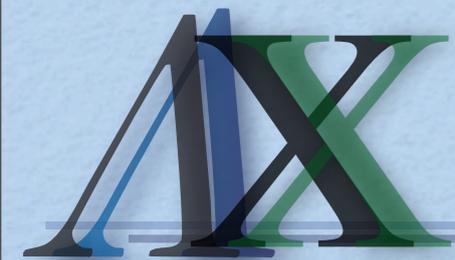


$\Lambda(1405)$ and $X(3872)$ as Multiquark Systems

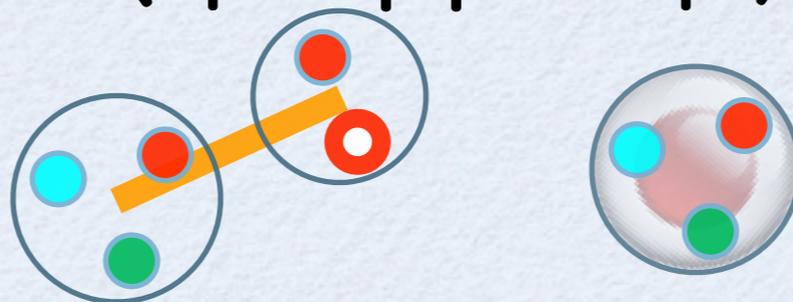
Sachiko Takeuchi

(Japan College of Social Work)



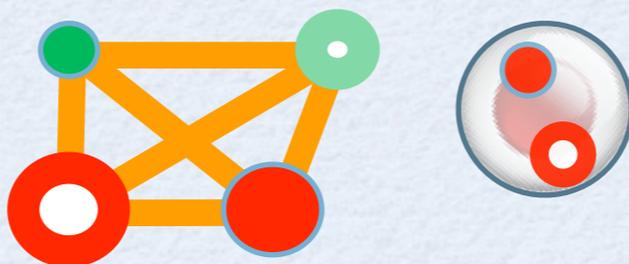
Contents

- $\Lambda(1405)$ by a $(q^3 - q\bar{q} + q^3)$ quark model



- What are the difference between the quark model and the chiral unitary model?

- $X(3872)$ by a $((q\bar{q})^2 + q\bar{q})$ quark model



Exotic states

- Multi-hadron systems ?
 - nuclei, hypernuclei
 - mesic nuclei
 - non qqq baryons
 - meson-baryon systems
 - pentaquarks or $q^3-(q\bar{q})$ systems
 - non- $q\bar{q}$ mesons
 - $(q\bar{q})^2$, $(q\bar{q})^3$, glueball, $(q\bar{q}g)$ systems

extra $q\bar{q}$

Exotic states

- Pentaquarks ($q^4\bar{q}$)

- Θ^+ , Ξ , ...

- negative-parity Λ^* → This talk

S.T. and K Shimizu, P.R. C76, 035204(07)

- $(q\bar{q})^2$ Mesons

- X(3872) → This talk

- $D_{s0}^*(2317)^\pm$, $D_{s1}^*(2460)^\pm$

- $f_0(600)$ $f_0(980)$ $a_0(980)$ $\kappa(800)$?

Adding $(q\bar{q})$ is important because of the parity

Ref. Particle Data Group

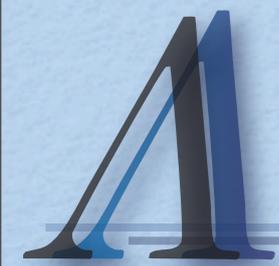
Λ (1405) peak

in a q^3 - $q\bar{q}$ scattering
with a q^3 pole

Sachiko Takeuchi

(Japan College of Social Work)

Kiyotaka Shimizu (Sophia Univ)



$\Lambda(1405)$

- Models for $q^4\bar{q}$
 - QCD-SR
 - Lattice
 - How to extract signals from the continuum?
 - parity
 - comparison to the other channels



$\Lambda(1405)$

- Models (cont.)
 - baryon-meson
 - Chiral unitary model Oset Ramos, Jido Oller *et al*
 - quark models
 - q^3 quark model ($0s^20p$) Isgur Karl
 - $q^4\bar{q}$ quark model ($0s^5$) Hogaasen Sorba
 - $q^4\bar{q}$ quark model (solved) Nemura *et al*
 - q^3 with the meson cloud Arima *et al*
 - $q^3-q\bar{q} + q^3$ quark model This talk

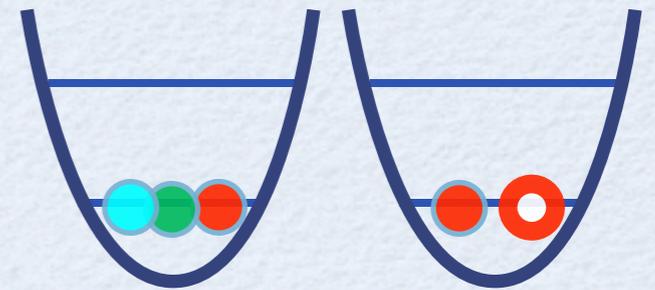
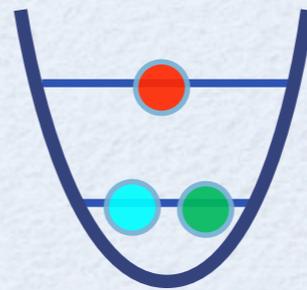
Λ $(q^4\bar{q})(0s)^5$ v.s. $q^3(0s)^20p$?

● Negative parity Baryons' mass from quark models

- $q^3 \sim 1600\text{MeV}$

- $q^3+q\bar{q} \sim (940 + 500\sim 600)\text{MeV}$

- $q^4\bar{q} \sim (940 + 500\sim 600)\text{MeV} + K + V$



$$K < 3/2 \hbar \omega_q$$

$$V < 0$$

$q^4\bar{q}$ for $\Lambda(1405)$??



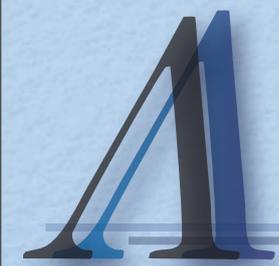
$(q^4\bar{q})(0s)^5$ v.s. $q^3(0s)^20p$?

- Flavor-singlet P-wave q^3 state ?
 - Observed Λ_8 - Λ_1 splitting
 - Observed large LS splitting $\Lambda(1405)$ - $\Lambda(1520)$
 - These two facts are difficult to reproduce...



$(q^4 \bar{q})(0s)^5$ v.s. $q^3(0s)^2 0p$?

- Flavor-singlet P-wave q^3 state ?
 - Observed $\Lambda_8 - \Lambda_1$ splitting
 - Observed large LS splitting
 - These two facts are difficult to reproduce...
- S-wave $q^4 \bar{q}$ state ?
 - CMI $(\lambda \cdot \lambda)(\sigma \cdot \sigma)$ can be strongly attractive in some states of $T=0$ $J^P = 1/2^-$
 - but also in $T=1$ $1/2^-$ Light Σ^* ?



$\Lambda(1405)$ is a resonance!

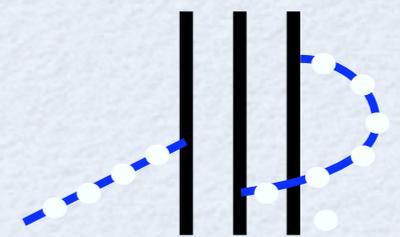
- Treating $\Lambda(1405)$ as a resonance in the B-M scattering is absolutely necessary.

- Chiral unitary model

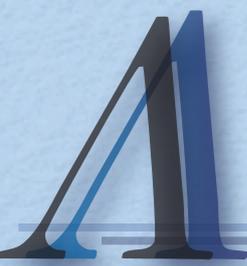
- $\Lambda(1405)$ appears as a resonance in the BM scattering. [Oset Ramos NPA635\(98\)99](#)

- Self energy of meson field

- Mass of the q^3 state reduces considerably.



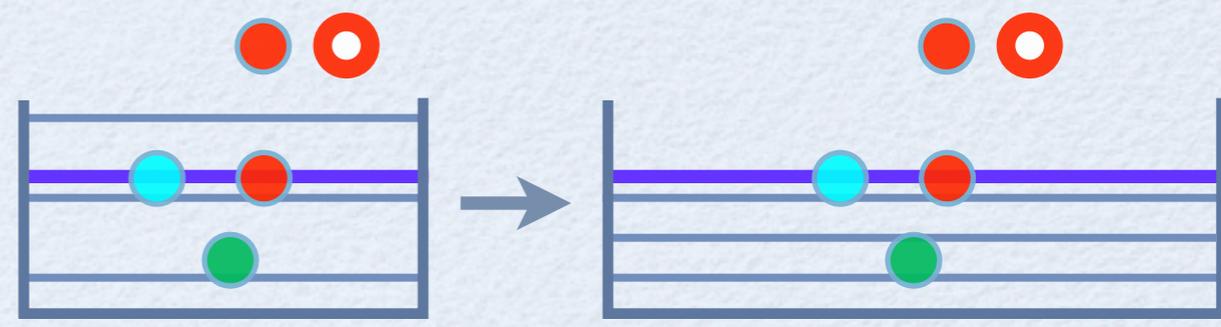
[Arima Matsui Shimizu PRC49\(94\)2831](#)



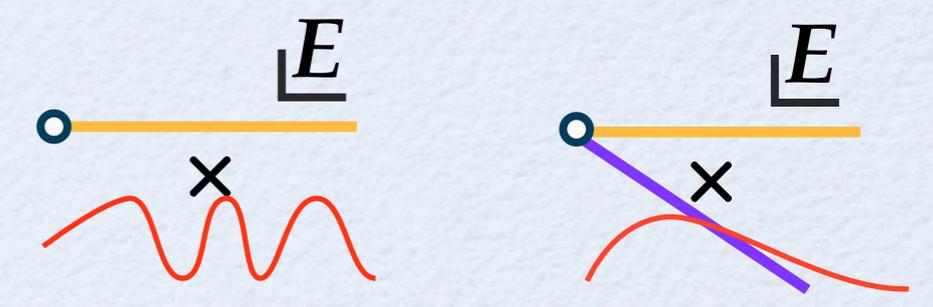
$\Lambda(1405)$ is a resonance!

How to extract signals from the continuum? (in the quark models)

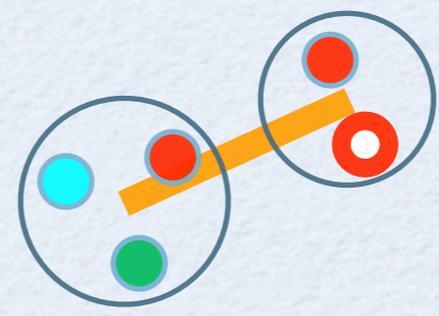
- solved models
- change model space



- complex scaling method



- configuration-restricted models
- quark cluster model



Oka Yazaki

Baryon-meson scattering (QCM)

- From Schrödinger eq for quarks:

$$(H_q - E)\phi = 0$$

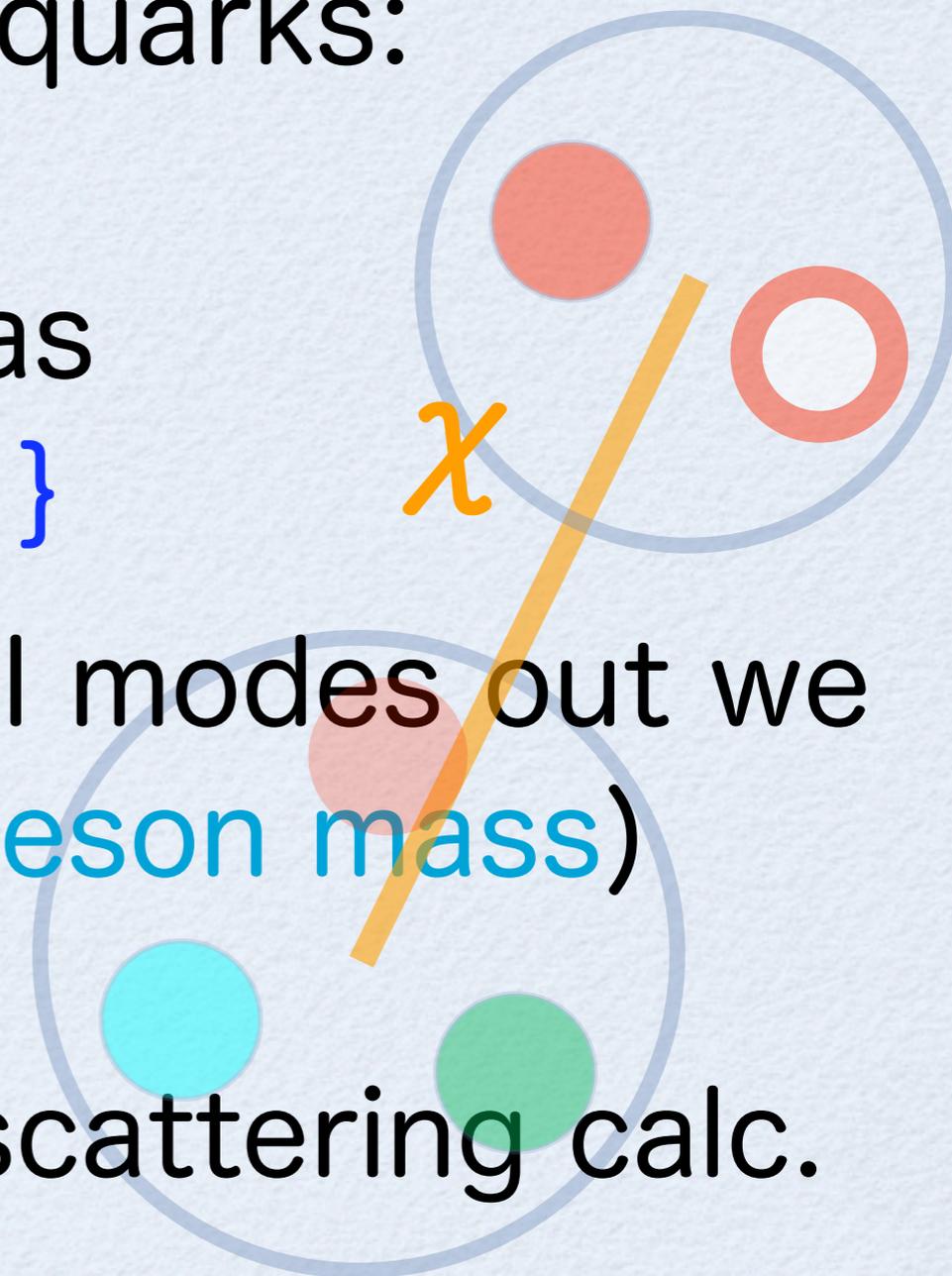
- Assuming wave function as

$$\Psi = A_q \{ \phi_B \phi_M \chi \}$$

- By integrating the internal modes out we get RGM eq (using real meson mass)

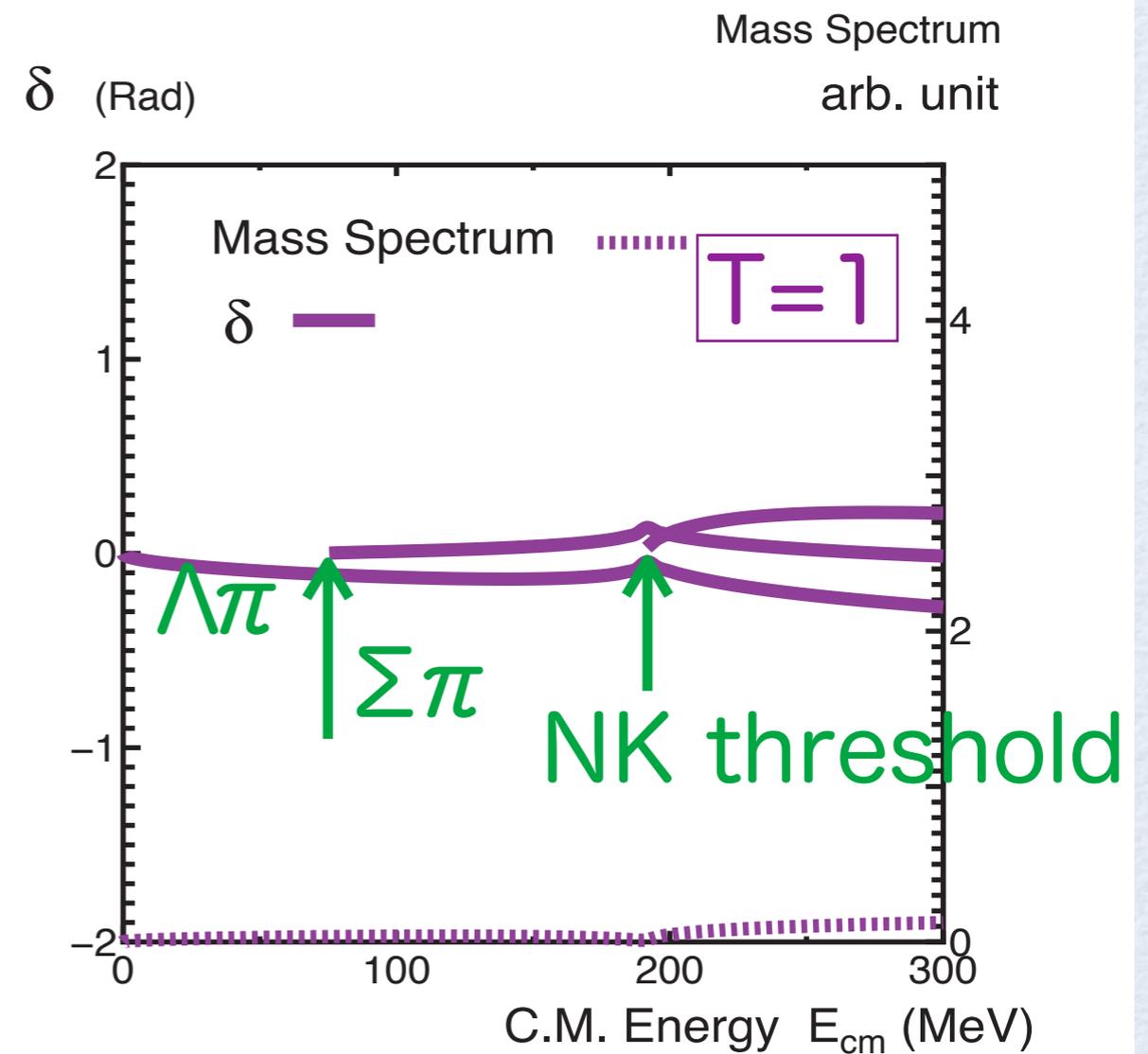
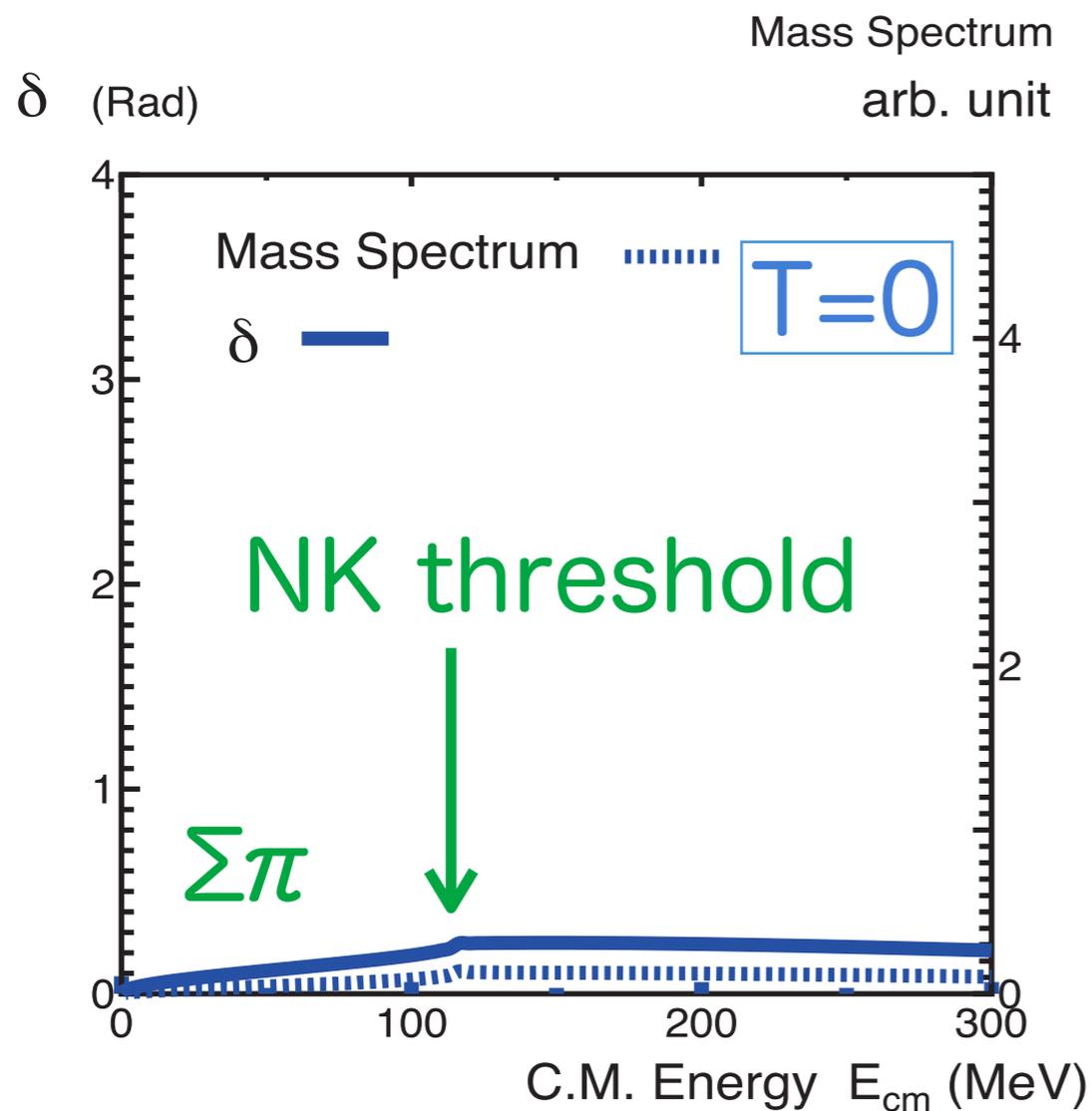
$$(H - EN)\chi = 0$$

- 3-channel coupled QCM scattering calc.
for $m_u \neq m_s$



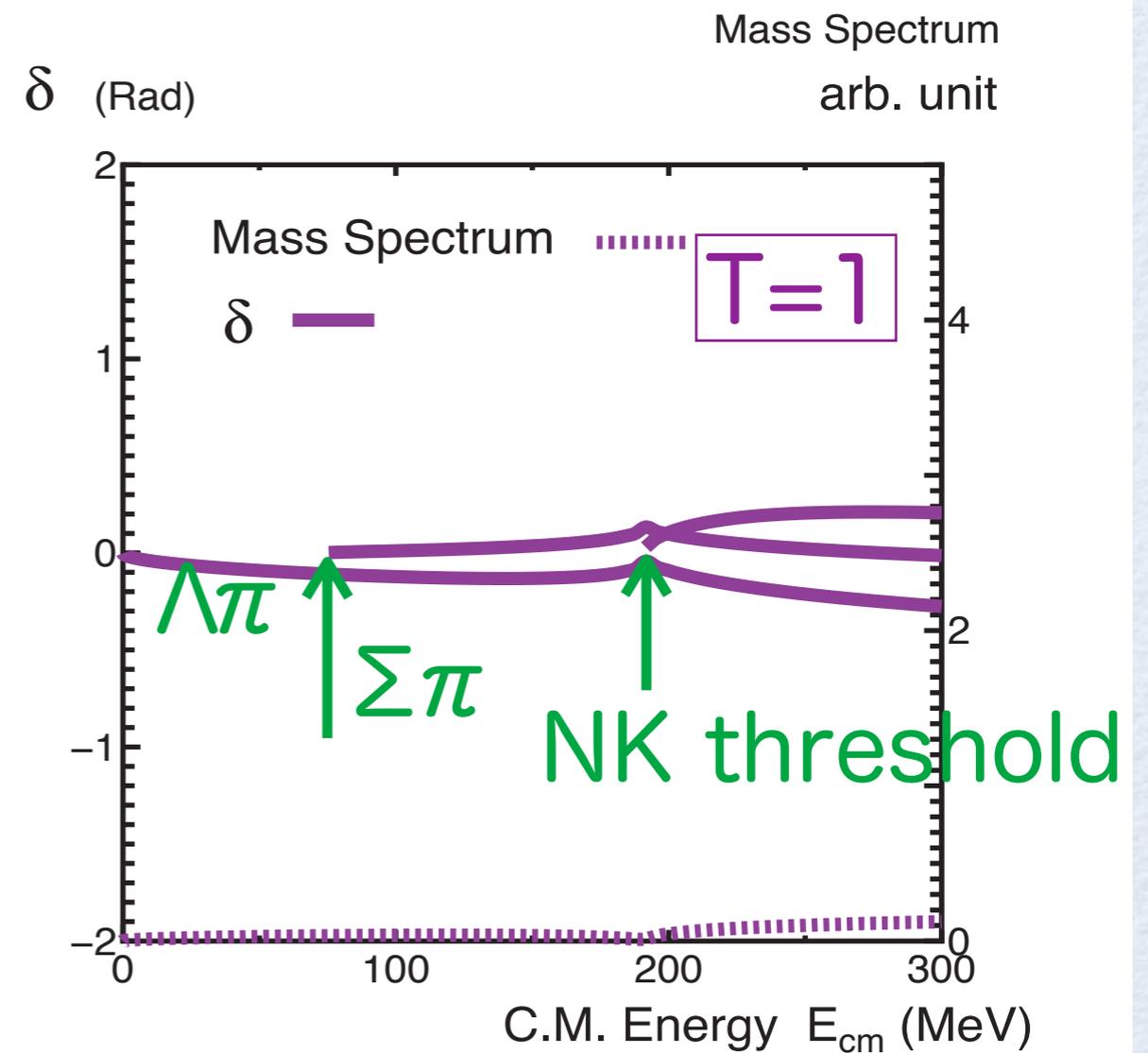
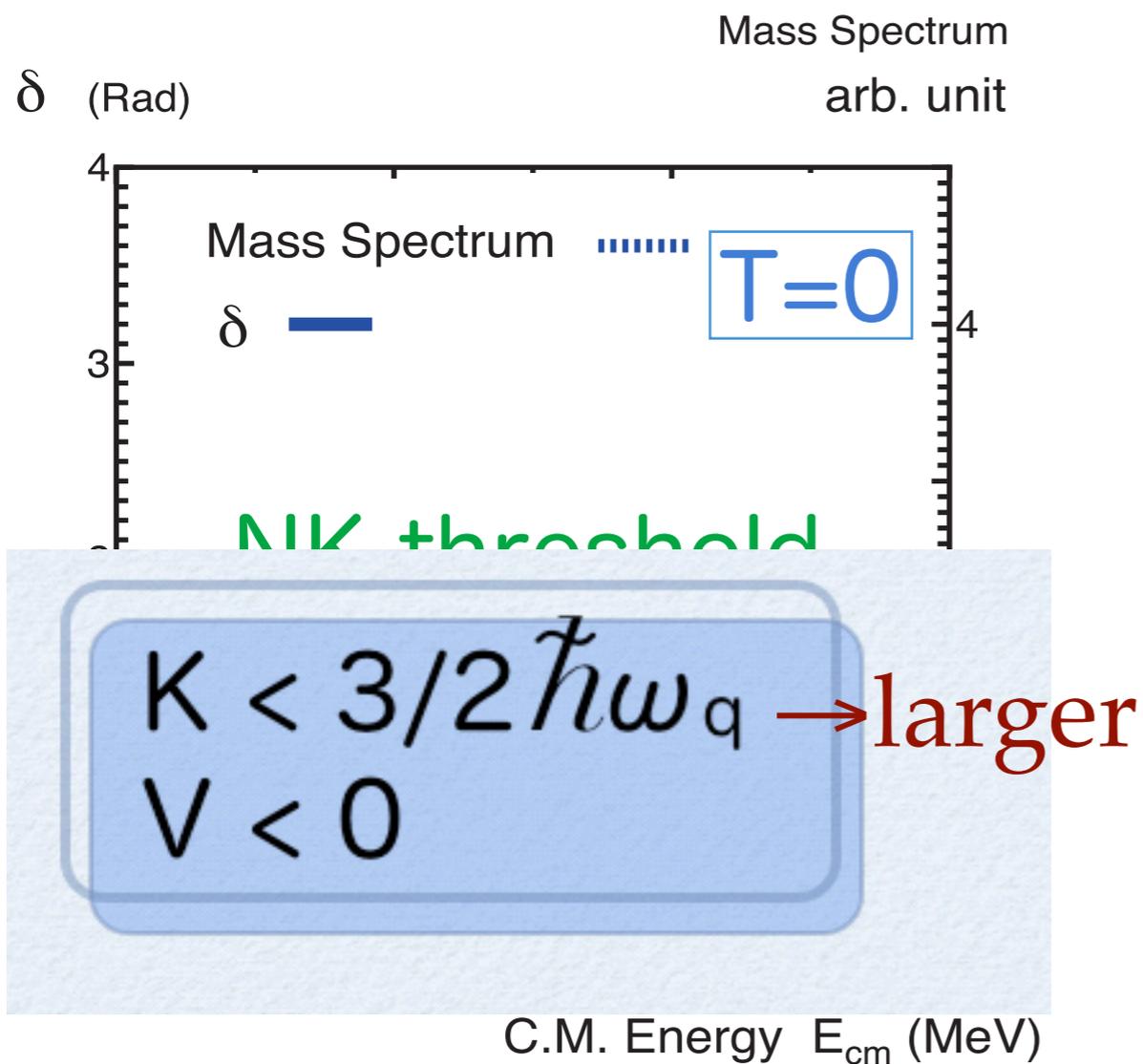
No peak is found for $q^4\bar{q}$!!

- Reduced mass of $\Sigma\pi$ is small \rightarrow Kinetic term is large \rightarrow Short range attraction is suppressed.
- No attraction in the $N\bar{K}$ channel.



No peak is found for $q^4\bar{q}$!!

- Reduced mass of $\Sigma\pi$ is small \rightarrow Kinetic term is large \rightarrow Short range attraction is suppressed.
- No attraction in the $N\bar{K}$ channel.



Channel dep of V_{BM} ($T=0$)

Short range part of V_{BM} by the $(\lambda.\lambda\sigma.\sigma)$ model

	$\Sigma \pi$	$N\bar{K}$	$\Lambda \eta$	ΞK
$\Sigma \pi$	$\frac{-16}{3}$	$\frac{116\sqrt{7}}{21}$	$\frac{16\sqrt{105}}{105}$	0
$N\bar{K}$		0	$\frac{28\sqrt{15}}{15}$	0
$\Lambda \eta$			$\frac{112}{15}$	$\frac{-40\sqrt{70}}{21}$
ΞK				$\frac{-160}{21}$

Table:
Matrix elements,
 $-\langle \lambda.\lambda\sigma.\sigma \rangle$

Channel dep of V_{BM} ($T=0$)

Short range part of V_{BM} by the $(\lambda.\lambda\sigma.\sigma)$ model

	$\Sigma \pi$	$N\bar{K}$	$\Lambda \eta$	ΞK
$\Sigma \pi$	-5.33	14.61	-1.56	0
$N\bar{K}$		0	7.23	0
$\Lambda \eta$			7.47	-15.94
ΞK				-7.62

Table:
Matrix elements,
 $-\langle \lambda.\lambda\sigma.\sigma \rangle$



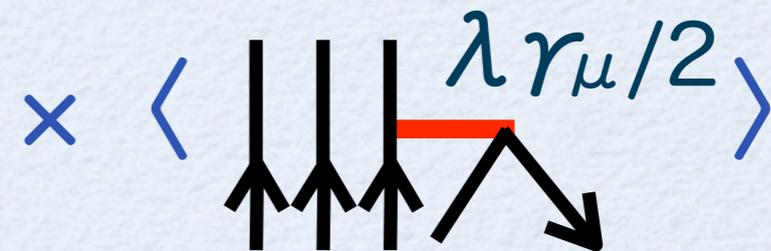
With q^3 -pole ...

- $\Lambda(1405) = \alpha |q^3\rangle + \beta |q^3 - q\bar{q}\rangle$

- Transition potential is:

$$\langle q^3 | V | q^3 - q\bar{q} \rangle = | \Lambda_1 q^3 (0s)^2 0p \rangle \langle \text{BM } q^4 \bar{q} (0s)^5 |$$

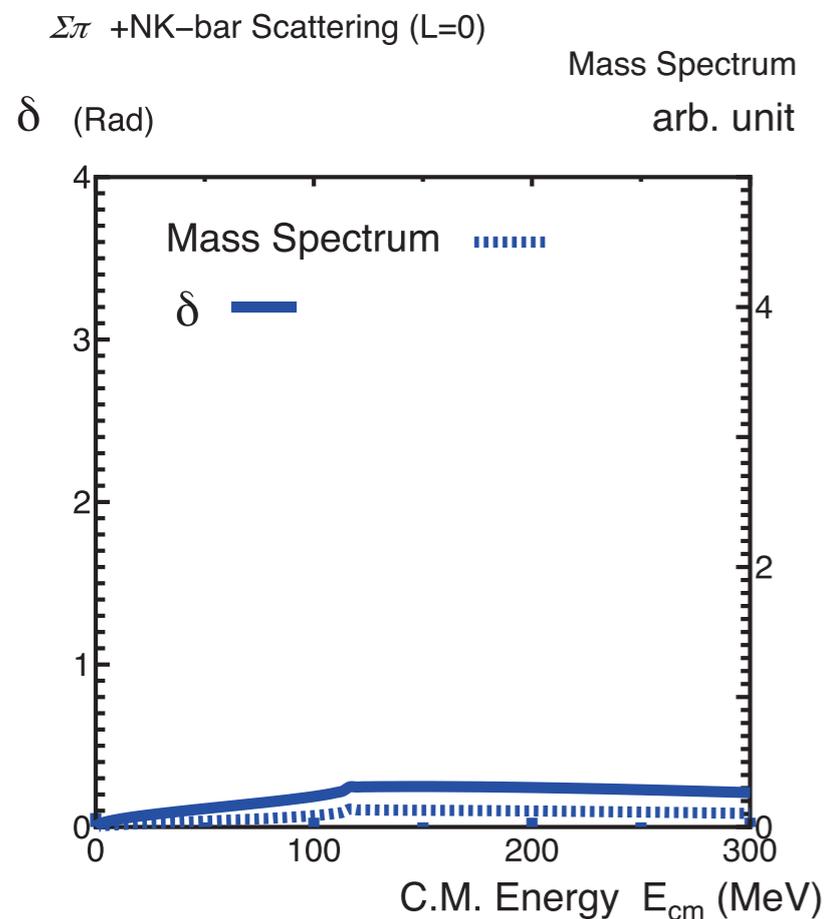
$\Lambda_1 1/2^-$		$\Sigma_8 1/2^-$	
$\Sigma \pi$	145	$\Lambda \pi$	-32
$N\bar{K}$	-85	$\Sigma \pi$	-51
$\Lambda \eta$	53	$N\bar{K}$	60
(in MeV)		$\Sigma \eta$	2



q^3 - $q\bar{q}$ scattering with q^3 -pole

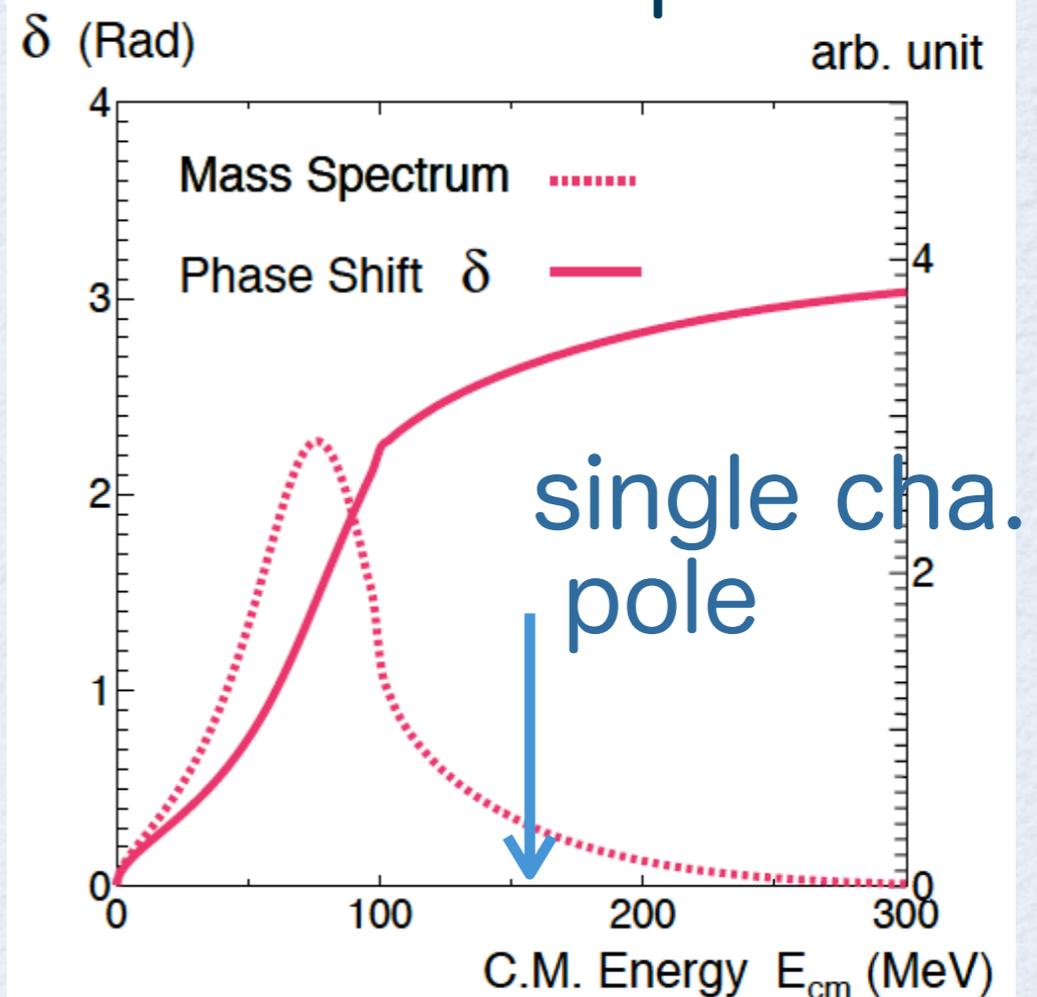
- q^3 -pole at $\Sigma\pi + 160\text{MeV}$ ($\sim 1490\text{MeV}$) gives a resonance at $\sim 1405\text{MeV}$!

$\Sigma\pi + NK$



+ pole \rightarrow

$\Sigma\pi + NK + \text{pole}$

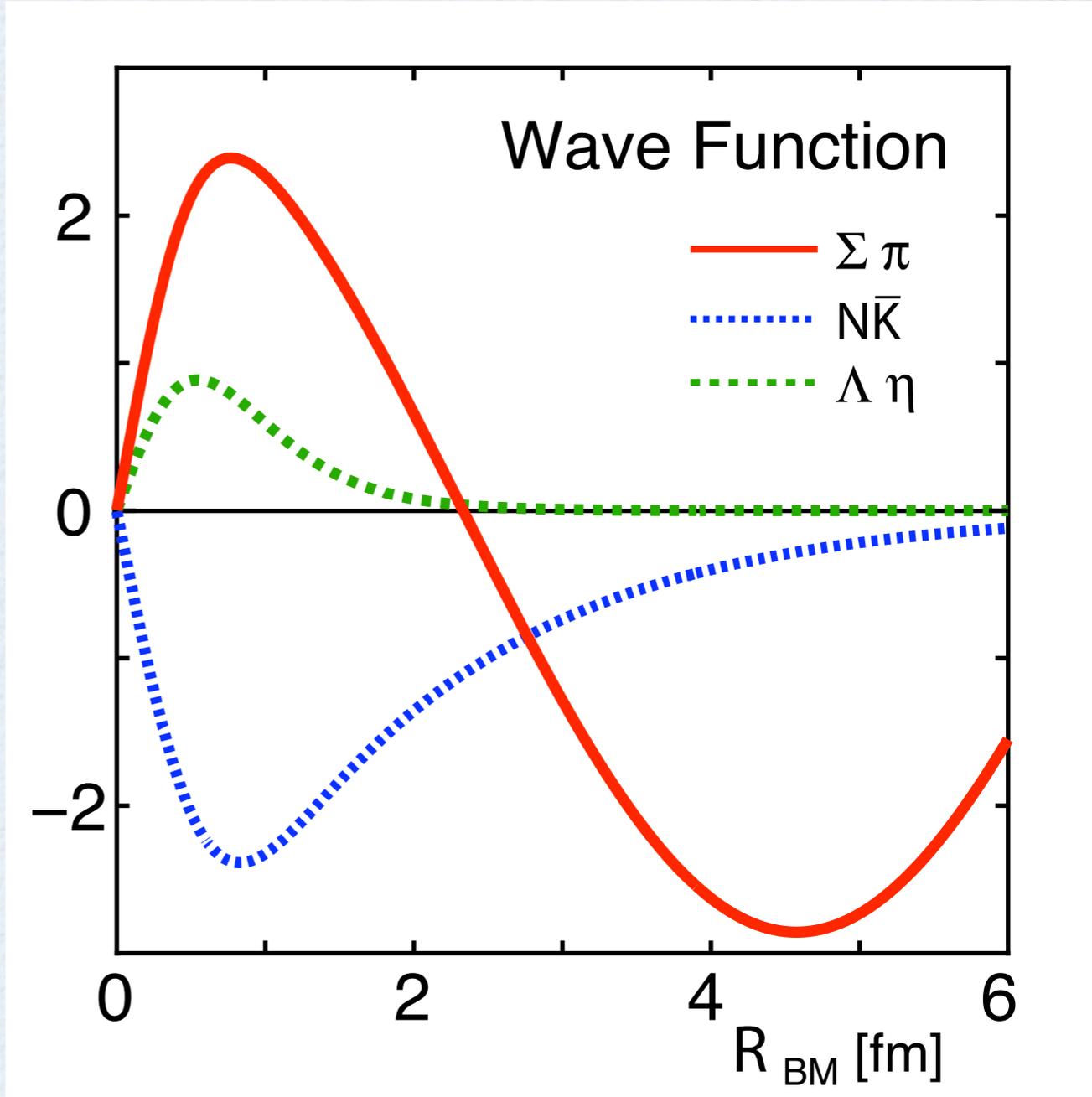


A wave functions at resonance

- Contribution of the q^3 -pole is large.

$$|\psi|^2$$

NK : $q^3 = 1 : 2.8$



- Can this be observed...?

Scattering Observables

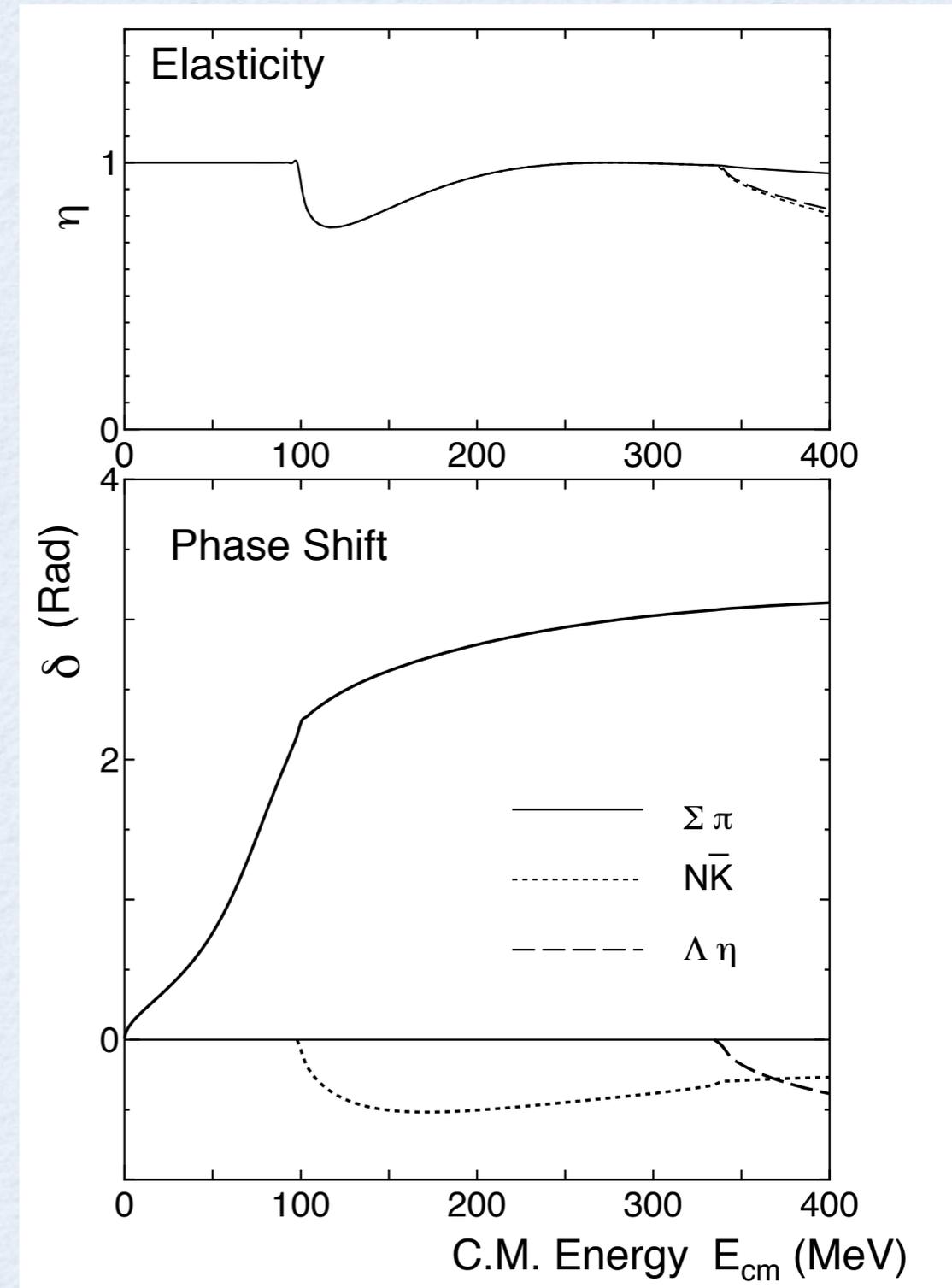
• mixing of $\Sigma\pi$ and NK is strong at the threshold.

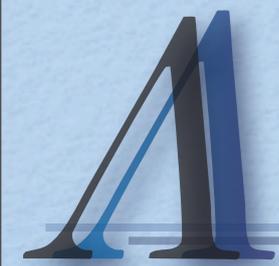
• NK scattering length :

$$-0.75 + i 0.38 \text{ fm}$$

$$\text{Exp. } (-1.70 \pm 0.07) + i(0.68 \pm 0.04)$$

Martin NPB179(81)33





Summary

- $\Lambda(1405)$ resonance can be described by a $(|q^3\rangle + |q^3-q\bar{q}\rangle)$ system.
- Interaction for $|q^3-q\bar{q}(\Sigma\pi)\rangle$ is attractive, but not for $|q^3-q\bar{q}(NK)\rangle$.
- Kinetic energy suppress the short-range attraction of $|q^3-q\bar{q}(\Sigma\pi)\rangle$.
- Without the mixing of $|q^3\rangle$, no peak appears.
- With the mixing of $|q^3\rangle$, $\Lambda(1405)$ -like peak appears!

Quark model v.s. Chiral unitary model

Quark model can reproduce the peak, but so does the chiral unitary model.

Quark model:

- quarks, no attraction between $N\bar{K}$, non-relativistic, q^3 pole

Chiral Unitary model:

- no internal structure, large attraction between $N\bar{K}$, semi-relativistic, no q^3 pole



Channel dep of V_{BM} ($T=0$)

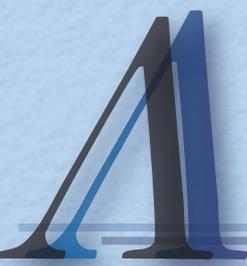
- Short range part of V_{BM}
 - Difference is found in the $N\bar{K}$ diagonal part.

— $\langle \lambda.\lambda \sigma.\sigma \rangle$

- No $N\bar{K}$ diagonal attraction : need something to make a peak just below the $N\bar{K}$ threshold.

$\langle F.F \rangle$

- $N\bar{K}$ diagonal attraction makes a peak just below the $N\bar{K}$ threshold.



Channel dep of V_{BM} ($T=0$)

Short range part of V_{BM} by the

(F.F) model
(WT-term)

	$\Sigma \pi$	$N\bar{K}$	$\Lambda \eta$	ΞK
$\Sigma \pi$	-8	$\sqrt{6}$	0	$-\sqrt{6}$
$N\bar{K}$		-6	$3\sqrt{2}$	0
$\Lambda \eta$			0	$-3\sqrt{2}$
ΞK				-6

Table:
Matrix elements,
 $\langle F.F \rangle$

Channel dep of V_{BM} ($T=0$)

- Short range part of V_{BM}
 - Difference is found in the $N\bar{K}$ diagonal part.

No attraction

Attraction

$-\langle \lambda.\lambda \sigma.\sigma \rangle$

$\langle F.F \rangle$

	$\Sigma \pi$	$N\bar{K}$	$\Lambda \eta$
$\Sigma \pi$	-5.33	14.61	-1.56
$N\bar{K}$		0	7.23
$\Lambda \eta$			7.47

	$\Sigma \pi$	$N\bar{K}$	$\Lambda \eta$
$\Sigma \pi$	-8	2.45	0
$N\bar{K}$		-6	4.24
$\Lambda \eta$			0

A Simplified model - kinematics

- non-rela

- $p^2/2\mu$

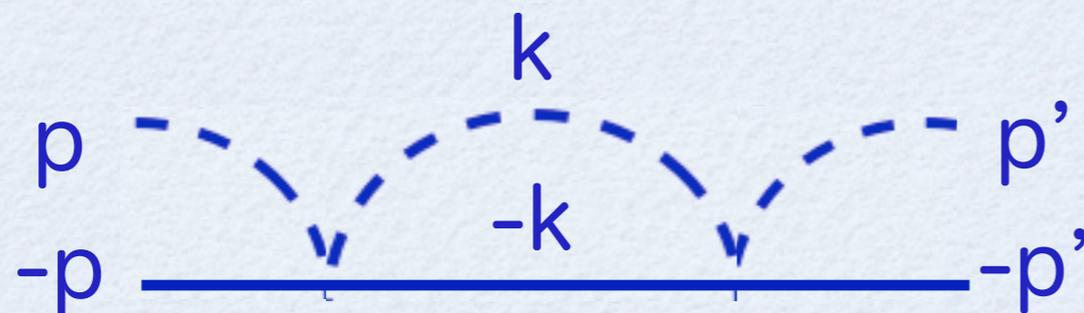
- semi-rela

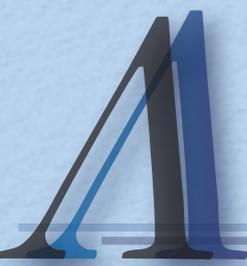
- propagator is

$$G_P^{(0)} = \frac{1}{(2\pi)^2 m} \int k^2 dk \frac{mM}{\omega\Omega} \frac{1}{\sqrt{s} - \omega - \Omega + i\epsilon}$$

with $\Omega = \sqrt{M^2 + k^2}$ and $\omega = \sqrt{m^2 + k^2}$

produces the $\Sigma \pi$ channel effective repulsion





Simplified model - int

- separable int with gaussian cut-off
- strength is the same as Oset-Ramos.
- two types of channel dependence:

— $\langle \lambda.\lambda \sigma.\sigma \rangle$

$\langle F.F \rangle$

	$\Sigma \pi$	$N\bar{K}$	$\Lambda \eta$		$\Sigma \pi$	$N\bar{K}$	$\Lambda \eta$
$\Sigma \pi$	-5.333	14.61	-1.56	$\Sigma \pi$	-8	2.45	0
$N\bar{K}$		0	7.23	$N\bar{K}$		-6	4.24
$\Lambda \eta$							

Cancelled by the kinetic energy part in the propergator



The situation is...

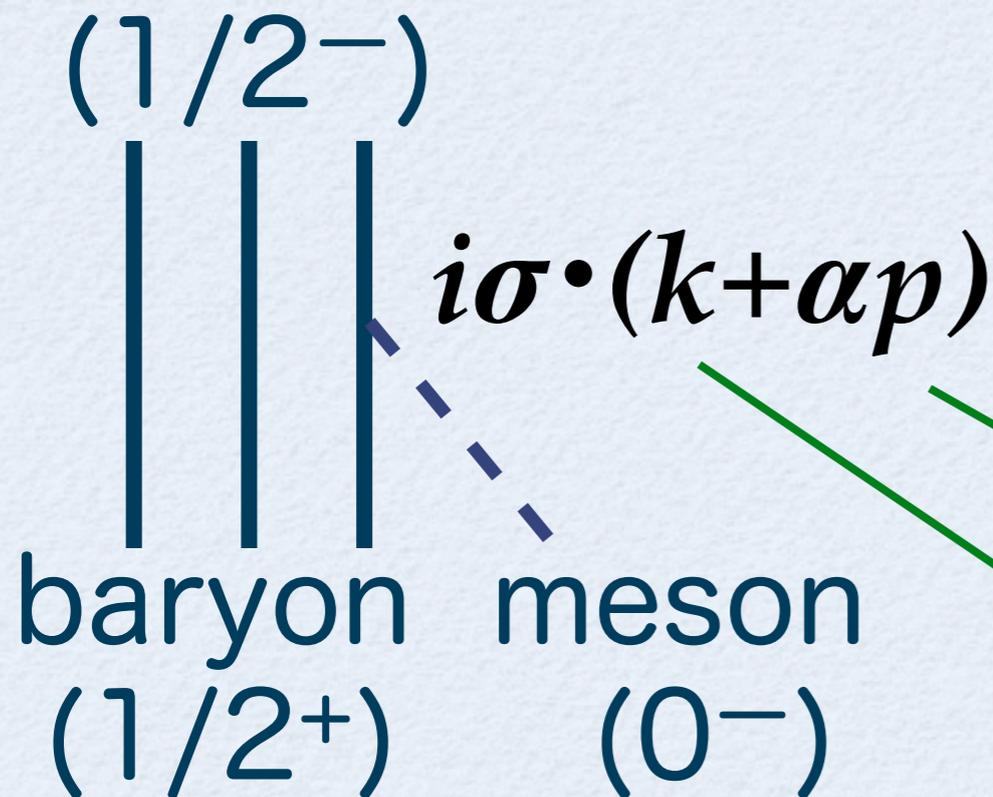
To understand the situation, we perform simplified baryon meson scattering problems such as

- scattering of baryon and meson without internal structure.
- semi-relativistic kinematics
- interaction is F.F like or $\lambda\lambda\sigma$ -like and separable.
- a 'q³-pole' couples to the continuum.

Simplified model - q^3 pole

- Flavor singlet transition for FF model

$$|1_{BM}\rangle = \sqrt{\frac{3}{8}}|\Sigma\pi\rangle - \frac{1}{2}|N\bar{K}\rangle + \sqrt{\frac{1}{8}}|\Lambda\eta\rangle + \frac{1}{2}|EK\rangle$$



Matrix element

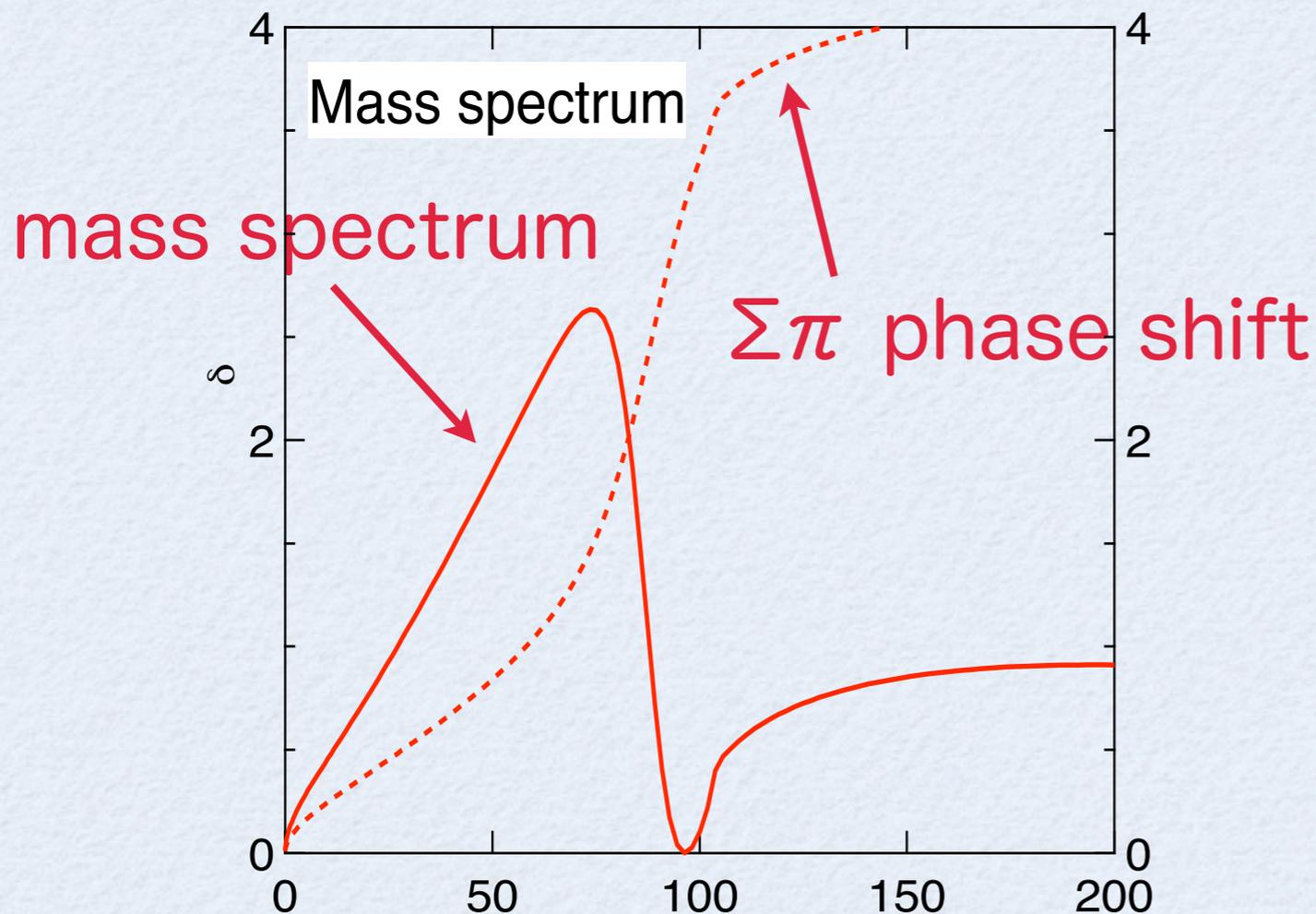
$$\langle B' 1/2^- | O | B 1/2^+ M \rangle \propto \begin{pmatrix} 1 \\ k^2 \end{pmatrix} \times \exp[-(bk)^2/6]$$



Simplified

Chiral-Unitary-like

- semi-rela, $\langle F.F \rangle$, no pole, energy-dep



NK scattering length
 $= -2.09 + 0.55 i$
(c.f. $-2.53 + 1.26 i$
for Oset Ramos original)

Exp. (-1.70 ± 0.07)
 $+ i(0.68 \pm 0.04)$

Simplified

Chiral-Unitary-like

- semi-rela, $\langle F.F \rangle$, no pole, energy-dep

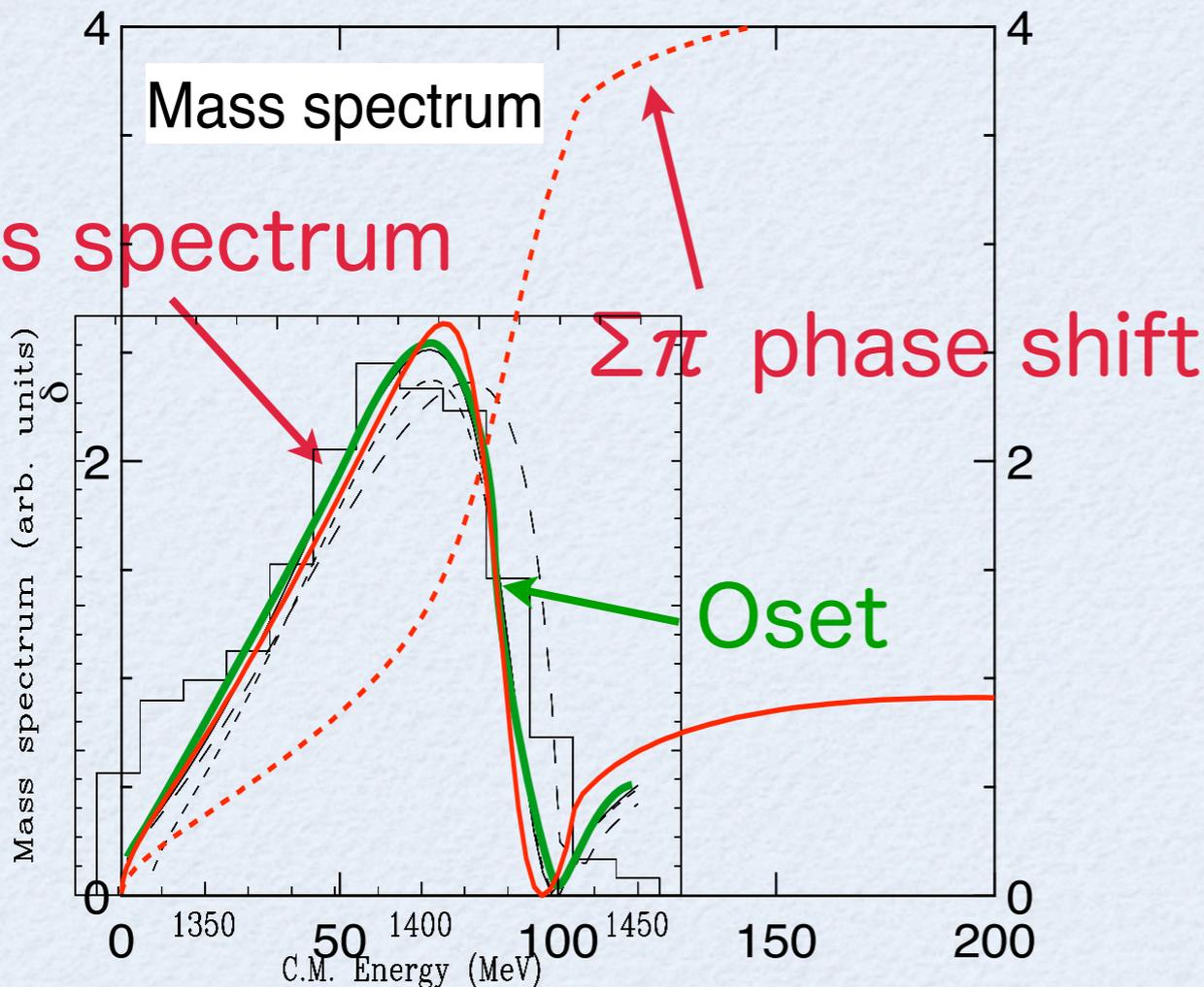


FIG. 2. The $\pi\Sigma$ mass distribution around the $N(1405)$ resonance from eq. (27). Short-dashed line: results in isospin basis. Long-dashed line: results omitting the $\eta\Sigma^0$, $\eta\Lambda$ channels. Red line: results with the full basis of physical states. Experimental data from [17].

NK scattering length
 $= -2.09 + 0.55 i$
 (c.f. $-2.53 + 1.26 i$
 for Oset Ramos original)

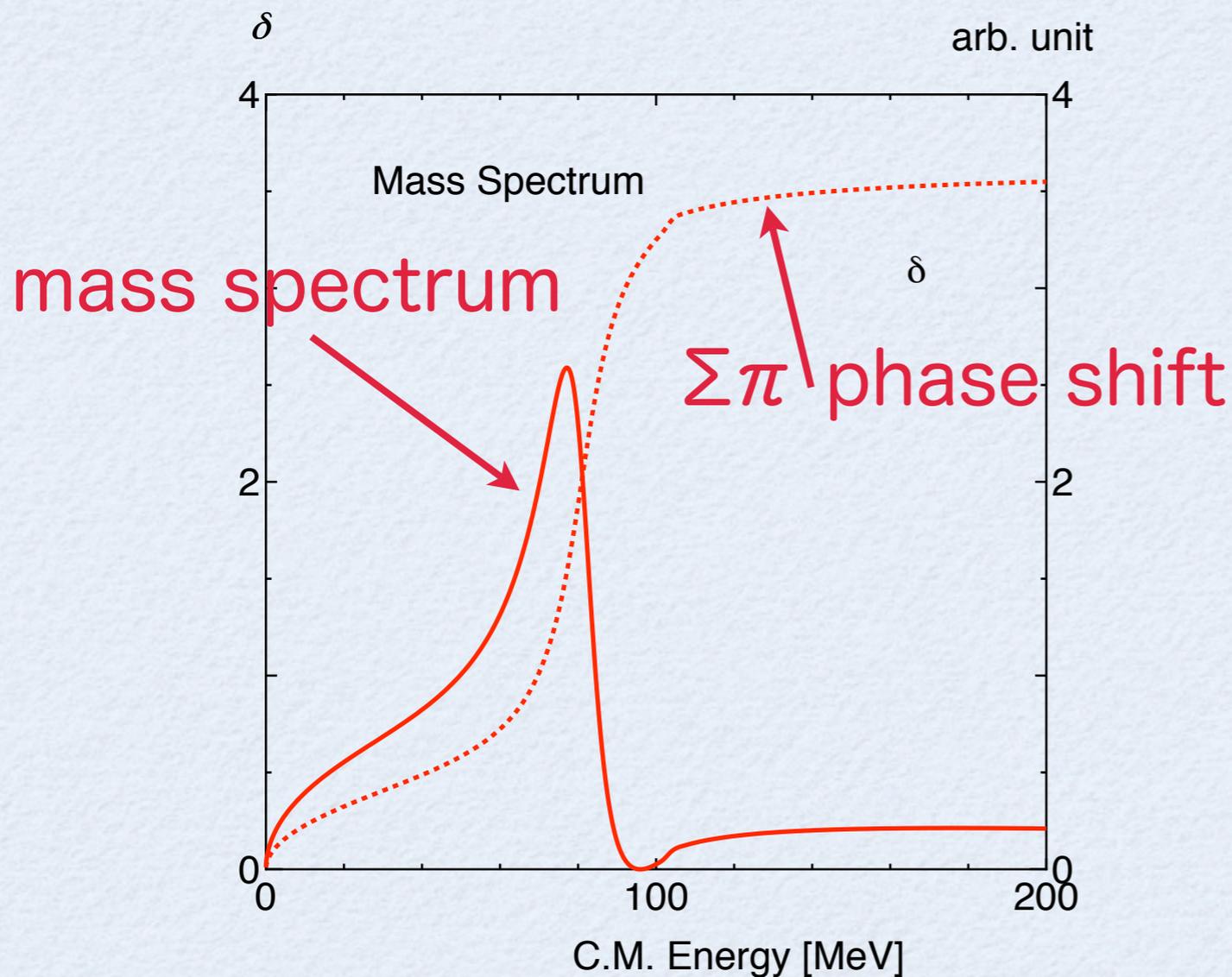
Exp. (-1.70 ± 0.07)
 $+ i(0.68 \pm 0.04)$



Simplified

Chiral-Unitary-like

- semi-rela, $\langle F.F \rangle$, no pole



NK scattering length
 $= -1.93 + 0.25 i$
(c.f. $-2.53 + 1.26 i$
for Oset Ramos original)

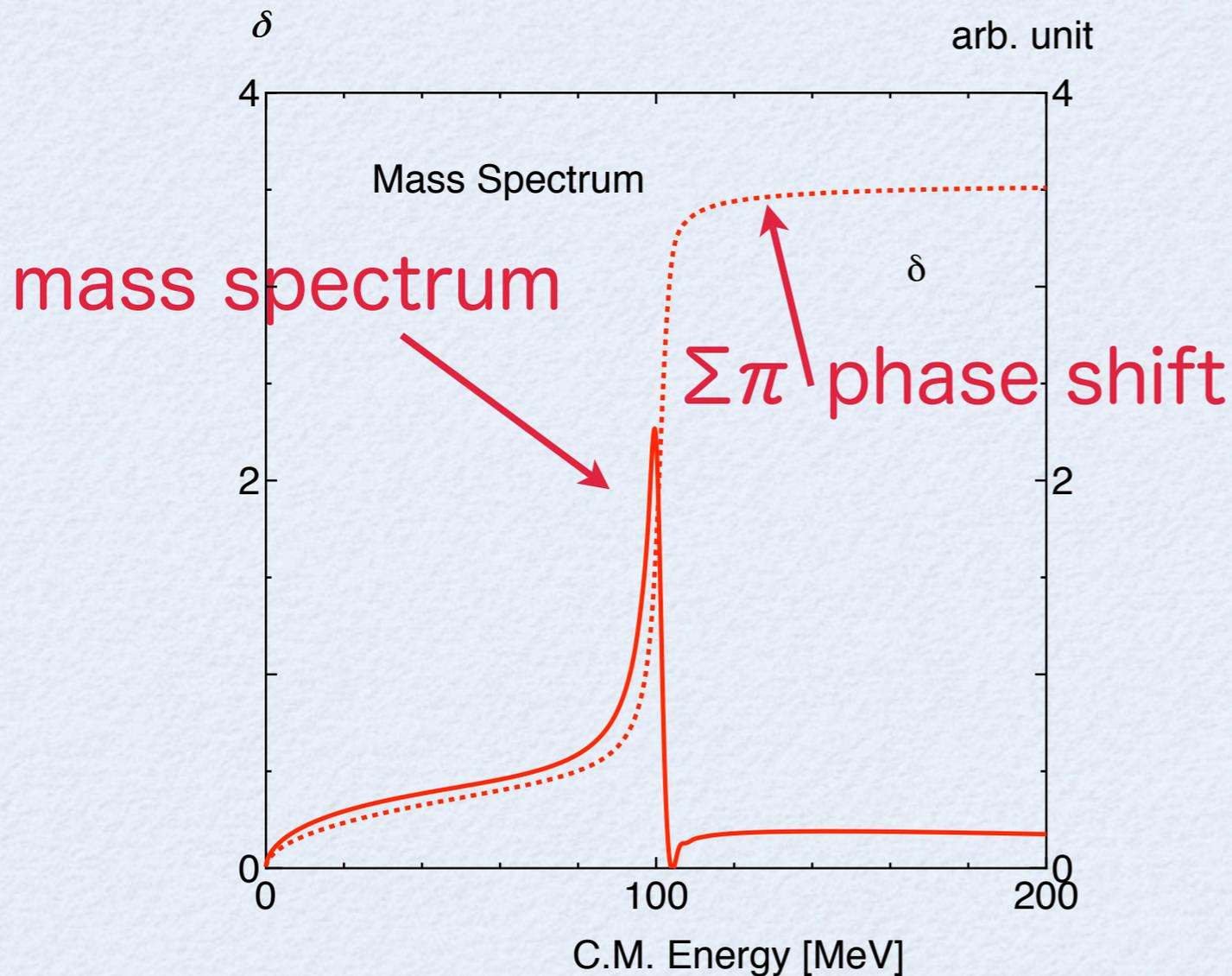
Exp. (-1.70 ± 0.07)
 $+ i(0.68 \pm 0.04)$



Simplified

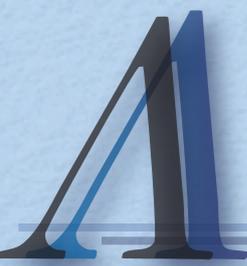
Chiral-Unitary-like

- semi-rela, $\langle F.F \rangle$, no pole (lower energy cut off)



$$\text{NK scattering length} = -4.20 + 1.14 i$$

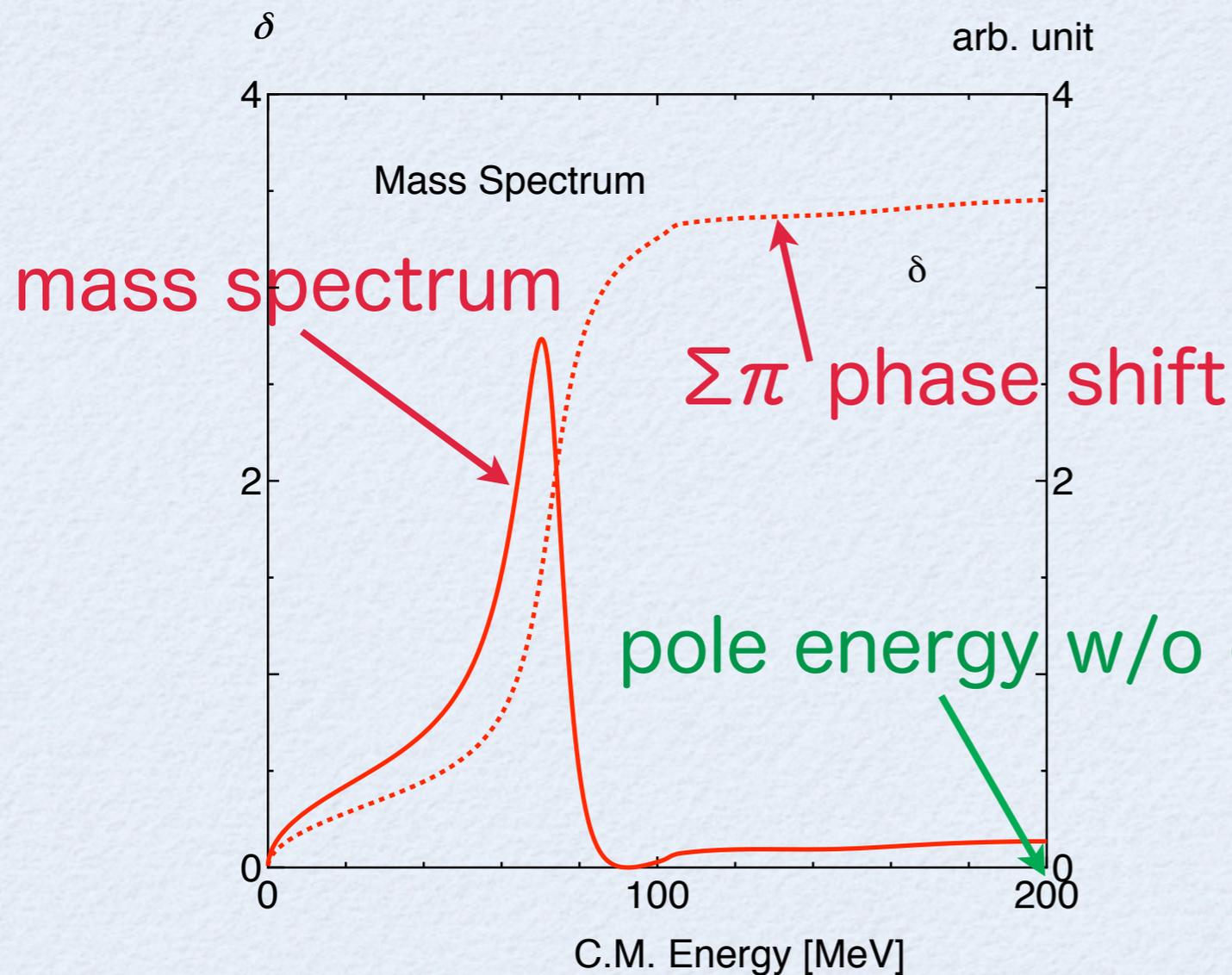
$$\text{Exp. } (-1.70 \pm 0.07) + i(0.68 \pm 0.04)$$



Simplified

Chiral-Unitary-like

- semi-rela, $\langle F.F \rangle$, with pole (kk-coupling)



(lower energy cut off)

$$\text{NK scattering length} = -1.06 + 0.17 i$$

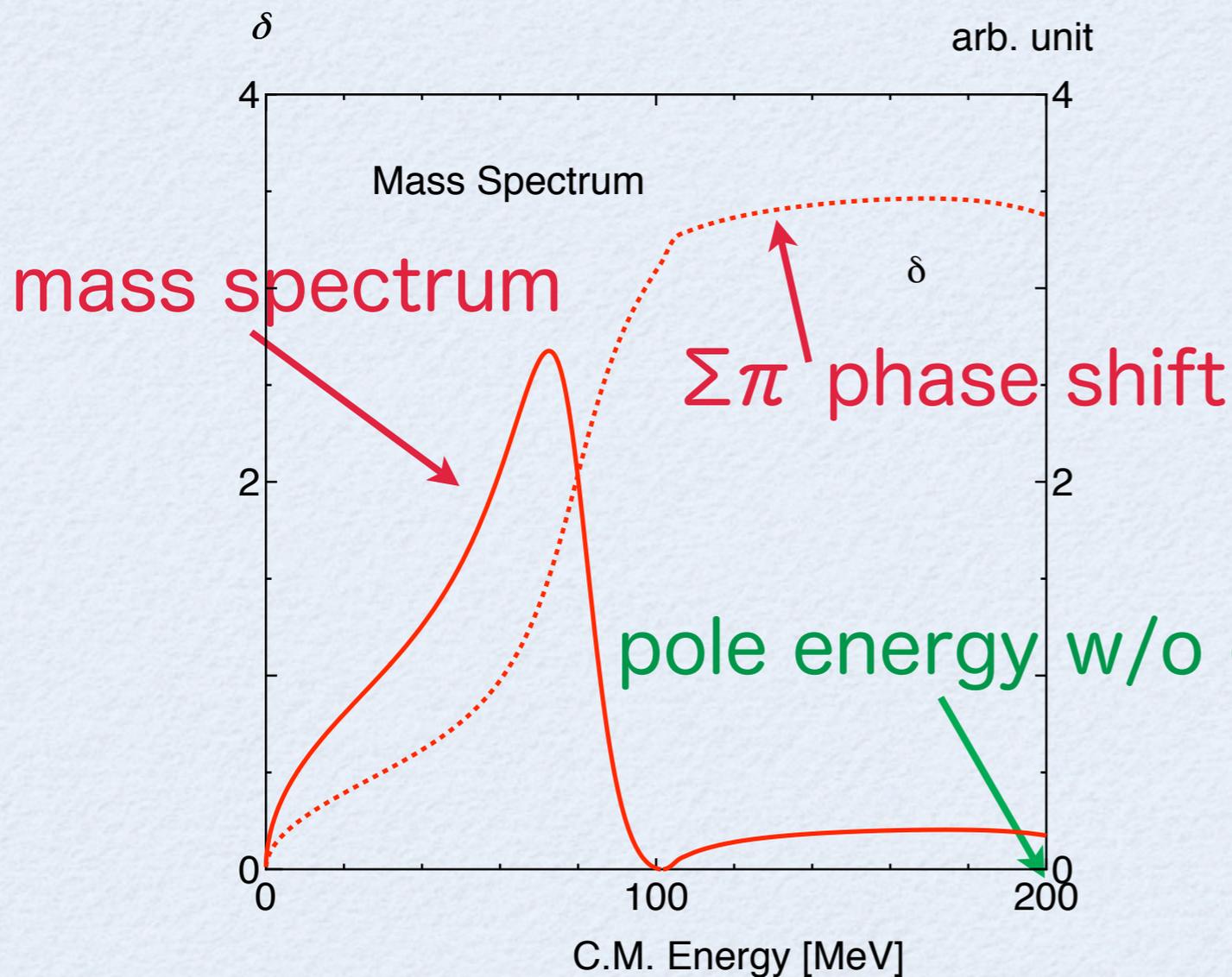
$$\text{Exp. } (-1.70 \pm 0.07) + i(0.68 \pm 0.04)$$



Simplified

Chiral-Unitary-like

- semi-rela, $\langle F.F \rangle$, with pole (1-coupling)



(lower energy cut off)

$$\text{NK scattering length} = -1.68 + 0.42 i$$

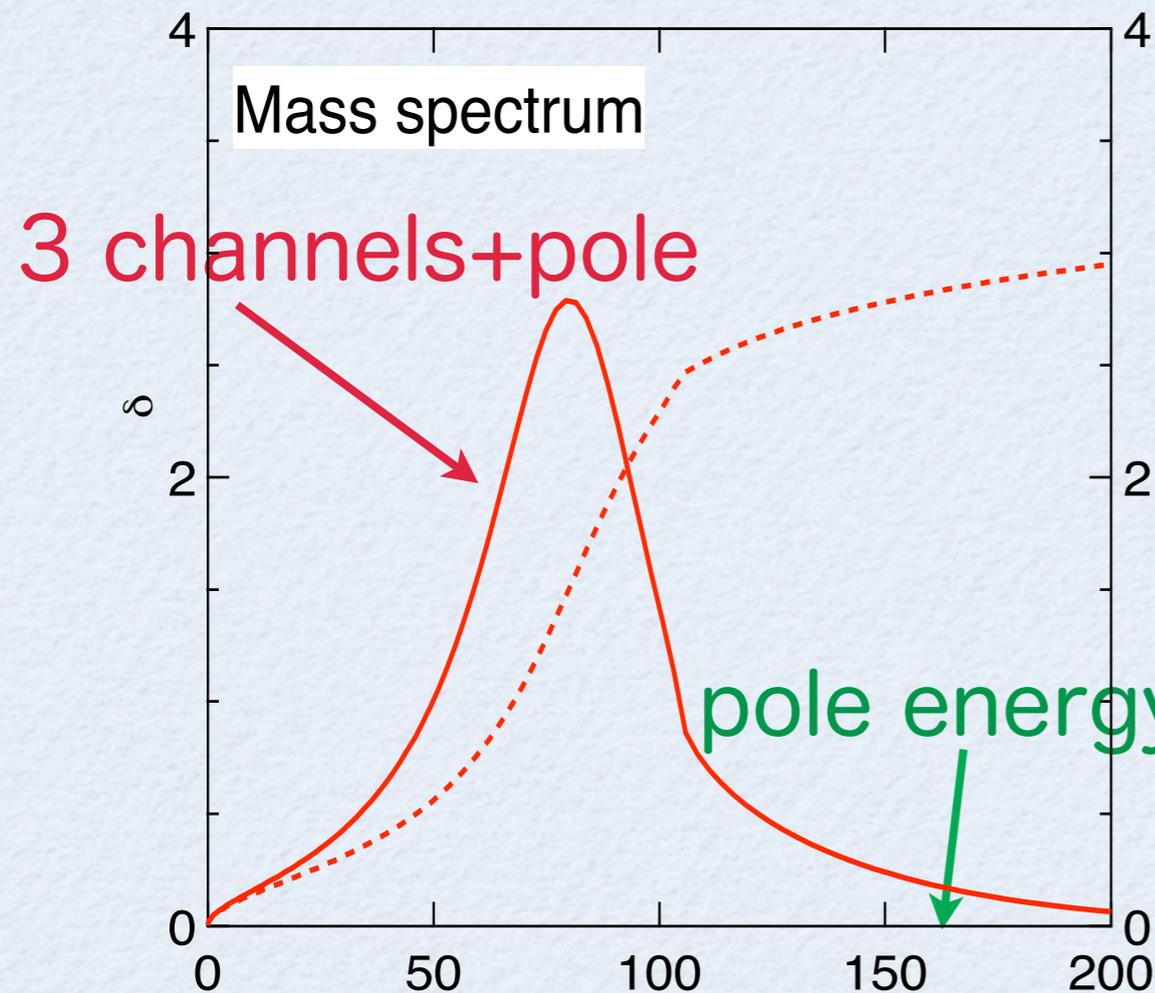
$$\text{Exp. } (-1.70 \pm 0.07) + i(0.68 \pm 0.04)$$



Simplified

color-magnetic-like

- nonrela, $-\langle \lambda.\lambda \sigma.\sigma \rangle$, with pole (1-coupling)



NK scattering length
 $= -0.63 + 0.33 i$
 (c.f. $-0.75 + 0.38 i$
 for the original QCM)

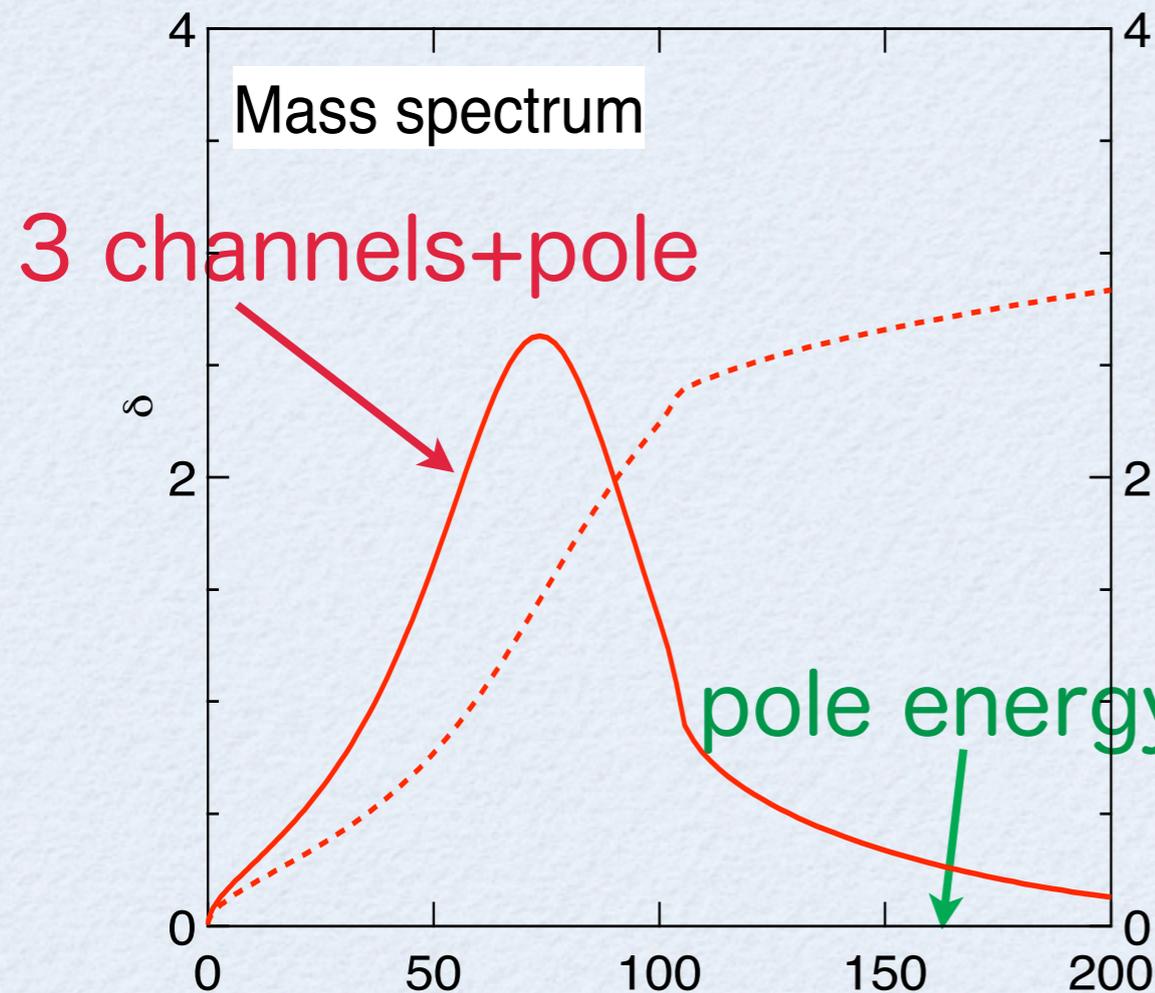
Exp. (-1.70 ± 0.07)
 $+ i(0.68 \pm 0.04)$



Simplified

color-magnetic-like

- semirela, $-\langle \lambda.\lambda \sigma.\sigma \rangle$, with pole (1-coupling)



NK scattering length
 $= -0.66 + 0.35 i$
 (c.f. $-0.75 + 0.38 i$
 for the original QCM)

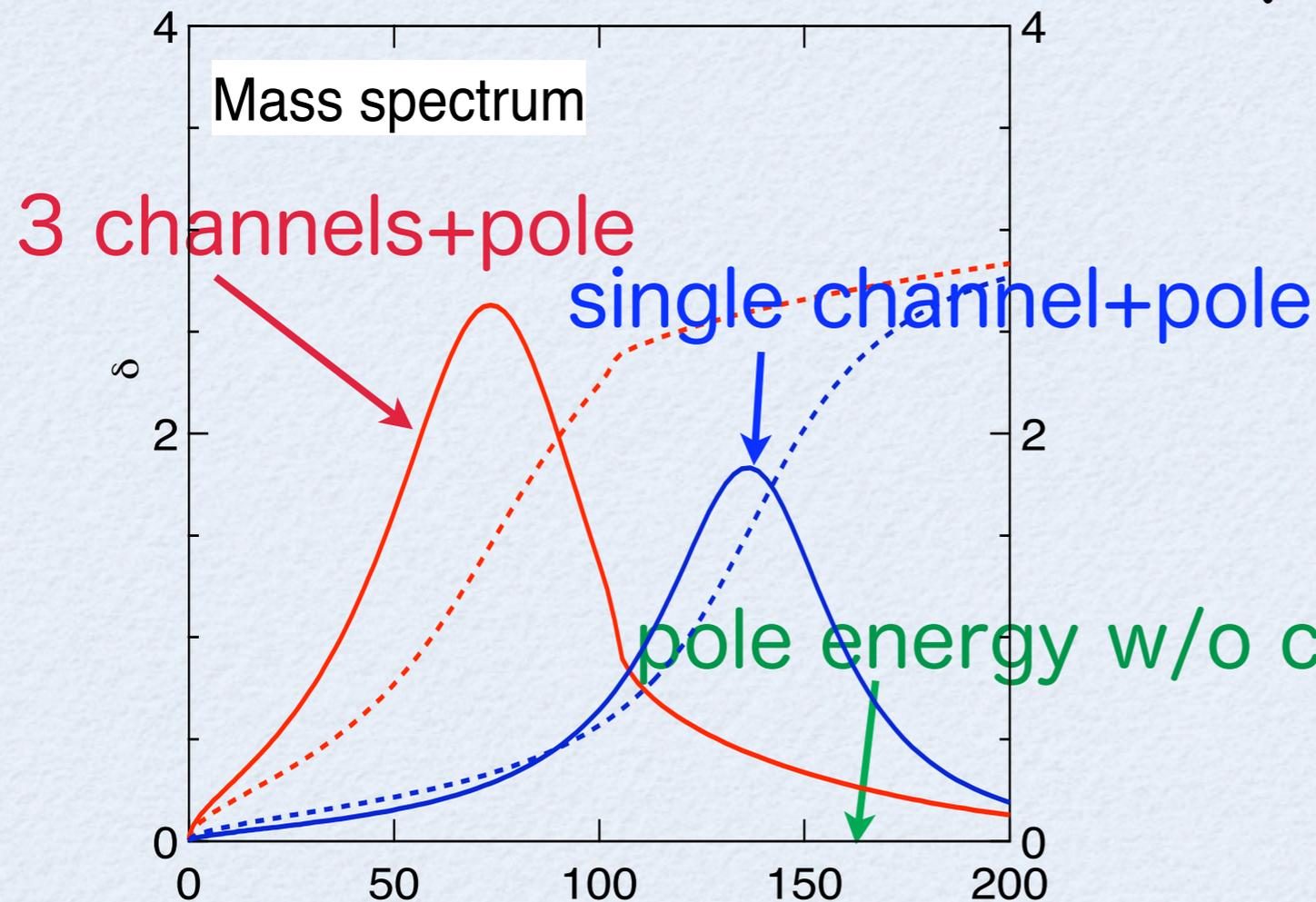
Exp. (-1.70 ± 0.07)
 $+ i(0.68 \pm 0.04)$



Simplified

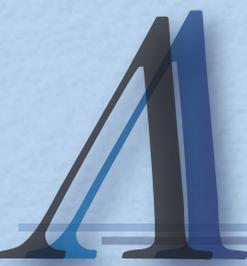
color-magnetic-like

- semirela, $-\langle \lambda.\lambda \sigma.\sigma \rangle$, with pole (1-coupling)



NK scattering length
 $= -0.68 + 0.46 i$
 (c.f. $-0.75 + 0.38 i$
 for the original QCM)

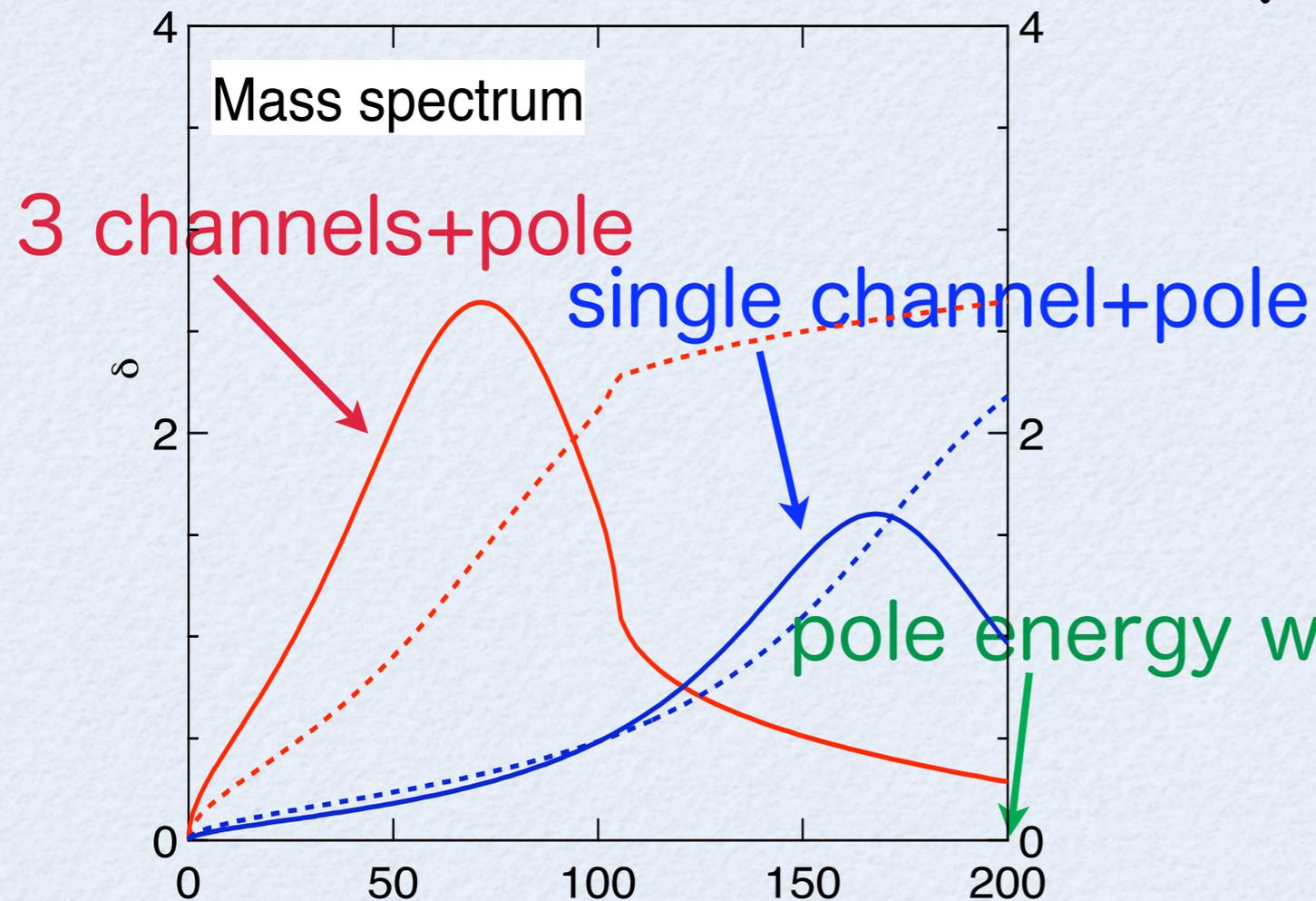
Exp. (-1.70 ± 0.07)
 $+ i(0.68 \pm 0.04)$



Simplified

color-magnetic-like

- semirela, $-\langle \lambda.\lambda \sigma.\sigma \rangle$, with pole (1-coupling)



NK scattering length
 $= -0.66 + 0.35 i$
 (c.f. $-0.75 + 0.38 i$
 for the original QCM)

Exp. (-1.70 ± 0.07)
 $+ i(0.68 \pm 0.04)$

Quark model v.s. Chiral unitary model

-  To have an internal structure is not important to obtain $\Lambda(1405)$ peak.
-  Kinematics is not important.
-  For the color-magnetic-like potential, one needs 'q³-pole'.
-  For FF-type potential, one may not need the 'q³-pole'. but the NKbar scattering length seems to become better.
-  The width of the peak is affected largely by the coupling of 'q³-pole'.



... and Outlook

- Other Baryon resonances ?
- Production and decay process ?
- More $(q\bar{q})$ -rich states ?

X(3872):

$(q\bar{q})-(c\bar{c})$ $(c\bar{q})-(q\bar{c})$ molecule

Sachiko Takeuchi

(Japan College of Social Work)

V.E. Lyubovitskij, Th. Gutsche, Amand Faessler
(Institut für Theoretische Physik, Univ Tübingen)

X Multiquark exotic systems

- Multiquark exotic systems
 - non qq̄ baryons
 - meson-baryon systems (Pentaquarks)
 - Θ^+ , Ξ , ...
 - negative-parity Λ ($q^3 - q\bar{q} + q^3$)
 - non- $q\bar{q}$ mesons
 - $(q\bar{q})^2$ systems *Barnea et al*
 - scalar mesons $< 1\text{ GeV}$
 - $qQ\bar{q}\bar{Q}$, $qs\bar{q}\bar{Q}$, systems

X Multiquark exotic systems

- Multiquark exotic systems
 - non qq̄ baryons
 - meson-baryon systems (Pentaquarks)
 - Θ^+ , Ξ , ...
 - negative-parity Λ ($q^3 - q\bar{q} + q^3$)
 - non- $q\bar{q}$ mesons
 - $(q\bar{q})^2$ systems *Barnea et al*
 - scalar mesons $< 1\text{ GeV}$
 - $qQ\bar{q}\bar{Q}$, $qs\bar{q}\bar{Q}$, systems



X Non $q\bar{q}$ meson candidates

••• $(q\bar{q})^2$ Mesons?

Refs. Particle Data Group

W.-M. Yao et al., J of Phys., G 33(2006)1

- $qc\bar{q}\bar{c}$?

$X(3872)$, Y 's, $Z(4430)^\pm$

- $qs\bar{q}\bar{c}$?

$D_{s0}^*(2317)^\pm$, $D_{s1}^*(2460)^\pm$,

$D_{s1}^*(2536)^\pm$, $D_{s2}^*(2573)^\pm$

- $qs\bar{q}\bar{s}$ or K^+K^- ?

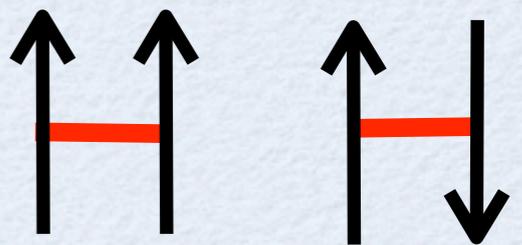
$a_0(980)$, $f_0(980)$, $X(1576)$

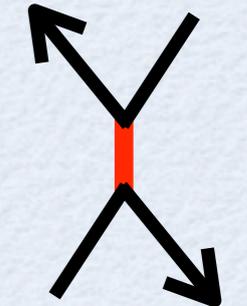
X

Hamiltonian for quarks

- H = Nonrela Kin + linear Conf + OGE + Ins + π, σ exch

- OGE

$$V_{\text{ele}} = \sum_{i < j} -\pi\alpha_s \frac{\lambda_i \cdot \lambda_j}{4} \frac{1}{2} \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} \right) \delta^3(\mathbf{r}_{ij})$$


$$V_{\text{CMI}} = \sum_{i < j} -\pi\alpha_s \frac{\lambda_i \cdot \lambda_j}{4} \sigma_i \cdot \sigma_j \frac{2\xi_i \xi_j}{3m_u^2} \delta^3(\mathbf{r}_{ij})$$


$$V_{\text{OGE}}^{(a)} = \sum_{i < j} \frac{1}{24} \left(\frac{16}{3} + \lambda_i \cdot \lambda_j \right) (3 + \sigma_i \cdot \sigma_j) \mathcal{P}_{ij} \pi\alpha_s \frac{1}{4mm'} \delta^3(\mathbf{r}_{ij})$$

X

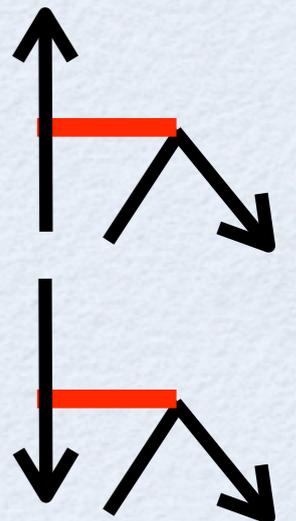
Hamiltonian for quarks

- H = Nonrela Kin + linear Conf + OGE + Ins + π, σ exch
- $q(q\bar{q}) \rightarrow q, \bar{q}(q\bar{q}) \rightarrow \bar{q}q$ transfer interaction

$$V_{i;j\bar{k}} = \lambda_i \cdot \lambda_{\bar{k}j} \frac{\alpha_s}{4} \frac{\pi}{m_a^2} \left[\left(\frac{\mathbf{k}}{2m_a} - \frac{\mathbf{p}_i + \mathbf{p}'_i + i\boldsymbol{\sigma}_i \times \mathbf{k}}{2m_i} \right) \cdot \boldsymbol{\sigma}_{\bar{k}j} \right] \delta_{\bar{k}j}^f$$

- consider only btw $(0s)^4$ and $(0p)$

$$V_{tr} = |(q\bar{q})^2(0s)^4\rangle V_{OGE} \langle q^2(0p)|$$



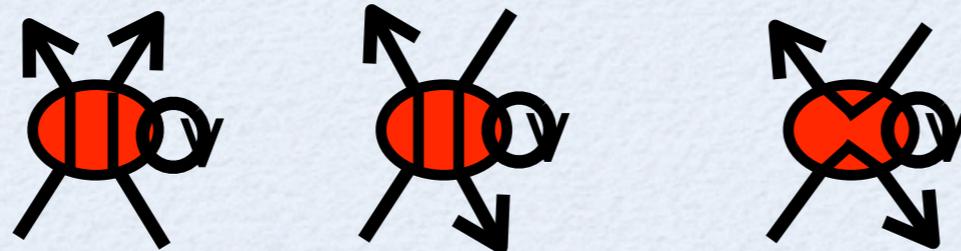
X

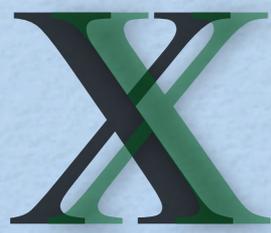
Hamiltonian for quarks

- **Ins** (affects only light quark pairs.)

$$V_{\text{INS}} = \sum_{i < j} \frac{V_0}{2} \xi_i \xi_j \left(1 + \kappa \frac{3}{32} \lambda_i \cdot \lambda_j + \frac{9}{32} \lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j \right) \mathcal{P}'_{ij} \delta^3(\mathbf{r}_{ij})$$

$$V_{\text{INS}}^{(a)} = \sum_{i < j} -\frac{V_0}{2} \xi_i \xi_j \mathcal{P} \mathcal{P}'' \left(1 - \frac{3}{32} \lambda_i \cdot \lambda_j + \frac{9}{32} \lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j \right) \delta^3(\mathbf{r}_{ij})$$





Realistic Calc. - mesons

$$m_u = 313 \text{ MeV}$$

$$m_s = 600 \text{ MeV}$$

$$m_c = 1250 \text{ MeV}$$

$$a_{\text{conf}} = 172.4 \text{ MeV/fm}$$

$$\alpha_s = 0.73$$

$$V_{0,\text{ins}} = -143 \text{ MeV/fm}^3 \quad (p_{\text{III}}=0.4)$$

$$\xi_{\text{su}}=1$$

$$\xi_{\text{cu}}=0.586$$

$$\xi_{\text{cs}}=0.489$$

$$\xi_{\text{cc}}=0.198$$

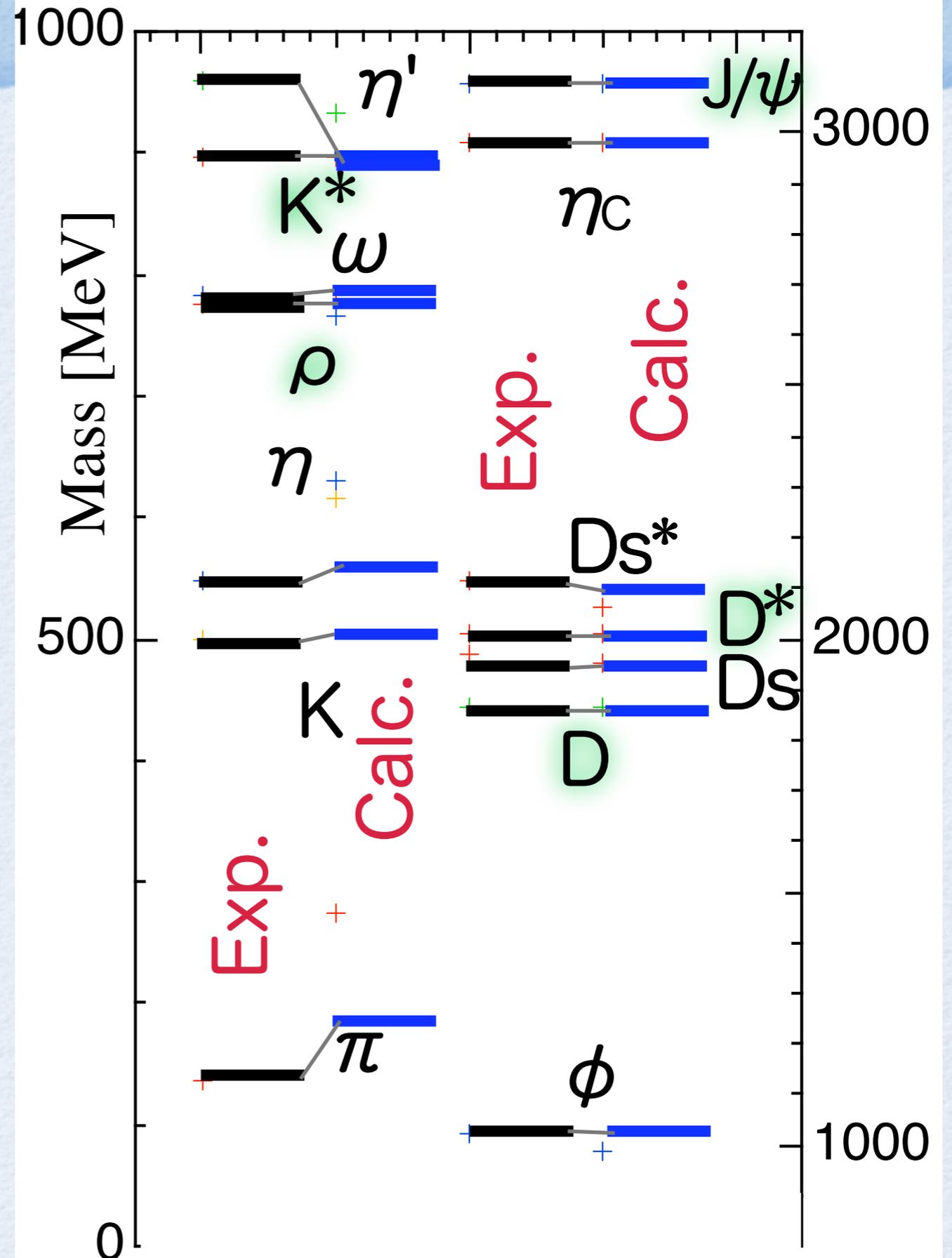
$$\Lambda_g = 3.3 \text{ fm}^{-1}$$

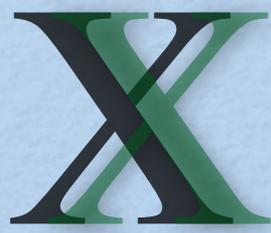
$$g_8^2/4\pi = 0.69$$

$$m_\sigma = 675 \text{ MeV}$$

$$\Lambda_\sigma = 5.3 \text{ fm}^{-1}$$

$$\Lambda_\pi = 1.1 \text{ fm}^{-1}$$





Realistic Calc. - mesons

$$m_u = 313 \text{ MeV}$$

$$m_s = 600 \text{ MeV}$$

$$m_c = 1250 \text{ MeV}$$

$$a_{\text{conf}} = 172.4 \text{ MeV/fm}$$

$$\alpha_s = 0.73$$

$$V_{0,\text{ins}} = -143 \text{ MeV/fm}^3 \quad (p_{\text{III}}=0.4)$$

$$\xi_{\text{su}}=1$$

$$\xi_{\text{cu}}=0.586$$

$$\xi_{\text{cs}}=0.489$$

$$\xi_{\text{cc}}=0.198$$

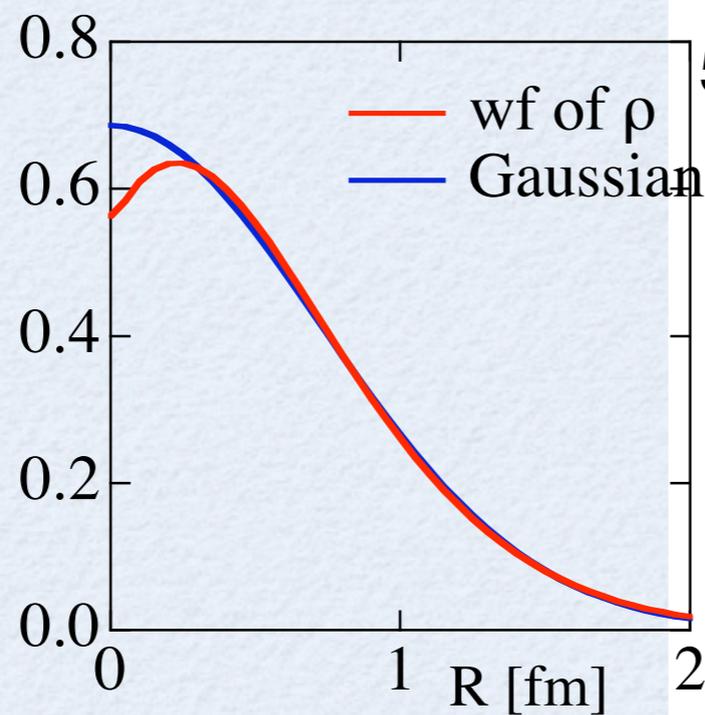
$$\Lambda_g = 3.3 \text{ fm}^{-1}$$

$$g_8^2/4\pi = 0.69$$

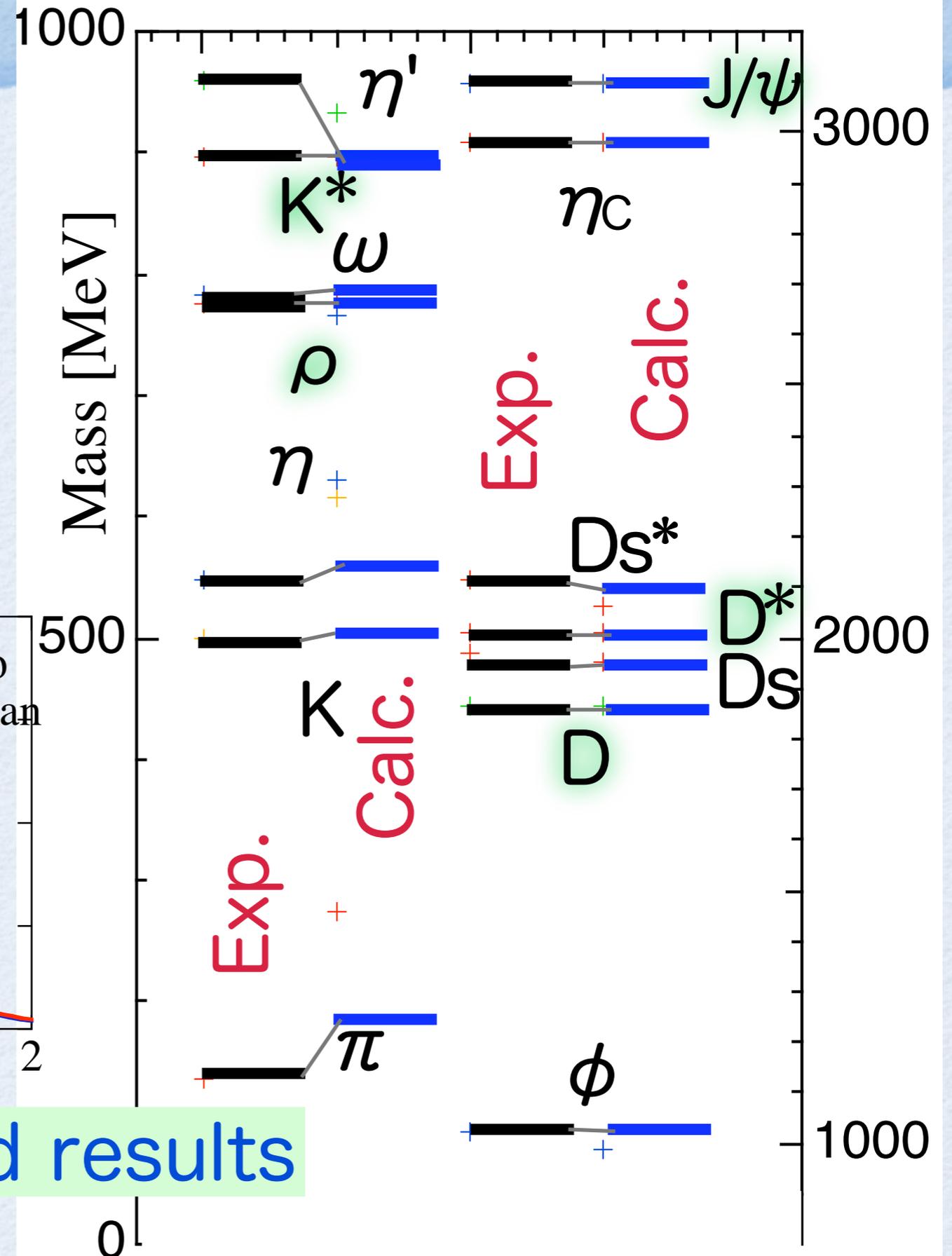
$$m_\sigma = 675 \text{ MeV}$$

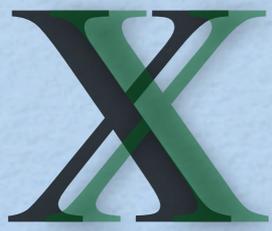
$$\Lambda_\sigma = 5.3 \text{ fm}^{-1}$$

$$\Lambda_\pi = 1.1 \text{ fm}^{-1}$$



Solved results



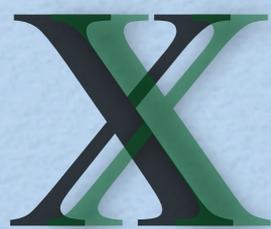


Estimate by $(0s)^4$

Effects of the interaction on $q\bar{q}$ pairs

Rough sizes are obtained from $N\Delta$, and $\eta'-\eta$ mass differences.

Color	Spin	Flavor	CMI	OgE-a	Ins	E[MeV]	States
1	0	1	-16	0	12	84	η_1
1	0	8	-16	0	-6	-327	$\pi \eta_8$
1	1	1	16/3	0	0	63	ω
1	1	8	16/3	0	0	63	ρ
8	0	1	2	0	3/4	41	
8	0	8	2	0	-3/8	15	
8	1	1	-2/3	9/2	9/4	97	
8	1	8	-2/3	0	-9/8	-34	In $J^{PC} = 0^{++}, 1^{+-}, 1^{++}, 2^{++}$

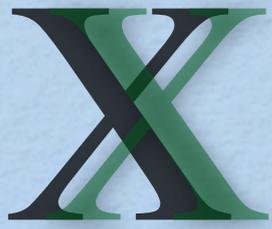


Estimate by $(0s)^4$

Effects of the interaction on $q\bar{q}$ pairs

Rough sizes are obtained from $N\Delta$, and $\eta'-\eta$ mass differences.

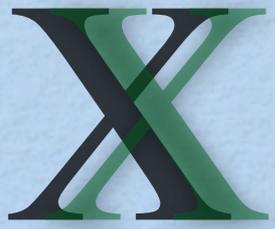
Color	Spin	Flavor	CMI	OgE-a	Ins	E[MeV]	States
1	0	1	-16	0	12	84	η_1
1	0	8	-16	0	-6	-327	$\pi \eta_8$
1	1	1	16/3	0	0	63	ω
1	1	8	16/3	0	0	63	ρ
8	0	1	2	0	3/4	41	
8	0	8	2	0	-3/8	15	
8	1	1	-2/3	9/2	9/4	97	
8	1	8	-2/3	0	-9/8	-34	In $J^{PC} = 0^{++}, 1^{+-}, 1^{++}, 2^{++}$



Estimate by $(0s)^4$

- Difference between the flavor singlet and octet pairs comes from annihilating diagrams.
- Not yet obtained from Lattice or QCDSR.

Color	Spin	Flavor	CMI	OgE-a	Ins	E[MeV]	State
8	1	1	-2/3	9/2	9/4	97	
8	1	8	-2/3	0	-9/8	-34	In $J^{PC} = 0^{++}, 1^{+-}, 1^{++}, 2^{++}$



X(3872) facts

X(3872)

- $M(X) = 3871.4 \pm 0.6 \text{ MeV}$
- $\Gamma < 2.3 \text{ MeV}$
- $I^G(J^{PC}) = 0^?(1^{++})$ $I=0$ as No X^-
- found in $B^\pm \rightarrow K^\pm X$, $p\bar{p} \rightarrow X$
- decay mode $X \rightarrow J/\psi \pi \pi$, $J/\psi \pi \pi \pi$, $J/\psi \gamma$
- $\Gamma(X \rightarrow J/\psi \gamma) / \Gamma(X \rightarrow J/\psi \pi^2) = 0.14 \pm 0.05$

X

X(3872) facts

X(3872) Threshold

- $J/\psi \omega = 3879.5 \text{ MeV}$

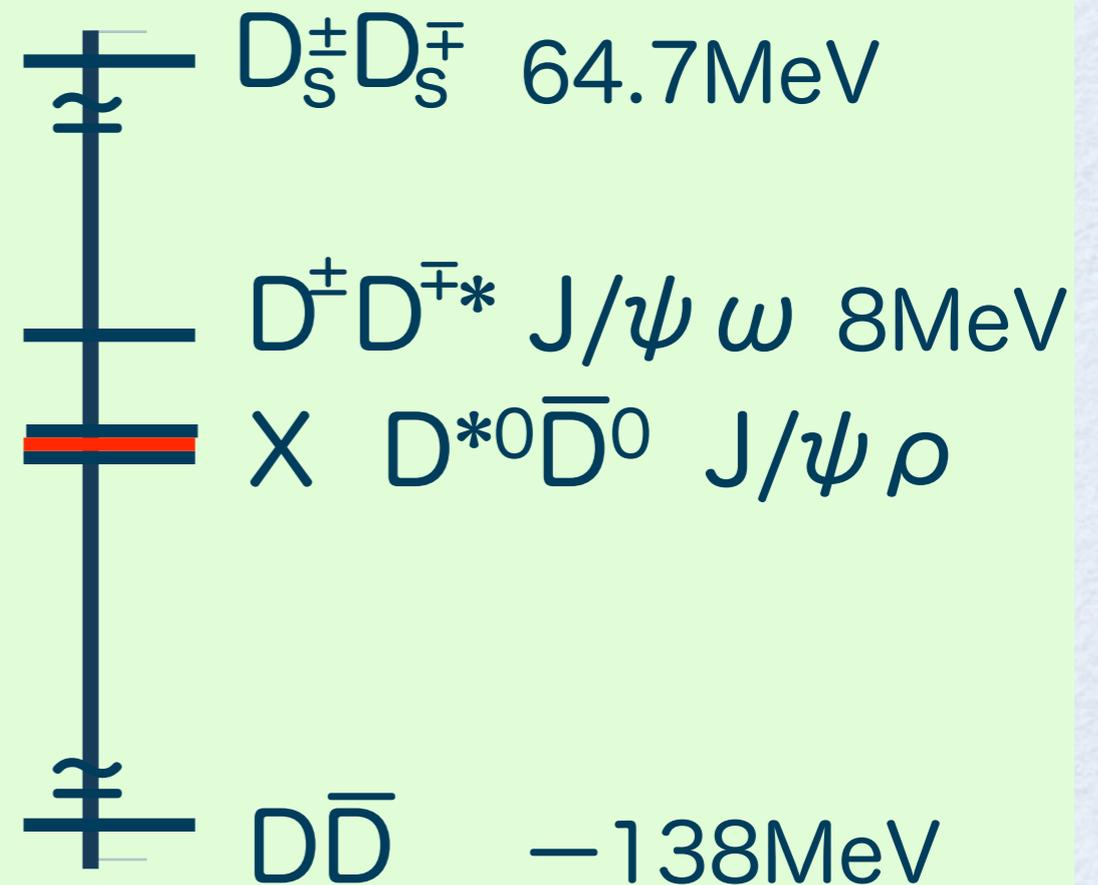
- $D^\pm D^{*\mp} = 3879.1 \text{ MeV}$

- $J/\psi \rho = 3872.7 \text{ MeV}$

- $D^0 \bar{D}^{*0} = 3871.3 \text{ MeV}$

isospin violated.

$u\bar{u}$ rather than $I=0,1$?



X Non $q\bar{q}$ meson candidates?

●●● $(q\bar{q})^2$ Mesons?

● $qc\bar{q}\bar{c}$?

$X(3872)$

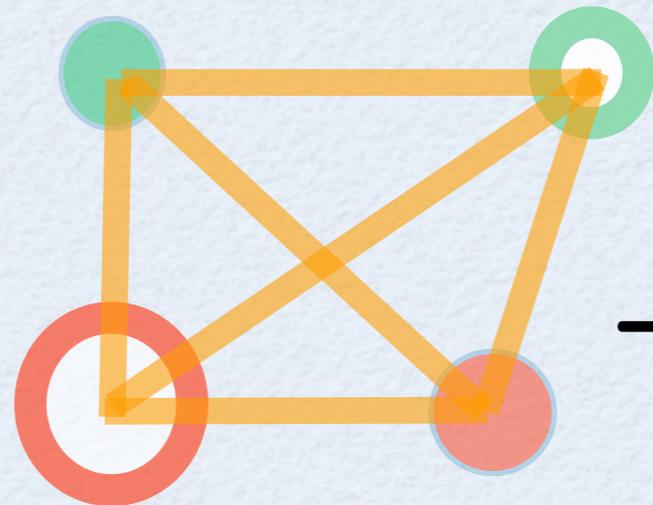
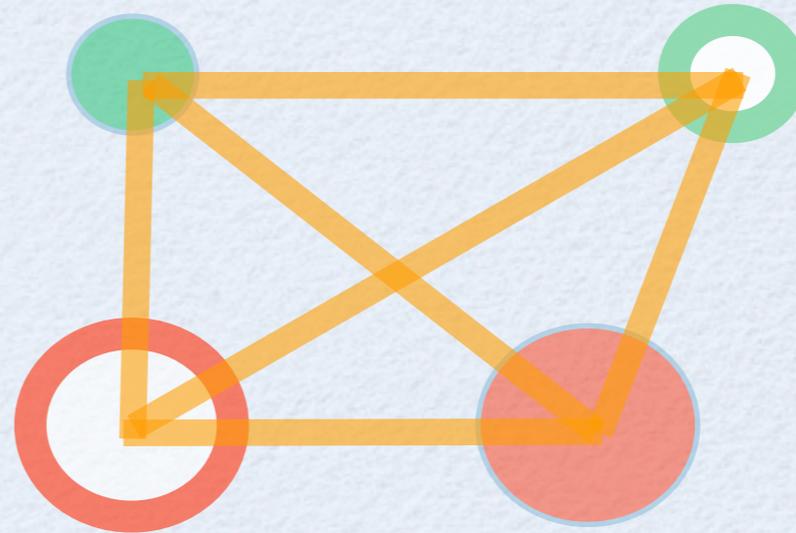
● $qs\bar{q}\bar{c}$?

$D_{s2}^*(2573)^\pm$

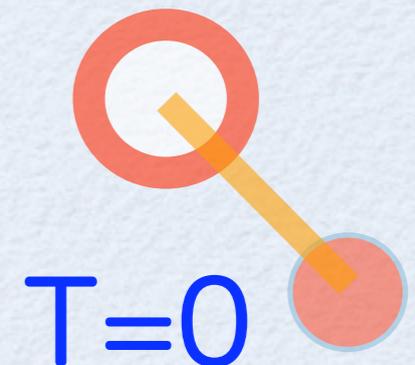
$D_{s1}^*(2536)^\pm$

$D_{s1}^*(2460)^\pm$

$D_{s0}^*(2317)^\pm$



+



$T=0$

P-wave

X

Realistic Calc. - $q\bar{q}c\bar{c}$

Stochastic variational approach

$$\Psi = \sum c_{k,m} \psi_m^c \psi^f \psi^\sigma \psi_k^{orb}$$

$$\psi_m^c = (\psi^c(1)\psi^c(3))(\psi^c(2)\psi^c(4)), (\psi^c(1)\lambda^a\psi^c(3))(\psi^c(2)\lambda^a\psi^c(4))$$

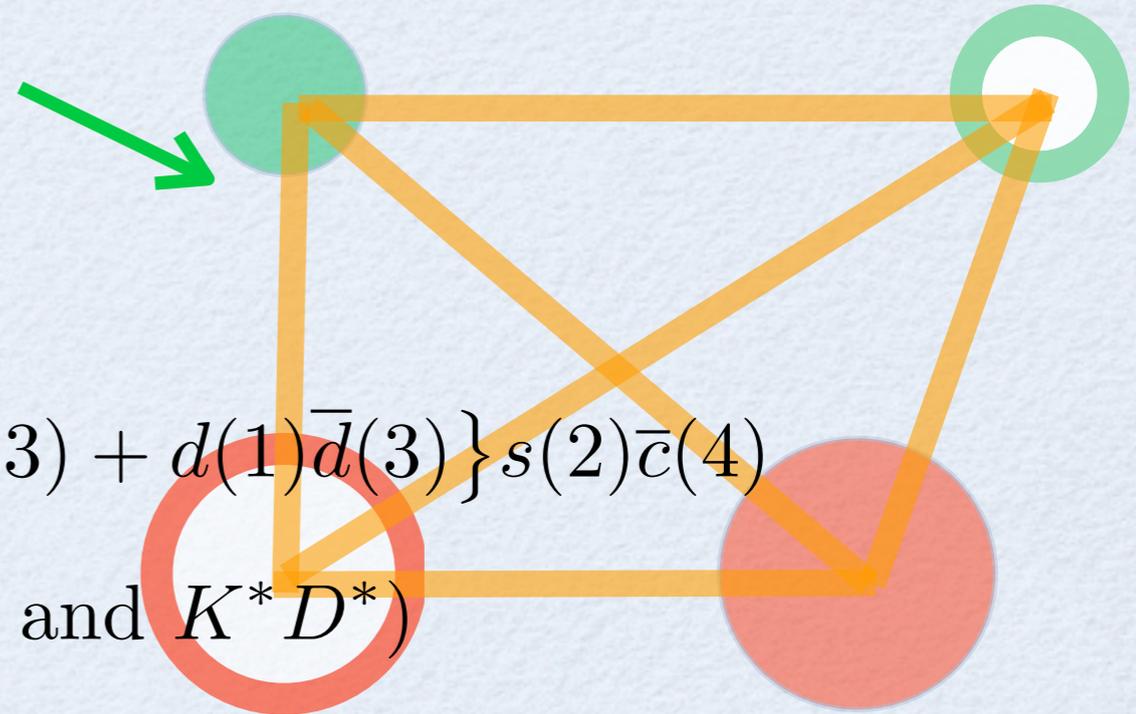
$$\psi^f = u(1)c(2)\bar{d}(3)\bar{c}(4), \frac{1}{\sqrt{2}} \{u(1)\bar{u}(3) + d(1)\bar{d}(3)\}c(2)\bar{c}(4)$$

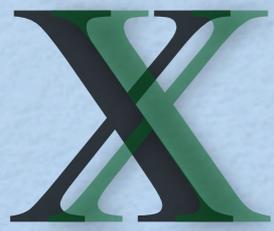
$$\psi^{orb} = \sum_k c_k \exp \left[- \sum_{i<j} \beta_{ij}^{(k)} r_{ij}^2 \right]$$

$$\psi^\sigma = |(11)1\rangle \quad (X : J/\psi, \rho)$$

$$\psi^f = u(1)s(2)\bar{d}(3)\bar{c}(4), \frac{1}{\sqrt{2}} \{u(1)\bar{u}(3) + d(1)\bar{d}(3)\}s(2)\bar{c}(4)$$

$$\psi^\sigma = |(00)J\rangle, |(11)J\rangle \quad (D_{sJ} : KD \text{ and } K^*D^*)$$

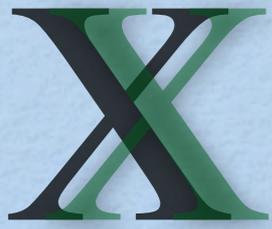




Realistic Calc. - $q\bar{q}c\bar{c}$

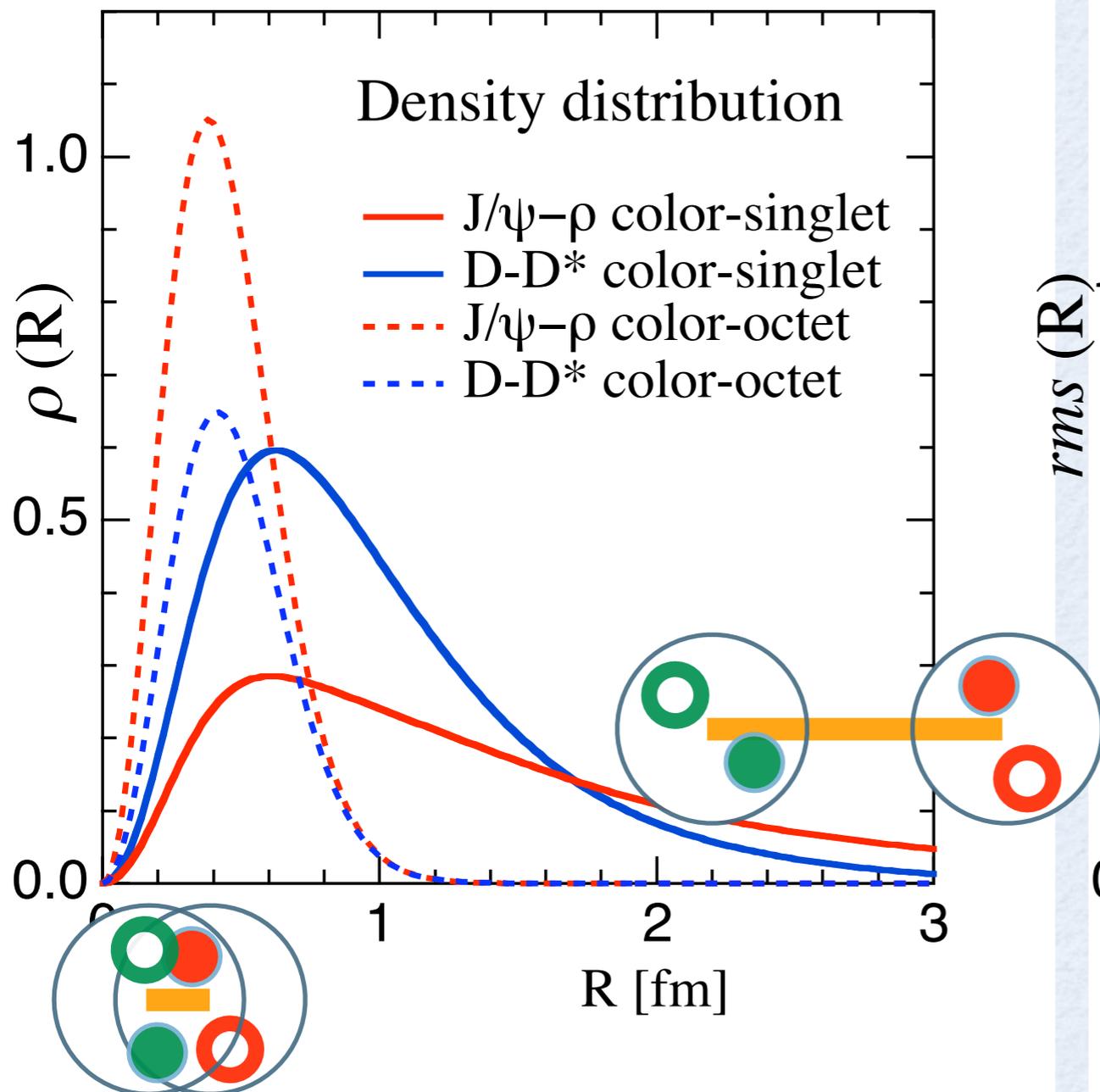
● Binding Energy: X (only $qc\bar{q}\bar{c}$ compo)

$I J^{PC}$	weaker meson-exch	stronger meson-exch
$11^{++} (J/\psi \rho)$	5 MeV	26 MeV
$01^{++} (J/\psi \omega)$	Not Bound	5 MeV

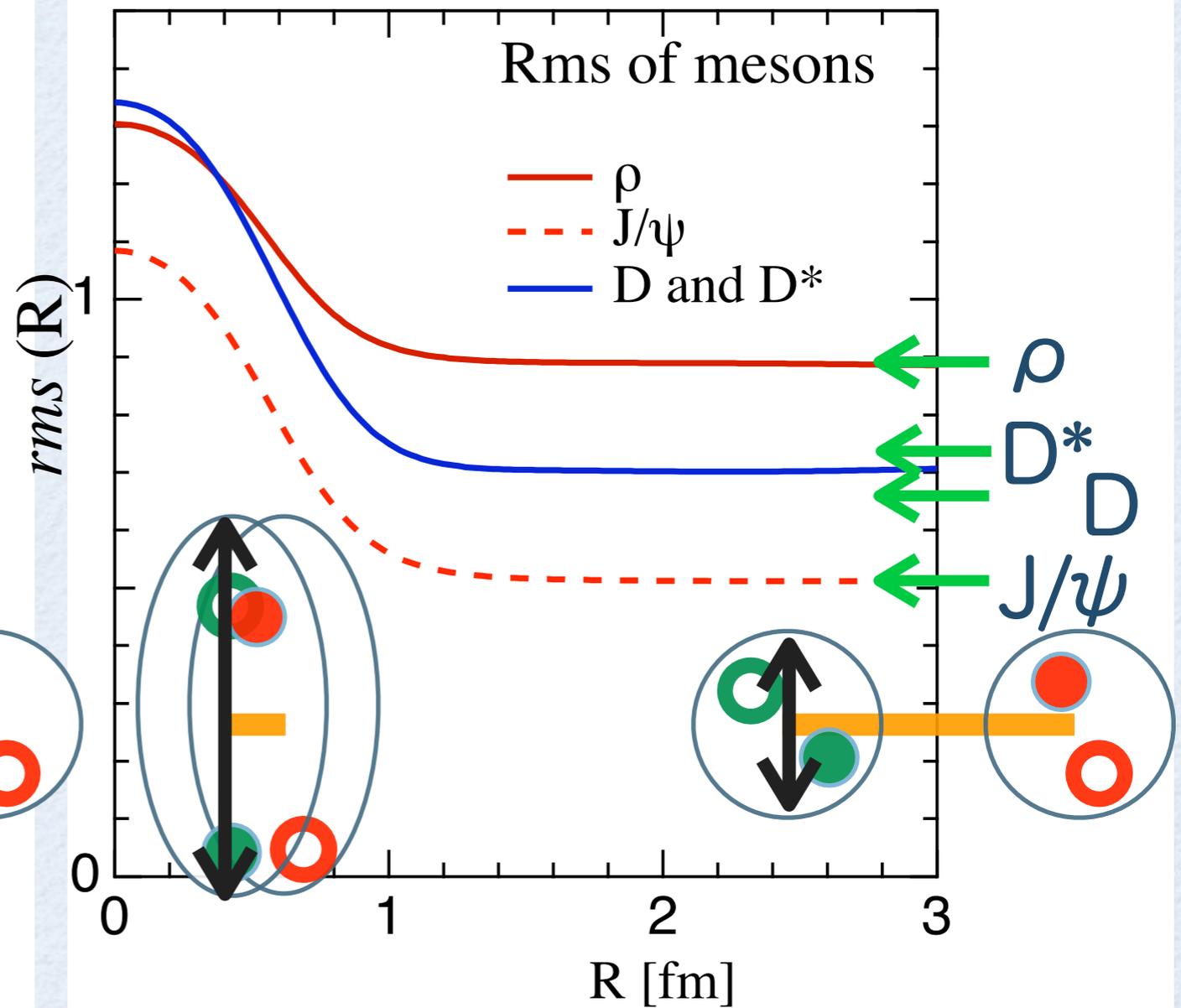


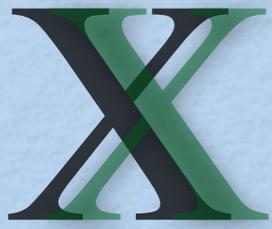
Density distri & rms

 $\langle \delta (R_{mm'} - X) \rangle$



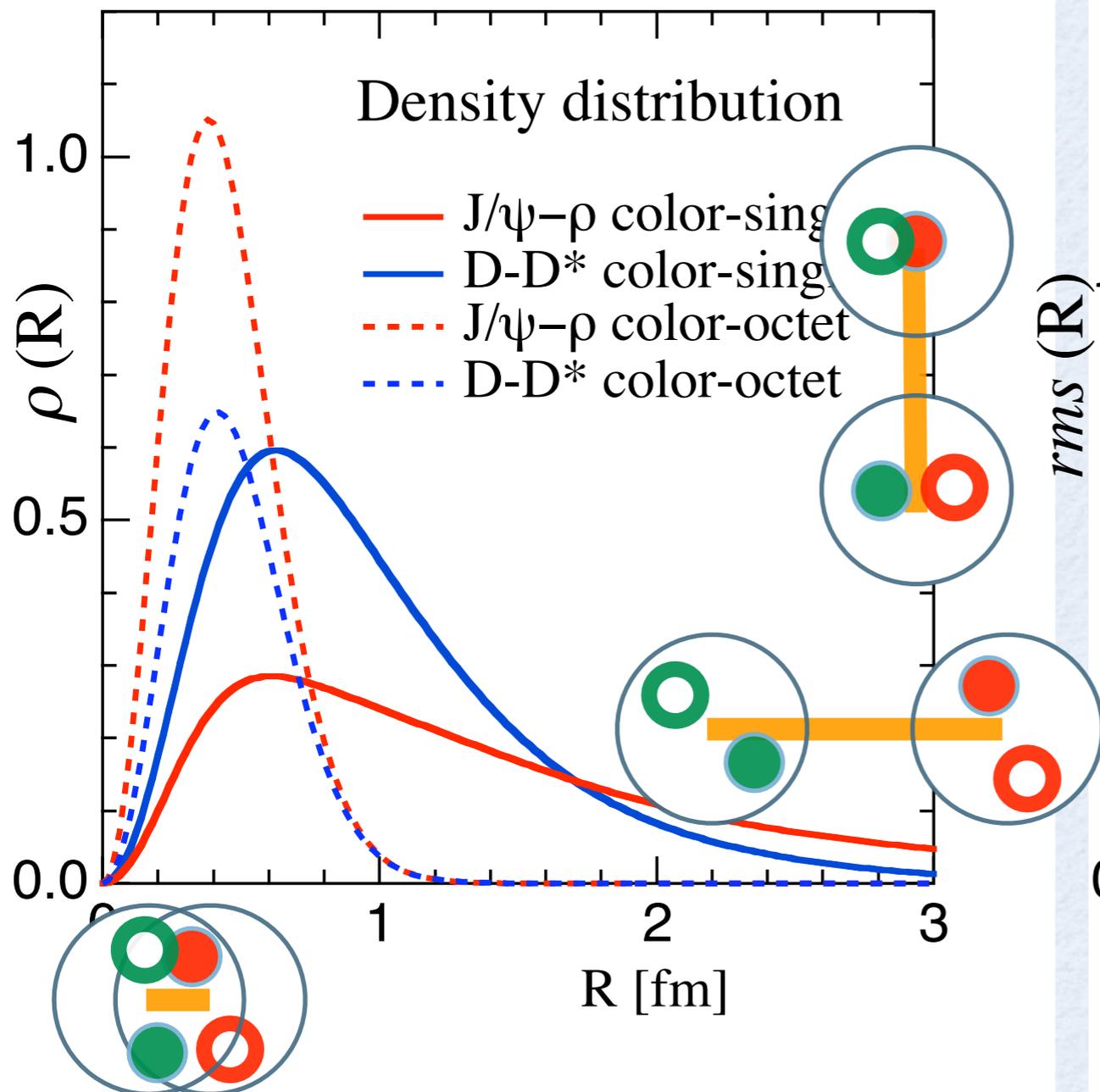
$\sqrt{\langle \delta (R_{mm'} - X) r_{ij}^2 \rangle}$



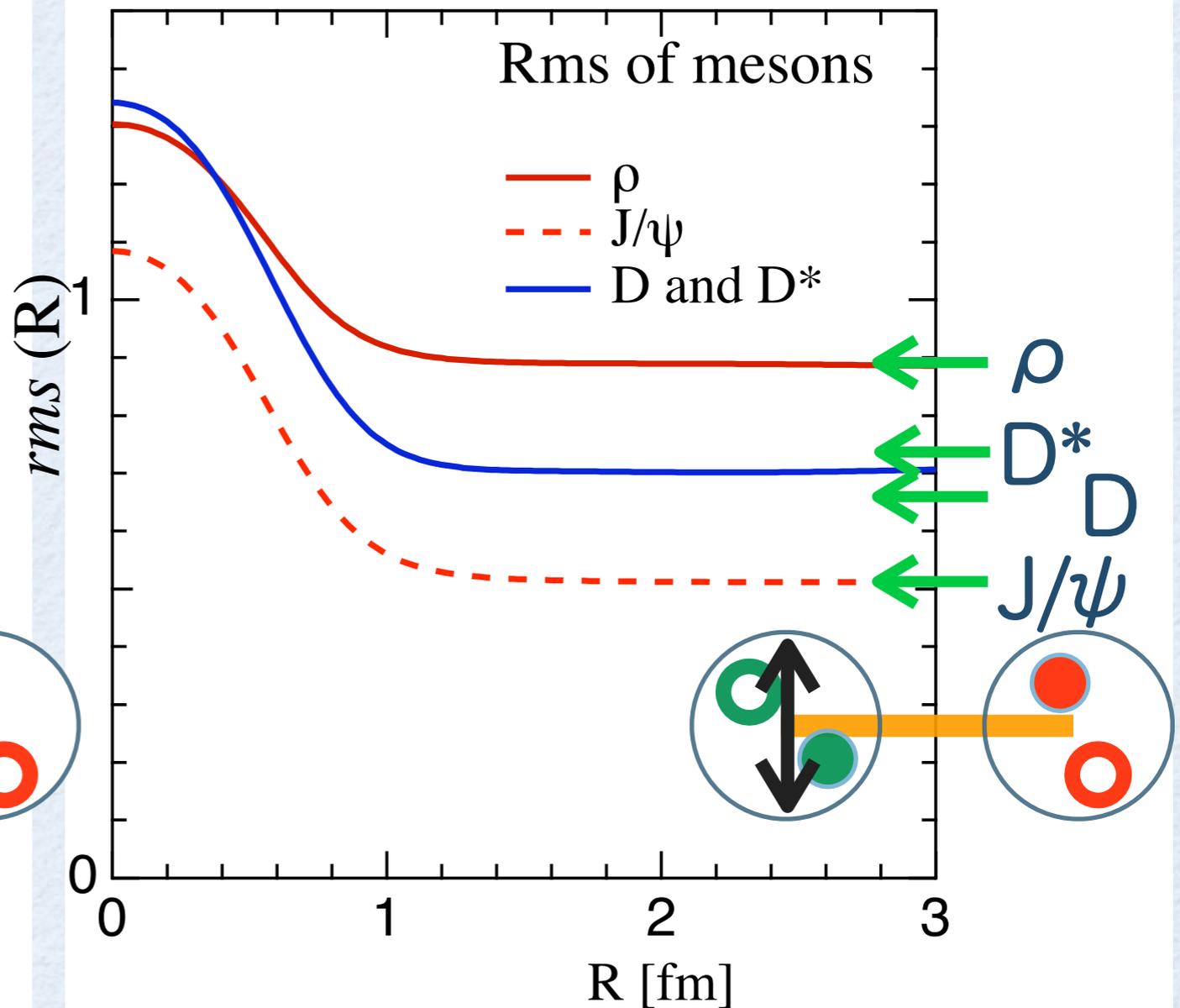


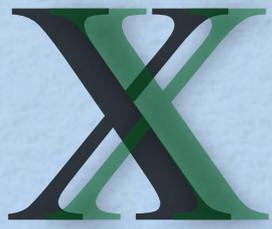
Density distri & rms

 $\langle \delta (R_{mm'} - X) \rangle$



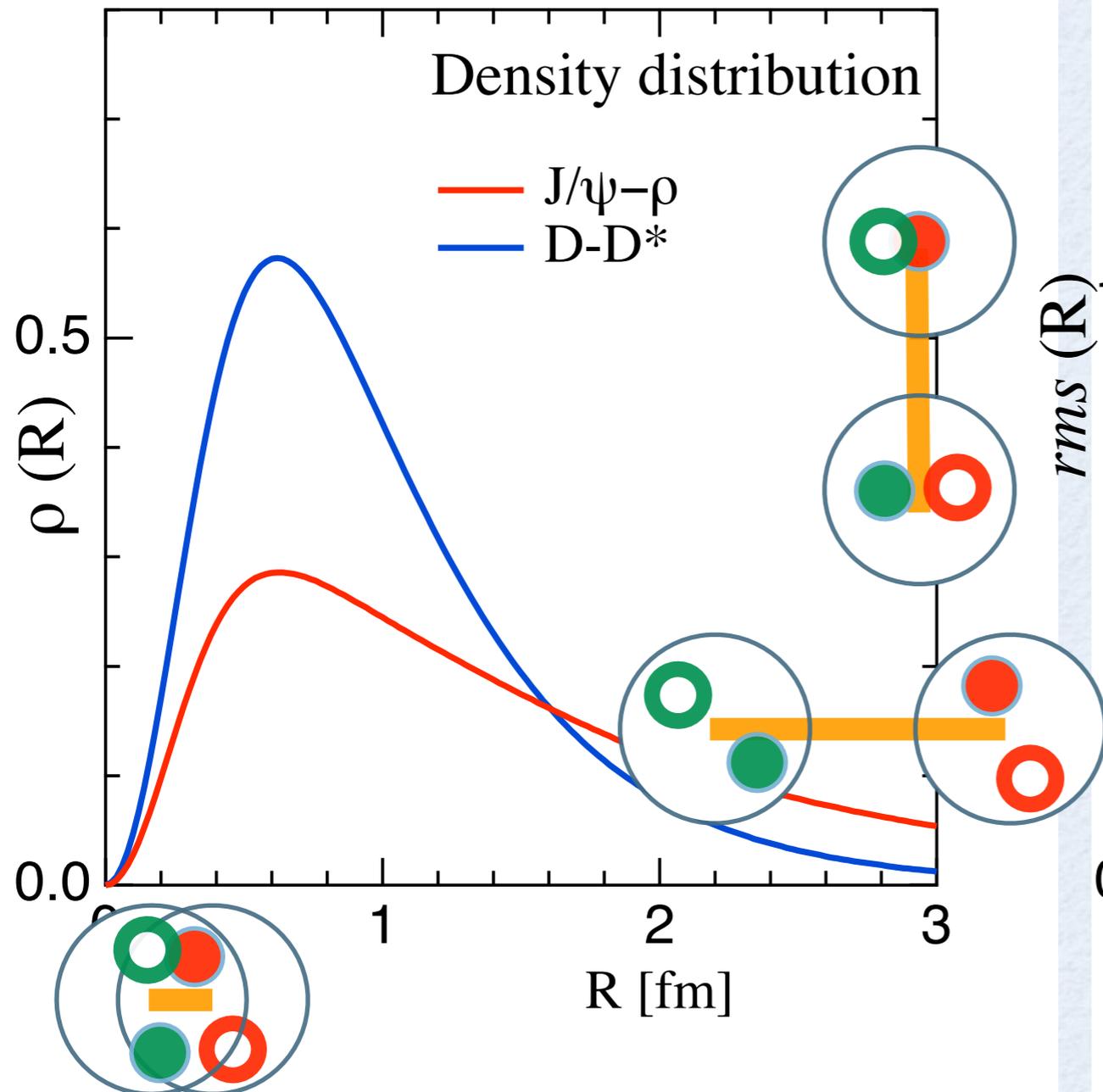
$\sqrt{\langle \delta (R_{mm'} - X) r_{ij}^2 \rangle}$



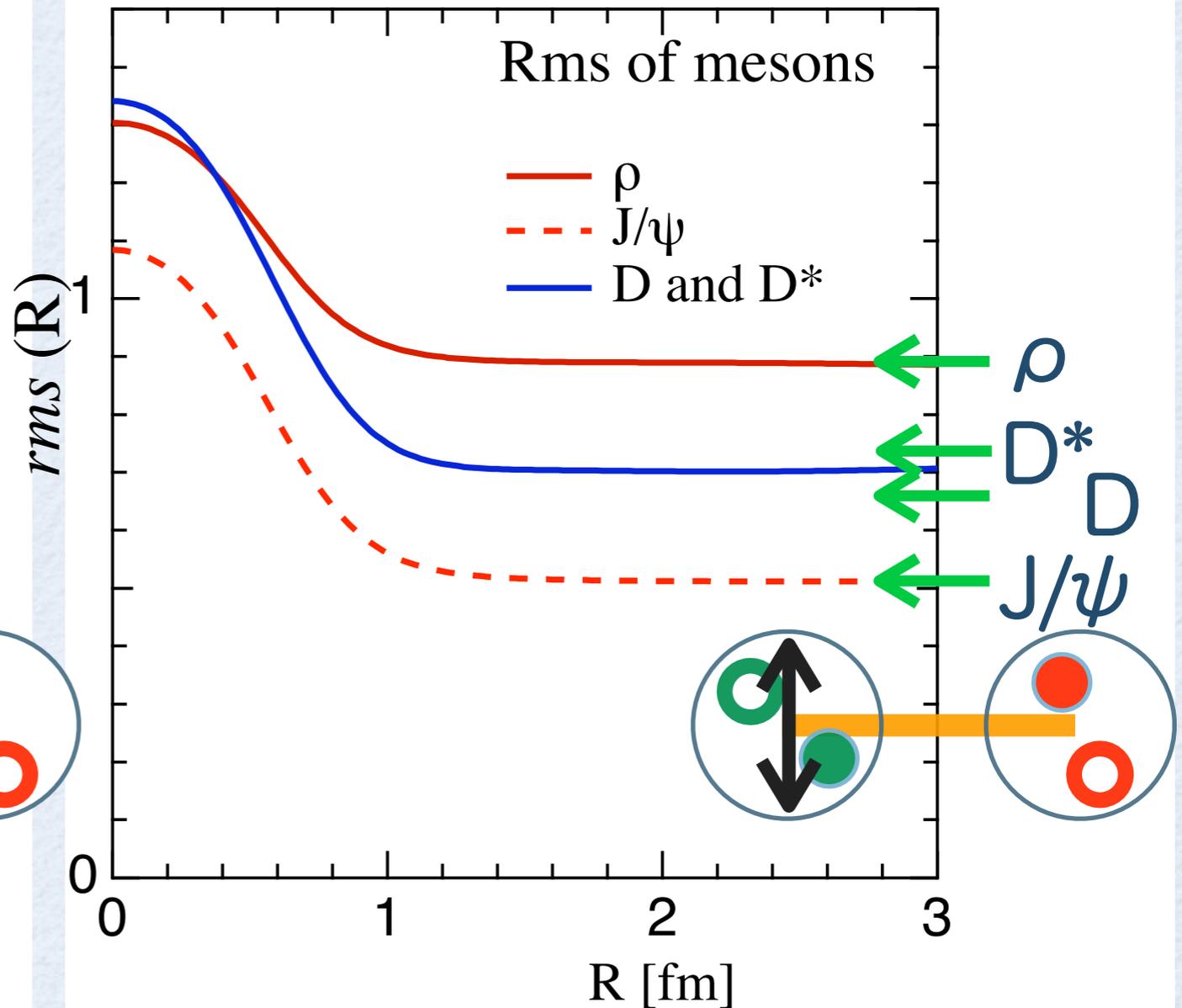


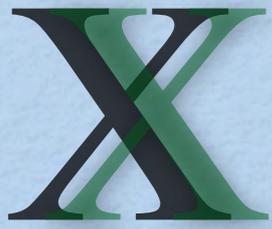
Density distri & rms

$$\langle \delta (R_{mm'} - X) \rangle$$



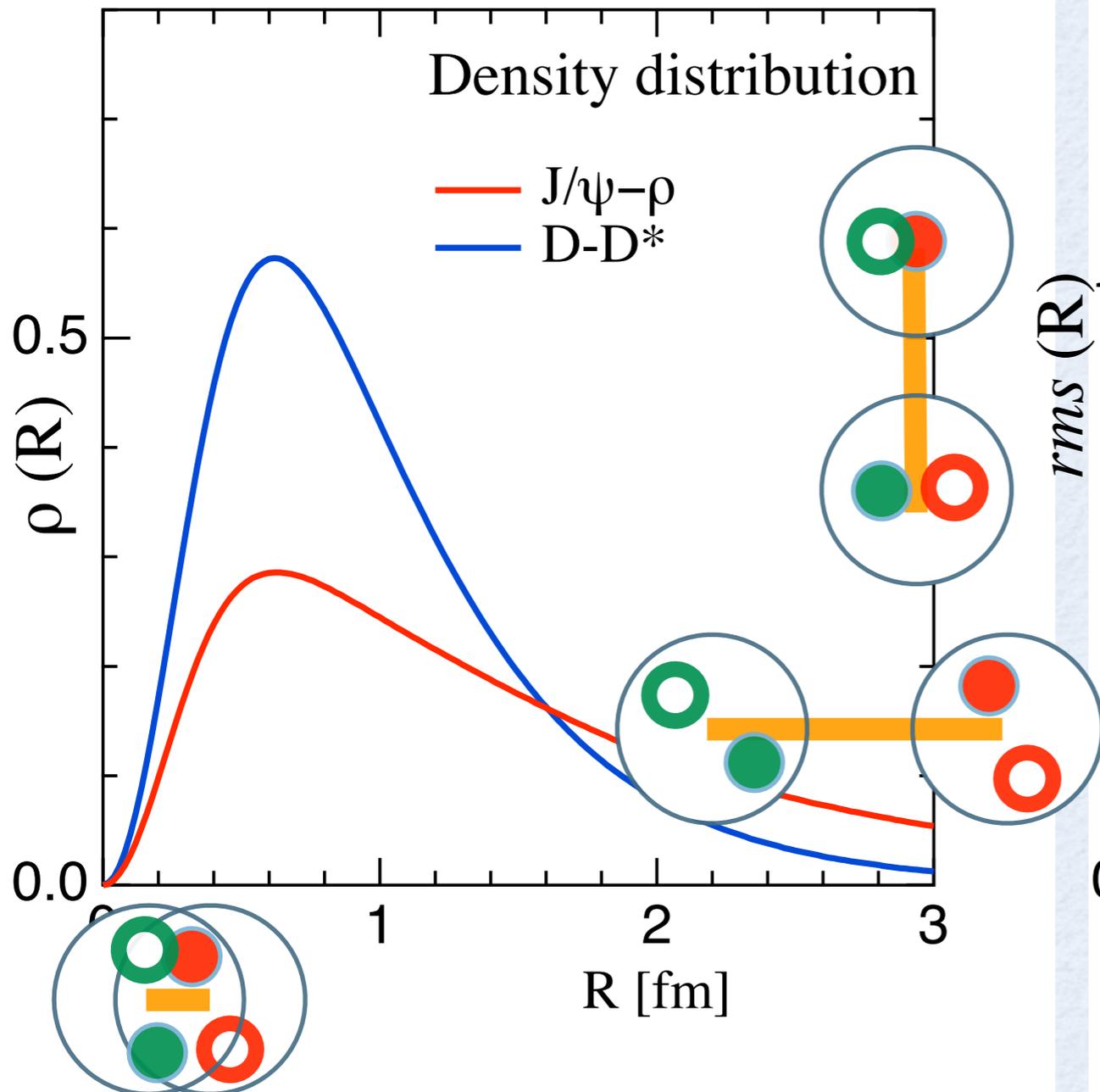
$$\sqrt{\langle \delta (R_{mm'} - X) r_{ij}^2 \rangle}$$



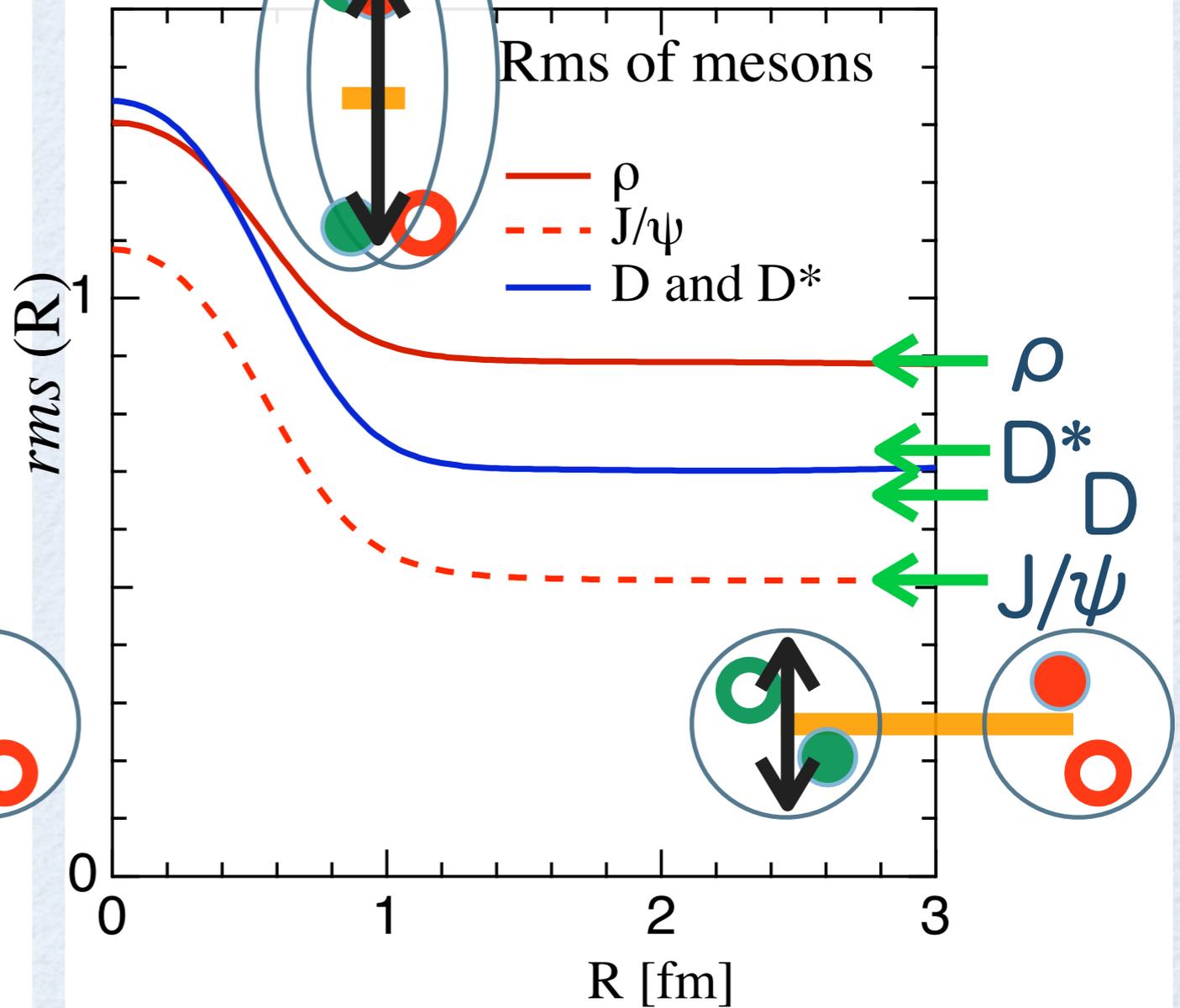


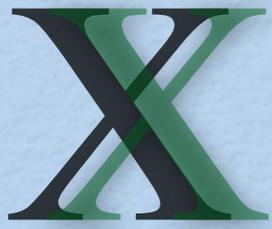
Density distri & rms

$\langle \delta (R_{mm'} - X) \rangle$



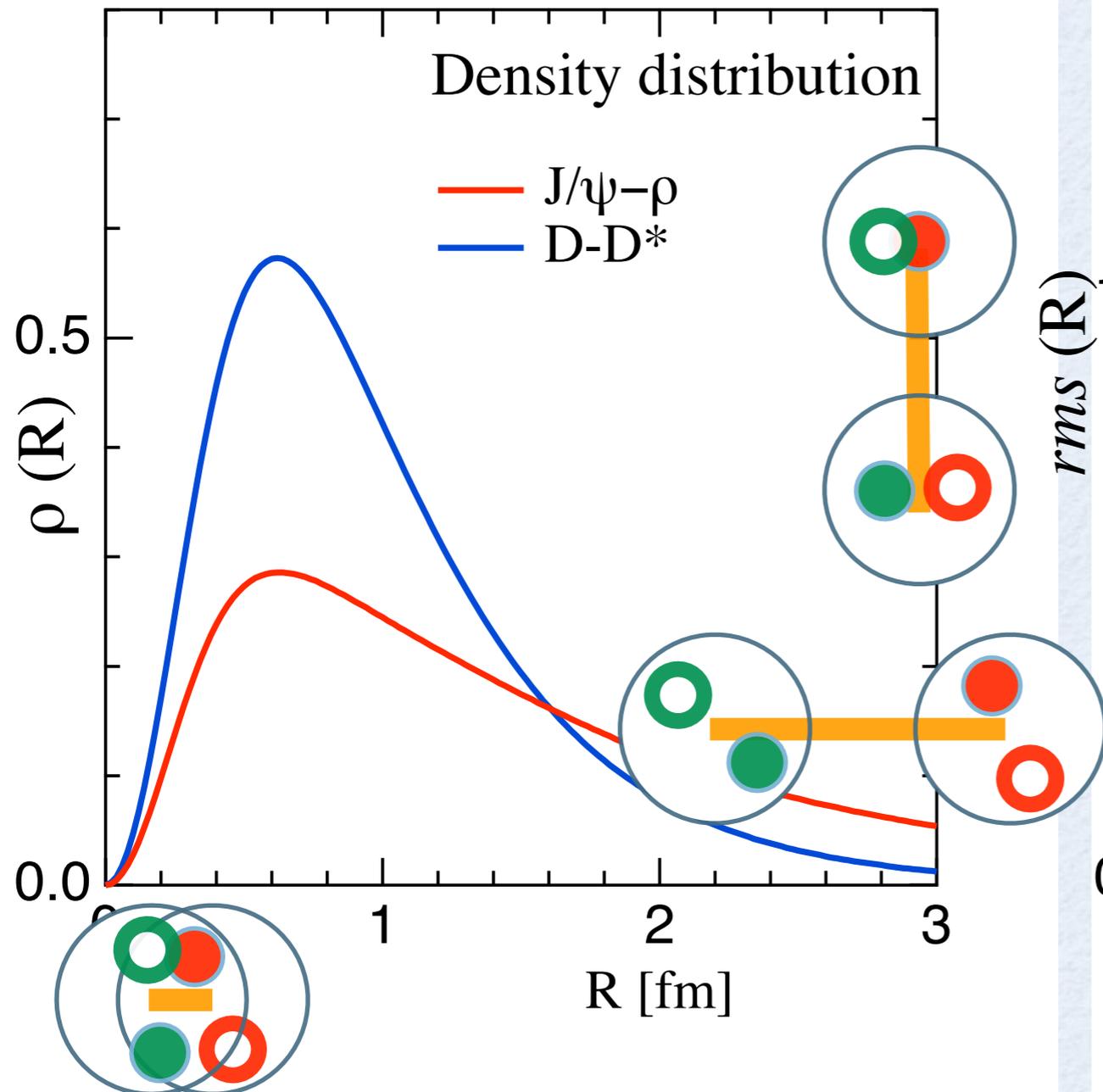
$\sqrt{\langle \delta (R_{mm'} - X) r_{ij}^2 \rangle}$



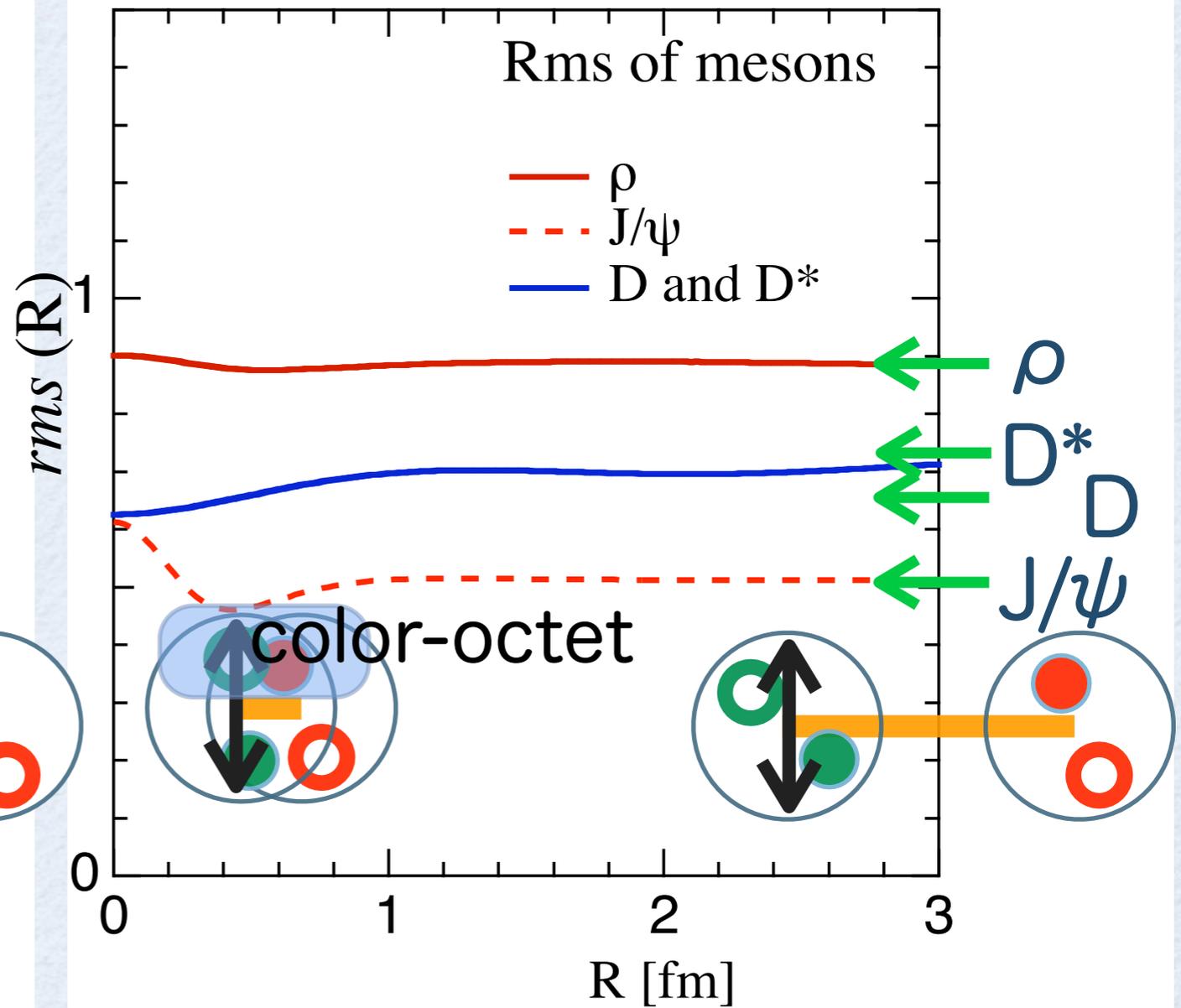


Density distri & rms

 $\langle \delta (R_{mm'} - X) \rangle$



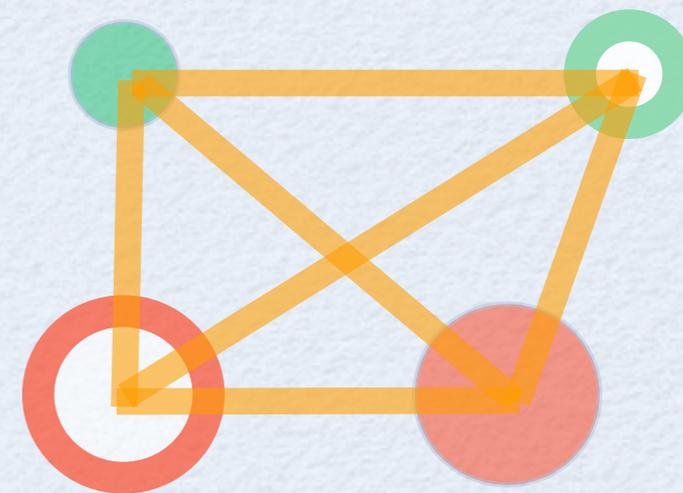
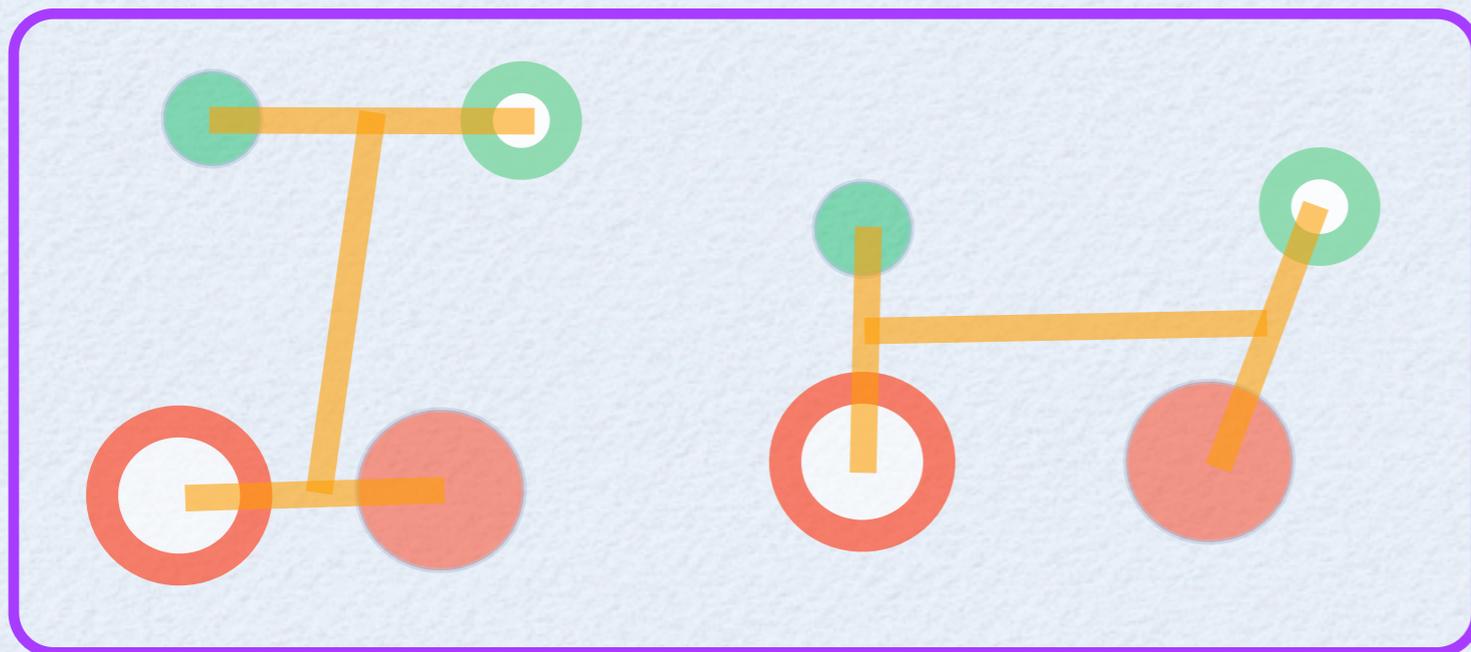
$\sqrt{\langle \delta (R_{mm'} - X) r_{ij}^2 \rangle}$



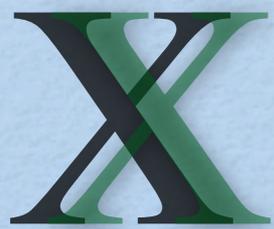
X

Effects of multiquark

- When only correlations between $u\bar{u}$ & $c\bar{c}$ or $u\bar{c}$ & $c\bar{u}$ are included, what happens?



No correlations among more than 3 quarks
→ two-meson-like configuration



Effects of multiquark

Binding Energy

IJ^{PC}	weaker meson-exch	stronger meson-exch	$J/\psi \rho$	DD^*
$11^{++} (J/\psi \rho)$	5 MeV	26 MeV	0.33	0.85
$\bigcirc - \bigcirc$ config	Not Bound	9 MeV	0.26	0.89

17 MeV difference: effects from correlations among more than 3 quarks

X Realistic Calc. $q\bar{q}c\bar{c}+c\bar{c}$

- Binding Energy: X ($qc\bar{q}\bar{c} + c\bar{c}$)
- I=0 becomes comparable to I=1 !

I J ^{PC}	stronger meson-exch
0 1 ⁺⁺ (J/ ψ ω)	5 MeV
0 1 ⁺⁺ (J/ ψ ω) +q ² pole at 3950 MeV	20MeV more bound (pole amp 0.1)

↑ Godfrey et.al. calc by (S \bar{C})

X

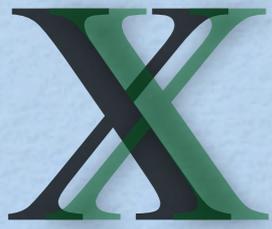
Summary for X(3872)

● $qc\bar{q}\bar{c}$ ($IJ^{PC}=01^{++}, 11^{++}$)

X(3872) : can be explained as a shallow bound state just below the DD^* threshold.

I=0 state has a repulsion from the OGE annihilation diagram and attraction from the $c\bar{c}$ coupling. So, it seems $I=0 \sim I=1$.

ambiguity: int. strength, size of $c\bar{c}$, E_{pole}



$$m_u \neq m_d$$

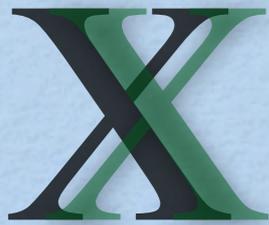
- The threshold difference between $D^0 D^{*0}$ and $D^+ D^{*-}$ enhances the $u\bar{u}$ component of X .
- estimate by a toy model

$$H =$$

H_0	0	v	v
0	$H_0 + 2\Delta m_q$	$-v$	v
v	$-v$	$E_{I=0}$	0
v	v	0	$E_{I=1}$

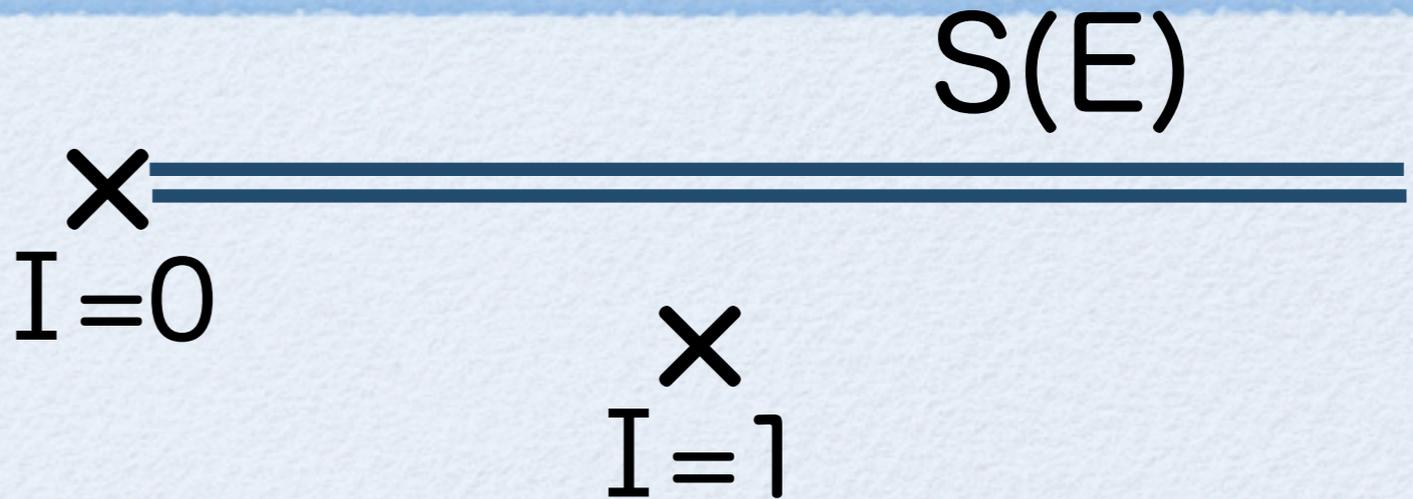
$$\psi =$$

$u\bar{c}c\bar{u}$ conti
$d\bar{c}c\bar{d}$ conti
$I=0$
$I=1$

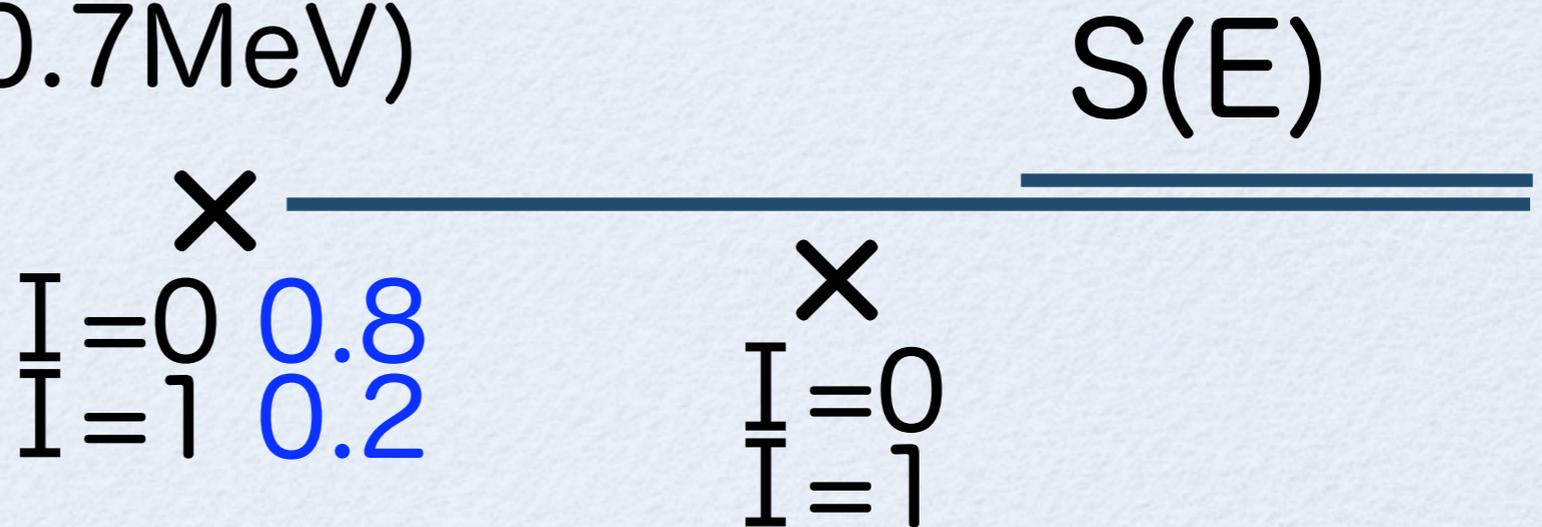


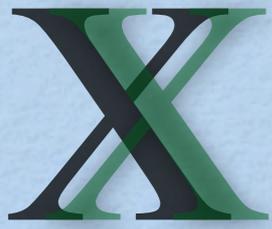
$$m_u \neq m_d$$

- $m_u = m_d$



- $m_u \neq m_d$
 - (Bound by 0.7MeV)





$$m_u \neq m_d$$

- $m_u \neq m_d$
- (resonance 1.2MeV above threshold)

	\times	\times	$S(E)$
$I=0$	0.7	$I=0$	
$I=1$	0.3	$I=1$	

- The threshold difference between $D^0 D^{*0}$ and $D^+ D^{*-}$ mixes $I=1$ and 0
 $\rightarrow J/\psi \pi^2$ and $J/\psi \pi^3$?

X more $q\bar{q}$ meson candidates

• $(q\bar{q})^2$ Mesons?

• $qs\bar{q}\bar{c}$?

$D_{s2}^*(2573)^\pm$

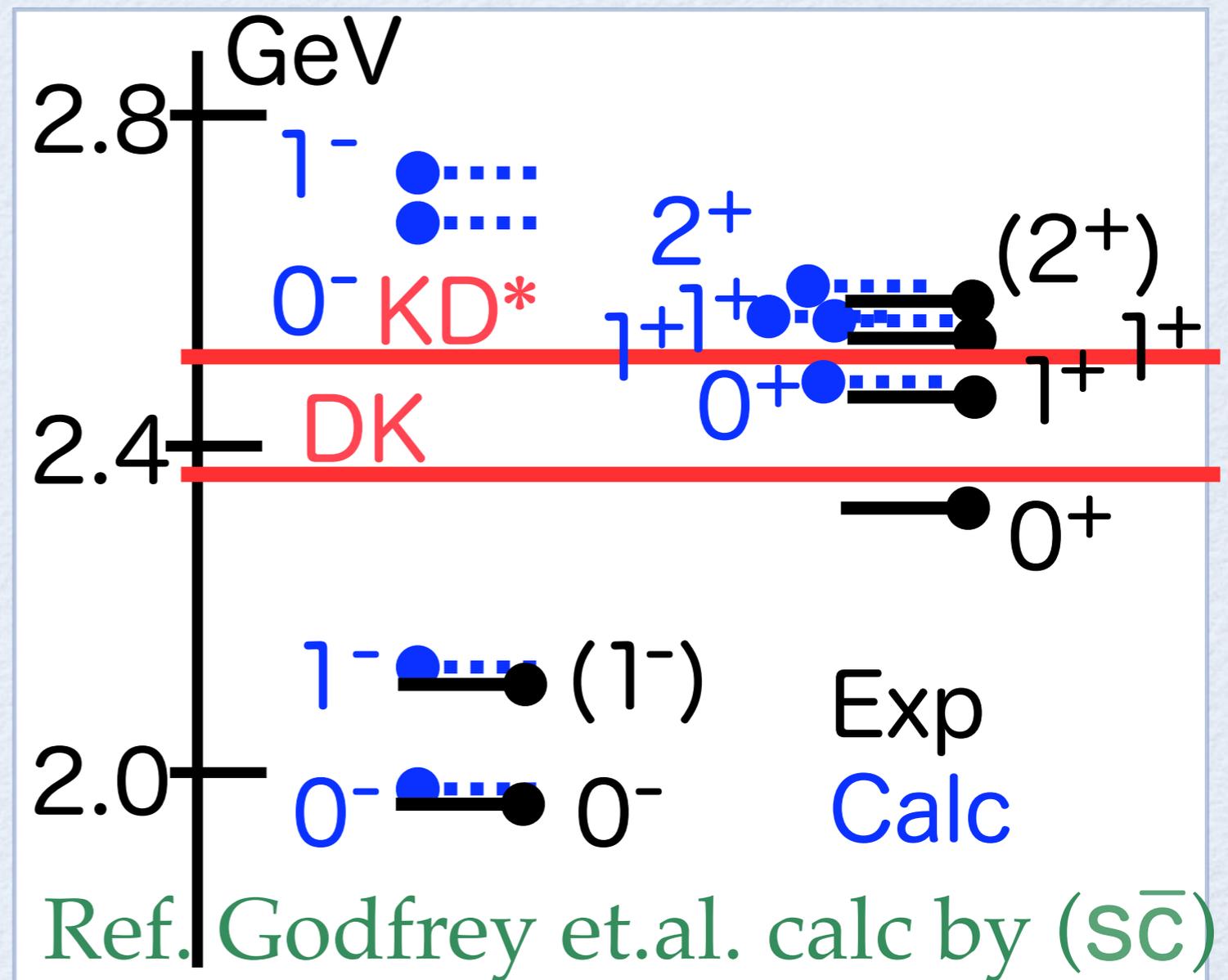
$D_{s1}^*(2536)^\pm$

$D_{s1}^*(2460)^\pm$

$D_{s0}^*(2317)^\pm$

• lighter than

$s\bar{c}$ p-wave mass by 160 or 90 MeV.



X more $q\bar{q}$ meson candidates

$(q\bar{q})^2$ Mesons?

● $qs\bar{q}\bar{c}$?

$D_{s2}^*(2573)^\pm$

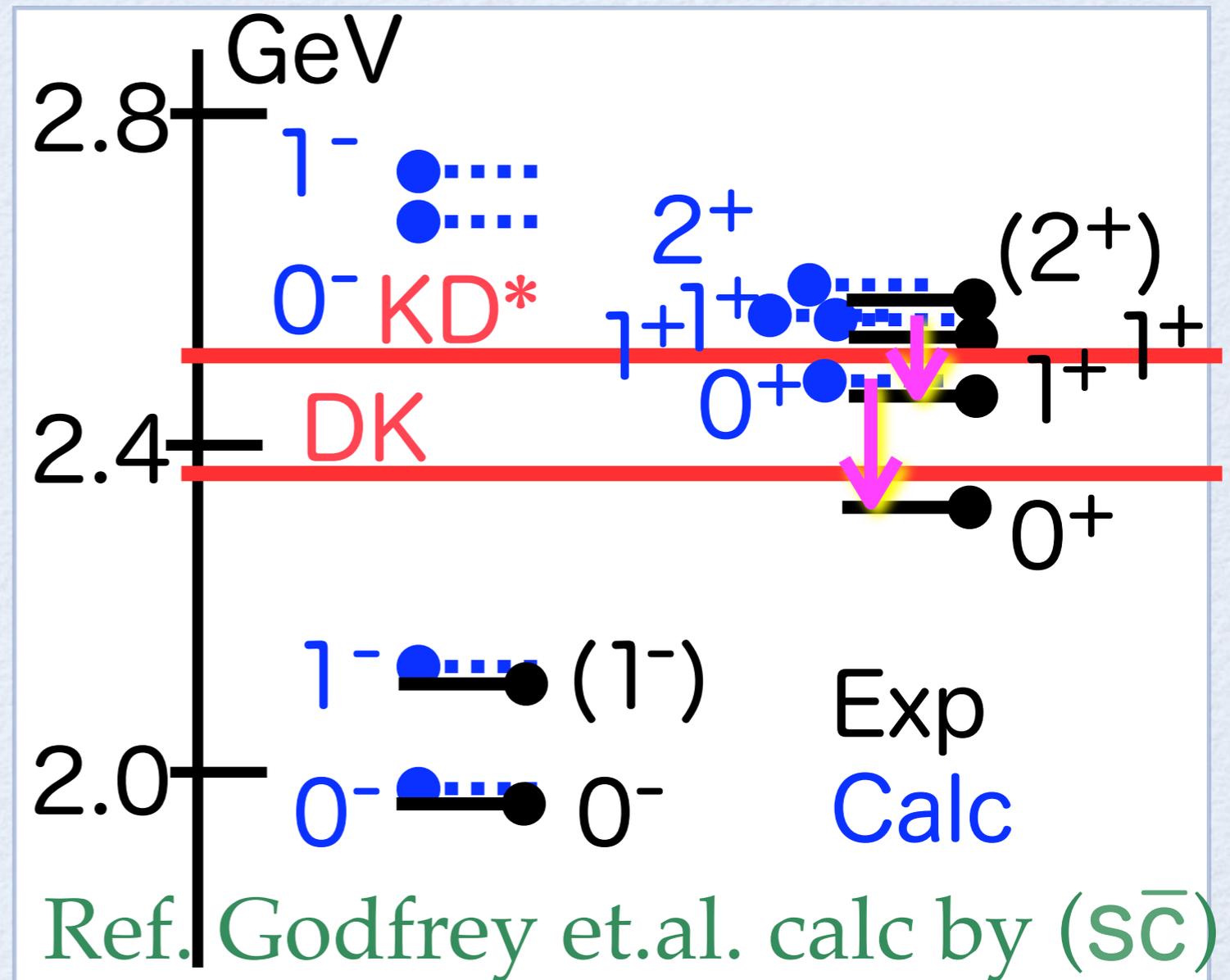
$D_{s1}^*(2536)^\pm$

90 MeV $D_{s1}^*(2460)^\pm$

160 MeV $D_{s0}^*(2317)^\pm$

● lighter than

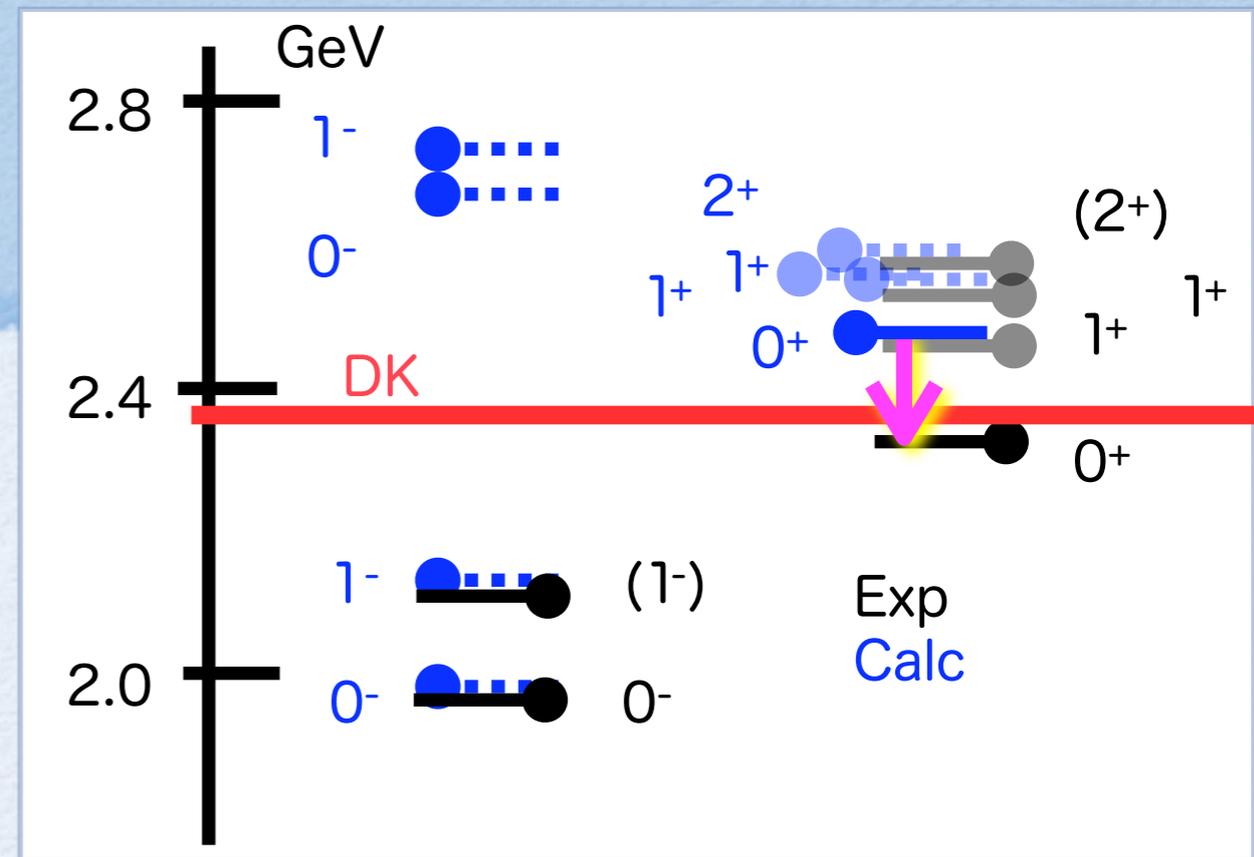
$s\bar{c}$ p-wave mass by 160 or 90 MeV.



X

$D_{s0}^*(2317)^\pm$

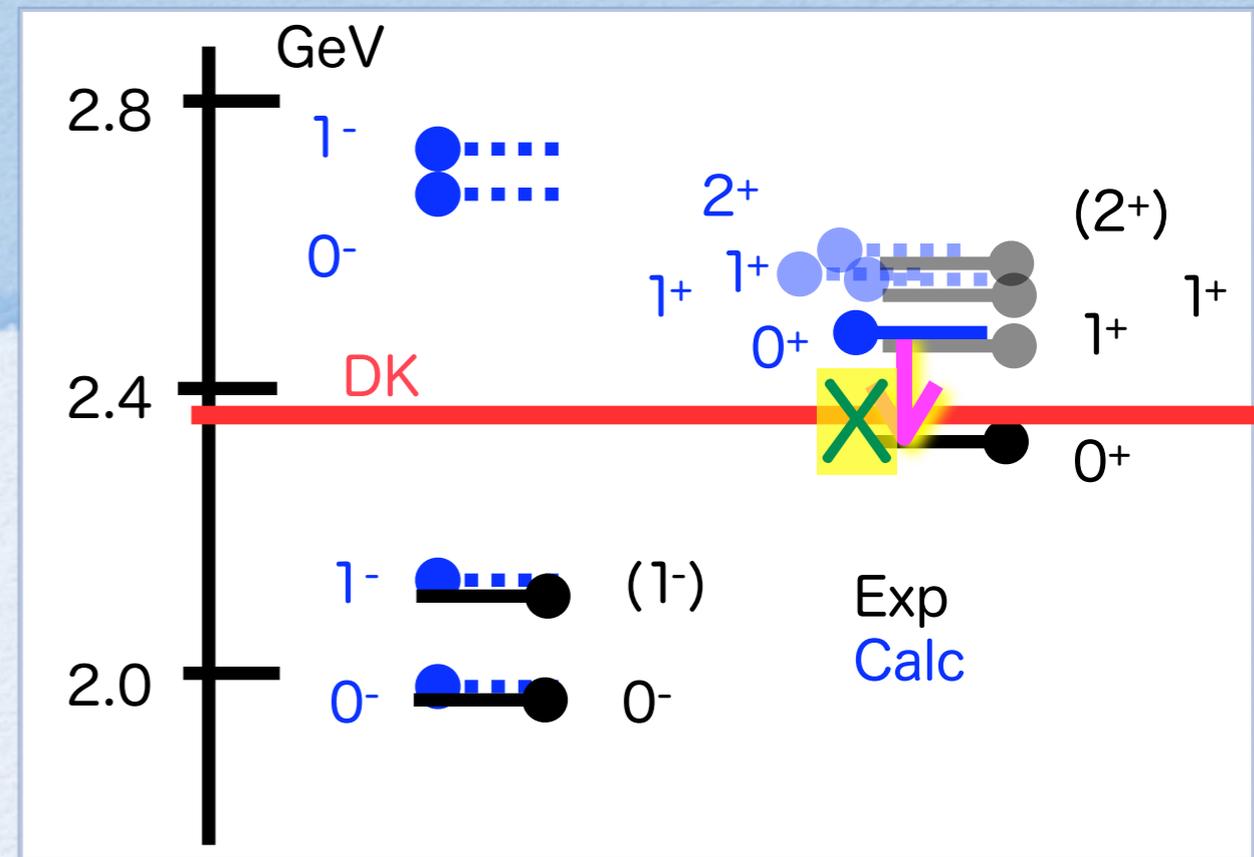
 $J=0$ $D_{s0}^*(2317)^\pm$



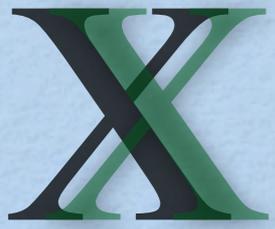
- $qs\bar{q}\bar{c}$ couples to D - K , D^* - K^* , η - D_s , ω - D_s^*
- Without a pole, attractive but **No bound**.
- Adding a $s\bar{c}$ pole at 2480 MeV makes the state **bound by 3 MeV** (pole amp 0.03).
- but should be 40 to 50 MeV more bound.

X $D_{s0}^*(2317)^\pm$

$J=0$ $D_{s0}^*(2317)^\pm$



- $qs\bar{q}\bar{c}$ couples to D - K , D^* - K^* , η - D_s , ω - D_s^*
- Without a pole, attractive but **No bound**.
- Adding a $s\bar{c}$ pole at 2480 MeV makes the state **bound by 3 MeV** (pole amp 0.03).
- but should be 40 to 50 MeV more bound.



Outlook

- Tetraquark systems may exist.
- A shallow bound state is a two-meson molecule with a multiquark component at the center, which gives an attraction for the binding. $\rightarrow X(3872)$
- To describe a deeply bound state ($\sim 40\text{MeV}$), an extra attraction or some other mechanism is necessary.