MASSES OF HEAVY TETRAQUARKS IN THE RELATIVISTIC QUARK MODEL

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Ebert, Faustov, Galkin (EFG) — Phys. Lett. B (2006) 214-219 [hep-ph/0512230]
Ebert, Faustov, Galkin, Lucha (EFGL) — arXiv:0706.3853 [hep-ph]

Table 1: New Charmonium states

state	mass (MeV)	width (MeV)	production/decay mode	comments	exp
h_c	$3524.4 \pm 0.6 \pm 0.4$	–	$\psi(2S) \to \pi^0 h_c \to (\gamma\gamma)(\gamma\eta_c)$	$\approx$ CQM	CLEO
	$3525.8 \pm 0.2 \pm 0.2$	< 1	$p\bar{p} \to h_c \to \gamma\eta_c$		E835
η_c'	$3654 \pm 6 \pm 8$	< 55	$B \to K\eta_c' \to KK_SK\pi$	$\approx$ CQM	Belle
	$3642.9 \pm 3.1 \pm 1.5$	$6.3 \pm 12.4 \pm 4.0$	$e^+e^- \to \eta_c' J/\psi$		CLEO
	$3630.8 \pm 3.4 \pm 1.0$	$17.0 \pm 8.0 \pm 2.5$	$\gamma\gamma \to \eta_c' \to K_SK\pi$		BaBar
	avg $= 3637.7 \pm 4.4$				
$X(3872)$	$3872.0 \pm 0.6 \pm 0.5$	< 2.3	$B \to KX \to K\pi\pi J/\psi$	molecule,	Belle
	3873.4 ± 1.4	–	$B \to KX \to K\pi\pi J/\psi$	cusp ,	BaBar
	–	–	$B \to X \to \pi\pi\pi J/\psi$	tetraquark	Belle
	–	–	$B \to X \to \gamma J/\psi$		Belle
	$3871.3 \pm 0.7 \pm 0.4$	–	$p\bar{p} \to X \to \pi\pi J/\psi$		CDF
	$3871.8 \pm 3.1 \pm 3.0$	–	$p\bar{p} \to X \to \pi\pi J/\psi$		DØ
	avg $= 3871.9 \pm 0.5$				
$X(3940)$	$3943 \pm 6 \pm 6$	< 52	$e^+e^- \to J/\psi X \to J/\psi D\bar{D}^*$	χ_{c1}', η_c'' / ?	Belle
$Y(3940)$	$3943 \pm 11 \pm 13$	$87 \pm 22 \pm 26$	$B \to KY \to K\pi\pi\pi J/\psi$	tetraquark / ?	Belle
$Z(3930)$	$3931 \pm 4 \pm 2$	$20 \pm 8 \pm 3$	$\gamma\gamma \to Z \to D\bar{D}$	χ_{c2}' / $\approx$ CQM	Belle
$Y(4260)$	$4259 \pm 8 \pm 4$	$88 \pm 23 \pm 5$	$e^+e^- \to \gamma_{ISR} Y \to \gamma_{ISR} J/\psi\pi\pi$	hybrid?,	BaBar
	$4283^{+17}_{-16} \pm 4$	$73^{+39}_{-25} \pm 5$	$e^+e^- \to \gamma_{ISR} Y \to \gamma_{ISR} J/\psi\pi\pi$	tetraquark	CLEO
	$4247 \pm 12^{+17}_{-32}$	$108 \pm 19 \pm 10$	$e^+e^- \to \gamma_{ISR} Y \to \gamma_{ISR} J/\psi\pi\pi$		Belle
$Y(4360)$	$4361 \pm 9 \pm 9$	$74 \pm 15 \pm 10$	$e^+e^- \to \gamma_{ISR} Y \to \gamma_{ISR} \psi'\pi\pi$	tetraquark/?	Belle
$Z(4433)$	$4433 \pm 4 \pm 1$	44^{+17+30}_{-13-11}	$B^+ \to \psi'\pi^+ K$	tetraquark/?	Belle
$Y(4660)$	$4664 \pm 11 \pm 5$	$48 \pm 15 \pm 3$	$e^+e^- \to \gamma_{ISR} Y \to \gamma_{ISR} \psi'\pi\pi$	tetraquark/?	Belle

Table 2: Experimental and theoretical spectrum of charmonium ($c\bar{c}$) states. († input).

State ($n^{2S+1}L_J$)	Experiment	BGS	EFG	ELQ	CP-PACS (Lattice)
$J/\psi(1^3S_1)$	3096.87 ± 0.04	3090	3096	3096.9†	3085 ± 1
$\eta_c(1^1S_0)$	2979.2 ± 1.3	2982	2979	2979.9†	3013 ± 1
$\psi'(2^3S_1)$	3685.96 ± 0.09	3672	3686	3686.0†	3777 ± 40
$\eta_c'(2^1S_0)$	3637.7 ± 4.4	3630	3633	3638†	3739 ± 46
$\psi(3^3S_1)$	4040 ± 10	4072	4088	4040†	–
$\eta_c(3^1S_0)$	$3943 \pm 6 \pm 6$?	4043	3991	3943†	–
$\psi(4^3S_1)$	4415 ± 6	4406	–	–	–
$\eta_c(4^1S_0)$		4384	–	–	–
$\chi_2(1^3P_2)$	3556.18 ± 0.13	3556	3556	3556.2†	3503 ± 24
$\chi_1(1^3P_1)$	3510.51 ± 0.12	3505	3510	3510.5†	3472 ± 9
$\chi_0(1^3P_0)$	3415.3 ± 0.4	3424	3424	3415.3†	3408 ± 10
$h_c(1^1P_1)$	3525 ± 1	3516	3525	3524.4†	3474 ± 10
$\chi_2(2^3P_2)$	3931 ± 5	3972	3972	3931†	4030 ± 180
$\chi_1(2^3P_1)$	$3943 \pm 6 \pm 6$?	3925	3929	3920.5	4067 ± 105
$\chi_0(2^3P_0)$		3852	3854	3881.4	4008 ± 122
$h_c(2^1P_1)$		3934	3945	3919.0	4053 ± 95
$\psi_3(1^3D_3)$		3806	3815	3868.3	–
$\psi_2(1^3D_2)$		3800	3811	3830.6	–
$\psi(1^3D_1)$	3769.9 ± 2.5	3785	3798	3769.9†	–
$\eta_{c2}(1^1D_2)$		3799	3811	3838.0	–

BGS – Barnes, Godfrey, Swanson (2005), EFG – Ebert, Faustov, Galkin (2003)
ELQ – Eichten, Lane, Quigg (2006), CP-PACS – Okamato et al. (2002)

- $\boldsymbol{X}(3872)$

The best studied of the new charmonium states. Known properties:

World average mass: $M(X(3872)) = 3871.9 \pm 0.5$ MeV

- ≈ 142 MeV above $D\bar{D}$ threshold (3729.4 MeV), but X has narrow width $\Gamma(X(3872)) < 2.3$ MeV $\Longrightarrow$ unnatural parity $P = (-1)^{J+1}$
- Very close to the following thresholds:

$D^0\bar{D}^{*0}$	—	3871.3 MeV
$\rho J/\psi$	—	3872.7 MeV
$D^{\pm}D^{*\pm}$	—	3879.5 MeV
$\omega J/\psi$	—	3879.6 MeV

Observed decay modes:

- $X(3872) \to \rho J/\psi \to \pi\pi J/\psi \Longrightarrow$ isovector state $(I = 1)$
- $X(3872) \to \omega J/\psi \to \pi\pi\pi J/\psi \Longrightarrow$ isoscalar state $(I = 0)$

$$\frac{Br(X \to \pi^+\pi^-\pi^0 J/\psi)}{Br(X \to \pi^+\pi^- J/\psi)} = 1.0 \pm 0.4 \pm 0.3$$

$\Longrightarrow$ strong isospin violation effects

- Mass difference of X produced in B^+ and B^0 decays: $\Delta M(X) = 2.7 \pm 1.3 \pm 0.2$ MeV
- $X(3872) \to \gamma J/\psi \Longrightarrow$ positive charge conjugation parity $C = +$
- Angular correlations in $\pi\pi J/\psi$ favour $J = 1 \Longrightarrow J^{PC} = 1^{++}$ state

Main theoretical interpretations

- Charmonium:

$\chi'_{c1}(2^3P_1)$ – the only possible candidate for 1^{++} state, but
⋆ predicted $M(\chi'_{c1}) = 3920 - 3940$ MeV is more than 50 MeV larger
⋆ large decay rate into $I = 1$ channel: $X(3872) \to \rho J/\psi \to \pi\pi J\psi$
⋆ predicted $\Gamma(\chi'_{c1}) = 15 - 130$ MeV is too large ($\Gamma(X(3872)) < 2.3$ MeV)

- $D^0\bar{D}^{*0}$ molecule (Swanson, Voloshin, Close, Page, . . .)

$D^0\bar{D}^{*0}$ molecules — bound by short range, colourless meson (π) exchange forces $\Longrightarrow$
⋆ $M(X)$ must be lower than $D^0\bar{D}^{*0}$ threshold (binding energy must be negative $\Longrightarrow$ very small)
⋆ large size (loosely bound state)
⋆ 1^{++} – lowest S-wave bound state
⋆ the only bound state: no radial or orbital excitations
⋆ absence of $D_s\bar{D}_s^*$ molecule

- Cusp at $D^0\bar{D}^{*0}$ threshold (Bugg)

Cusps occur in amplitudes at thresholds $\Longrightarrow$ bumps in cross sections slightly above threshold
⋆ width of typical enhancements due cusps are $O(\Lambda_{QCD})$ which is $\gg \Gamma(X)$
⋆ bumps should be at many thresholds
⋆ no radial or orbital excitations

- Tetraquark (diquark-antidiquark) (Maiani et al., EFG)

Tetraquarks — diquark-antidiquark pairs in colour $\bar{3}$ and 3 configurations bound by colour forces $\Longrightarrow$

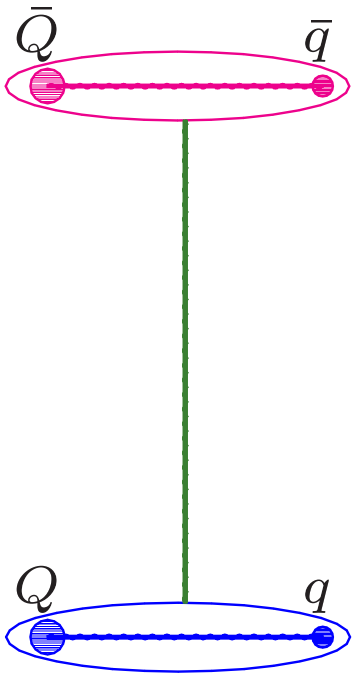
⋆ typical hadronic size

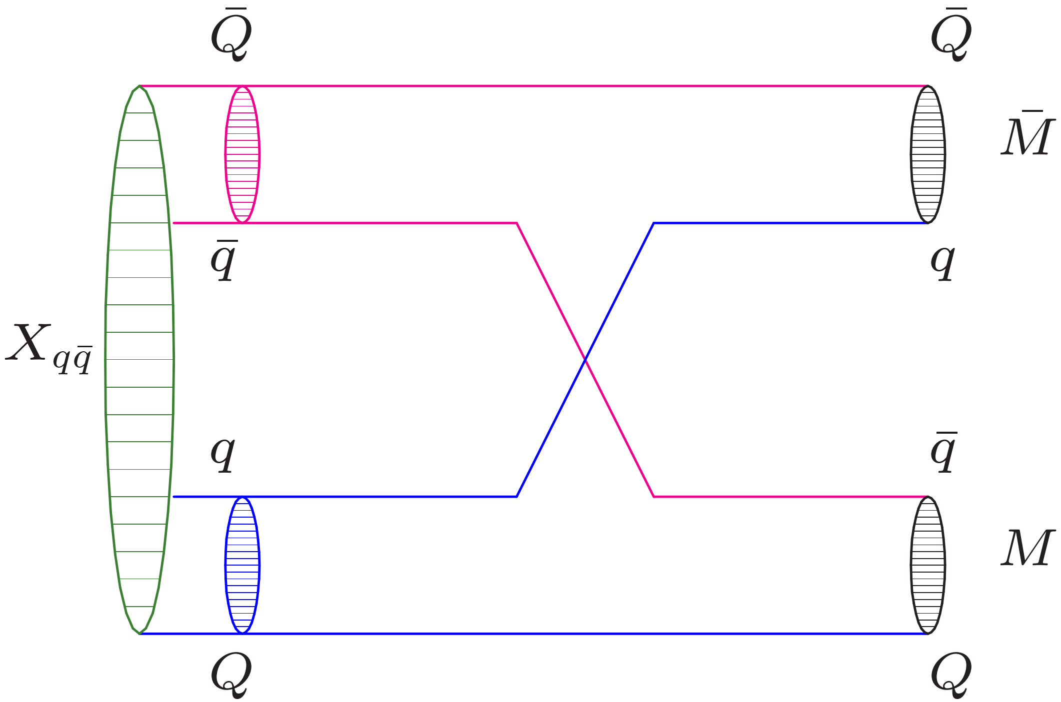
⋆ X should be split into two states $[cu][\bar{c}\bar{u}]$ and $[cd][\bar{c}\bar{d}]$ with $\Delta M \sim 7$ MeV

⋆ existence of charged partners $X^+ = [cu][\bar{c}\bar{d}]$, $X^- = [cd][\bar{c}\bar{u}]$

⋆ existence of tetraquarks with open $X_{s\bar{q}} = [cs][\bar{c}\bar{q}]$ and hidden $X_{s\bar{s}} = [cs][\bar{c}\bar{s}]$ strangeness

⋆ rich spectroscopy — radial and orbital excitations between diquarks





$(Q = c, b \qquad q = u, d$ or $s)$

Both scalar (asymmetric in flavour $[cq]_{S=0} = [cq]$) and axial vector (symmetric in flavour $[cq]_{S=1} = \{cq\}$) diquarks are considered

The $[cq][\bar{c}\bar{q}']$ ground states:

- ⋆ Two states with $J^{PC} = 0^{++}$:
  (C is defined for $q = q'$)

$$X(0^{++}) = [cq]_{S=0}[\bar{c}\bar{q}']_{S=0}$$

$$X(0^{++\prime}) = [cq]_{S=1}[\bar{c}\bar{q}']_{S=1}$$

- ⋆ Three states with $J = 1$:

$$X(1^{++}) = \frac{1}{\sqrt{2}}([cq]_{S=1}[\bar{c}\bar{q}']_{S=0} + [cq]_{S=0}[\bar{c}\bar{q}']_{S=1})$$

$$X(1^{+-}) = \frac{1}{\sqrt{2}}([cq]_{S=0}[\bar{c}\bar{q}']_{S=1} - [cq]_{S=1}[\bar{c}\bar{q}']_{S=0})$$

$$X(1^{+-\prime}) = [cq]_{S=1}[\bar{c}\bar{q}']_{S=1}$$

- ⋆ One state with $J^{PC} = 2^{++}$:

$$X(2^{++}) = [cq]_{S=1}[\bar{c}\bar{q}']_{S=1}$$

- Doubly heavy tetraquarks $(QQ')(\bar{q}\bar{q}')$

• explicitly exotic states with heavy flavour number equal to 2
$\Longrightarrow$ their observation would be a direct proof of existence of multiquark states

• estimates of the production rates of such tetraquarks indicate that they could be produced and detected at present (SELEX, Tevatron, RHIC) and future facilities (LHC, LHCb, ALICE).

• we considered the doubly heavy $(QQ')(\bar{q}\bar{q}')$ tetraquark ($Q = b, c$ and $q = u, d, s$) as the bound system of the heavy diquark (QQ') and light antidiquark $(\bar{q}\bar{q}')$

RELATIVISTIC QUARK MODEL

Quasipotential equation of Schrödinger type:

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p},\mathbf{q};M)\Psi_M(\mathbf{q})$$

$\mathbf{p}$ - relative momentum of quarks
M - bound state mass ($M = E_1 + E_2$)
μ_R - relativistic reduced mass:

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}$$

$b(M)$ - on-mass-shell relative momentum in cms:

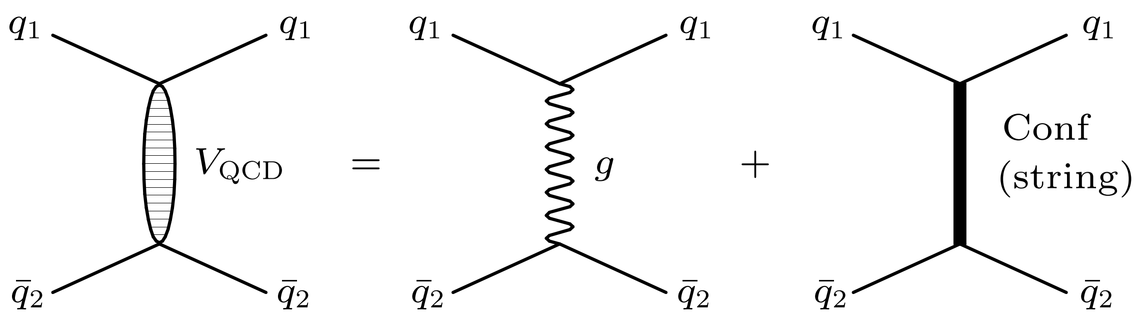
$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}$$

$E_{1,2}$ - center of mass energies:

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}$$

- Parameters of the model fixed from meson sector

- $q\bar{q}$ **quasipotential**



$$V(\mathbf{p},\mathbf{q};M) = \bar{u}_1(p)\bar{u}_2(-p)\left\{\frac{4}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu + V^V_{\rm conf}(\mathbf{k})\Gamma_1^\mu\Gamma_{2;\mu} + V^S_{\rm conf}(\mathbf{k})\right\}u_1(q)u_2(-q)$$

$\mathbf{k} = \mathbf{p} - \mathbf{q}$
$D_{\mu\nu}(\mathbf{k})$ - (perturbative) gluon propagator
$\Gamma_\mu(\mathbf{k})$ - effective long-range vertex with Pauli term:

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu,$$

κ - anomalous chromomagnetic moment of quark,

$$u^{\lambda}(p) = \sqrt{\frac{\epsilon(p)+m}{2\epsilon(p)}} \begin{pmatrix} 1 \\ \dfrac{\boldsymbol{\sigma}\mathbf{p}}{\epsilon(p)+m} \end{pmatrix} \chi^{\lambda},$$

with $\epsilon(p) = \sqrt{\mathbf{p}^2 + m^2}$.

- Lorentz structure of $V_{\rm conf} = V^V_{\rm conf} + V^S_{\rm conf}$

  In nonrelativistic limit

$$\left.\begin{array}{rcl} V^V_{\rm conf} & = & (1-\varepsilon)(Ar+B) \\ V^S_{\rm conf} & = & \varepsilon(Ar+B) \end{array}\right\} \quad \text{Sum :} \quad (Ar+B)$$

ε - mixing parameter

Parameters A, B, κ, ε and quark masses fixed from analysis of meson masses and radiative decays:

$\varepsilon = -1$ from heavy quarkonium radiative decays ($J/\psi \to \eta_c + \gamma$) and HQET

$\kappa = -1$ from fine splitting of heavy quarkonium ^3P_J states and HQET

$(1+\kappa) = 0 \Longrightarrow$ vanishing long-range chromomagnetic interaction !

Freezing of α_s for light quarks (Simonov, Badalyan)

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln \frac{\mu^2 + M_0^2}{\Lambda^2}}, \quad \beta_0 = 11 - \frac{2}{3} n_f, \quad \mu = \frac{2m_1 m_2}{m_1 + m_2},$$

$$M_0 = 2.24\sqrt{A} = 0.95 \text{ GeV}$$

Quasipotential parameters:

$A = 0.18$ GeV^2, $B = -0.30$ GeV,

$\Lambda = 0.413$ GeV (from M_ρ)

Quark masses:

$m_b = 4.88$ GeV $m_s = 0.50$ GeV

$m_c = 1.55$ GeV $m_{u,d} = 0.33$ GeV

- **Heavy tetraquarks in diquark-antidiquark picture**

(Qq)-interaction: $V_{Qq} = \frac{1}{2} V_{Q\bar{q}}$

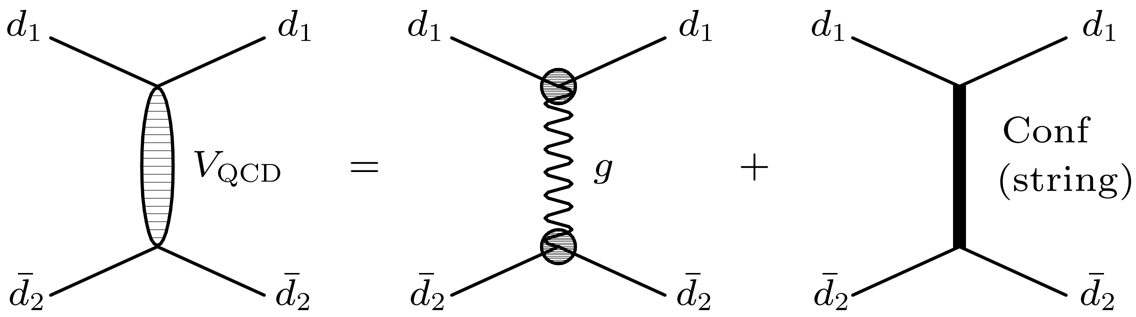
$$V(\mathbf{p}, \mathbf{q}; M) = \bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p}, \mathbf{q}; M)u_1(q)u_2(-q),$$

where

$$\mathcal{V}(\mathbf{p}, \mathbf{q}; M) = \frac{2}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma_1^\mu \gamma_2^\nu + \frac{1}{2} V^V_{\rm conf}(\mathbf{k})\Gamma_1^\mu \Gamma_{2;\mu} + \frac{1}{2} V^S_{\rm conf}(\mathbf{k})$$

$(d_1\bar{d}_2)$-interaction: $d = (Qq)$

$$V(\mathbf{p}, \mathbf{q}; M) = \frac{\langle d_1(P)|J_\mu|d_1(Q)\rangle}{2\sqrt{E_{d_1}E_{d_1}}} \frac{4}{3}\alpha_S D^{\mu\nu}(\mathbf{k}) \frac{\langle d_2(P')|J_\nu|d_2(Q')\rangle}{2\sqrt{E_{d_2}E_{d_2}}}$$
$$+\psi^*_{d_1}(P)\psi^*_{d_2}(P')\left[J_{d_1;\mu}J^\mu_{d_2}V^V_{\rm conf}(\mathbf{k}) + V^S_{\rm conf}(\mathbf{k})\right]\psi_{d_1}(Q)\psi_{d_2}(Q'),$$



$J_{d,\mu}$ – effective long-range vector vertex of diquark:

$$J_{d;\mu} = \begin{cases} \dfrac{(P+Q)_\mu}{2\sqrt{E_d E_d}} & \text{for scalar diquark} \\ -\dfrac{(P+Q)_\mu}{2\sqrt{E_d E_d}} + \dfrac{i\mu_d}{2M_d}\Sigma_\mu^\nu k_\nu & \text{for axial vector diquark } (\mu_d = 0) \end{cases}$$

μ_d - total chromomagnetic moment of axial vector diquark
diquark spin matrix: $(\Sigma_{\rho\sigma})_\mu^\nu = -i(g_{\mu\rho}\delta_\sigma^\nu - g_{\mu\sigma}\delta_\rho^\nu)$
$\mathbf{S}_d$ - axial vector diquark spin: $(S_{d;k})_{il} = -i\varepsilon_{kil}$

$\psi_d(P)$ – diquark wave function:

$$\psi_d(p) = \begin{cases} 1 & \text{for scalar diquark} \\ \varepsilon_d(p) & \text{for axial vector diquark} \end{cases}$$

$\varepsilon_d(p)$ – polarization vector of axial vector diquark

$\langle d(P)|J_\mu|d(Q)\rangle$ – vertex of diquark-gluon interaction:

$$\langle d(P)|J_\mu(0)|d(Q)\rangle = \int \frac{d^3p\, d^3q}{(2\pi)^6} \bar{\Psi}^d_P(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi^d_Q(\mathbf{q}) \Rightarrow F(k^2)$$

Γ_μ – two-particle vertex function of the diquark-gluon interaction:

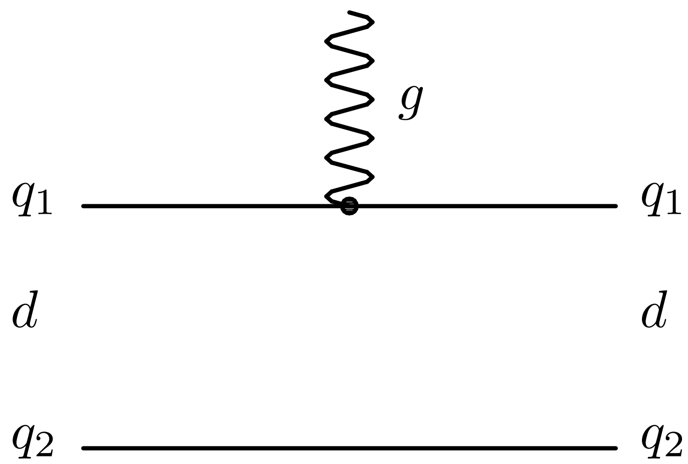


Figure 1: The vertex function Γ of the diquark-gluon interaction in the impulse approximation. The gluon interaction only with one quark is shown.

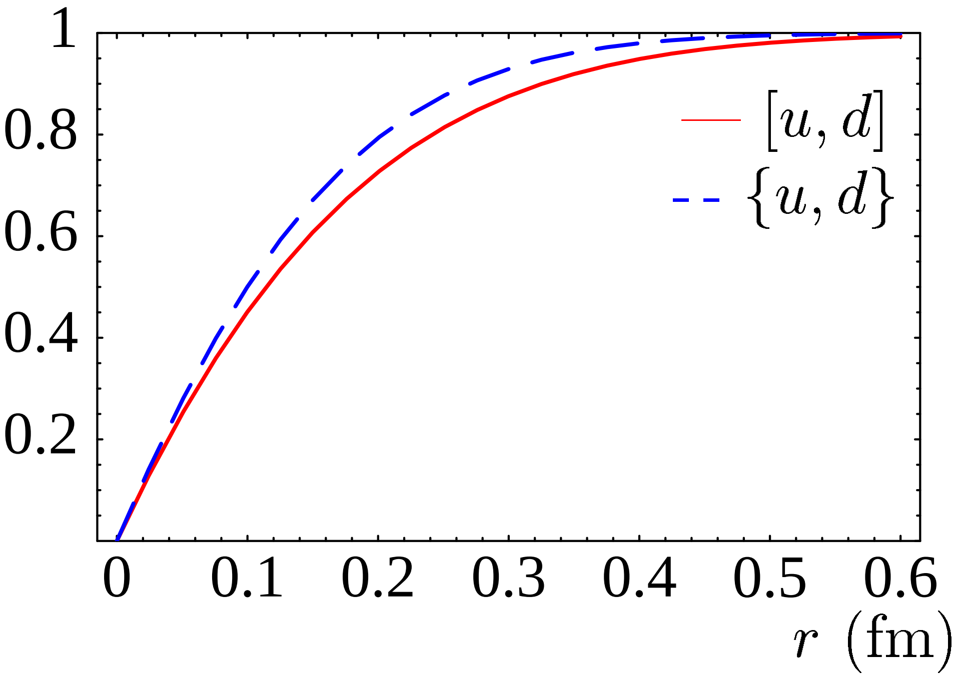
Table 3: Masses of light ground state diquarks (in MeV). S and A denotes scalar and axial vector diquarks antisymmetric $[q, q']$ and symmetric $\{q, q'\}$ in flavour, respectively.

Quark content	Diquark type	Mass				
		our RQM	Ebert et al. NJL	Burden et al. BSE	Maris BSE	Hess et al. Lattice
$[u, d]$	S	710	705	737	820	694(22)
$\{u, d\}$	A	909	875	949	1020	806(50)
$[u, s]$	S	948	895	882	1100	
$\{u, s\}$	A	1069	1050	1050	1300	
$\{s, s\}$	A	1203	1215	1130	1440	

Table 4: Masses of heavy-light and doubly heavy diquarks (MeV).

Quark content	Diquark type	Mass	
		$Q = c$	$Q = b$
$[Q, q]$	S	1973	5359
$\{Q, q\}$	A	2036	5381
$[Q, s]$	S	2091	5462
$\{Q, s\}$	A	2158	5482
$[Q, c]$	S		6519
$\{Q, c\}$	A	3226	6526
$\{Q, b\}$	A	6526	9778

The form factors $F(r)$ for the scalar $[u, d]$ (solid line) and axial vector $\{u, d\}$ (dashed line) diquarks:



The form factors $F(r)$ for $\{c, q\}$ (red line) and $\{b, q\}$ (blue line) axial vector diquarks.

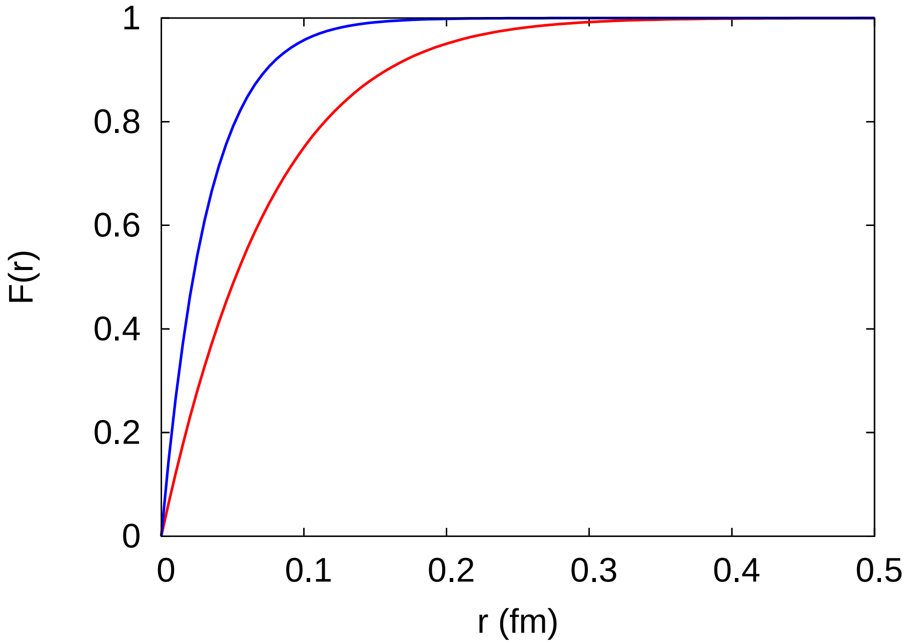


Table 5: Masses of the ground state heavy baryons (in MeV).

Baryon	$I(J^P)$	Theory						Experiment
		our (2005)	Capstick Isgur	Roncaglia et al.	Savage	Jenkins	Mathur* et al.	PDG
Λ_c	$0(\frac{1}{2}^+)$	2297	2265	2285			2290	2286.46(14)
Σ_c	$1(\frac{1}{2}^+)$	2439	2440	2453			2452	2453.76(18)
Σ_c^*	$1(\frac{3}{2}^+)$	2518	2495	2520	2518		2538	2518.0(5)
Ξ_c	$\frac{1}{2}(\frac{1}{2}^+)$	2481		2468			2473	2471.0(4)
Ξ_c'	$\frac{1}{2}(\frac{1}{2}^+)$	2578		2580	2579	2580.8(2.1)	2599	2578.0(2.9)
Ξ_c^*	$\frac{1}{2}(\frac{3}{2}^+)$	2654		2650			2680	2646.1(1.2)
Ω_c	$0(\frac{1}{2}^+)$	2698		2710			2678	2697.5(2.6)
Ω_c^*	$0(\frac{3}{2}^+)$	2768		2770	2768	2760.5(4.9)	2752	2768.3(3.0)†
Λ_b	$0(\frac{1}{2}^+)$	5622	5585	5620			5672	5620.2(1.6)
Σ_b	$1(\frac{1}{2}^+)$	5805	5795	5820		5824.2(9.0)	5847	5807.5(3.6)‡
Σ_b^*	$1(\frac{3}{2}^+)$	5834	5805	5850		5840.0(8.8)	5871	5829.0(3.3)‡
Ξ_b	$\frac{1}{2}(\frac{1}{2}^+)$	5812		5810		5805.7(8.1)	5788	5774(26)⋆
Ξ_b'	$\frac{1}{2}(\frac{1}{2}^+)$	5937		5950		5950.9(8.5)	5936	
Ξ_b^*	$\frac{1}{2}(\frac{3}{2}^+)$	5963		5980		5966.1(8.3)	5959	
Ω_b	$0(\frac{1}{2}^+)$	6065		6060		6068.7(11.1)	6040	
Ω_b^*	$0(\frac{3}{2}^+)$	6088		6090		6083.2(11.0)	6060	

* error estimates of lattice calculations — ∼50 MeV for charmed, ∼100 MeV for bottom baryons

† BaBar 2006; ‡ CDF 2006; ⋆ D0 2007

HEAVY TETRAQUARKS

The potential of the heavy diquark-antidiquark interaction

$$V(r) = V_{\rm Coul}(r) + V_{\rm conf}(r) + \frac{1}{E_1E_2}\Bigg\{\mathbf{p}\left[V_{\rm Coul}(r)+V^V_{\rm conf}(r)\right]\mathbf{p} - \frac{1}{4}\Delta V^V_{\rm conf}(r) + V'_{\rm Coul}(r)\frac{\mathbf{L}^2}{2r}$$
$$+\frac{1}{r}\left[V'_{\rm Coul}(r) + \frac{\mu_d}{4}\left(\frac{E_1}{M_1}+\frac{E_2}{M_2}\right)V'^V_{\rm conf}(r)\right]\mathbf{L}(\mathbf{S}_1+\mathbf{S}_2)$$
$$+\frac{\mu_d}{4}\left(\frac{E_1}{M_1}-\frac{E_2}{M_2}\right)\frac{V'^V_{\rm conf}(r)}{r}\mathbf{L}(\mathbf{S}_1-\mathbf{S}_2)$$
$$+\frac{1}{3}\left[\frac{1}{r}V'_{\rm Coul}(r) - V''_{\rm Coul}(r) + \frac{\mu_d^2}{4}\frac{E_1E_2}{M_1M_2}\left(\frac{1}{r}V'^V_{\rm conf}(r) - V''^V_{\rm conf}(r)\right)\right]\left[\frac{3}{r^2}(\mathbf{S}_1\mathbf{r})(\mathbf{S}_2\mathbf{r}) - \mathbf{S}_1\mathbf{S}_2\right]$$
$$+\frac{2}{3}\left[\Delta V_{\rm Coul}(r) + \frac{\mu_d^2}{4}\frac{E_1E_2}{M_1M_2}\Delta V^V_{\rm conf}(r)\right]\mathbf{S}_1\mathbf{S}_2\Bigg\},$$

where

$$V_{\rm Coul}(r) = -\frac{4}{3}\alpha_s\frac{F_1(r)F_2(r)}{r}$$

$\mathbf{S}_{1,2}$ – spins of diquark and antidiquark; $\mathbf{L}$ – orbital momentum.

The diquark-antidiquark model of heavy tetraquarks predicts existence of the $SU(3)$ nonet of states with hidden charm or beauty ($Q = c, b$):

- four tetraquarks ($[Qq][\bar{Q}\bar{q}]$, $q = u, d$) with neither open nor hidden strangeness, which have electric charges 0 or ± 1 and isospin 0 or 1

- four tetraquarks ($[Qs][\bar{Q}\bar{q}]$ and $[Qq][\bar{Q}\bar{s}]$, $q = u, d$) with open strangeness ($S = \pm 1$), which have electric charges 0 or ± 1 and isospin $\frac{1}{2}$

- one tetraquark ($[Qs][\bar{Q}\bar{s}]$) with hidden strangeness and zero electric charge.

In our model we neglect the mass difference of u and d quarks and electromagnetic interactions – thus corresponding tetraquarks will be degenerate in mass. More detailed analysis predicts that such mass differences can be of few MeV.

The (non)observation of such states will be a crucial test of the tetraquark model.

Table 6: Masses of hidden charm tetraquark states (in MeV).

State J^{PC}	Diquark content	Tetraquark mass $cq\bar{c}\bar{q}$	$cs\bar{c}\bar{s}$	$cs\bar{c}\bar{q}/cq\bar{c}\bar{s}$
$1S$				
0^{++}	$S\bar{S}$	3812	4051	3922
$1^{+\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	3871	4113	3982
0^{++}	$A\bar{A}$	3852	4110	3967
1^{+-}	$A\bar{A}$	3890	4143	4004
2^{++}	$A\bar{A}$	3968	4209	4080
$1P$				
1^{--}	$S\bar{S}$	4244	4466	4350

Table 7: Thresholds for open charm decays and nearby hidden-charm thresholds.

Channel	Threshold (MeV)	Channel	Threshold (MeV)	Channel	Threshold (MeV)
$D^0\bar{D}^0$	3729.4	$D_s^+D_s^-$	3936.2	$D^0D_s^\pm$	3832.9
D^+D^-	3738.8	$\eta' J/\psi$	4054.7	$D^\pm D_s^\mp$	3837.7
$D^0\bar{D}^{*0}$	3871.3	$D_s^\pm D_s^{*\mp}$	4080.0	$D^{*0}D_s^\pm$	3975.0
$\rho J/\psi$	3872.7	$\phi J/\psi$	4116.4	$D^0D_s^{*\pm}$	3976.7
$D^\pm D^{*\mp}$	3879.5	$D_s^{*+}D_s^{*-}$	4223.8	$K^{*\pm}J/\psi$	3988.6
$\omega J/\psi$	3879.6			$K^{*0}J/\psi$	3993.0
$D^{*0}\bar{D}^{*0}$	4013.6			$D^{*0}D_s^{*\pm}$	4118.8

Table 8: Masses of hidden bottom tetraquark states (in MeV).

State J^{PC}	Diquark content	Tetraquark mass $bq\bar{b}\bar{q}$	Tetraquark mass $bs\bar{b}\bar{s}$	Tetraquark mass $bs\bar{b}\bar{q}/bq\bar{b}\bar{s}$
$1S$				
0^{++}	$S\bar{S}$	10471	10662	10572
$1^{+\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	10492	10682	10593
0^{++}	$A\bar{A}$	10473	10671	10584
1^{+-}	$A\bar{A}$	10494	10686	10599
2^{++}	$A\bar{A}$	10534	10716	10628
$1P$				
1^{--}	$S\bar{S}$	10807	11002	10907

Table 9: Thresholds for open bottom decays.

Channel	Threshold (MeV)	Channel	Threshold (MeV)	Channel	Threshold (MeV)
$B\bar{B}$	10558	$B_s^+B_s^-$	10739	BB_s	10649
$B\bar{B}^*$	10604	$B_s^{\pm}B_s^{*\mp}$	10786	B^*B_s	10695
$B^*\bar{B}^*$	10650	$B_s^{*+}B_s^{*-}$	10833	$B^*B_s^*$	10742

Table 10: Comparison of theoretical predictions for the masses of charm diquark-antidiquark states $cq\bar{c}\bar{q}$ (in MeV) and possible experimental candidates.

State	Diquark	Theory			Experiment	
J^{PC}	content	EFG	Maiani et al.	Maiani et al. ($cs\bar{c}\bar{s}$)	state	mass
$1S$						
0^{++}	$S\bar{S}$	3812	3723			
1^{++}	$(S\bar{A}-\bar{S}A)/\sqrt{2}$	3871	3872†		$X(3872)$ $X(3876)$	3871.9 ± 0.5 $3875.4\pm0.7^{+1.2}_{-2.0}$
1^{+-}	$(S\bar{A}+\bar{S}A)/\sqrt{2}$	3871	3754			
0^{++}	$A\bar{A}$	3852	3832			
1^{+-}	$A\bar{A}$	3890	3882			
2^{++}	$A\bar{A}$	3968	3952		$Y(3943)$	$3943\pm11\pm13$
$1P$						
1^{--}	$S\bar{S}$	4244		4330 ± 70	$Y(4260)$	$4259\pm8^{+2}_{-6}$ $4247\pm12^{+17}_{-32}$
1^{--}	$(S\bar{A}-\bar{S}A)/\sqrt{2}$	4284			$Y(4260)$	$4283^{+17}_{-16}\pm4$
1^{--}	$A\bar{A}$	4278				
1^{--}	$A\bar{A}$	4350			$Y(4360)$	$4361\pm9\pm9$
$2S$						
$1^{+\pm}$	$(S\bar{A}\pm\bar{S}A)/\sqrt{2}$	4431			$Z(4433)$	$4433\pm4\pm1$
0^{++}	$A\bar{A}$	4434				
1^{+-}	$A\bar{A}$	4461	$\sim$ 4470			
$2P$						
1^{--}	$S\bar{S}$	4666			$Y(4660)$	$4664\pm11\pm5$

† input

Table 11: Masses M of heavy-diquark (QQ')–light-antidiquark $(\bar{q}\bar{q})$ states. T is the lowest threshold for decays into two heavy-light $(Q\bar{q})$ mesons and $\Delta = M - T$. All values are given in MeV.

System	State	$Q = Q' = c$			$Q = Q' = b$			$Q = c$, $Q' = b$		
	$I(J^P)$	M	T	Δ	M	T	Δ	M	T	Δ
$(QQ')(\bar{u}\bar{d})$										
	$0(0^+)$							7239	7144	95
	$0(1^+)$	3935	3871	64	10502	10604	-102	7246	7190	56
	$1(1^+)$							7403	7190	213
	$1(0^+)$	4056	3729	327	10648	10558	90	7383	7144	239
	$1(1^+)$	4079	3871	208	10657	10604	53	7396	7190	206
	$1(2^+)$	4118	4014	104	10673	10650	23	7422	7332	90
$(QQ')(\bar{u}\bar{s})$										
	$\frac{1}{2}(0^+)$							7444	7232	212
	$\frac{1}{2}(1^+)$	4143	3975	168	10706	10693	13	7451	7277	174
	$\frac{1}{2}(1^+)$							7555	7277	278
	$\frac{1}{2}(0^+)$	4221	3833	388	10802	10649	153	7540	7232	308
	$\frac{1}{2}(1^+)$	4239	3975	264	10809	10693	116	7552	7277	275
	$\frac{1}{2}(2^+)$	4271	4119	152	10823	10742	81	7572	7420	152
$(QQ')(\bar{s}\bar{s})$										
	$0(1^+)$							7684	7381	303
	$0(0^+)$	4359	3936	423	10932	10739	193	7673	7336	337
	$0(1^+)$	4375	4080	295	10939	10786	153	7683	7381	302
	$0(2^+)$	4402	4224	178	10950	10833	117	7701	7525	176

SUMMARY

- Masses of heavy tetraquarks with hidden and open charm and bottom were calculated in the diquark-antidiquark picture.
- Dynamical approach based on the relativistic quark model was used, where both diquark and tetraquark masses were obtained by numerical solution of the quasipotential equation with the corresponding relativistic potentials.
- The diquark size was taken into account with the help of the diquark-gluon form factor in terms of diquark wave functions.
- No free adjustable parameters were introduced.
- $X(3872)$ can be the 1^{++} neutral charm tetraquark state. If it is really a tetraquark, one more neutral and two charged tetraquark states must exist with close masses.
- $Y(4260)$, $Y(4360)$ and $Y(4660)$ can be the 1^{--} P-wave tetraquark states.
- Charged $Z(4433)$ can be the 1^{+-} $2S$-wave tetraquark state.
- The ground states of tetraquarks with hidden bottom are predicted to have masses below the open bottom threshold and thus should be narrow.
- All the $(cc)(\bar{q}\bar{q}')$ tetraquarks are predicted to be above the decay threshold into the open charm mesons.
- Only the $I(J^P) = 0(1^+)$ state of $(bb)(\bar{u}\bar{d})$ was found to lie below the BB^* threshold.

The (non)observation of these additional states will be an important test of the tetraquark model.

BACKUP SLIDES

Tetraquark model of Maiani et al.

Main assumptions:

- Tetraquarks are diquark-antidiquark bound states $[cq][\bar{c}\bar{q}']$

- Scalar (asymmetric in flavour $[cq]_{S=0} = [cq]$) and axial vector (symmetric in flavour $[cq]_{S=1} = \{cq\}$) diquarks are considered

- Nonrelativistic constituent quark model is used

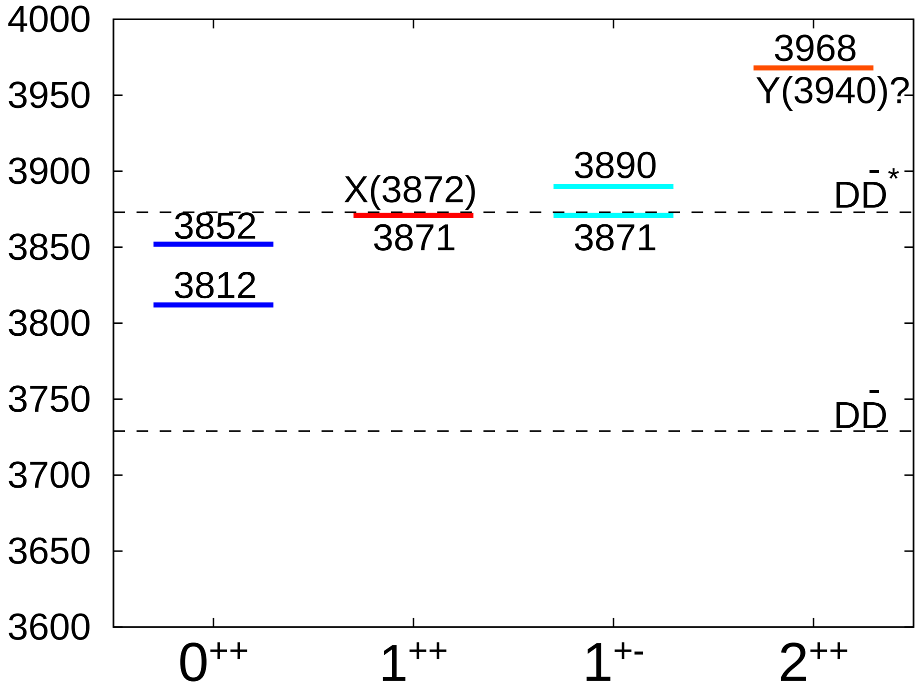
- Mass spectrum is described in terms of:

⋆ constituent diquark mass ($m_{[cq]} = 1933$ MeV fixed from $X(3872)$ mass)
⋆ spin-spin interactions between all quarks (strength k_{ij} $[i, j = q, Q, \bar{q}, \bar{Q}]$ are determined phenomenologically from meson and baryon spectroscopy)

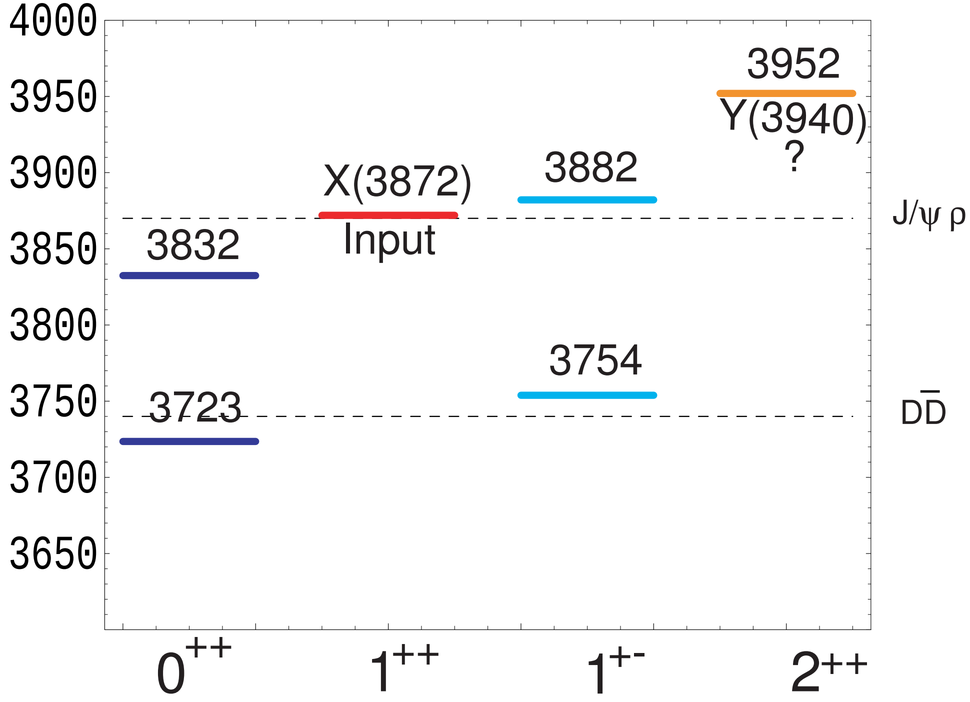
$$H = 2m_{[cq]} + \sum_{i<j} 2k_{ij}(S_i \cdot S_j)$$

- $X(3872)$ is identified with $J^{PC} = 1^{++}$ state with symmetric spin distribution

$$[cq]_{S=1}[\bar{c}\bar{q}]_{S=0} + [cq]_{S=0}[\bar{c}\bar{q}]_{S=1}$$



EFG



Maiani et al.