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Effective $U_A(1)$ symmetry breaking interactions

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Abstract. We review the phenomenological consequences of various effective $U_A(1)$ symmetry breaking interactions. In particular we look at the baryon spectra. We comment on the recent conjecture that the chiral symmetry might be restored in the higher regions of hadron spectra in the light of our results for scalar mesons and the spectral sum rules.

The so-called "$U_A(1)$ problem" consists of (i) the fact that the sum of the eighth and ninth pseudoscalar meson $\eta(550)$, $\eta^{'}(960)$ masses lies (far) above the flavour $SU(3)$ mass relations prediction of two kaon masses ($2\, m_K$) and (ii) that their mixing angle is far from being the "ideal" one. These two facts imply a large *explicit* $U_A(1)$ symmetry breaking, that is believed to be induced by instantons in QCD. These instanton effects can be described by 't Hooft's $U_A(1)$-symmetry breaking quark flavour determinant effective interaction [1]

$$\begin{aligned}\mathcal{L}_{tH}^{(6)} &= -K_{tH}\left[\det\left(\bar{\psi}(1+\gamma_5)\psi\right) + \det\left(\bar{\psi}(1-\gamma_5)\psi\right)\right] \\ &= -2K_{tH}\Re e\left(\det\bar{\psi}(1+\gamma_5)\psi\right). \end{aligned} \tag{1}$$

Phenomenological consequences of the determinant effective interaction in spinless meson channels of either parity have been studied in Ref. [2], where new effects for the scalar mesons were reported. In the same place the strength K of the 't Hooft interaction was also fixed in terms of the pseudoscalar (PS) meson properties as

$$-12K_{tH}\langle\bar{q}q\rangle^3 = f_\eta^2\left[m_{\eta'}^2 + m_\eta^2 - 2m_K^2\right]. \tag{2}$$

Equivalent results for scalar mesons have been reported in Ref. [3] using directly the instanton-induced (II) interaction with a finite spatial range.

There is another $U_A(1)$ symmetry breaking effective interaction that is proportional to the squared imaginary part of the determinant

$$\begin{aligned}\mathcal{L}_{VW}^{(12)} &= K_{VW}\left[\det\left(\bar{\psi}(1+\gamma_5)\psi\right) - \det\left(\bar{\psi}(1-\gamma_5)\psi\right)\right]^2 \\ &= -4K_{VW}\left(\Im m\det\bar{\psi}(1+\gamma_5)\psi\right)^2. \end{aligned} \tag{3}$$

as well as the analogues of the above two with antisymmetric Pauli tensors inserted between the Dirac spinors [4].

1 Mesons

The role of $U_A(1)$ symmetry breaking in meson spectra has been extensively studied over the past 10 years [2–7]; a brief review of the field can be found in Ref. [8]. There scalar meson spectra and the V - A spectral sum rules were discussed. Considering the fact that the recent parity-doubling/chiral symmetry restoration (χSR) conjecture [9] uses several methods and/or results obtained or used in the aforementioned studies, this seems like a good place to make several comments of direct relevance to this conjecture, rather than to review again some established facts. This is by no means to be understood as a polemic, but rather as a part of an academic dialogue: It is an observation on matters not discussed by Glozman that could potentially have serious implications for the viability of this idea.

1. **An explicit counterexample** to the claim that asymptotic restoration of chiral $SU_L(N_f) \times SU_R(N_f)$ symmetry implies parity doubling in meson spectra is the original form of the second Weinberg sum rule: Even with as strong an assumption as the vanishing of the zeroth moment of the spectral density difference (that later proved to be false in QCD), Weinberg still had to assume another ("KSFR") relation (connecting the widths of the vector and axial-vector states) before he could turn his assumption into a prediction of the vector/axial-vector meson mass ratio (that turned out to be $1/\sqrt{2}$ rather than unity, as conjectured by Glozman!). Now, one may object that these are only the ground state mesons, that are not subject of the χSR conjecture. But, as one increases the number of states in the spectra, the number of new "radially excited state KSFR" relations, that are necessary to calculate the V/A mass ratios, also grows. Clearly, more is necessary for parity doubling of vector (V) and axial-vector (A) mesons than mere asymptotic equality of spectral functions. If that were not enough, it has been shown [4] that the second spectral sum rule is not only sensitive to $SU_L(N_f) \times SU_R(N_f)$ chiral symmetry breaking (χSB), but rather to nonconservation of the "larger" (enveloping) $U_L(2N_f) \times U_R(2N_f)$ *current algebra*. Thus, one may have χSR and still have a nonvanishing second spectral sum. In other words, spectral functions do *not* depend only on *chiral* symmetry, as assumed by Glozman, but also on higher current algebras.
2. **Technical objections** to the way $U_A(1)$ symmetry breaking was treated. (i) $U_A(1)$ symmetry breaking depends (sensitively) on the number of (light) flavours, c.f. Ref. [2]. Glozman uses two-flavour mass formulas for "realistic" purposes (to compare with experimental spectra), instead of the three flavour ones. This is inadmissible and only hides other shortcomings of this scheme: (a) Too few flavour singlets are predicted: some observed states must be assigned to glueballs, so it is not clear which states are radial excitations; (b) not all of the suggested mass differences may serve as measure of $U_A(1)$ symmetry breaking: In this regard we have shown in Ref. [6] that the scalar meson mass difference may vanish even with 't Hooft force turned on, depending on the strength of the vector/axial-vector interaction, i.e. on the admixing of pseudo-vector component to the pseudoscalar mesons. (ii) An oversimplified assignment of mesons to chiral symmetry irreducible representations (irreps)

has been made in Ref. [9]: mixings of pseudoscalar (PS) and axial vector (A), scalar (S) and vector (V), V and tensor (T), A and pseudotensor (PT) have been ignored. As we have shown in Ref. [6], mixing of PS and A mesons substantially changes the scalar meson mass difference that was used as a measure of $U_A(1)$ symmetry restoration by Glozman. Indeed the said mass difference can be made arbitrarily close to zero, even in the presence of $U_A(1)$ symmetry breaking interaction, by means of changing the amount of the PS-A mixing. Other mixings, mentioned above, may well have similarly dramatic effects.

3. **Objections to the identification procedure** for chiral multiplets. Even if the experimental interpretation in Ref. [9] were correct, the conclusion that chiral symmetry restoration-induced parity doublets have already been observed would still be invalid. Rather, the "observed" parity doubling would be purely accidental. This is so because members of *different* chiral multiplets have been compared (see below) in Glozman's putative scheme [9]. That is, of course, inadmissible: chiral restoration implies parity doubling within the same chiral multiplet, not among members of two different multiplets (which doubling may indeed occur, but only due to random coincidence) [10]. In a logically consistent check of chiral restoration one must first positively identify the purported members of chiral multiplets going from the bottom up, i.e. starting with the ground state and then matching corresponding excited states. One must not start at some high-lying set of (accidental) parity doublets and then move down until one arbitrarily declares victory and all states lying below that arbitrary line as being beyond the reach of the conjecture, as was done in Ref. [9]. As one moves down in mass from the alleged parity doubled chiral partners in Glozman's proposed scheme, one finds that some chiral multiplets are incomplete: for example, the (well established) $\pi(1300)$ state does not have a scalar partner in the observed spectrum, according to Glozman's scheme. This proves that a misidentification of chiral multiplets has taken place, which fact negates all claims relating to chiral symmetry restoration in meson spectra at high masses.

2 Baryons

More recently, significant effects due to the instanton-induced interaction have been reported in baryon spectroscopy [11]. In this note we wish to give a simple explanation of the 't Hooft quark flavour-determinant effective interaction's effects in baryon spectroscopy. We confirm the results of earlier studies [11], with one distinction: we have no free parameters to adjust in our calculation because we take the value of the 't Hooft coupling constant K as constrained above by the meson spectra.

The effective two-body 't Hooft interaction leads to the following two-quark potential

$$\begin{aligned} V_{12} &= 4K\langle\bar{q}q\rangle_0 P^{\bar{3}}_{12}\left(1+\gamma^5{}_1\gamma^5{}_2\right)\delta(\mathbf{r}_1-\mathbf{r}_2) \\ P^{\bar{3}}_{12} &= \left[\frac{1}{3}-\frac{1}{4}\boldsymbol{\lambda}_1\cdot\boldsymbol{\lambda}_2\right]. \end{aligned} \tag{4}$$

The flavour dependence of this potential is proportional to the $\bar{\mathbf{3}}$ projection operator $P^{\bar{3}}_{12}$, i.e., it only operates in the flavour antisymmetric state. Note, however, that in the $q\bar{q}$ channels the same flavour factor is *not exactly* a flavour singlet projector any more.

The 't Hooft interaction also leads to the following three-quark potential

$$
\begin{aligned}
V_{123} &= 12KP^{1}_{123}\left(1+\sum_{i<j}^{3}\gamma^{5}{}_{i}\gamma^{5}{}_{j}\right)\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_3-\mathbf{r}_2) \\
12P^{1}_{123} &= \left[\frac{4}{9}-\frac{1}{3}\sum_{i<j}^{3}\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j + d^{abc}\boldsymbol{\lambda}^{a}_{1}\boldsymbol{\lambda}^{b}_{2}\boldsymbol{\lambda}^{c}_{3}\right]. \qquad (5)
\end{aligned}
$$

As can be seen from Eq. (5) the flavour dependence of the 't Hooft three-quark potential is just the flavour SU(3) singlet projection operator P^{1}_{123} [8] for three quarks. Thus the 't Hooft three-quark potential contributes only in the flavour singlet q^3 channel, as already noticed in Ref. [11]. As the lowest lying flavour singlet is necessarily a P-wave state (due to the Pauli principle) and the spatial part of the three-body potential Eq. (5) contains two Dirac delta functions, its matrix element is zero.

We use the constituent quark model [12] with the harmonic oscillator Hamiltonian to calculate the basic effects of the 't Hooft interaction. This model is clearly rather simple, but should be adequate for the purpose of identifying the qualitative features and making first estimates of the 't Hooft interaction effects in baryons. In the following we shall keep only the leading-order ($\mathcal{O}(1)$) terms in the nonrelativistic [NR] expansion, i.e. we do not keep the spin dependent parts. In this spirit we have also neglected the strong-hyperfine ("Breit") interaction in the constituent quark Hamiltonian [12], that is believed to be an important part of the (extended) constituent quark model, but that also suffers from several shortcomings, an excessively large coupling constant being one. We shall show that some of the best known effects associated with the strong Breit force are reproduced by the 't Hooft interaction.

With these assumptions we can calculate the three-quark system spectra in different flavour channels. But as the 't Hooft potential is a contact term, one cannot separate the resulting Schrödinger equation exactly. So, we must use some approximate method, e.g. perturbation theory. We find the following 't Hooft potential flavour space matrix elements

$$\langle V\rangle_1 = 12K\langle\bar{q}q\rangle_0\langle\delta(\mathbf{r}_1-\mathbf{r}_2)\rangle_1 \qquad (6)$$

$$\langle V\rangle_8 = 6K\langle\bar{q}q\rangle_0\langle\delta(\mathbf{r}_1-\mathbf{r}_2)\rangle_8 \qquad (7)$$

$$\langle V\rangle_{10} = 0 \qquad (8)$$

The reason for the last line is that the flavour singlet **1** three-quark system cannot have a completely symmetric spin-spatial wave function, the way the ground state octet and decimet do, due to the Pauli principle and the complete antisymmetry of the singlet's flavour- and colour wave functions.

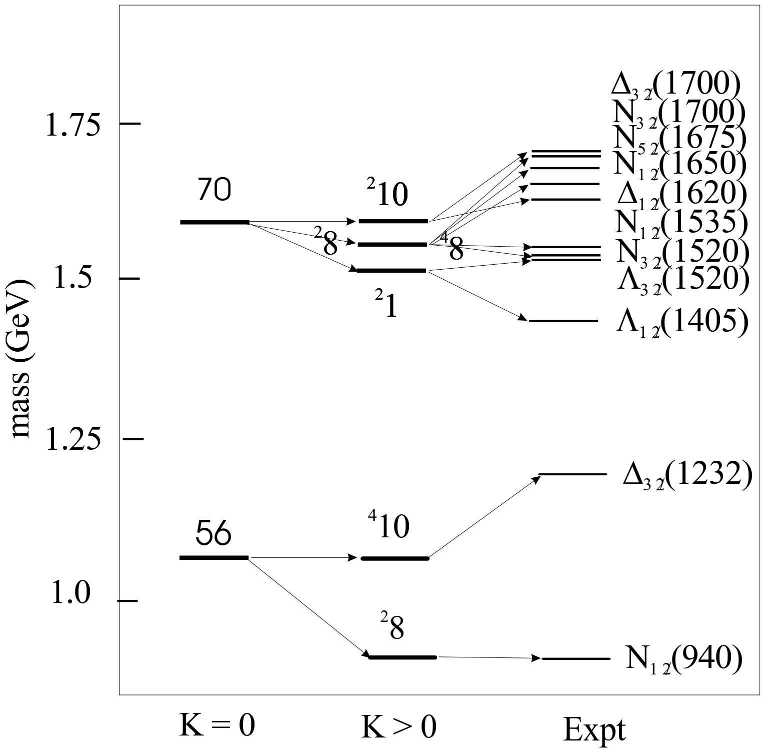

Fig. 1. Baryon mass spectrum as calculated in the nonrelativistic quark model with harmonic oscillator confinement without (K = 0) and with (K ≠ 0) 'tHooft interaction for the two lowest lying shells (N = 0,1); the prediction for the Roper resonance (N = 2) is not shown. Also shown are the observed baryon states (expt).

In the first approximation with all spin-spin interactions neglected,

$$\begin{aligned}\langle\Psi_S(N=0)|\delta(\mathbf{r}_1-\mathbf{r}_2)|\Psi_S(N=0)\rangle &= \langle\delta(\mathbf{r}_1-\mathbf{r}_2)\rangle_{\mathbf{8}\subset\mathbf{56}} = \langle\delta(\mathbf{r}_1-\mathbf{r}_2)\rangle_{\mathbf{10}\subset\mathbf{56}} \\ &\equiv I = \left(\frac{m_q\omega}{\sqrt{2\pi}}\right)^{3/2} \end{aligned} \tag{9}$$

$$\begin{aligned}\langle\Psi_P(N=1)|\delta(\mathbf{r}_1-\mathbf{r}_2)|\Psi_P(N=1)\rangle &= \langle\delta(\mathbf{r}_1-\mathbf{r}_2)\rangle_{\mathbf{1}\subset\mathbf{70}} = \frac{1}{2}\sum_M\langle\psi^\lambda_{1M}|\delta(\mathbf{r}_1-\mathbf{r}_2)|\psi^\lambda_{1M}\rangle \\ &= \frac{1}{2}I = \frac{1}{2}\left(\frac{m_q\omega}{\sqrt{2\pi}}\right)^{3/2} \end{aligned} \tag{10}$$

$$\begin{aligned}\langle\Psi'_S(N=2)|\delta(\mathbf{r}_1-\mathbf{r}_2)|\Psi'_S(N=2)\rangle &= \langle\delta(\mathbf{r}_1-\mathbf{r}_2)\rangle_{\mathbf{8}\subset\mathbf{56}} \\ &= \frac{5}{4}I = \frac{5}{4}\left(\frac{m_q\omega}{\sqrt{2\pi}}\right)^{3/2}, \end{aligned} \tag{11}$$

where ω= 500 MeV is the oscillator frequency in the model, and the constituent quark mass, $m_q = 313 MeV$, is approximately one third of the nucleon's. Thus we find the following energy shifts

$$\delta E_{\mathbf{1}\subset\mathbf{70}}(N=1) = \delta E_{\mathbf{8}\subset\mathbf{56}}(N=0) = 6K\langle\bar{q}q\rangle_0 I \tag{12}$$

$$\delta E_{\mathbf{8}\subset\mathbf{70}}(N=1) = 3K\langle\bar{q}q\rangle_0 I \tag{13}$$

$$\delta E_{\mathbf{8}\subset\mathbf{56}}(N=2) = \frac{15}{2}K\langle\bar{q}q\rangle_0 I \tag{14}$$

$$\delta E_{\mathbf{10}} = 0. \tag{15}$$

Inserting into Eq. (2) the experimental value for the ps meson masses and decay constants, as well as the quark condensate $\langle\bar{q}q\rangle = -(225\text{MeV})^3$, the 't Hooft coupling constant becomes $K = 390\text{GeV}^{-5}$, we find the baryon spectrum shown in Fig. 1. The second radially excited state (the Roper resonance) mass moves down by about 130MeV, but is still too large to be visible in Fig. 1. There one can see that about one third of the observed positive parity ground state $\mathbf{8}-\mathbf{10}$ mass splitting and about one half of the observed negative parity $\mathbf{1}-\mathbf{10}$ mass splitting are reproduced by 't Hooft interaction. Admittedly, one cannot describe the fine structure (LS splitting) of the spectra (as yet), but that ought to be possible with the inclusion of spin-dependent forces. In particular these results show that 't Hooft's interaction causes a significant part (at least a half) of the $\Lambda_0(1405)$ and $\Lambda_0(1520)$'s mass shifts to anomalously low masses compared with other P-wave baryons. [Remember that $N^*(1535)$ and $N^*(1520)$ ought to be about 130 MeV lighter than the corresponding Λ_0's, due to one strange quark in the latter, in the absence of 't Hooft's interaction.] This mass shift was first pointed out in Ref. [11]. Finally, the mystery of the Roper resonance's abnormally low mass now seems within the reach of rational explanation starting from QCD.

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