Light and heavy baryon masses: (2) The $1/N_c$ expansion method

Fl. Stancu

University of Liége

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Collab: F.Buisseret (UMH), N. Matagne (Giessen), C. Semay (UMH)

Outline

1. **The $1/N_c$ expansion method in baryon spectroscopy**
2. **Short Quark Model (QM) Reminder**
3. **Compatibility: $1/N_c$ expansion and QM mass formula**

The $1/N_c$ expansion method in baryon spectroscopy

- Large N_c QCD: 't Hooft and Witten
- The mass operator
- Light baryons: The $1/N_c$ expansion and the wave function of excited states
- Heavy baryons: the combined $1/N_c$ and $1/m_Q$ expansion

Baryonic mass formula in large N_c QCD versus quark model
C. Semay, F. Buisseret, N. Matagne and F.S. PRD75:096001(2007)

Mass formula for strange baryons in large N_c QCD versus quark model
C. Semay, F. Buisseret and F.S. PRD75:096001(2007)

Charm and bottom baryon masses in the combined $1/N_c$ and $1/m_Q$ expansion versus quark model
C. Semay, F. Buisseret and F.S. arXiv:0808.3349

't Hooft's idea: Large N_c QCD, N_c colours

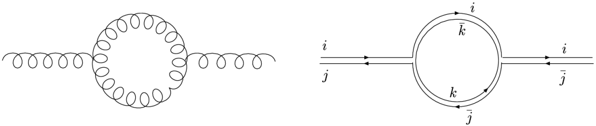
$N_c \to \infty$ colours: exact SU(2N$_f$) symmetry

Witten: Large but finite N_c in baryons: $\to$ $1/N_c$ expansion method for mass, magnetic moments, axial current, decay widths, etc.

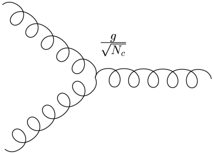
- **systematic and predictive**
- **Model independent predictions**
- **Comparison with quark model: introduce the band number N**

't Hooft (1974) Double line notation

Gluon vacuum polarization: $\mathcal{O}(1)$



Coupling constant $g \rightarrow \frac{g}{\sqrt{N_c}}$ **has a finite limit for large** N_c



a quark loop $\mathcal{O}(N_c^{-1})$**, nonplanar diagrams** $\mathcal{O}(N_c^{-2})$

Gervais, Sakita (1986) Dashen, Manohar (1993)

SU(2N$_f$) is an exact contracted symmetry for $N_c \to \infty$
For finite N_c $\to$ correction of order $1/N_c$.
Example: N$_f$ = 3

$$SU(6) \supset SU(2) \times SU(3)$$

$SU(6)$ operators:

$$S^i = q^\dagger (S^i \times \mathbb{1}) q \quad (3,1)$$

$$T^a = q^\dagger (\mathbb{1} \otimes T^a) q \quad (1,8)$$

$$G^{ia} = q^\dagger (S^i \otimes T^a) q \quad (3,8)$$

For excited states O(3) $\otimes$ SU(2N$_f$) $\to$ also ℓ_i needed

Large N_c operator analysis. Observables:

$$\mathcal{O} = \sum_i c_i \, \mathcal{O}_i + \sum_i b_i \, \overline{B}_i$$

An n-body operator acts on n quark lines

$$\mathcal{O}_i = \frac{1}{N_c^{n-1}} \mathcal{O}_\ell{}^{(k)} \cdot \mathcal{O}_{\mathcal{SF}}{}^{(k)}$$

Generators of O(3) (ℓ_i) and of SU(6) (S_i, T_a, G_{ia})

c_i,b_i **: reduced matrix elements which encode QCD dynamics, fitted from data**

N. B. Matrix elements of $\mathcal{O}_{\mathcal{SF}}$ can carry nontrivial N_c dependence, $\overline{B}_i$ are SU(3)- flavour breaking

Widths: Generic large N_c counting rules $\Gamma \sim \mathcal{O}(N_c^0)$

Example: SU(4) algebra and nonstrange ground state mass

$$
\begin{gathered}
[S_i, T_a] = 0, \qquad [S_i, G_{ja}] = i\varepsilon_{ijk} G_{ka}, \qquad [T_a, G_{ib}] = i\varepsilon_{abc} G_{ic}, \\
[S_i, S_j] = i\varepsilon_{ijk} S_k, \qquad [T_a, T_b] = i\varepsilon_{abc} T_c, \\
[G_{ia}, G_{jb}] = \frac{i}{4}\delta_{ij}\varepsilon_{abc} T_c + \frac{i}{4}\delta_{ab}\varepsilon_{ijk} S_k.
\end{gathered} \tag{1}
$$

Operator identities, Dashen, Jenkins, Manohar, PRD51:3697

$$
\{S_i, S_i\} + \{T_a, T_a\} + \{G_{ia}, G_{ia}\} = \frac{3}{2} N_c (N_c + 4)
$$

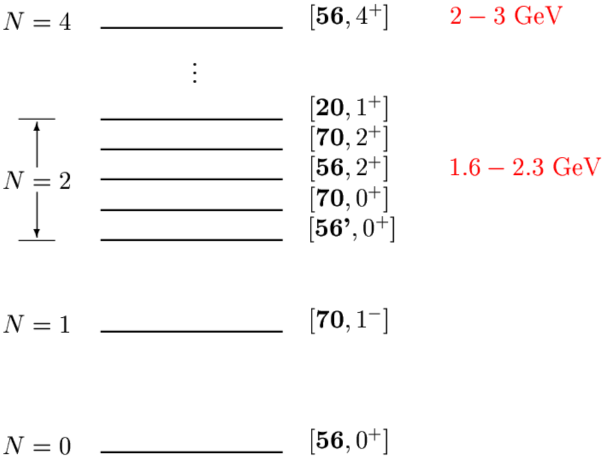
$$
\{T_a, T_a\} = \{S_i, S_i\}
$$

$$
M = c_1 N_c + c_4 \frac{1}{N_c} S^2 + \mathcal{O}\left(\frac{1}{N_c^3}\right) \tag{2}
$$

Fit to N and Δ: c_1 = 289 MeV, c_4 = 292 MeV

The baryon spectrum $N_c = 3$

SU(6) multiplets



Symmetric states $[N_c] \to [56]$ for $N_c = 3$ are easy

Mixed symmetric states $[N_c - 1, 1] \to [70]$ for $N_c = 3$ more complicated $\to$ New theoretical developments

The $[56, 4^+]$ baryon masses in the $1/N_c$ expansion

N.Matagne and F.S. PRD71:014010(2005)

$$M = \sum_{i=1}^{3} c_i O_i + \sum_{i=1}^{3} b_i \bar{B}_i + \mathcal{O}(1/N_c^2)$$

Operator	Fitted coef. (MeV)
$O_1 = N_c \mathbb{1}$	$c_1 =$ 736 $\pm$ 30
$O_2 = \frac{1}{N_c} l_i S_i$	$c_2 =$ 4 $\pm$ 40
$O_3 = \frac{1}{N_c} S_i S_i$	$c_3 =$ 135 $\pm$ 90
$\bar{B}_1 = -\mathcal{S}$	$b_1 =$ 110 $\pm$ 67
$\bar{B}_2 = \frac{1}{N_c} l_i G_{i8} - \frac{1}{2\sqrt{3}} O_2$	
$\bar{B}_3 = \frac{1}{N_c} S_i G_{i8} - \frac{1}{2\sqrt{3}} O_3$	

$\chi^2_{\text{dof}} = 0.26$

$\frac{\langle O_3 \rangle}{\langle O_2 \rangle} = \frac{\langle \bar{B}_3 \rangle}{\langle \bar{B}_2 \rangle}$ for all states

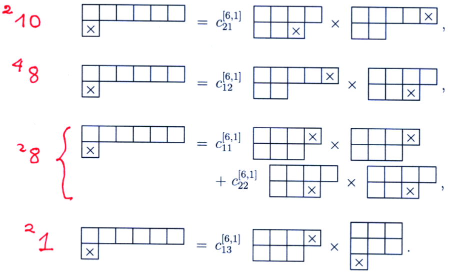
Standard approach for mixed symmetry spin-flavour states

Carlson, Carone, Goity, Lebed, PRD59:114008(1994)

Truncated wave function

Decoupling: core and excited quark

$$S^i = s^i + S^i_c,\ T^a = t^a + T^a_c,\ G^{ia} = g^{ia} + G^{ia}_c$$



Example of core+excited quark separation method

$[70, 1^-]$ multiplet, fit to 7 experimental masses

Problems: a) Many more independent operators: 13 for SU(4)
b) The core always has $S_c = I_c \rightarrow$ information on isospin lost

N.Matagne, F.S. PRD77:054026

Operator	Approximate wave function (MeV)	Exact wave function (MeV)
$O_1 = N_c 1$	$c_1 = 211 \pm 23$	$c_1 = 299 \pm 20$
$O_2 = \ell^i s^i$	$c_2 = 3 \pm 15$	$c_2 = 3 \pm 15$
$O_3 = \frac{1}{N_c} s^i S_c^i$	$c_3 = -1486 \pm 141$	$c_3 = -1096 \pm 125$
$O_4 = \frac{1}{N_c} S_c^i S_c^i$	$c_4 = 1182 \pm 74$	$c_4 = 1545 \pm 122$
$O_5 = \frac{1}{N_c} \ell^a T_c^a$	$c_5 = -1508 \pm 149$	$c_5 = 417 \pm 79$
χ^2_{dof}	1.56	1.56

The $N=1$ band revisited *N.Matagne and Fl. Stancu, Nucl. Phys. A in print*

Operator	Fit 1 (MeV)	Fit 2 (MeV)	Fit 3 (Mev)
$O_1 = N_c \mathbb{1}$	481 ± 5	482 ± 5	484 ± 4
$O_2 = \ell^i s^i$	-31 ± 26	-20 ± 23	-12 ± 20
$O_3 = \frac{1}{N_c} S^i S^i$	161 ± 16	149 ± 11	163 ± 16
$O_4 = \frac{1}{N_c} T^a T^a$	169 ± 36	170 ± 36	141 ± 27
$O_5 = \frac{15}{N_c} \ell^{(2)ij} G^{ia} G^{ja}$	-29 ± 31		-34 ± 30
$O_6 = \frac{3}{N_c} \ell^i T^a G^{ia}$	32 ± 26	35 ± 26	
$O_7 = \frac{3}{N_c^2} S^i T^a G^{ia}$			
$\chi^2_{\rm dof}$	0.43	0.68	0.94

Spin-spin and isospin-isospin contributions in the $N = 1$ band

$$O_3 = \frac{1}{N_c} S_i S_i, \quad O_4 = \frac{1}{N_c} T_i T_i$$

	Part. contrib. (MeV)					
	c_1O_1	c_2O_2	c_3O_3	c_4O_4	c_5O_5	c_6O_6
$^2N_{\frac{1}{2}}$	1444	10	40	42	0	-8
$^4N_{\frac{1}{2}}$	1444	26	201	42	-31	-20
$^2N_{\frac{3}{2}}$	1444	-5	40	42	0	4
$^4N_{\frac{3}{2}}$	1444	10	201	42	25	-8
$^4N_{\frac{5}{2}}$	1444	-16	201	42	-6	12
$^2\Delta_{\frac{1}{2}}$	1444	-10	40	211	0	-40
$^2\Delta_{\frac{3}{2}}$	1444	5	40	211	0	20

Short Quark Model (QM) Reminder

Light baryons

$$H = \sum_i^3 \sqrt{p_i^2 + m_i^2} + V_Y; \quad V_Y = a \sum_i^3 |\vec{x_i} - \vec{x_T}|$$

m_i – quark mass; a – string tension; $\vec{x_T}$ – Toricelli point

Good approximation for the Y-junction

$$V_Y \rightarrow V_0 = \frac{a}{2}[\sum_i^3 |\vec{x_i} - \vec{R}| + \frac{1}{2} \sum_{i<j} |\vec{x_i} - \vec{x_j}|]$$

Replace H by

$$H_0 = \sum_i^3 \sqrt{p_i^2 + m_i^2} + V_0$$

$\vec{R}$ – position of the center of mass motion

Approximate solution: auxiliary field technique (*Yu.Simonov*)

SR kinetic energy → NR kinetic energy
linear confinement → quadratic confinement (h. o.)
Minimization with respect to auxiliary fields

$$H_0|\psi_0\rangle = M_0|\psi_0\rangle$$

$$m_i = 0$$

$$\mu_0 \simeq [\frac{a}{3}Q(N+3)]^{1/2}, \quad Q = 1/2 + \sqrt{3}/4$$

$$M_0 = 6\mu_0$$

N. B. N is the same as in the SU(6) classification of baryons
Regge trajectories : M_0^2 = 12 a (N+3)

Corrections to confinement

Coulomb-type

$$\Delta M_{OGE} = -2\alpha_s \sum_{i<j} \langle \frac{1}{|\vec{x_i} - \vec{x_j}|} \rangle = -2\alpha_s \frac{aQ}{\sqrt{3}\mu_0}$$

Self-energy

$$\Delta M_{QSE} = -\frac{3fa}{2\pi\mu_0}, \quad f \in [3, 4]$$

Final mass formula

$$M_0^2 = 2\pi\sigma(N+3) - \frac{4\pi\sigma\alpha_s}{\sqrt{3}} - \frac{12f\sigma}{2+\sqrt{3}}$$

with scaling

$$12aQ = 2\pi\sigma$$

Compatibility: $1/N_c$ expansion and QM mass formula

The $1/N_c$ expansion versus quark model

The $1/N_c$ expansion

$$M = c_1 N_c + \mathcal{O}(1/N_c)$$

The quark model formula

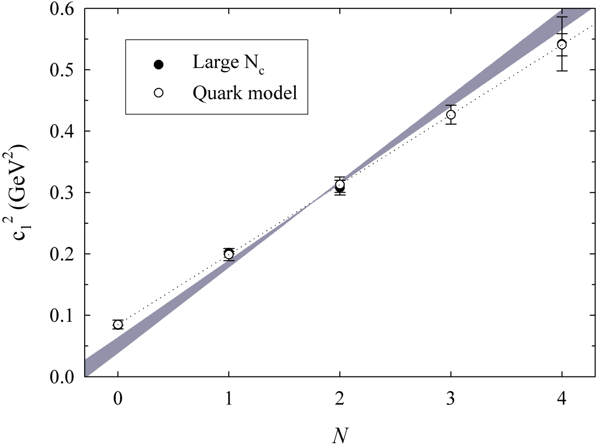
$$M_0^2 = 2\pi\sigma(N+3) - \frac{4\pi\sigma\alpha_s}{\sqrt{3}} - \frac{12 f\sigma}{2+\sqrt{3}}$$

Take $N_c = 3$ and compare $(3c_1)^2$ with M_0^2

Comparison with $1/N_c$ expansion: $c_1^2 = (M_0/N_c)^2$

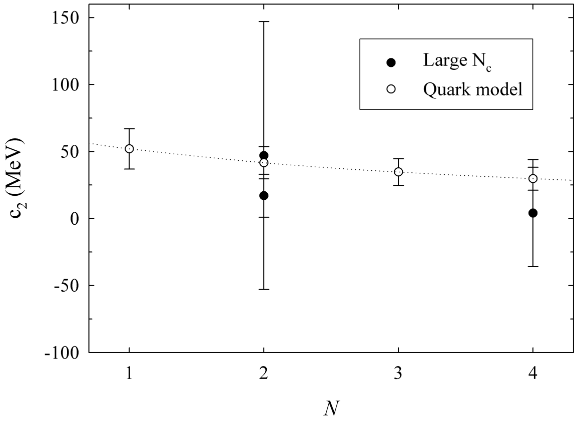
Fit: $\sigma = 0.163 \pm 0.004$ MeV, $\alpha_s = 0.40 \pm 0.05$, $f = 3.50 \pm 0.12$

Shaded area: $\sigma \in [0.17, 0.20]$



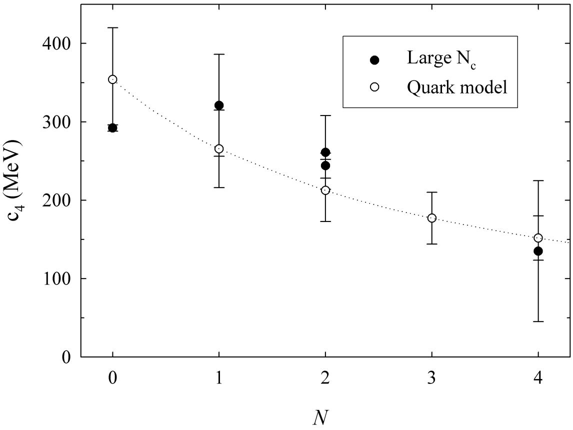
Auxiliary fields method: $c_2 \sim \mu_0^{-2} \rightarrow \ c_2 = c_2^0/(N+3)$
(SPIN-ORBIT)

Fitted: $c_2^0 = 208 \pm 60$ MeV



The coefficient $c_4 \sim \mu_0^{-2} \rightarrow \; c_4 = c_4^0/(N+3)$ (SPIN-SPIN)

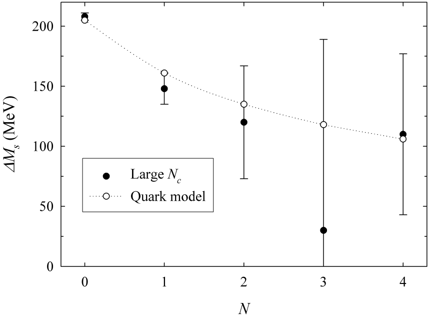
Fitted: $c_4^0 = 1062 \pm 198$ MeV



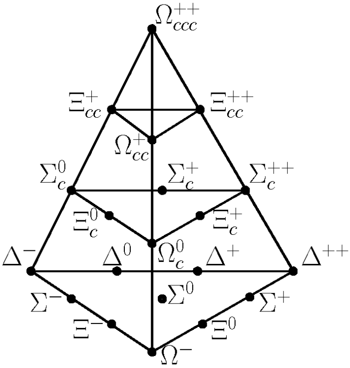
The SU(3) breaking: $m_s \neq m_{u,d}$

$$n_s\ \Delta M_s = \sum b_i \overline{B}_i$$

N is a good clasification number, Regge slope unchanged

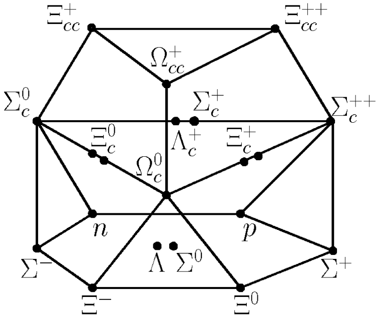


HEAVY BARYONS: SU(4) Symmetric 20-dim, J=3/2



Undiscovered yet: double and triple charm

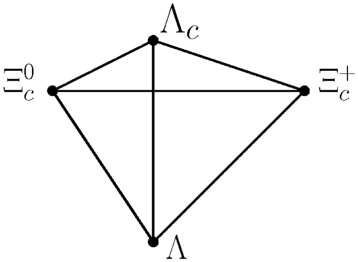
HEAVY BARYONS: SU(4) Mixed Symmetric 20-dim, J=1/2



Seen at SELEX: $\Xi_{cc(d)}^{+}(3520)$**, No evidence BABAR, BELLE**

Unobserved yet: $\Xi_{cc(u)}^{++}$, $\Omega_{cc(s)}^{+}$

HEAVY BARYONS: SU(4) Antisymmetric 4-dim, J=1/2



HEAVY BARYONS

Charm and bottom baryons in the combined $1/N_c$ and $1/m_Q$ expansion versus quark model
C. Semay, F. Buisseret and F.S. hep-ph:0808.3349

Extension of the $1/N_c$ operator analysis in the heavy quark spin-flavor symmetry: SU(6) $\times$ SU(2)$_c$ $\times$ SU(2)$_b$ $\rightarrow$ separate spin symmetry for each heavy flavor

Exact SU(3): ground state mass formula to order $1/m_Q^2$

E. Jenkins, PRD54:4515(1996); ibid. PRD77:034012(2008)

N_Q - the number of heavy quarks

$$M^{(1)} = m_Q N_Q + \Lambda_{qq} + \lambda_Q + \lambda_{qqQ}$$

Light pair:

$$\Lambda_{qq} = c_0 N_c + \frac{c_2}{N_c} J_{qq}^2$$

Heavy-quark corrections

$$\lambda_Q = N_Q \frac{1}{2m_Q} \left(c_0^{'} + \frac{c_2^{'}}{N_c^2} J_{qq}^2 \right)$$

Heavy-quark spin-symmetry violation

$$\lambda_{qqQ} = 2 \frac{c_2^{''}}{N_c m_Q} \vec{J}_{qq} \cdot \vec{J}_Q$$

Broken SU(3): dominant contribution N_c^0

$$M = M^{(1)} + \epsilon\Lambda_\chi T^8,$$

T^8 - SU(3) generator
$\epsilon\Lambda_\chi$= **Measure of the SU(3)-flavor breaking factor**

$$\Xi_Q - \Lambda_Q = \frac{\sqrt{3}}{2}\left(\epsilon\Lambda_\chi\right)$$

$\epsilon\Lambda_\chi = 0.206$ GeV, consistent with average for $Q = c, b$
$\Lambda_\chi \sim 1$ GeV, chiral symmetry breaking scale parameter.

The coefficients are functions of $1/N_c$ and a QCD scale Λ

$$c_0 = \Lambda,$$

$$c_2 \sim \Lambda,$$
$$c_0^{'} \sim c_2^{'} \sim c_2^{''} \sim \Lambda^2.$$

The mass formula gives the mass combinations at dominant order

$$\Lambda_Q = m_Q + N_c\Lambda,$$

$$\frac{1}{3}(\Sigma_Q + 2\Sigma_Q^*) - \Lambda_Q = 2\frac{\Lambda}{N_c},$$

$$\Sigma_Q^* - \Sigma_Q = \frac{3}{2}\left(\frac{2\Lambda^2}{N_c m_Q}\right),$$

$$\frac{1}{3}(\Lambda_Q+2\Xi_Q)-\frac{1}{4}\left[\frac{5}{8}(2N + 3\Sigma + \Lambda + 2\Xi) - \frac{1}{10}(4\Delta + 3\Sigma^* + 2\Xi^* + \Omega)\right] = m_Q.$$

Compatibility of the two approaches

New parameters in QM

Fixed parameters	Fitted parameters
$k_1 = 0.930$	$m_c = 1.252$ GeV
$\alpha_1 = 0.7\alpha_0$	$m_b = 4.612$ GeV

k_1, α_1 see previous studies
m_c and m_b fitted to Λ_c and Λ_b exp masses

Compatibility: Example: the dominant term

$$c_0 = \frac{1}{3}\, M_1|_{N=0} = \frac{4}{3}\mu_1 - \frac{2}{9}\sqrt{\frac{k_1 a}{3}}(\alpha_0 + 2\sqrt{2}\alpha_1) - \frac{fa}{3\pi\mu_1}$$

Parameter	$1/N_c$ expansion	Quark model
Λ	0.324 GeV	0.333 GeV
m_c	1.315 GeV	1.252 GeV
m_b	4.642 GeV	4.612 GeV
$\epsilon\Lambda_\chi$	0.206 GeV	0.170 GeV

PREDICTIONS

Baryon	Theoretical mass	Experiment
$\Xi_c(nsc)$	2.433 GeV	2.469 GeV
$\Xi_b(nsb)$	5.767 GeV	5.793 GeV

CONCLUSIONS

Large N_c approach supports quark model assumptions

-relativistic kinetic energy
-Y-junction for confinement
-N is a good classification number $-->$ Regge trajectories

- LIGHT BARYONS
  -spin-spin or isospin-isospin is the dominant interaction
  -negligible spin-orbit
  -spin-dependent contributions vanish at high energy
- HEAVY BARYONS
  -ground state: compatibility of parameters

-QM can give some dynamical info about $c_i,\ b_i$