optic fibres. The teleportation achieved an astonishing 90% fidelity for 45^0 polarized photons with each measurement run lasting for 28 hours. The results are really amazing, considering that the experiment was carried out in real world conditions. Without Bob performing the EOM polarization rotation the observed fidelity dropped to 59%, deviating from the expected 50% due to statistical fluctuations, proving that indeed teleportation has taken place [1].

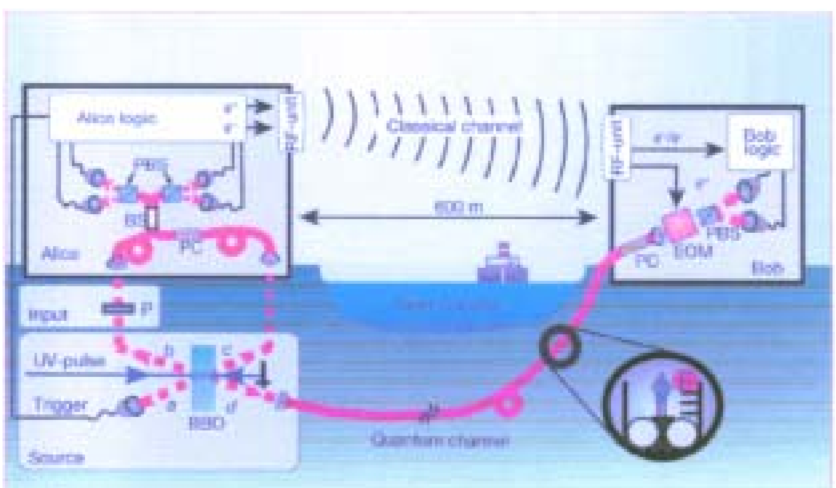

Figure 2 [1]: c and d are the entangled photon pair, a and b are the singlet photon states, the latter used as the unknown initial state $|\psi_1\rangle$.

6 Conclusion

Quantum entanglement is far from being only a theoretical obscurity. As we have seen it has profound implications in future communications and computing. Teleportation is the key to quantum parallel computing, albeit not even a single-processor quantum computer has, to this day[4], been made, which could measure up to the standards of present day desktop computers, not even mentioning supercomputers. The theory is solid, but the practical difficulties are enormous. Quantum computing and communication has its theoretical advantages and is worth pursuing with modern technology. Dense coding will help with communication error reducing. In this field of technology is teleportation's place and future, sadly not in star-trek like transportation devices. As Anton Zeilinger, the director of Vienna branch of the Institute of Quantum Optics and Quantum Information and the head of the team that successfully achieved the Vienna experiment put in an interview [8]:

'... We are talking about quantum phenomena here; we have no idea how we could produce these with larger objects. And even if it was possible, the problems involved would be huge. Firstly: for physical reasons, the original has to be completely isolated from its environment for the transfer to work. There has to be a total vacuum for it to work. And it is a well-known fact that this is not particularly healthy for human beings. Secondly, you would take all the properties from a person and transfer them onto another. This means producing a being who no longer has any hair colour, no eye colour, nix. A man without qualities! This is not only unethical: it's so crazy that it's impossible to imagine....'

[4]an unveiling of a 16-bit quantum computing system is announced on 13th February 2007 by the D-Wave company

References

[1] R. Ursin, T. Jennewein et al., *Quantum Teleportation Across the Danube*, Nature **430**, 849 (2004).

[2] W. K. Wooters, *Quantum Entanglement as a Quantifiable Resource*, Phil. Trans. R. Soc. London A **356**, 1717 (1998).

[3] A. Einstein, B. Podolsky and E. Rosen, *Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?*, Phys. Rev. Lett. **47**, 777 (1935)

[4] N. Bohr, *Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?*, Phys. Rev. Lett. **48**, 696 (1935)

[5] J. S. Bell, *On the Einstein-Podolsky-Rosen Paradox*, Physics **1**, 195 (1964)

[6] C. H. Bennett et al., *Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels*, Phys. Rev. Lett. **70**, 1895 (1993)

[7] A. S. Kholevo *Some estimates of the information transmitted by quantum communication channel*, Probl. Peredachi Inform. **9**, 3 (1973) (rus) [Probl. Inf. Transm. (USSR) **9**, 177 (1973)]

[8] signandsight website - *Anton Zeilinger, Mathias Plüss, Regina Hügli: Spooky action and beyond*, http://www.signandsight.com/features/614.html, visited on 31th January, 2007.