Light and heavy baryon masses (1) The quark model

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Introduction

- Description of baryons
  - Quark Model: Model-dependent
    Hamiltonian dynamics
  - Large N_c QCD: $N_c \to \infty$, model-independent
    Group theory
  - Compatibility of both approaches
- Light baryons
- Heavy baryons (One heavy quark c or b)
  - New experimental data

Large N_c expansion

- When $N_c \to \infty$, exact $SU(2N_f)$ symmetry
  - Baryons: N_c quarks
- Large but finite N_c
  - $SU(2N_f)$ broken, $1/N_c$ expansion
- Mass formula $M = \sum_i c_i \hat{O}_i$
  - $1/N_c^2$ neglected
  - c_i to be fitted. Contain the QCD dynamics.

Quark model ?

Large N_c expansion (II)

- Ground state baryons (N and Δ)

$$M = c_1 N_c 1 + c_4 \frac{S^2}{N_c} + O(N_c^{-3})$$

- Excited baryons
  - Labelled by an integer N, quantum of excitation

    Harmonic oscillator picture

    $N = 0$ for ground state baryons $\quad P = (-1)^N$

$$\Rightarrow c_i = c_i(N)$$

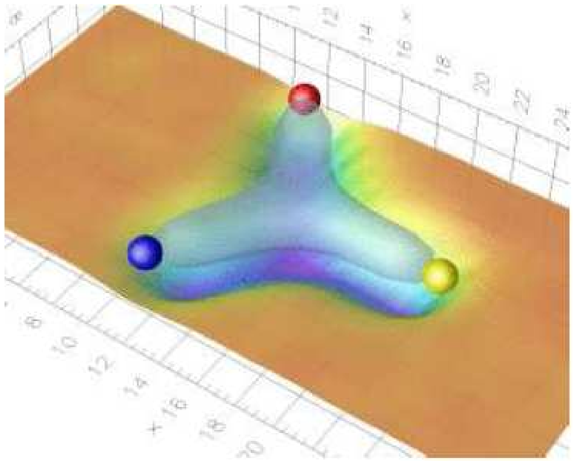
See Fl. Stancu's talk

Quark Model for baryons

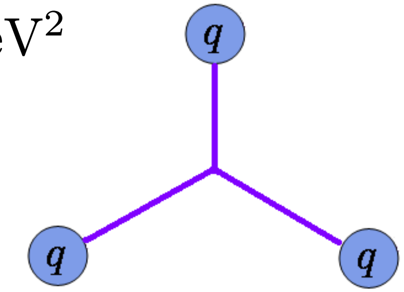
Semirelativistic Hamiltonian (I)

- Dominant order: $H = \sum_i \sqrt{\vec{p}_i^{\,2} + m_i^2} + a|\vec{x}_i - \vec{x}_Y|$
  - Spinless Salpeter Hamiltonian
  - Y-junction as long-range potential
- Lattice QCD

F. Bissey *et al.*, Phys. Rev. D **76**, 114512 (2007) [hep-lat/0606016]

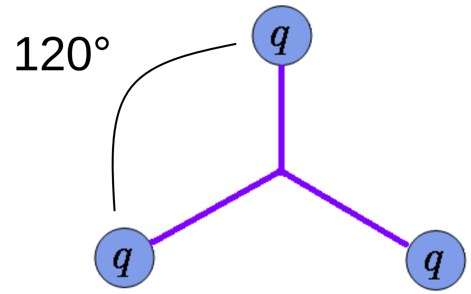


$a \approx 0.16 - 0.2\ \mathrm{GeV}^2$



Semirelativistic Hamiltonian (II)

- Meeting point
  - Toricelli point
  - Center of mass $\vec{R}$ in good approximation



B. Silvestre-Brac *et al*., Eur. Phys. J. C **32**, 385 (2003) [hep-ph/0309247].

$$H = \sum_i \sqrt{\vec{p}_i^{\,2} + m_i^2} + a|\vec{x}_i - \vec{R}|$$

- Validity of replacing the Toricelli point by the center of mass?

Semirelativistic Hamiltonian (III)

System	L	J^P	N	Conf.	V_Y	V_G	V_G
				σ	0.2	0.2	$0.2\,r$
					Toricelli	CM	
nnn	0	$1/2^+$	1		2.032	2.104	2.036
$(x=1)$			2		2.785	2.898	2.804
	1	$1/2^-$	1			2.567	2.483
bbb	0	$3/2^+$	1		0.810	0.839	0.812
$(x=1)$			2		1.111	1.156	1.118
	1	$1/2^-$	1			1.024	0.990
snn	0	$1/2^+$	1		1.933	2.004	1.937
$(x=1.667)$			2		2.626	2.758	2.666
	1	$1/2^-$	1			2.437	2.356
bnn	0	$1/2^+$	1		1.752	1.854	1.769
$(x=15.76)$			2		2.310	2.548	2.432
	1	$1/2^-$	1			2.256	2.153
nss	0	$1/2^+$	1		1.826	1.894	1.831
$(x=0.6)$			2		2.474	2.595	2.509
	1	$1/2^-$	1			2.296	2.219
nbb	0	$1/2^+$	1		1.313	1.373	1.329
$(x=0.063)$			2		1.645	1.718	1.662
	1	$1/2^-$	1			1.574	1.523

- $q\,q\,q'$

$$x = \frac{m_{q'}}{m_q}$$

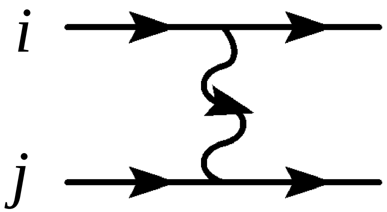
- Scaling $r(x)$ known

$$r(1) = 0.95$$

$$r(\infty) = 0.93$$

OGE interactions (I)

- Short distances: One gluon exchange becomes dominant



$$V_{ij}(r_{ij}) = -\frac{2}{3}\frac{\alpha_s}{r_{ij}} + \text{ corrections}$$

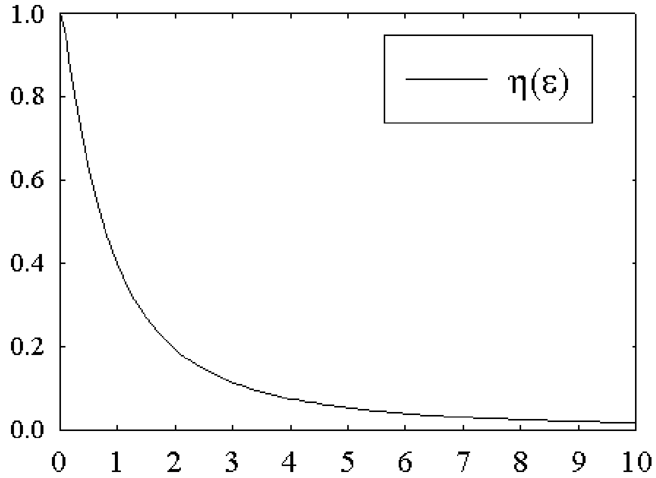
  - α_s strong coupling constant ($\alpha_s \approx 0.2 - 0.4$)
    - Remains small once confinement is separated
    - Can be added in perturbation

$$\Delta M_{oge} = -\frac{2\alpha_s}{3} \sum_{i<j} \left\langle \frac{1}{|\vec{x}_i - \vec{x}_j|} \right\rangle$$

Quark self-energy

Yu. A. Simonov, Phys. Lett. B **515**, 137 (2001)]

- Nonperturbative interactions between the color magnetic moment of the quark and the vacuum
  - $\Delta M_{qse} = -\frac{f\,a}{\pi}\sum_i \frac{\eta(m_i/\delta)}{2\mu_i}$ $\quad f \in [3,4], \quad \delta \approx 1 \text{ GeV}$
  - $\mu_i = \langle\sqrt{\vec{p}_i^{\,2} + m_i^2}\rangle$ quark kinetic energy
  - Relevant for light quarks



Further corrections

- Spin-independent formalism
  - OGE short-range
    - Spin-orbit, spin-spin, … terms

- Neglected in the present approach
  - Qualitatively

    Yu. A. Simonov, hep-ph/9911237

$$V_{ij}^{SD} \propto \frac{1}{\mu_i \mu_j}$$

The auxiliary field technique

B. Silvestre-Brac, C. Semay, and F. Buisseret,
J. Phys. A **41**, 275301 (2008) [arXiv:0802.3601];
arXiv:0806.2020.

Auxiliary fields (I)

- Three-body Hamiltonian $H = \sum_i \sqrt{\vec{p}_i^{\,2} + m_i^2} + a|\vec{x}_i - \vec{R}|$
- How to get analytical relations ?
  - Auxiliary field technique

$$H \to H(\mu_j, \nu_j) = \sum_j \frac{\vec{p}_j^{\,2} + m_j^2}{2\mu_j} + \frac{a^2(\vec{x}_j - \vec{R})^2}{2\nu_j} + \frac{\mu_j}{2} + \frac{\nu_j}{2}$$

  - Elimination

$$\delta_{\mu_k} H(\mu_j, \nu_j) = 0, \quad \mu_k^s = \sqrt{\vec{p}_k^{\,2}}$$ Quark kinetic energy

$$\delta_{\nu_k} H(\mu_j, \nu_j) = 0, \quad \nu_k^s = a|\vec{x}_k - \vec{R}|$$ String energy

  - By definition $H(\mu_k^s, \nu_k^s) = H$

Auxiliary fields (II)

- Let us see the auxiliary fields as real variational parameters

$$H(\mu_j,\nu_j)=\sum_j \frac{\vec{p}_j^{\,2}+m_j^2}{2\mu_j}+\frac{a^2(\vec{x}_j-\vec{R})^2}{2\nu_j}+\frac{\mu_j}{2}+\frac{\nu_j}{2}$$

  - Harmonic oscillator
  - Relative coordinates

$$\vec{R}=\frac{\mu_1\vec{x}_1+\mu_2\vec{x}_2+\mu_3\vec{x}_3}{\mu_1+\mu_2+\mu_3} \qquad \vec{\xi}\propto\vec{x}_1-\vec{x}_2 \qquad \vec{\eta}\propto\frac{\mu_1\vec{x}_1+\mu_2\vec{x}_2}{\mu_1+\mu_2}-\vec{x}_3$$

Mass formula (I)

- Two identical quarks ($m_1 = m_2$, $\mu_1 = \mu_2$, $\nu_1 = \nu_2$)

$$M(\mu, \mu_3, \nu, \nu_3) = \omega_\xi (N_\xi + 3/2) + \omega_\eta (N_\eta + 3/2)$$
$$+\mu + \nu + \frac{\mu_3 + \nu_3}{2} + \frac{m^2}{\mu} + \frac{m_3^2}{2\mu_3}$$
$$\omega_\xi = \frac{a}{\sqrt{\mu\nu}}, \quad \omega_\eta = \frac{a}{\sqrt{2\mu + \mu_3}} \sqrt{\frac{\mu_3}{\mu\nu} + \frac{2\mu}{\mu_3 \nu_3}}$$

- Harmonic oscillator quanta of excitation $N_{\xi/\eta} = 2n_{\xi/\eta} + \ell_{\xi/\eta}$
- Knowledge of $\langle \xi^2 \rangle, \quad \langle \eta^2 \rangle$
  - OGE

Mass formula (II)

- Elimination of the auxiliary fields

$$\delta_\mu M(\mu, \mu_3, \nu, \nu_3) = 0, \quad \delta_{\mu_3} M(\mu, \mu_3, \nu, \nu_3) = 0$$

$$\delta_\nu M(\mu, \mu_3, \nu, \nu_3) = 0, \quad \delta_{\nu_3} M(\mu, \mu_3, \nu, \nu_3) = 0$$

  - Optimal values $\mu_{k,0} \approx \langle \mu_k^s \rangle$, etc.
    - « Mean field approximation »
  - Many cases where analytical solutions are found
- Property: $M(\mu_0, \mu_{3,0}, \nu_0, \nu_{3,0}) > M$
  - Overestimation of the masses

F. Buisseret and V. Mathieu, Eur. Phys. J. A **29**, 343 (2006) [hep-ph/0607083].

Light baryons

C. Semay, F. Buisseret, N. Matagne, and Fl. Stancu,
Phys. Rev. D **75**, 096001 (2007) [hep-ph/0702075].

C. Semay, F. Buisseret, and Fl. Stancu,
Phys. Rev. D **76**, 116005 (2007) [arxiv:0708.3291].

Nonstrange baryons (I)

- Small u and d current mass: $m_{\mathrm{u}} = m_{\mathrm{d}} = 0$
  - Three massless quarks ($m = m_3 = 0$, $\mu = \mu_3$, $\nu = \nu_3$)

$$M(\mu) = \frac{a}{\mu}(N_\xi + N_\eta + 3) + 3\mu$$

  - Single quantum number $N = N_\xi + N_\eta$
    - Same as Large N_c

  - Elimination of μ ⟹ $M_0 = 6\mu_0 = \sqrt{12a(N+3)}$
- Baryon Regge trajectories: $M^2 \propto N$

$$P = (-1)^N$$

Nonstrange baryons (II)

- Experiment: Meson Regge slope = Baryon Regge slope = 2 π σ
  - Our slope = 12 a
  - Artefact of the method ⟹ scaling $a = \frac{\pi\sigma}{6}$
    - Physical string tension σ
- OGE $\Delta M_{oge} = -\frac{2\alpha_s}{3}\sum_{i<j}\left\langle\frac{1}{|\vec{x}_i - \vec{x}_j|}\right\rangle \approx -\frac{\pi\alpha_s\sigma}{3\sqrt{3}\mu_0}$
- QSE $\Delta M_{qse} = -\frac{f\sigma}{4\mu_0}$

$$M^2 = 2\pi\sigma(N+3) - \frac{4}{\sqrt{3}}\pi\sigma\alpha_s - \frac{12}{(2+\sqrt{3})}f\sigma$$

Strange baryons (I)

- Let us assume there is one s quark

$$M(\mu,\mu_s,\nu,\nu_s)=\omega_\xi(N_\xi+3/2)+\omega_\eta(N_\eta+3/2)+\mu+\nu+\frac{\mu_s+\nu_s}{2}+\frac{m_s^2}{2\mu_s}$$

  - The auxiliary fields cannot be analytically eliminated in general
  - Power expansion in m_s
  - Ansatz $\omega_\xi-\omega_\eta\approx 0$
    - When $n_s = 0$ or 3, $\omega_\xi=\omega_\eta$

(Power expansion and Ansatz →) Analytical solution

$$M = M_q + n_s\Theta\frac{m_s^2}{\mu_0}$$

M_q: Nonstrange baryon

$n_s\Theta\frac{m_s^2}{\mu_0}$: Strange quarks

Strange baryons (II)

- Strange quark term $\Theta = \Theta(K, \dots)$

$$\Theta = \frac{1}{2} - \frac{\alpha_s \pi \sigma}{36\sqrt{3}\mu_0^2} + \frac{f\sigma}{12}\left(\frac{3}{4\mu_0^2} + \frac{\beta}{\delta^2}\right)$$

Confinement

OGE

QSE

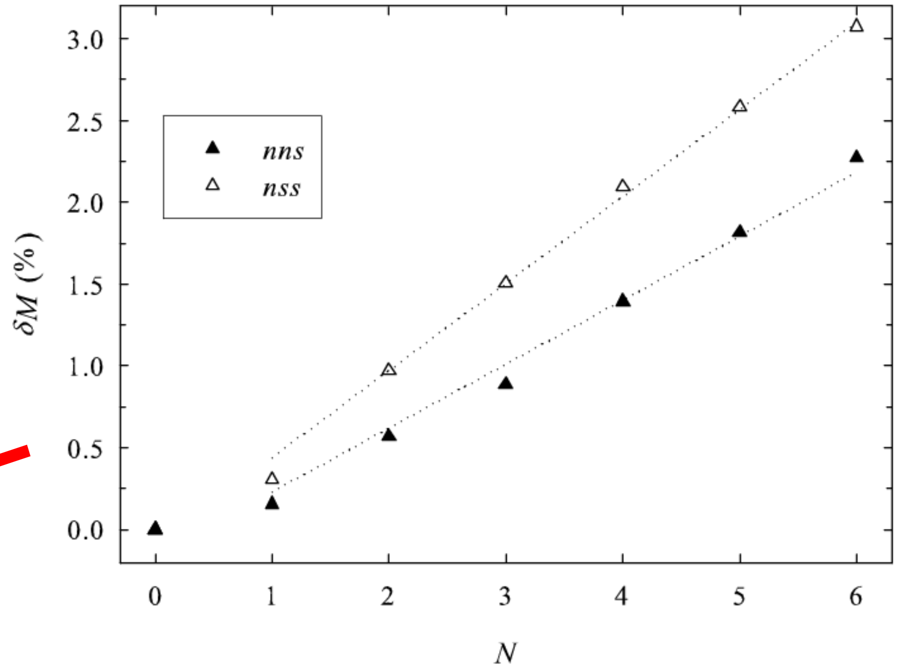
$$\eta(\epsilon) \approx 1 - \beta\epsilon^2, \quad \beta = 2.85$$

- Regge slope unchanged
- Interecept increases with n_s and m_s

Strange baryons (III)

- Is N really a good quantum number?
  - Numerical minimization of $M(\mu, \mu_s, \nu, \nu_s)$
    - Typical values for parameters
    - Fixed N
    - Measure of the variation of M for $0 \leq N_\eta \leq N$

OK



Mass spectrum

- Nonstrange baryons, $m = 0$
  - $\sigma = 0.175\ \text{GeV}^2$, $\alpha_s = 0.4$, $f = 3.5$, $\delta = 1.0$ GeV

N	Multiplet	$\bar{M}_{\text{exp}}$ (GeV)	Model (GeV)
0	$[56, 0^+]$: N(939),. . . ,Ω(1672)	1.173±0.001	1.171
1	$[70, 1^-]$: Λ(1405),. . . , Λ(1830)	1.657±0.029	1.538
2	$[56, 2^+]$: N(1680),. . . , Σ(2030)	1.878±0.022	1.845
3	$[70, 3^-]$: Λ(2100),. . . , N(2250)	2.193±0.091	2.112
4	$[56, 4^+]$: N(2220),. . . ,Δ(2420)	2.351±0.101	2.351
5	$[70, 5^-]$: N(2600)	2.638±0.097	2.570
6	$[56, 6^+]$: N(2700), Δ(2950)	2.885±0.148	2.773

Charm and bottom baryons

C. Semay, F. Buisseret, and Fl. Stancu, arXiv:0808.3349.

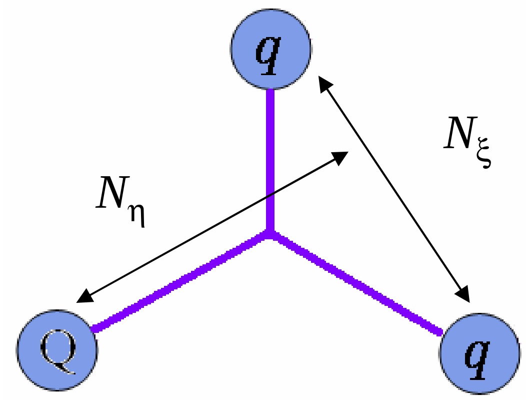
Experimental data

- In 2007: New heavy baryons, $\Xi_b, \Sigma_b, \Sigma_b^*$

One c quark	One b quark	
$\Lambda_c = 2286.46 \pm 0.14$ MeV,	$\Lambda_b = 5620.2 \pm 1.6$ MeV,	
$\Sigma_c = 2453.56 \pm 0.16$ MeV,	$\Sigma_b = 5811.5 \pm 1.7$ MeV,	Nonstrange
$\Sigma_c^* = 2518.0 \pm 0.8$ MeV,	$\Sigma_b^* = 5832.7 \pm 1.8$ MeV,	
$\Xi_c = 2469.5 \pm 0.3$ MeV,	$\Xi_b = 5792.9 \pm 3.0$ MeV.	
$\Xi_c' = 2576.9 \pm 2.1$ MeV,		$n_s = 1$
$\Xi_c^* = 2646.4 \pm 0.9$ MeV,		
$\Omega_c = 2697.5 \pm 2.6$ MeV,		$n_s = 2$
$\Omega_c^* = 2768.3 \pm 3.0$ MeV.		

Mass formula (I)

- Heavy baryon
  - One heavy quark m_Q
  - Two light ones, $m = 0$

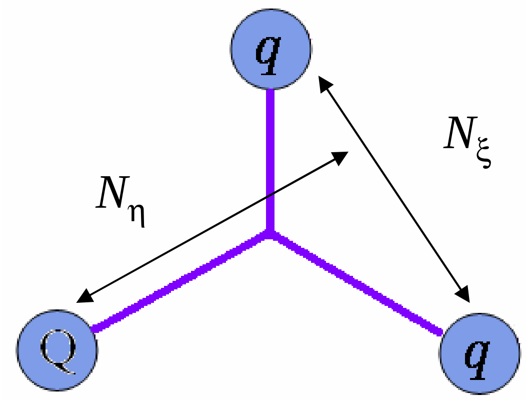


$$M(\mu, \mu_Q, \nu, \nu_Q) = \omega_\xi(N_\xi + 3/2) + \omega_\eta(N_\eta + 3/2) + \mu + \nu + \frac{\mu_Q + \nu_Q}{2} + \frac{m_Q^2}{2\mu_Q}$$

- 1/ m_Q expansion to eliminate the auxiliary fields
  - $\mu_Q \approx m_Q, \quad \nu_Q \propto 1/m_Q$

Mass formula (II)

- Analytical result



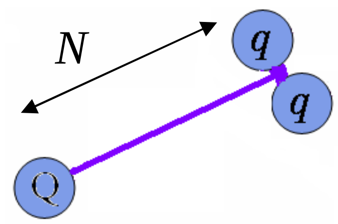
$$M_{qqQ} = m_Q + 4\mu_1 + \frac{a}{2m_Q} G(N_\eta, N_\xi),$$

$$\mu_1 = \sqrt{\frac{a(N_\eta + N_\xi + 3)}{2}},$$

$$G(N_\eta, N_\xi) = \sqrt{2N_\xi + 3}\left(\sqrt{2(N_\eta + N_\xi + 3)} - \sqrt{2N_\xi + 3}\right)$$

Minimal mass for $N_\xi = 0$, $N_\eta = N$

Heavy quark – diquark picture for excited states



$$F(N) = G(N, 0)$$

Mass formula (III)

- As for light baryons, rescaling of a, $a = \frac{\pi\sigma}{6}$
  - Regge trajectories

$$(M - m_Q)^2 \approx \tfrac{4\pi\sigma}{3} N \approx 1.3\pi\sigma N$$

  - Lower slope than light baryons
  - Mesons
    - Light $q\bar{q}$ $M^2 \approx 2\pi\sigma N$
    - Heavy $Q\bar{q}$ $(M - m_Q)^2 \approx \pi\sigma N$

OGE contribution

- Running strong coupling constant
  - $\alpha_s(qq) \neq \alpha_s(Qq)$
  - Simple choice $\alpha_1 = \alpha_s(Qq) = b\,\alpha_s(qq) = b\,\alpha_0$
  - Previous study, b = 0.7 OK.

C. Semay and B. Silvestre-Brac, Phys. Rev. D **52**, 6553 (1995).

- Mass term

$$\Delta M_{oge} = -\frac{2}{3}\alpha_0\sqrt{\frac{\pi\sigma}{18}}\left[1+\frac{\sqrt{3\pi\sigma}}{48m_Q}\left(\sqrt{\frac{2(2N+3)}{N+3}}-1\right)\right] - \frac{4}{3}\alpha_1\sqrt{\frac{\pi\sigma}{3(N+3)}}\left[1-\frac{\sqrt{\pi\sigma}}{4m_Q}\frac{1}{\sqrt{2(N+3)}}\right].$$

QSE contribution

- Heavy quark $\Delta M_{qse} \propto m_Q^{-3} \approx 0$
- Mass term from two light quarks

$$\Delta M_{qse} = -\frac{fa}{\pi \mu_1}\left[1 + \frac{\pi\sigma}{24\mu_1 m_Q} F(N)\right].$$

- Recall that $\mu_1 = \sqrt{\frac{\pi\sigma(N+3)}{12}}$

$$F(N) = \sqrt{2N+3}(\sqrt{2(N+3)} - \sqrt{2N+3})$$

Strangeness contribution

- Power expansion in m_s

$$n_s\,\Delta M_{1s} = n_s\,\frac{m_s^2}{\mu_1}\left[\frac{1}{2} - \frac{1}{12\mu_1}\left(\alpha_0\sqrt{\frac{\pi\sigma}{18}} + 2\alpha_1\sqrt{\frac{\pi\sigma}{3(N+3)}}\right) + \frac{f\sigma}{12}\left(\frac{3}{4\mu_1^2} + \frac{\beta}{\delta^2}\right)\right]$$

Flux tubes

OGE

QSE

Partial summary

- Aim

  To compare large N_c QCD and quark model

  This talk

- Ingredients
  - Semirelativistic kinetic energy
  - Y-junction confinement
  - One gluon exchange
  - Quark self-energy
  - Spin-independent

- Analytical results
  - Auxiliary field method
  - Linear confinement $\Longrightarrow$ Harmonic oscillator
  - Harmonic oscillator quantum of excitation N
    - Link with Large N_c
- N is a good quantum number
  - Light baryons: Total quantum of excitation
  - Heavy baryons: Heavy quark – Light diquark
  - Regge trajectories