Pion electro-production in a dynamical model including quasi-bound tree-quark states.

Bojan Golli and Simon Širca (Ljubljana)

Manuel Fiolhais (Coimbra)

Programme:

- The method: coupled channel approach for the K matrix
- Application to scattering in P11 and P33 partial waves focusing in particular on the N(1440) and $\Delta(1600)$.
- Preliminary results for pion photo-production in the region of the Roper resonance.

Aims

- Construct a method to include quark-model states into a dynamical calculation respecting unitarity and incorporating proper asymptotic states.
- Establish a link between quark models and models using effective Lagrangians
- Calculate meson scattering and electro-production in a unified scheme
- Understand the large width of N(1440), underestimated in CQM calculation by a factor 2 – 3,
- . . . as well as the peculiar behaviour of the scattering amplitudes, which is far from the familiar Breit-Wigner shape.

The model

The meson field linearly couples to the quark core; no meson self-interaction

$$H = H_{\text{quark}} + \int dk \sum_{lmt} \left\{ \omega_k \, a^{\dagger}_{lmt}(k) a_{lmt}(k) + \left[V_{lmt}(k) a_{lmt}(k) + V_{lmt}(k)^{\dagger} \, a^{\dagger}_{lmt}(k) \right] \right\}$$

$V_{lmt}(k)$ may induce also radial excitations of the quark core, e.g. $1s \to 2s$ transitions.

Constructing the K matrix (Chew-Low type approach)

$$K^{JT}_{\pi N \pi N}(k, k_0) = -\pi \sqrt{\frac{\omega_k E_N}{k\, W}} \langle \Psi^N_{JT}(W) || V(k) || \Psi_N \rangle \, .$$

The principal-value (PV) state:

$$|\Psi^N_{JT}(W)\rangle = \sqrt{\frac{\omega_0 E_N}{k_0 W}} \left\{ \left[a^{\dagger}(k_0) |\Psi_N\rangle \right]^{JT} - \frac{\mathcal{P}}{H - W} \, [V(k_0)|\Psi_N\rangle]^{JT} \right\},$$

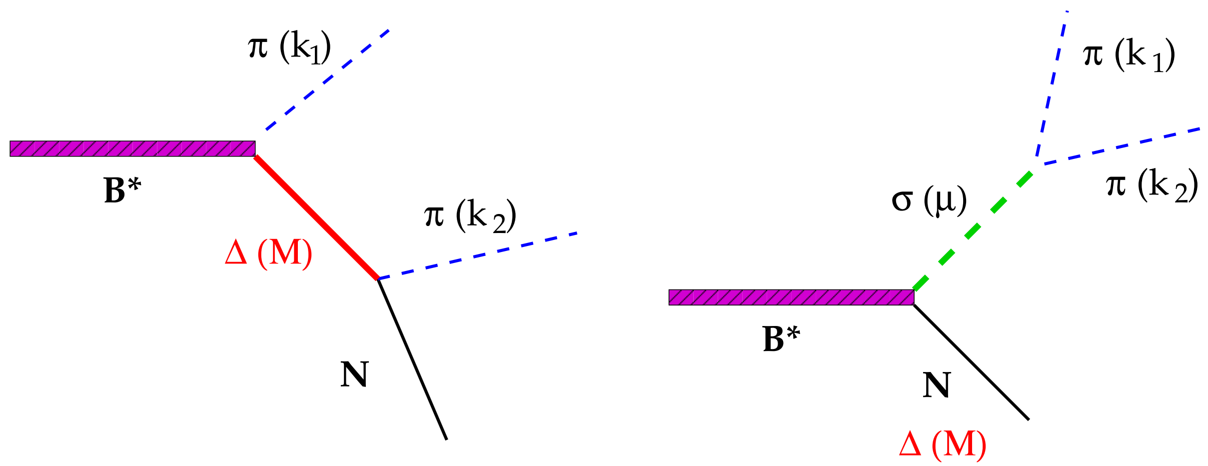
Kinematics

$$\omega_0 = W - E_N = \frac{W^2 - M_N^2 + m_\pi^2}{2W} \, , \quad k_0 = \sqrt{\omega_0^2 - m_\pi^2} \, , \quad E_N = \sqrt{M_N^2 + k_0^2} \, .$$

Assumption about the two-pion channels

Cascade decay:

$$\pi N \longrightarrow B^* \longrightarrow \pi\Delta \longrightarrow \pi\pi N \qquad\qquad \pi N \longrightarrow B^* \longrightarrow \sigma N \longrightarrow (2\pi)^{l=0}_{T=0} N$$



$$\omega_1 = W - E = \frac{W^2 - M^2 + m_\pi^2}{2W}, \qquad \omega_\mu = W - E_N = \frac{W^2 - M_N^2 + \mu^2}{2W},$$

$$k_1 = \sqrt{\omega_1^2 - m_\pi^2}, \qquad E = \sqrt{M^2 + k_1^2} \qquad k_\mu = \sqrt{\omega_\mu^2 - m_\mu^2} \qquad E_N = \sqrt{M_N^2 + k_\mu^2}.$$

$$M_N + m_\pi < M < W - m_\pi \qquad 2m_\pi < \mu < W - M_N$$

Decay through the Roper channel, $\pi N \rightarrow B^* \rightarrow \pi R \rightarrow \pi\pi N$.

Neglecting ρN, ηN ... channels

The multi-channel K matrix

The PV state of the $\pi\Delta$ channel

$$|\Psi^{\Delta}_{JT}(W,M)\rangle = \sqrt{\frac{\omega_1 E}{k_1 W}}\left\{\left[a^\dagger(k_1)|\widetilde{\Psi}_\Delta(M)\rangle\right]^{JT} - \frac{\mathcal{P}}{H-E}\left[V(k_1)|\widetilde{\Psi}_\Delta(M)\rangle\right]^{JT}\right\}.$$

Normalization

$$\langle\Psi^{P}_{\alpha}(W)|\Psi^{P}_{\beta}(W')\rangle = \delta(W-W')\delta_{\alpha\beta}(1+\mathbf{K}^2)_{\alpha\alpha}\,.$$

Orthonormal states

$$|\widetilde{\Psi}^{\alpha}(W)'\rangle = \sum_{\beta}\left[\mathbf{1}+\mathbf{K}^2\right]^{-1/2}_{\beta,\alpha}|\Psi^{\beta}(W)\rangle$$

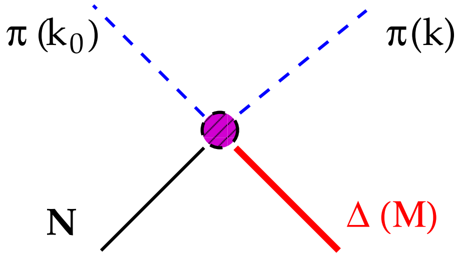
The intermediate Δ state:

$$\langle\widetilde{\Psi}_\Delta(M)|\widetilde{\Psi}_\Delta(M')\rangle = \delta(M-M')\,.$$

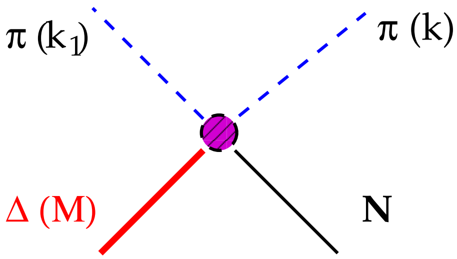
$$|\widetilde{\Psi}_\Delta(M)\rangle \approx w_\Delta(M)\left\{|\Phi_\Delta\rangle - \int\frac{dk\;\mathcal{V}_{N\Delta}(k,k_2)}{\omega_k+E_N(k)-M}[a^\dagger(k)|\Phi_N\rangle]^{\frac{3}{2}\frac{3}{2}} - \int\frac{dk\;\mathcal{V}_{\Delta\Delta}(k)}{\omega_k+E_\Delta(k)-M}[a^\dagger(k)|\Phi_\Delta\rangle]^{\frac{3}{2}\frac{3}{2}}\right\}$$

$$w_\Delta(M)^2 \approx \frac{1}{\pi}\frac{\frac{1}{2}\Gamma_\Delta}{(M_\Delta-M)^2+(\frac{1}{2}\Gamma_\Delta)^2}$$

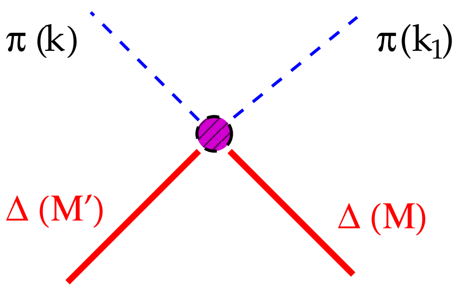
The inelastic elements of the K matrix:



$$K^{JT}_{\pi\Delta\pi N}(k, k_0, M) = -\pi\sqrt{\frac{\omega_k E}{k\,W}}\langle\Psi^N_{JT}(W)\|V(k)\|\widetilde{\Psi}_\Delta(M)\rangle\,.$$



$$K^{JT}_{\pi N\pi\Delta}(k, k_1, M) = -\pi\sqrt{\frac{\omega_k E}{k\,W}}\langle\Psi^\Delta_{JT}(W, M)\|V(k)\|\Psi_N\rangle\,.$$



$$K^{JT}_{\pi\Delta\pi\Delta}(k, k_1, M', M) = -\pi\sqrt{\frac{\omega_k E}{k\,W}}\langle\Psi^\Delta_{JT}(W, M)\|V(k)\|\tilde{\Psi}_\Delta(M')\rangle\,.$$

Relation to the S and T matrix: $S = \frac{1 + iK}{1 - iK}\,, \qquad T = \frac{K}{1 - iK}\,,$

The unitarity requires the symmetry $K^{JT}_{\pi N\pi\Delta}(k_0, k_1) = K^{JT}_{\pi\Delta\pi N}(k_1, k_0)\,.$

Ansaetze for the channel principal value states

πN channel:

$$|\Psi^{N}_{JT}(W)\rangle = \sqrt{\frac{\omega_0 E_N(k_0)}{k_0 W}}\Big\{\sum_B c^N_B(W)|\Phi_B\rangle + \boxed{[a^\dagger(k_0)|\Psi_N(k_0)\rangle]^{JT}}$$
$$+\int \frac{dk \quad \chi^N_{JT}(k,k_0)}{\omega_k + E_N(k) - W}[a^\dagger(k)|\Psi_N(k)\rangle]^{JT} + \int dM' \int \frac{dk\ \chi^{\Delta N}_{JT}(k,k_0,M')}{\omega_k + E'(k) - W}[a^\dagger(k)|\widetilde{\Psi}_\Delta(M')\rangle]^{JT}\Big\}\,,$$

$\pi\Delta(E)$ channel:

$$|\Psi^{\Delta}_{JT}(W,M)\rangle = \sqrt{\frac{\omega_1 E(k_1)}{k_1 W}}\Big\{\sum_B c^\Delta_B(W,M)|\Phi_B\rangle + \boxed{[a^\dagger(k_1)|\widetilde{\Psi}_\Delta(M)\rangle]^{JT}}$$
$$+\int \frac{dk\ \chi^{N\Delta}_{JT}(k,k_1,M)}{\omega_k + E_N(k) - W}[a^\dagger(k)|\Psi_N(k)\rangle]^{JT} + \int dM' \int \frac{dk\ \chi^{\Delta}_{JT}(k,k_1,M',M)}{\omega_k + E'(k) - W}[a^\dagger(k)|\widetilde{\Psi}_\Delta(M')\rangle]^{JT}$$

Above the π threshold: $K_{NN}(W) = \pi\,\dfrac{\omega_0 E_N(k_0)}{k_0 W}\,\chi^N_{JT}(k_0,k_0)\,,$

Above the 2π threshold:

$$K_{\Delta N}(W,M) = \pi\sqrt{\frac{\omega_0 E_N(k_0)\omega_1 E(k_1)}{k_0 k_1 W^2}}\,\chi^{\Delta N}_{JT}(k_1,k_0,M)\,,$$
$$K_{N\Delta}(W,M) = \pi\sqrt{\frac{\omega_0 E_N(k_0)\omega_1 E(k_1)}{k_0 k_1 W^2}}\,\chi^{N\Delta}_{JT}(k_0,k_1,M)\,,$$
$$K_{\Delta\Delta}(W,M',M) = \pi\sqrt{\frac{\omega_1 E(k_1)\omega_1' E(k_1')}{k_1 k_1' W^2}}\,\chi^{\Delta}_{JT}(k_1',k_1,M',M)\,.$$

Including the sigma-nucleon channel

The effective Hamiltonian for the s-wave σ-mesons

$$H_\sigma = \int d\mu \int dk\, \omega_{\mu k}\, b_\mu^\dagger(k) b_\mu(k) + \bar{V}_\mu^\dagger(k) b_\mu^\dagger(k) + \bar{V}_\mu(k) b_\mu(k)\,, \qquad \omega_{\mu k}^2 = k^2 + \mu^2\,,$$

$$\bar{V}_\mu(k) = V_\mu(k)\, w_\sigma(\mu)\,, \qquad V_\mu(k) = G_\sigma \frac{k}{\sqrt{2\omega_{\mu k}}}\,, \qquad w_\sigma(\mu)^2 \approx \frac{1}{\pi} \frac{\frac{1}{2}\Gamma_\sigma}{(\mu - m_\sigma)^2 + \frac{1}{4}\Gamma_\sigma^2}$$

$$m_\sigma = 450\ \text{MeV}\,, \qquad \Gamma_\sigma = 550\ \text{MeV}$$

μ invariant mass of the 2π system.

PV state (Born approximation for the K matrix)

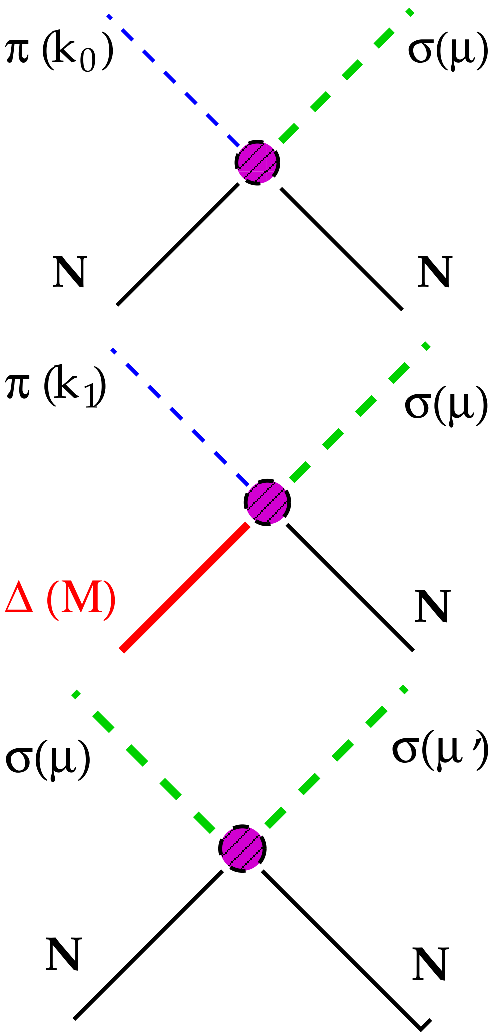
P11 partial waves

$$|\Psi^\sigma_{\frac{1}{2}\frac{1}{2}}(W,\mu)\rangle \approx \mathcal{N}_{\mu 0} \left\{ c_R^\sigma(W,\mu)|\Phi_R\rangle + c_N^\sigma(W,\mu)|\Phi_N\rangle + b_\mu^\dagger(k_{\mu 0})|\Phi_N\rangle \right\}\,,$$

P33 partial waves

$$|\Psi^\sigma_{\frac{3}{2}\frac{3}{2}}(W,\mu,M)\rangle \approx \mathcal{N}_{\mu 1} \left\{ c_\Delta^\sigma(W,\mu,M)|\Phi_\Delta\rangle + c_{\Delta^*}^\sigma(W,\mu,M)|\Phi_{\Delta^*}\rangle + b_\mu^\dagger(k_{\mu 1}) w_\Delta(M)|\Phi_\Delta\rangle \right\}\,,$$

P11 partial waves

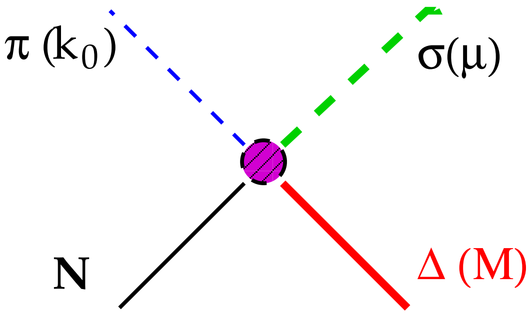


$$K^{\frac{1}{2}\frac{1}{2}}_{\sigma N}(W, \mu) = -\pi \mathcal{N}_{\mu 0} \langle \Psi^{N}_{\frac{1}{2}\frac{1}{2}}(W) | \widetilde{V}^{\mu}(k_{\mu 0}) | \Phi_N \rangle$$

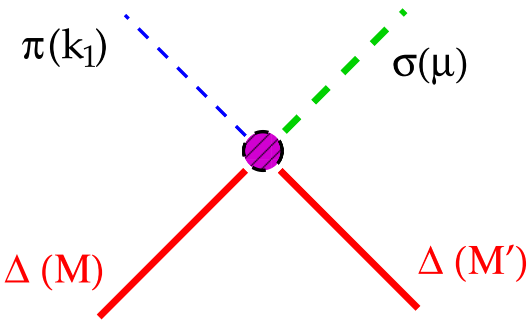
$$K^{\frac{1}{2}\frac{1}{2}}_{\sigma \Delta}(W, \mu, M) = -\pi \mathcal{N}_{\mu 0} \langle \Psi^{\Delta}_{\frac{1}{2}\frac{1}{2}}(W, M) | \widetilde{V}^{\mu}(k_{\mu 0}) | \Phi_N \rangle$$

$$K^{\frac{1}{2}\frac{1}{2}}_{\sigma \sigma}(W, \mu, \mu') = -\pi \mathcal{N}_{\mu 0} \langle \Psi^{\sigma}_{\frac{1}{2}\frac{1}{2}}(W, \mu') | \widetilde{V}^{\mu}(k_{\mu 0}) | \Phi_N \rangle$$

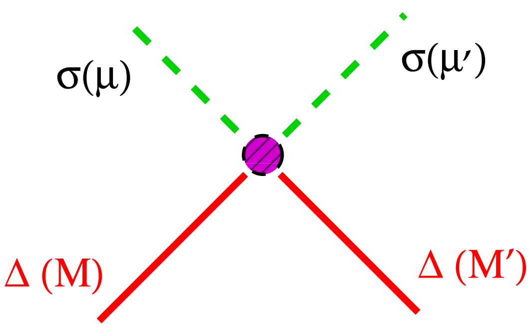
P33 partial waves



$$K^{\frac{3}{2}\frac{3}{2}}_{\sigma N}(W, M, \mu) = -\pi \mathcal{N}_{\mu 1} \, \langle \Psi^{N}_{\frac{3}{2}\frac{3}{2}}(W) | \widetilde{V}^{\mu}(k_{\mu 1}) | \widetilde{\Psi}_{\Delta}(M) \rangle$$



$$K^{\frac{3}{2}\frac{3}{2}}_{\sigma \Delta}(W, \mu, M, M') = -\pi \mathcal{N}_{\mu 1} \, \langle \Psi^{\Delta}_{\frac{3}{2}\frac{3}{2}}(W, M') | \widetilde{V}^{\mu}(k_{\mu 1}) | \widetilde{\Psi}_{\Delta}(M) \rangle$$



$$K^{\frac{3}{2}\frac{3}{2}}_{\sigma \sigma}(W, \mu, M, \mu', M') = -\pi \mathcal{N}_{\mu 1} \, \langle \Psi^{\sigma}_{\frac{3}{2}\frac{3}{2}}(W, \mu', M') | \widetilde{V}^{\mu}(k_{\mu 1}) | \widetilde{\Psi}_{\Delta}(M) \rangle$$

Integral equation for the K matrix

(Lippmann-Schwinger equation)

$$\chi^{N}_{JT}(k,k_0) = -\sum_B c^N_B(W) V_{NB}(k) + \mathcal{K}^{NN}(k,k_0) + \int dk' \frac{\mathcal{K}^{NN}(k,k')\chi^{N}_{JT}(k',k_0)}{\omega'_k + E_N(k') - W} + \int dk' \frac{\mathcal{K}^{N\Delta}_{M_\Delta}(k,k')\hat{\chi}^{\Delta N}_{JT}(k',k_0)}{\omega'_k + E_\Delta(k') - W}$$

$$\hat{\chi}^{\Delta}_{JT}(k,k_1) = -\sum_B \hat{c}^\Delta_B(W,M)\, V^{M'}_{\Delta B}(k) + \mathcal{K}^{\Delta\Delta}_{M'M}(k,k_1) + \int dk' \frac{\mathcal{K}^{\Delta\Delta}_{M'M_\Delta}(k,k')\hat{\chi}^{\Delta}_{JT}(k',k_1)}{\omega'_k + E_\Delta(k') - W} + \int dk' \frac{\mathcal{K}^{\Delta N}_{M'}(k,k')\hat{\chi}^{N\Delta}_{JT}(k',k_1)}{\omega'_k + E_N(k') - W}$$

$$\hat{\chi}^{\Delta N}_{JT}(k,k_0) = -\sum_B c^N_B(W) V^{m}_{\Delta B}(k) + \mathcal{K}^{\Delta N}_{M}(k,k_0) + \int dk' \frac{\mathcal{K}^{\Delta N}_{M}(k,k')\chi^{N}_{JT}(k',k_0)}{\omega'_k + E_N(k') - W} + \int dk' \frac{\mathcal{K}^{\Delta\Delta}_{MM_\Delta}(k,k')\hat{\chi}^{\Delta N}_{JT}(k',k_0)}{\omega'_k + E_\Delta(k') - W}$$

$$\hat{\chi}^{N\Delta}_{JT}(k,k_1) = -\sum_B \hat{c}^\Delta_B(W,M) V_{NB}(k) + \mathcal{K}^{N\Delta}_{M}(k,k_1) + \int dk' \frac{\mathcal{K}^{N\Delta}_{M_\Delta}(k,k')\chi^{\Delta}_{JT}(k',k_1)}{\omega'_k + E_\Delta(k') - W} + \int dk' \frac{\mathcal{K}^{NN}(k,k')\hat{\chi}^{N\Delta}_{JT}(k',k_1)}{\omega'_k + E_N(k') - W}$$

$$(W - M^0_B) c^N_B(W) = V_{NB}(k_0) + \int dk \frac{\hat{\chi}^{\Delta N}_{JT}(k,k_0)\, V_{\Delta B}(k)}{\omega_k + E_\Delta(k) - W} + \int dk \frac{\chi^{N}_{JT}(k,k_0)\, V_{NB}(k)}{\omega_k + E_N(k) - W}$$

$$(W - M^0_B) \hat{c}^\Delta_B(W,M) = V_{\Delta B}(k_1) + \int dk \frac{\chi^{N\Delta}_{JT}(k,k_1)\, V_{NB}(k)}{\omega_k + E_N(k) - W} + \int dk \frac{\hat{\chi}^{\Delta}_{JT}(k,k_1)\, V_{\Delta B}(k)}{\omega_k + E_\Delta(k) - W}$$

Determining the poles of the K matrix

Equation for the $c^{H}_{\mathcal{R}'}$ coefficients

$$\sum_{\mathcal{R}'} A_{\mathcal{R}\mathcal{R}'}(W) c^{H}_{\mathcal{R}'}(W, m_H) = \mathcal{V}^{M}_{H\mathcal{R}}(k_H)\,,$$

$$\mathbf{U}\mathbf{A}\mathbf{U}^{T} = \mathbf{D}\,, \qquad \mathbf{D} = \begin{bmatrix} Z_{\mathcal{R}}(W)(W - M_{\mathcal{R}}) & 0 & 0 \\ 0 & Z_{\mathcal{R}'}(W)(W - M_{\mathcal{R}'}) & 0 \\ 0 & 0 & Z_{\mathcal{R}''}(W)(W - M_{\mathcal{R}''}) \end{bmatrix}$$

$$\widetilde{\mathcal{V}}_{H\mathcal{R}} = \sum_{\mathcal{R}'} u_{\mathcal{R}\mathcal{R}'} \mathcal{V}_{H\mathcal{R}'}\,, \qquad \tilde{c}^{H}_{\mathcal{R}} = \frac{\widetilde{\mathcal{V}}_{H\mathcal{R}}}{Z_{\mathcal{R}}(W)(W - M_{\mathcal{R}})}\,.$$

$$\chi^{H'H} = -\sum_{\mathcal{R}} \widetilde{\mathcal{V}}_{H\mathcal{R}} \frac{1}{Z_{\mathcal{R}}(W)(W - M_{\mathcal{R}})} \widetilde{\mathcal{V}}_{H'\mathcal{R}}$$

Born approximation for the K matrix

$$\chi^{N}_{JT}(k, k_0) = -\sum_B c^N_B(W) V_{NB}(k) + \mathcal{K}^{NN}(k, k_0)$$
$$\hat{\chi}^{\Delta}_{JT}(k, k_1) = -\sum_B \hat{c}^{\Delta}_B(W, M)\, V^{M'}_{\Delta B}(k) + \mathcal{K}^{\Delta\Delta}_{M'M}(k, k_1)$$
$$\hat{\chi}^{\Delta N}_{JT}(k, k_0) = -\sum_B c^N_B(W) V^{m}_{\Delta B}(k) + \mathcal{K}^{\Delta N}_{M}(k, k_0)$$
$$\hat{\chi}^{N\Delta}_{JT}(k, k_1) = -\sum_B \hat{c}^{\Delta}_B(W, M) V_{NB}(k) + \mathcal{K}^{N\Delta}_{M}(k, k_1)$$

$$(W - M^0_B) c^N_B(W) = V_{NB}(k_0)$$
$$(W - M^0_B) \hat{c}^{\Delta}_B(W, M) = V_{\Delta B}(k_1)$$

Born approximation for the P11 and P33 partial waves

Assuming two (quasi) bound states: P11: nucleon and N(1440)
P33: $\Delta(1232)$ and $\Delta(1600)$

and two channels

$$\pi N \text{ channel} \quad |\Psi^N(W)\rangle \approx \mathcal{N}_0 \left[c_B^N(W)|\Phi_B\rangle + c_{B^*}^N(W)|\Phi_{B^*}\rangle + [a^\dagger(k_0)|\Phi_N\rangle]^{\frac{3}{2}\frac{3}{2}} \right]$$

$$\pi\Delta \text{ channel} \quad |\Psi^\Delta(W,M)\rangle \approx \mathcal{N}_1 \left[c_B^\Delta(W,M)|\Phi_B\rangle + c_{B^*}^\Delta(W,M)|\Phi_{B^*}\rangle + [a^\dagger(k_1) w_\Delta(M)|\Phi_\Delta\rangle]^{JT} \right]$$

Same 3-q radial structure for N and $\Delta(1232)$ and
same 3-q radial structure for {N(1440) and $\Delta(1600)$

Solution above the 2π threshold, neglecting background:

$$K_{ij} = a_i a_j \left[\frac{1}{M_B - W} + \frac{r_\omega^2}{M_{B^*} - W} \right], \qquad T_{ij} = \frac{a_i a_j}{\left[\frac{1}{M_B - W} + \frac{r_\omega^2}{M_{B^*} - W} \right]^{-1} - i\left[a_N^2 + \bar{a}_\Delta^2 \right]},$$

$$a_N = \sqrt{\pi}\mathcal{N}_0 \langle \Phi_B || V(k_0) || \Phi_N \rangle, \qquad r_\omega = \frac{g_{\pi NR}}{g_{\pi NN}},$$

$$\bar{a}_\Delta^2 = \int_{M_N + m_\pi}^{W - m_\pi} dM \, w_\Delta(M)^2 \, a_\Delta(W,M)^2, \qquad a_\Delta(W,M) = \sqrt{\pi}\mathcal{N}_1 \langle \Phi_B || V(k_1) || \Phi_\Delta \rangle$$

Solving the integral equation: separable kernels

Approximations

$$\frac{1}{\omega_k + \omega_k' - \omega_0 + E_B(\bar{k}) - E_N(k_0)} \approx \frac{\omega_0 + E_B(\bar{k}) - E_N(k_0)}{(\omega_k + E_B(\bar{k}) - E_N(k_0))(\omega_k' + E_B(\bar{k}) - E_N(k_0))}$$

$$\bar{k}^2 \approx \langle(\mathbf{k}_0 + \mathbf{k}_1)^2\rangle \approx k_0^2 + k_1^2\,, \qquad E_B(\bar{k}) + E_N(k_0) - \omega_0 \approx 2M_B$$

$$\mathcal{K}^{NN}(k,k') = \sum_i f_{NN}^{B_i} \frac{M_{Bi}}{E_N} (\omega_0 + \varepsilon_i^N) \frac{\mathcal{V}_{B_iN}(k')\mathcal{V}_{B_iN}(k)}{(\omega_k' + \varepsilon_i^N)(\omega_k + \varepsilon_i^N)}$$

$$\mathcal{K}_M^{N\Delta}(k,k') = \sum_i f_{N\Delta}^{B_i} \frac{M_{Bi}}{E} (\omega_1 + \varepsilon_i^N) \frac{\mathcal{V}_{B_iN}(k')\mathcal{V}_{B_i\Delta}(k)}{(\omega_k' + \varepsilon_i^N)(\omega_k + \varepsilon_i^\Delta(M))} = \mathcal{K}_M^{\Delta N}(k',k)$$

$$\mathcal{K}_{M'M}^{\Delta\Delta}(k,k') = \sum_i f_{\Delta\Delta}^{B_i} \frac{M_{Bi}}{E'} (\omega_1' + \varepsilon_i^\Delta(M)) \frac{\mathcal{V}_{B_i\Delta}(k)}{(\omega_k + \varepsilon_i^\Delta(M))} \frac{\mathcal{V}_{B_i\Delta}(k')}{(\omega_k' + \varepsilon_i^\Delta(M'))}$$

$$\varepsilon_i^N = \frac{M_{Bi}^2 - M_N^2 - m_\pi^2}{2E_N}\,, \qquad \varepsilon_i^\Delta(M) = \frac{M_{Bi}^2 - M^2 - m_\pi^2}{2E}\,,$$

Solution for the K matrix:

$$K_{hh'} = K_{hh'}(\text{resonant}) + K_{hh'}(\text{background}) = \pi\mathcal{N}_H\mathcal{N}_{h'} \left\{ \sum_B \frac{\mathcal{V}_{hB}\mathcal{V}_{h'B}}{(M_B - W)} + \mathcal{D}_{hh'} \right\}$$

Results for the Cloudy Bag Model

$$\langle \Phi_{B'} || V(k) || \Phi_B \rangle = r_q \, v(k) \, \langle J_{B'}, T_{B'} = J_{B'} || \sum_{i=1}^{3} \sigma_m^i \tau_t^i || J_B, T_B = J_B \rangle$$

$$v(k) = \frac{1}{2f} \frac{k^2}{\sqrt{12\pi^2 \omega_k}} \frac{\omega_{MIT}}{\omega_{MIT} - 1} \frac{j_1(kR_{bag})}{kR_{bag}} \, .$$

$$r_q = \begin{cases} 1 & \text{for} \quad B = B' = (1s)^3 \text{ configuration} \\ r_\omega = \left[\frac{\omega^1_{MIT}(\omega^0_{MIT}-1)}{\omega^0_{MIT}(\omega^1_{MIT}-1)} \right]^{1/2} = 0.457 & \text{for} \quad B = (1s)^3, B' = (1s)^2(2s)^1 \\ \frac{2}{3} + r_\omega^2 & \text{for} \quad B = B' = (1s)^2(2s)^1 \end{cases}$$

$R_{bag} = 0.83$ fm, $f = 76$ MeV

similar results for $0.75 \text{ fm} < R_{bag} < 1.0 \text{ fm}$

Free parameters: bare masses of the resonant states

$$M_R = 1510 \text{ MeV}, \quad M_\Delta = 1232 \text{ MeV}, \quad M_{\Delta^*} = 1770 \text{ MeV}$$

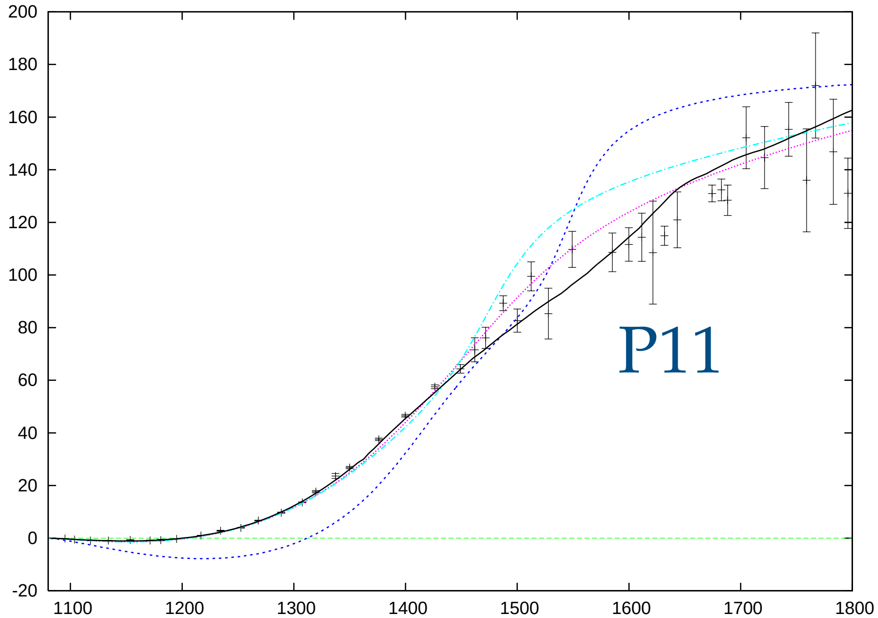
Parameters of the σ-channel: $G_\sigma = 0.8$, $m_\sigma = 450$ MeV , $\Gamma_\sigma = 550$ MeV

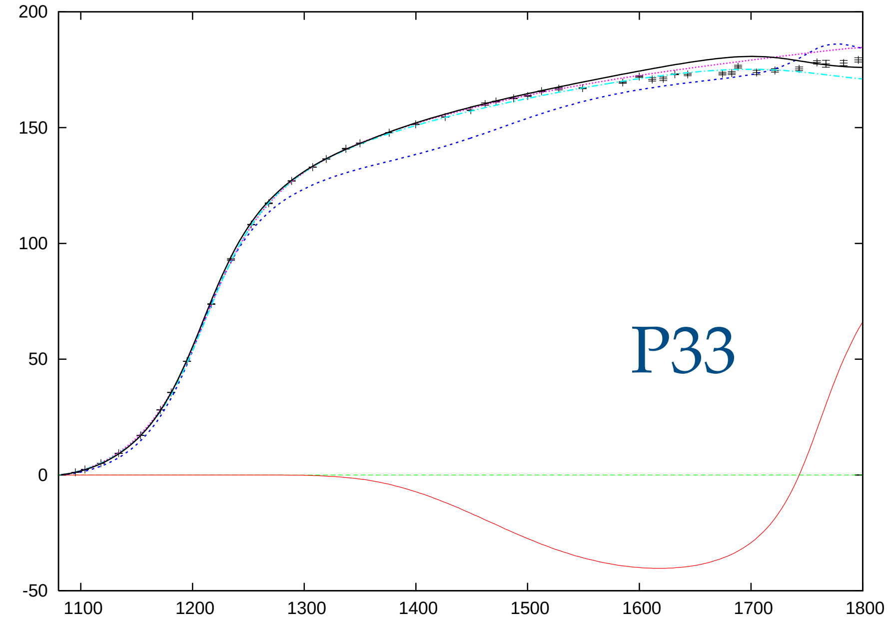
Results

——— ... only πN and $\pi\Delta$ channels

——— ... σN ($\sigma\Delta$) channel added:

Phase shift





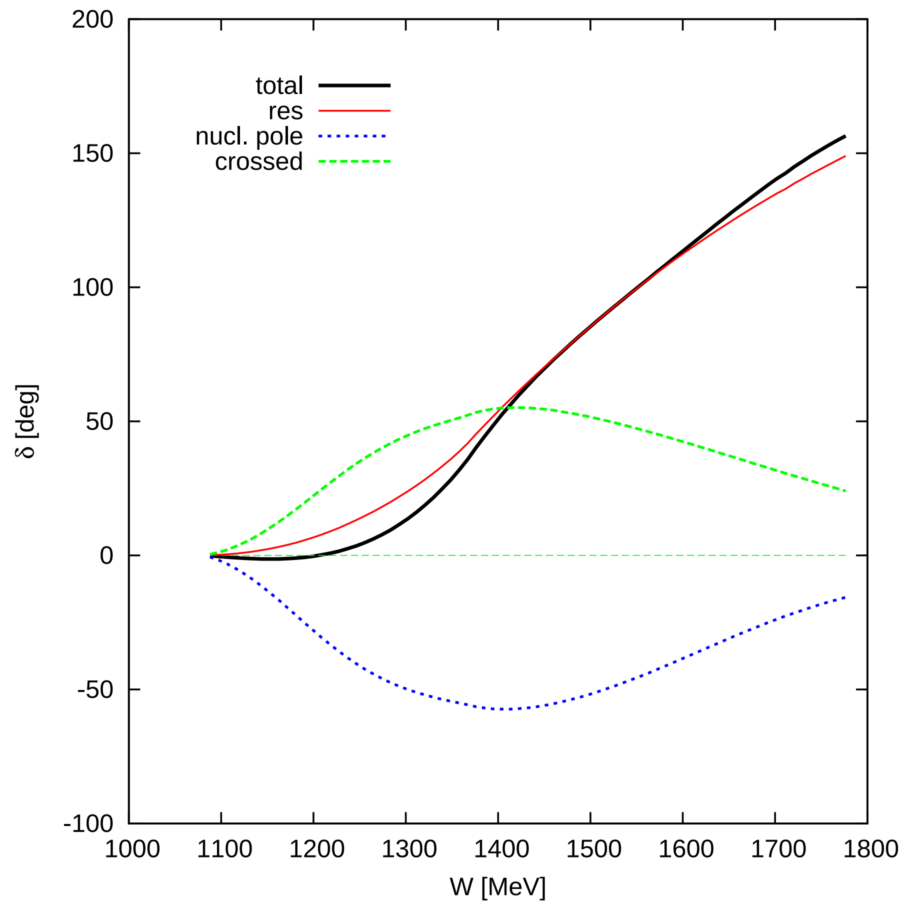
Born approximation

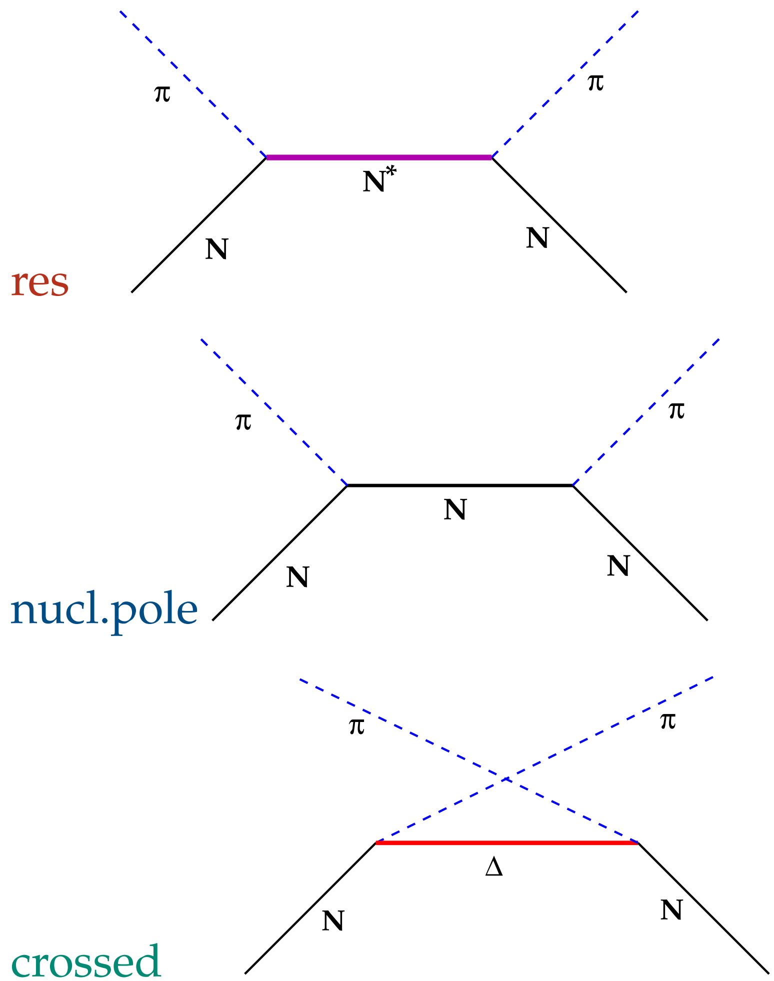
$$\frac{g_{\pi NR}}{g_{\pi NR}^{\rm quark}} = 1.6\,, \qquad \frac{g_{\pi N\Delta}}{g_{\pi N\Delta}^{\rm quark}} = 0.75 \qquad\qquad \frac{g_{\pi NR}}{g_{\pi NR}^{\rm quark}} = 0.80 \qquad \frac{g_{\pi N\Delta}}{g_{\pi N\Delta}^{\rm quark}} = 1.40$$

Full calculation

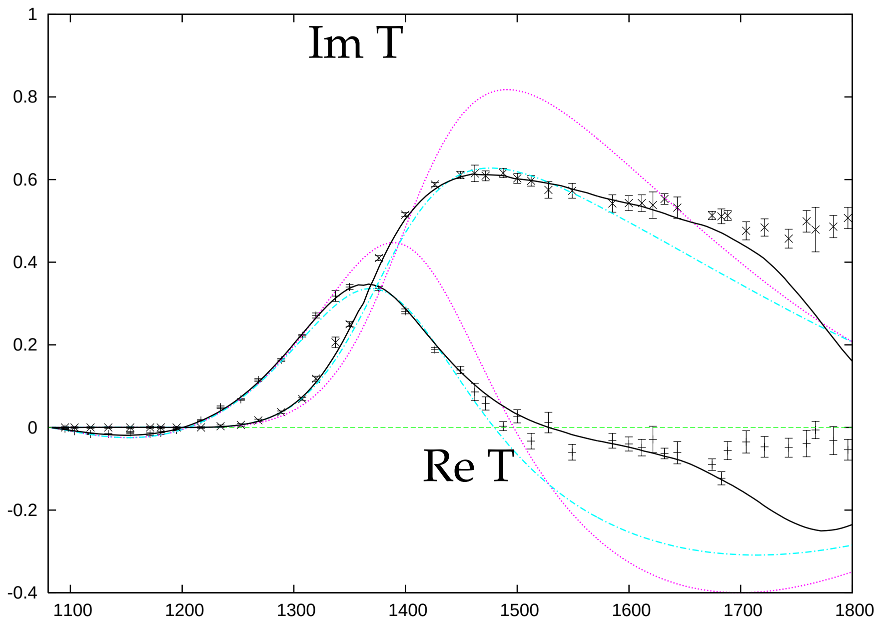
$$\frac{g_{\pi NR}}{g_{\pi NR}^{\rm quark}} = 1.0\,, \qquad \frac{g_{\pi N\Delta}}{g_{\pi N\Delta}^{\rm quark}} = 1.0 \qquad\qquad \frac{g_{\pi NR}}{g_{\pi NR}^{\rm quark}} = 1.0 \qquad \frac{g_{\pi N\Delta}}{g_{\pi N\Delta}^{\rm quark}} = 1.0$$

Resonant and background contributions to P11 phase shift

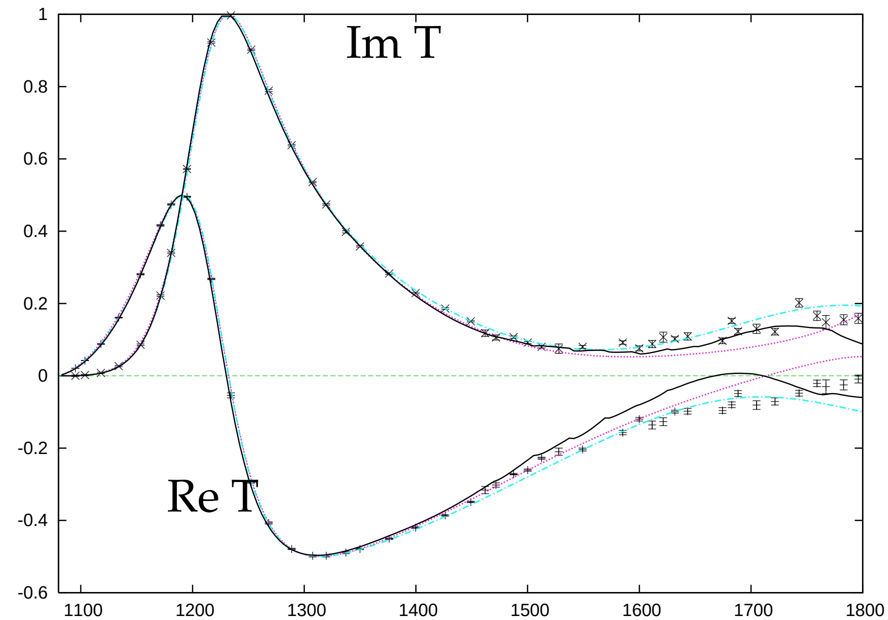




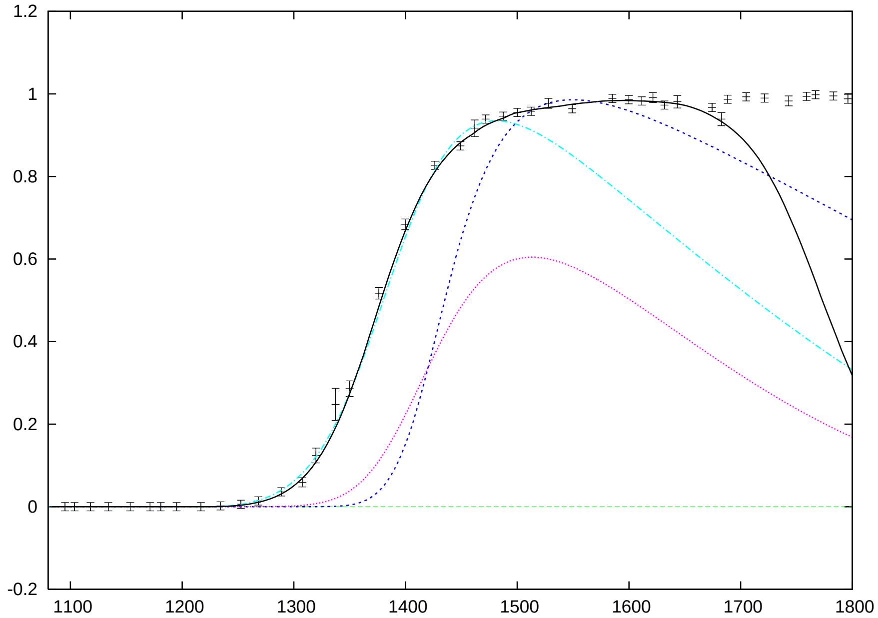
Scattering amplitude

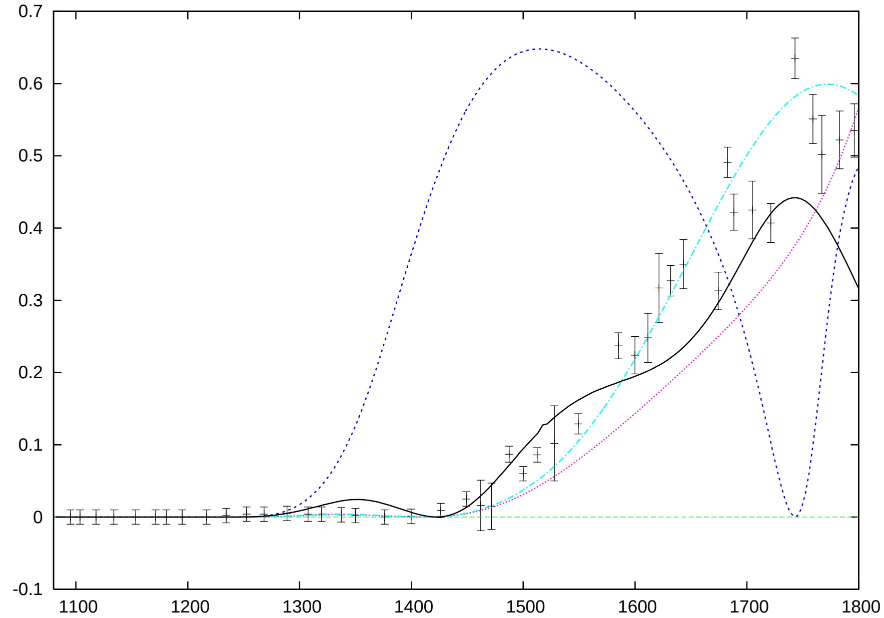


P11



P33





P11 P33

inelasticity

Parameter of the full calculation vs. Born approximation

P11 | P33

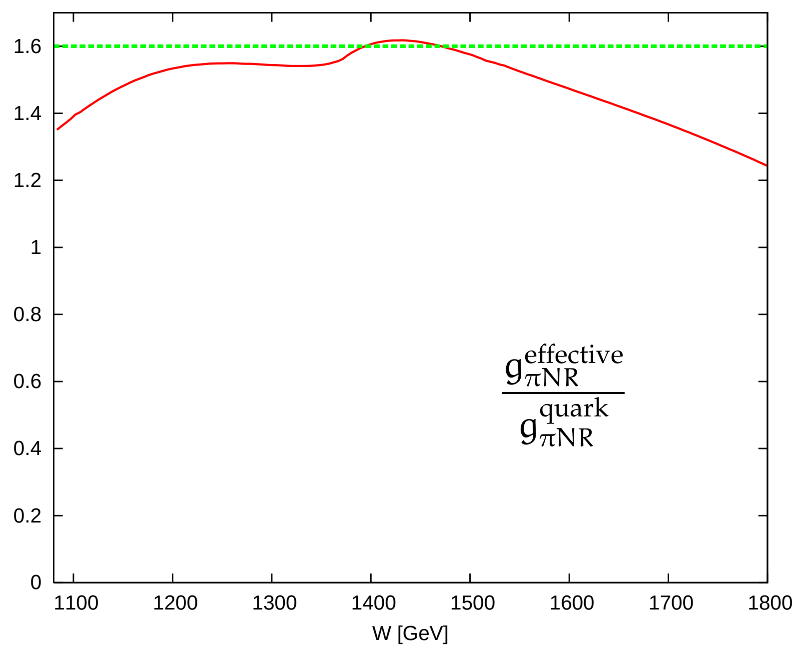
Born:

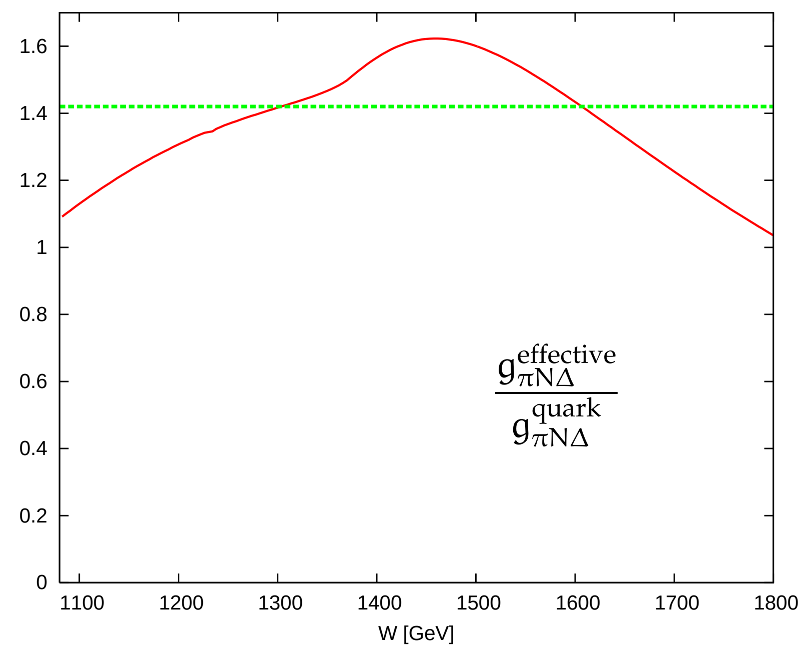
P11: $\dfrac{g_{\pi NR}}{g^{\text{quark}}_{\pi NR}} = 1.6\,, \quad \dfrac{g_{\pi N\Delta}}{g^{\text{quark}}_{\pi N\Delta}} = 0.75$

P33: $\dfrac{g_{\pi NR}}{g^{\text{quark}}_{\pi NR}} = 0.80 \quad \dfrac{g_{\pi N\Delta}}{g^{\text{quark}}_{\pi N\Delta}} = 1.40$

Full:

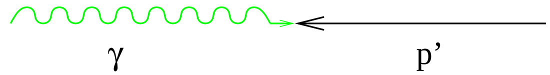
$$\frac{g_{\pi NR}}{g^{\text{quark}}_{\pi NR}} = 1.0\,, \quad \frac{g_{\pi N\Delta}}{g^{\text{quark}}_{\pi N\Delta}} = 1.0$$

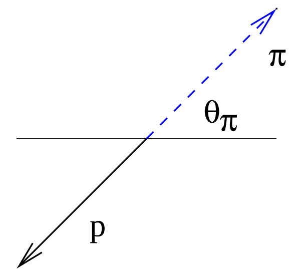




——— values used in the Born approximation for the K matrix

Electro-production amplitudes





Formally, the K matrix acquires a new channel γN.
Because the EM interaction is considerably weaker than the strong interaction, we assume

$$K_{\gamma N\,\gamma N} \ll K_{\gamma N\,\pi N} \ll K_{\pi N\,\pi N}$$

(and similarly for other channels). The Heitler-like equation for the electro-production amplitudes then reduces to

$$\mathcal{M}_N(W) = -\mathcal{M}_N^K(W) + i\Big[T_{\pi N\pi N}(W)\mathcal{M}_N^K(W) + \overline{T}_{\pi N\pi\Delta}(W,\bar{M})\overline{\mathcal{M}}_\Delta^K(W,\bar{M}) + \overline{T}_{\pi N\sigma N}(W,\bar{\mu}_\sigma)\overline{\mathcal{M}}_\sigma^K(W,\bar{\mu}_\sigma)\Big]$$

The T matrix for electro-production is related to the electro-production amplitudes by

$$T^{(JT)}_{\gamma N\pi N} = i\pi\frac{1}{\sqrt{2\pi}^3}\sum_m \sqrt{k_0 k_\gamma}\,\mathcal{M}_N(W,M_J,M_T,t,k_\gamma,\mu)\,Y_{1m}(\hat{r})\,C^{JM_J}_{\frac{1}{2}m_s 1m}C^{TM_T}_{\frac{1}{2}\frac{1}{2}1t}$$

The K-matrix type of electro-production amplitudes for both channels:

$$\mathcal{M}_N^K(W) = -\sqrt{\frac{\omega_\gamma}{k_0}}\langle\Psi^N_{N*}(W)|\tilde{V}_\gamma(\mu,\mathbf{k}_\gamma)|\Phi_N\rangle\,, \qquad \mathcal{M}_\Delta^K(W,M) = -\sqrt{\frac{\omega_\gamma}{k_0}}\langle\Psi^\Delta_{N*}(W,M)|\tilde{V}_\gamma(\mu,\mathbf{k}_\gamma)|\Phi_N\rangle\,,$$

$$\mathcal{M}_\sigma^K(W,\mu_\sigma) = -\sqrt{\frac{\omega_\gamma}{k_0}}\langle\Psi^\sigma_{N*}(W,\mu_\sigma)|\tilde{V}_\gamma(\mu,\mathbf{k}_\gamma)|\Phi_N\rangle$$

The EM interaction

$$V_\gamma(\mu,\mathbf{k}_\gamma) = \frac{1}{\sqrt{2\pi}^3}\tilde{V}_\gamma(\mu,\mathbf{k}_\gamma)\,, \qquad \tilde{V}_\gamma(\mu,\mathbf{k}_\gamma) = \frac{e_0}{\sqrt{2\omega_\gamma}}\int d\mathbf{r}\,\varepsilon_\mu\cdot\mathbf{j}(\mathbf{r})e^{i\mathbf{k}_\gamma\cdot\mathbf{r}}$$

Separation of amplitudes into the resonant and the background part

Because the K matrix elements contain poles, it convenient to separate the amplitudes as

$$\mathcal{M}_H^K = \sqrt{\frac{\omega_\gamma E_N^\gamma}{k_0 W}}\, g(W)\, K_{NH}\, \langle\Psi_{N*}^{(\mathrm{res})}(W)|\tilde{V}_\gamma|\Psi_N\rangle + \mathcal{M}_H^{K\,(\mathrm{non})} \qquad H = N, \Delta, \sigma$$

$$\mathcal{M}_H^{K\,(\mathrm{non})} = -\sqrt{\frac{\omega_\gamma E_N^\gamma}{k_0 W}}\left\{g(W)\, K_{NH}^{(\mathrm{bg})}\,\langle\Psi_{N^*}^{(\mathrm{res})}(W)|\tilde{V}_\gamma|\Psi_N\rangle + \sqrt{\frac{\omega_H E_H}{k_H W}}\,\left[c_N^H\langle\Psi_{N^*}^{(\mathrm{n.p.})}|\tilde{V}_\gamma|\Psi_N\rangle + \langle\Psi_{N*}^{H\,(\mathrm{dir})}|\tilde{V}_\gamma|\Psi_N\rangle\right]\right\}$$

Then

$$\mathcal{M}_N^{(\mathrm{res})} = \sqrt{\frac{\omega_\gamma E_N^\gamma}{k_0 W}}\, g(W)\,\langle\Psi_{N*}^{(\mathrm{res})}(W)|\tilde{V}_\gamma|\Psi_N\rangle\, T_{\pi N\pi N} = \sqrt{\frac{\omega_\gamma E_N^\gamma}{k_0 W}}\, g(W)\, A_{N*}\, T_{\pi N\pi N}$$

and

$$\mathcal{M}_N^{(\mathrm{non})} = \mathcal{M}_N^{K\,(\mathrm{non})} + i\left[T_{\pi N\pi N}\mathcal{M}_N^{K\,(\mathrm{non})} + \overline{T}_{\pi N\pi\Delta}\overline{\mathcal{M}}_\Delta^{K\,(\mathrm{non})} + \overline{T}_{\pi N\sigma N}\overline{\mathcal{M}}_\sigma^{K\,(\mathrm{non})}\right].$$

Electro-excitation amplitude (proportional to the corresponding EM transition form-factor)

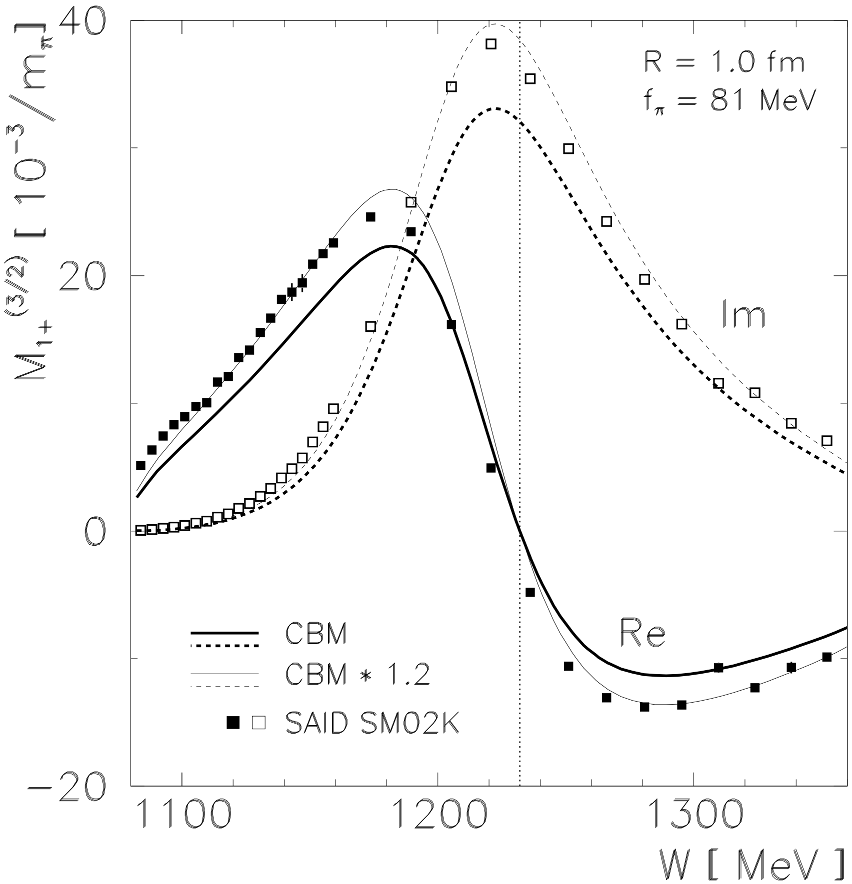
$$A_{N*} \equiv \langle\Psi_{N*}^{(\mathrm{res})}(W)|\tilde{V}_\gamma|\Psi_N\rangle$$

where

$$|\Psi_{N*}^{(\mathrm{res})}(W)\rangle = z_{N*}\left\{|\Phi_{N*}\rangle - \int\frac{dk\,\mathcal{V}_{NN^*}(k)}{\omega_k + E_N(k) - M}\,[a^\dagger(k)|\Psi_N\rangle]^{JT} - \int\frac{dk\,\mathcal{V}_{\Delta N^*}^{M_\Delta}(k)}{\omega_k + E_\Delta(k) - M}\,[a^\dagger(k)|\widehat{\Psi}_\Delta(M_\Delta)\rangle]^{JT}\right\} + \ldots$$

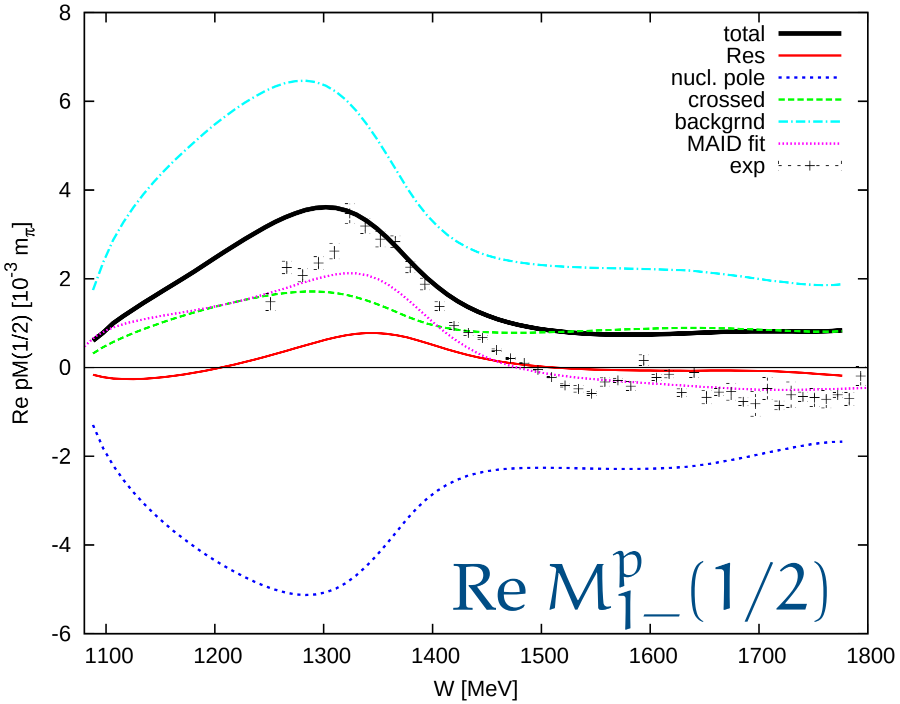
P33 photo-production amplitude in the region of the $\Delta(1232)$

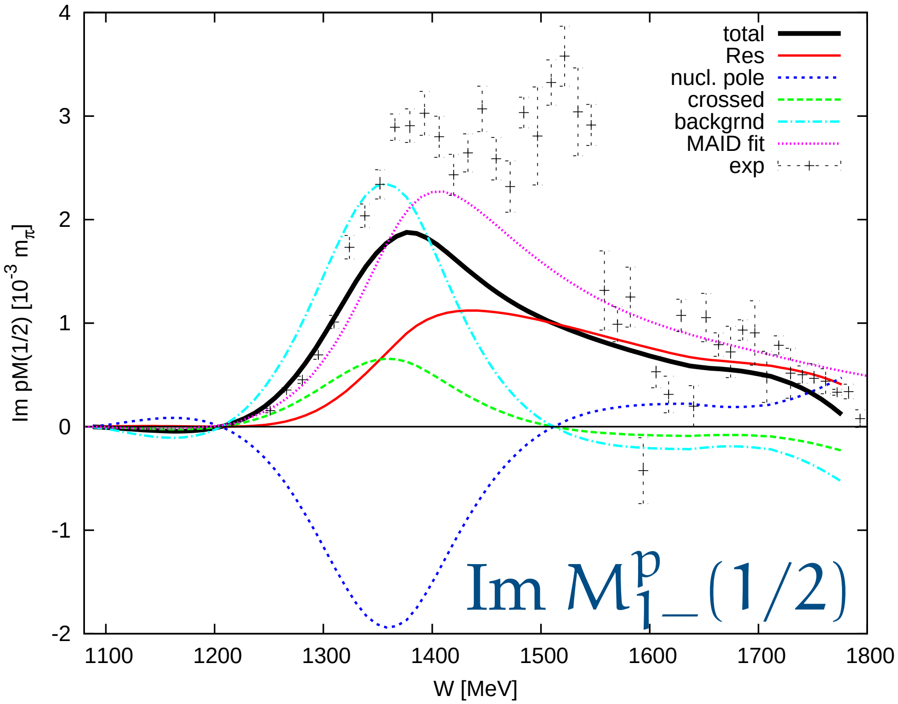
- dominated by the resonant contribution
- non-resonant part is less important
- the contribution from the pion cloud comparable to the contribution of the quark core

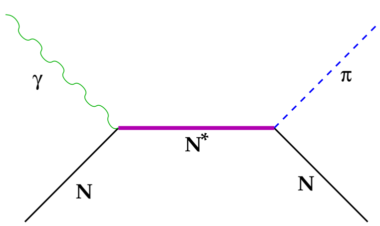


P11 photo-production amplitude in the region of the $N(1440)$

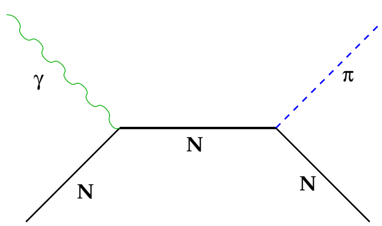
Preliminary results $p + \gamma \to p + \pi^0$



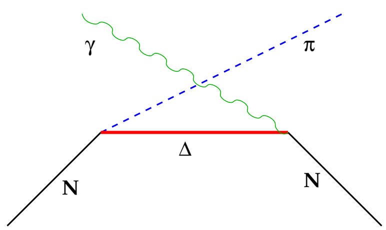




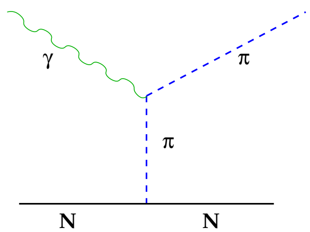
Resonant



nucl. pole



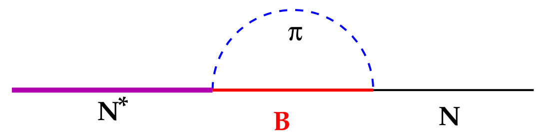
crossed

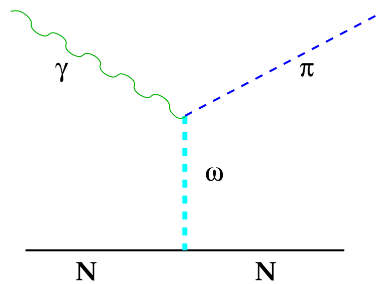


background (pion pole)

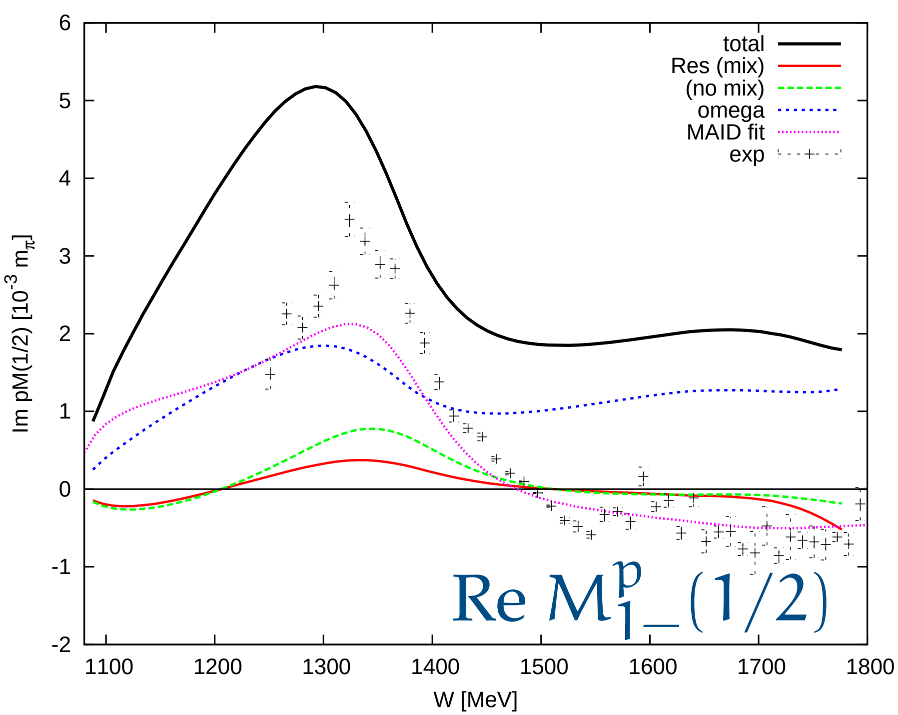
Open problems

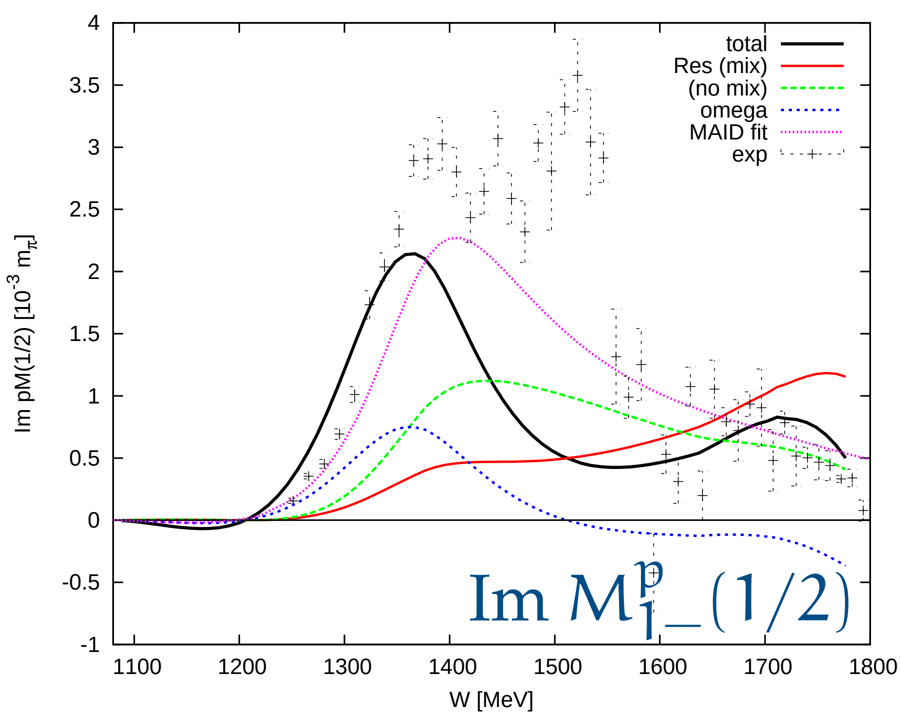
- Mixing of bare N ($(1s)^3$ configuration) in the resonant state





- ω-meson contribution at low energies





Conclusion

We have

- developed a framework to incorporate quark-model states into a multi-channel description of nucleon resonances,
- provided a mechanism that can considerably enlarge the width of resonances compared to their 'static' values,
- stressed the important role of the $\pi\Delta$ and the σN channels from the two-pion threshold to $W \sim 1700$ MeV,
- shown that background processes strongly influence the behaviour of the scattering and electro-production amplitudes