.

MINI-WORKSHOP BLED 2014: QUARQ MASSES AND HADRON SPECTRA

6-13 July 2014, Bled

Current Quark Masses in Multi-Quark Interactions

EPJA 49(2013)14, Phys.Rev. D88 (2013) 5, 054032

A. A. Osipov, B. Hiller, A. H. Blin
U. Coimbra, Portugal

IDEA OF SPONTANEOUS SYMMETRY BREAKING IN PARTICLE PHYSICS

The first works:

Y. Nambu, G. Jona-Lasinio,
Phys. Rev. 122 (1961) 345-358;
Phys. Rev. 124 (1961) 246-254.

T. Eguchi, PRD 14 (1976) 2755.
K. Kikkawa, Prog.Theor.Phys. 56 (1976) 947.

M.K. Volkov, D. Ebert, Sov. Jour. Nucl. Phys. 36 (1982) 1265.

Yoichiro Nambu
The Nobel Prize in Physics 2008

MAIN ASSUMPTIONS

1. Scales of nonperturbative QCD:

$$\Lambda_{QCD} \sim \Lambda_{conf} < \Lambda_{\chi SB} \approx 4\pi f_\pi .$$

In the regime $\Lambda_{conf} < \Lambda < \Lambda_{\chi SB}$ the induced effective interaction between quarks is of the form

$$L = \overline{q}\left(i\gamma^\mu \partial_\mu - m\right)q + \frac{\overline{G}}{\Lambda^2}\left(\overline{q}\Gamma q\right)^2 + \frac{\overline{\kappa}}{\Lambda^5}\left(\overline{q}\Gamma q\right)^3 + ...,$$

where Λ is determined by the chiral symmetry-breaking scale. If instantons are responsible for multi-quark interactions, than

$$\Lambda \sim \rho^{-1} \approx \left(0.33\ fm\right)^{-1}$$

2. Chiral symmetry restrictions

The color quark fields possess definite transformation properties with respect to the chiral flavor $U(3)_R \otimes U(3)_L$ global symmetry of the QCD Lagrangian with massless quarks

To study scalar and pseudoscalar modes it is convenient to introduce the U(3) Lie-algebra valued field:

$$\Sigma = \left(s_a - ip_a\right)\frac{\lambda_a}{2}, \quad s_a = \bar{q}\lambda_a q, \quad p_a = \bar{q}\lambda_a i\gamma_5 q.$$

and the external source χ, which generate the explicit symmetry breaking effects – future mass terms and mass dependent interactions, with the transformation properties:

$$\Sigma' = V_R \Sigma V_L^+, \quad \chi' = V_R \chi V_L^+.$$

MULTI-QUARK INTERACTIONS WITHOUT DERIVATIVES

$$L_i \propto \frac{\overline{g}_i}{\Lambda^{\gamma}} \chi^{\alpha} \Sigma^{\beta}$$

a). Dimensional arguments:

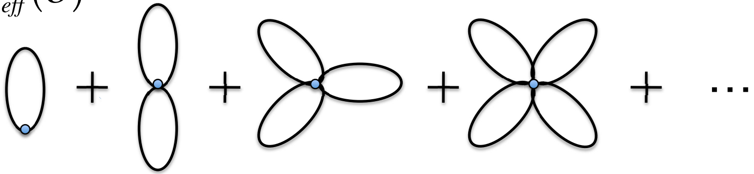
$$[\Lambda] = M, \quad [\chi] = M, \quad [\Sigma] = M^3, \quad [L_i] = M^4.$$

Therefore,

$$\alpha + 3\beta - \gamma = 4$$

b). The regime of dynamical chiral symmetry breaking: effective potential is

$$V_{eff}(\sigma) =$$



$$\sim \Lambda^3 \qquad \sim \frac{\Lambda^4}{\Lambda^2} \qquad \sim \frac{\Lambda^6}{\Lambda^5} \qquad \sim \frac{\Lambda^8}{\Lambda^8} \qquad \sim \frac{\Lambda^{2\beta}}{\Lambda^{\gamma}}$$

i.e. the leading contributions to the effective potential give the vertices with

$$2\beta - \gamma \geq 0$$

Combining both restrictions we come to the conclusion that only vertices with

$$\alpha + \beta \leq 4$$

must be taken into account at leading order.

1). $\alpha = 0, \beta = 1,2,3,4$ these are 4, 6 and 8-quark interactions:

where
$$L_{\text{int}} = L_{4q} + L_{6q} + L_{8q},$$

$$L_{4q} = \frac{\overline{G}}{\Lambda^2} Tr\left(\Sigma^+ \Sigma\right), \qquad L_{6q} = \frac{\overline{\kappa}}{\Lambda^5}\left(\det\Sigma + \det\Sigma^+\right),$$

$$L_{8q}^{(1)} = \frac{\overline{g}_1}{\Lambda^8}\left[Tr\left(\Sigma^+\Sigma\right)\right]^2, \quad L_{8q}^{(2)} = \frac{\overline{g}_2}{\Lambda^8} Tr\left(\Sigma^+\Sigma\Sigma^+\Sigma\right).$$

2). There are only six classes of vertices depending on external sources χ, they are:

$$\alpha = 1, \beta = 1,2,3; \quad \alpha = 2, \beta = 1,2; \quad \alpha = 3, \beta = 1.$$

This group contains 11 terms:

$$L_\chi = \sum_{i=0}^{10} L_i,$$

$$L_0 = -Tr\left(\Sigma^+ \chi + \chi^+ \Sigma\right),$$

$$L_1 = -\frac{\overline{\kappa}_1}{\Lambda} e_{ijk} e_{mnl} \Sigma_{im} \chi_{jn} \chi_{kl} + h.c.,$$

$$L_2 = \frac{\overline{\kappa}_2}{\Lambda^3} e_{ijk} e_{mnl} \chi_{im} \Sigma_{jn} \Sigma_{kl} + h.c.,$$

$$L_3 = \frac{\overline{g}_3}{\Lambda^6} Tr\left(\Sigma^+ \Sigma \Sigma^+ \chi\right) + h.c.,$$

$$L_4 = \frac{\bar{g}_4}{\Lambda^6} Tr\left(\Sigma^+\Sigma\right) Tr\left(\Sigma^+\chi\right) + h.c.,$$

$$L_5 = \frac{\bar{g}_5}{\Lambda^4} Tr\left(\Sigma^+\chi\Sigma^+\chi\right) + h.c.,$$

$$L_6 = \frac{\bar{g}_6}{\Lambda^4} Tr\left(\Sigma\Sigma^+\chi\chi^+ + \Sigma^+\Sigma\chi^+\chi\right),$$

$$L_7 = \frac{\bar{g}_7}{\Lambda^4} \left(Tr\,\Sigma^+\chi + h.c.\right)^2,$$

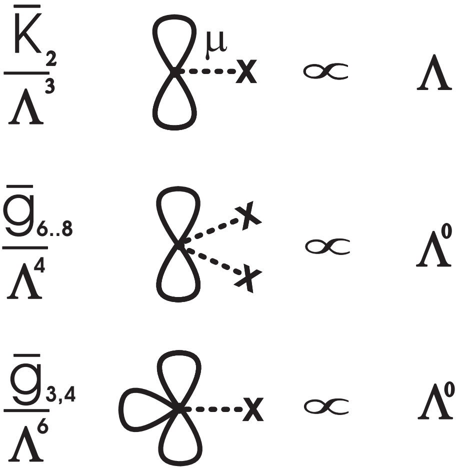
$$L_8 = \frac{\bar{g}_8}{\Lambda^4} \left(Tr\,\Sigma^+\chi - h.c.\right)^2,$$

$$L_9 = -\frac{\bar{g}_9}{\Lambda^2} Tr\left(\Sigma^+\chi\chi^+\chi\right) + h.c.,$$

$$L_{10} = -\frac{\bar{g}_{10}}{\Lambda^2} Tr\left(\chi^+\chi\right) Tr\left(\chi^+\Sigma\right) + h.c.$$

Explicit chiral symmetry breaking interactions

Put $\chi = \frac{\mu}{2}$ with $\mu = diag(\mu_u, \mu_d, \mu_s)$



.

N_c assignments:

$\Sigma \sim N_c$; $\Lambda \sim N_c^0 \sim 1$; $\chi \sim N_c^0 \sim 1$

• Then we get exactly that the diagrams which survive as $\Lambda \rightarrow \infty$ also surive as $N_c \rightarrow \infty$ and comply with the usual requirements:

• Leading quark contribution to the vacuum energy from $4q$ interactions known to be of order $N_c \rightarrow G \sim \frac{1}{N_c}$

• $U_A(1)$ anomaly contribution (´t Hooft interaction) is suppressed by one power of $\frac{1}{N_c} \rightarrow \kappa \sim \frac{1}{N_c^3}$.

• Zweig's rule violating effects are always of order $\frac{1}{N_c}$ with respect to leading contribution:e.g. $\rightarrow g_1 \sim \frac{1}{N_c^4}$.

.

- We have L_{4q} and L_0 of $\mathcal{O}(N_c)$ and all other terms in the Lagrangian of $\mathcal{O}(N_c^0)$.

- Non OZI-violating Lagrangian pieces scaling as $\mathcal{O}(N_c^0)$ represent NLO contributions with one internal quark loop in N_c counting. The coupling encodes the admixture of four quark component $\bar{q}q\bar{q}q$ to the leading $\bar{q}q$ at $N_c \to \infty$.

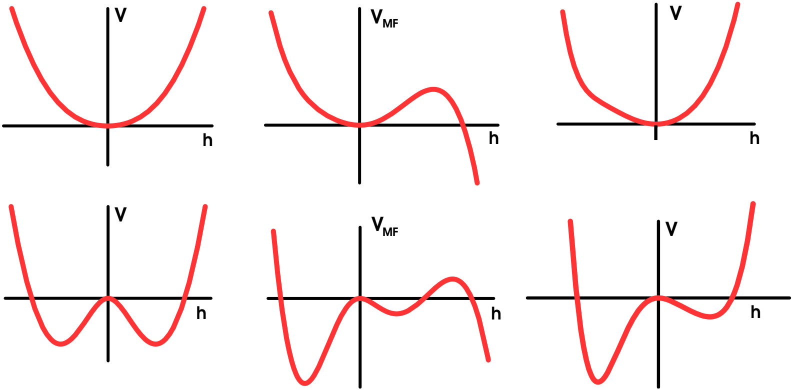
- Diagrams tracing Zweig's rule violation:
$\kappa, \kappa_1, \kappa_2, g_1, g_4, g_7, g_8, g_{10}$

- Diagrams with admixture of 4 quark and 2 quark states:
g_2, g_3, g_5, g_6, g_9.

- Only the phenomenology of terms L_0, G, κ, g_1, g_2 have been studied until now. We still have to understand the role of the remaining 10 terms to be consistent with the generic $\frac{1}{N_c}$ expansion of QCD.

8q and stability; Effective scalar potential V, $SU(3)$ chiral limit.

$\tau < 1$ $\boxed{4q}$ $\boxed{4q + 6q}$ $\boxed{4q + 6q + 8q}$



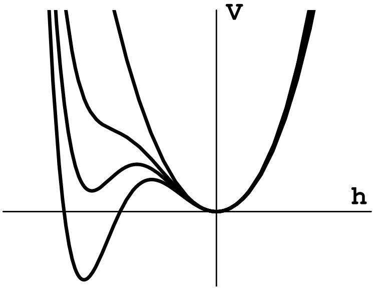
$\tau > 1$

χSB by $4q$ $\Uparrow$

$h \sim$ quark condensate. $\tau = \frac{N_c G \Lambda^2}{2\pi^2} \sim$ curvature at origin.

Effective potential V (closer look)

4q+6q+8q $\qquad \tau < 1$



$\Uparrow$

χSB by 6q

Figures from Osipov et al., Annals of Phys. 322 (2007) 2021.

.

$8q$-interactions may strongly affect *magnetic catalysis* **and** *thermodynamic observables*, **without changing the spectra at $T = \mu = 0, H = 0$.**

- G and g_1 dependence of SPA and masses of light 0^{-+} and 0^{++} mesons:

$$\xi = G + g_1(h_u^2 + h_d^2 + h_s^2)/4,$$

except 00, 08 and 88 states of scalar nonet.

$\rightarrow$ almost identical spectra can be obtained by changing G, g_1 and freezing all other parameters.

- But at finite T, μ or H:

$h_i(T, \mu, H)$ via gap equations $\longrightarrow$ ξ steered by g_1.

Magnetic Catalysis

• In $(2+1)$ and $(3+1)$ D a constant magnetic field H catalyzes the dynamical symmetry breaking $\Rightarrow$ generates fermion mass M_{dyn} even for $4q$ $G \to 0$. The symmetry is not restored at any arbitrarily large H.

- P. Klevansky and R. H. Lemmer, Phys. Rev. **39**, 3478 (1989).
- K. G. Klimenko, Theor. Math. Phys. **89**, 211 (1991);
- I. V. Krive and S. A. Naftulin, Sov. J. Nucl. Phys. **54**, 897 (1991);

• The zero-energy surface of the lowest Landau level plays a crucial role in the dynamics of fermion pairing, which is essentially $(1+1)$-D. Deep analogy with BSC. $M_{dyn} << $ Landau gap $\sim \sqrt{|eH|}$.

- V. P. Gusynin, V. A. Miransky, I. A. Shovkovy, PRL **73**, 3499 (1994).

• The existence of a zero-energy surface in the spectrum of a Dirac particle is ensured for any homogeneous magnetic field with a fixed direction by a quantum mechanical supersymmetry of the corresponding second-order Dirac Hamiltonian

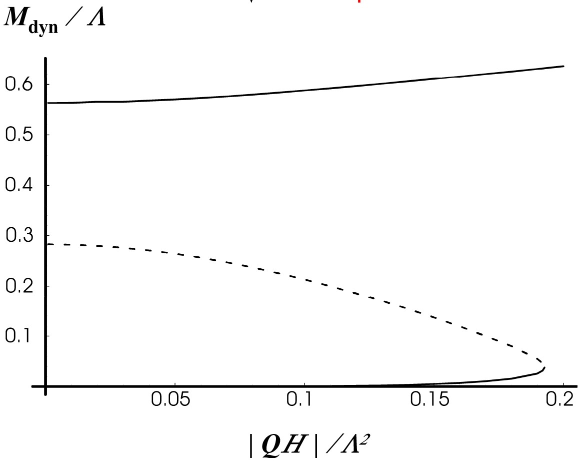
- R. Jackiw, Phys. Rev. D **29**, 2375 (1984);
- A. Barducci, R. Casalbuoni, L. Lusanna, N. Cimento A **35**, 377 (1976).

.

• **Our aim:** Having in mind that homogeneous magnetic fields can act as strong catalysts of chiral symmetry breaking, one might ask what is the effect caused by the strong interaction, when higher order multi-fermion interactions are present.

New effect: generation of other local minima through $6q + 8q$

$\Downarrow$ $\Leftarrow$ Deeper Minima $\Rightarrow$

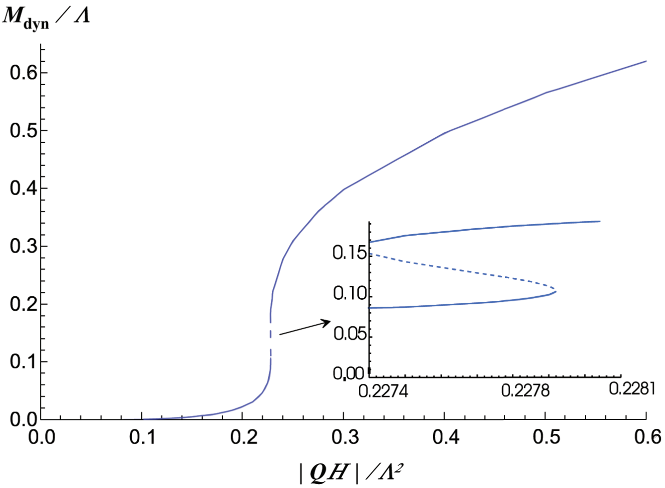


$\Rightarrow$ Deeper Minima $\Leftarrow$

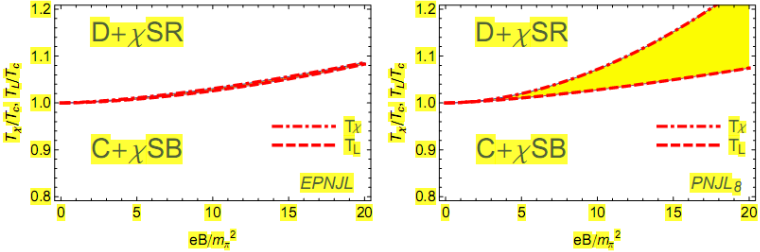
dashed curve: maxima

Fig. from Osipov et al., Phys. Lett. B 650 (2007) 262

Fig. from Hiller et al., SIGMA 4 (2008) 024.



Subcritical values of couplings: $G\Lambda^2 = 3, \kappa\Lambda^5 = -800, \lambda\Lambda^8 = 1667.$



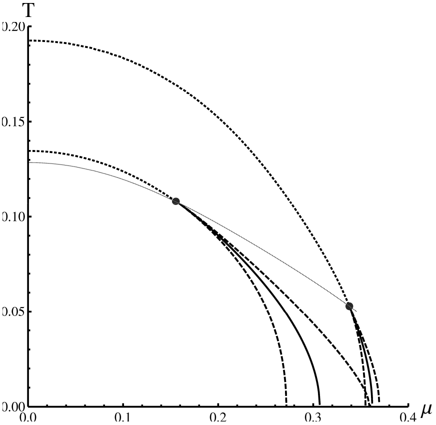
Yellow zone: Hot quark matter is deconfined and chiral symmetry still broken spontaneously.
Fig. from Gatto+Ruggieri, PRD 82 (2010) 054027

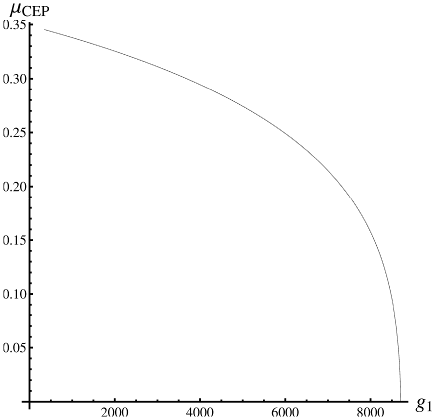
Thermal and medium effects

Phase Diagram for NJL

Figures from Hiller et al, Phys. Rev. D81 (2010)116005

8q decrease T_c and shift CEP to higher T, smaller μ.

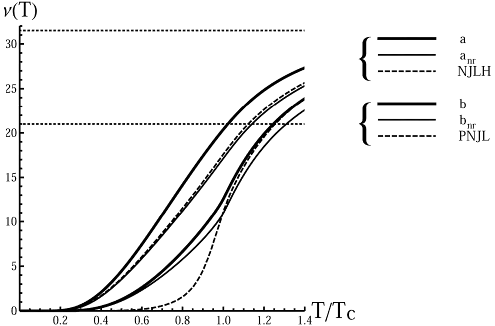




Right upper curve for: $g_1 = 1000 GeV^{-8}$, lower curve for: $g_1 = 8000 GeV^{-8}$;
thin line: CEP in T, μ diagram, depending on g_1;
Left: CEP (other view)

. **Number of effective degrees of freedom** at $\mu = 0$

- Bold lines: NJL, Pauli-Villars regulator on vacuum +thermal integrals,
- Thin lines: NJL, PV on vacuum only
- (upper): $g_1 = 1000 GeV^{-8}$ $T_c = 190 MeV$ (PV all), $T_c^{\infty} = 179 MeV$
- (lower) $g_1 = 8000 GeV^{-8}$ $T_c = 135 MeV$ (PV all), $T_c^{\infty} = 132 MeV$
- dashed curves: K. Fukushima, Phys. Rev. D77, 114028 (2008); lower curve (PNJL), $T_c^{\infty} = 204.8$; upper (NJL), $T_c^{\infty} = 171.6 MeV$, $3D$ cutoff
- $8q$: **additional source for suppression of degrees of freedom in NJL**



$$\nu(T) = \frac{P(T) - P(0)}{\frac{\pi^2 T^4}{90}}$$

The total Lagrangian is the sum

$$L = \bar{q} i \gamma^{\mu} \partial_{\mu} q + L_{\text{int}} + L_{\chi}$$

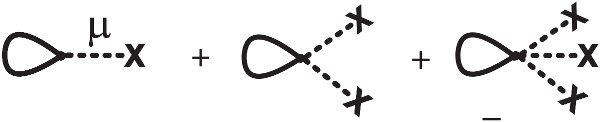
Putting $\chi = \mu / 2$, where $\mu = diag(\mu_u, \mu_d, \mu_s)$, we obtain a set of explicitly breaking chiral symmetry terms. This leads to the following mass dependent part of the NJL Lagrangian:

$$L_{\chi} \to L_{\mu} = -\bar{q} m q + \sum_{i=2}^{8} L_i^{'}$$

where the current quark mass matrix m is equal to

$$m = \mu + \frac{\bar{\kappa}_1}{\Lambda} (\det \mu) \mu^{-1} + \frac{\bar{g}_9}{4\Lambda^2} \mu^3 + \frac{\bar{g}_{10}}{4\Lambda^2} \left(Tr\, \mu^2 \right) \mu .$$

Current quark mass term



$$\Lambda^2 N_c \times \left(1 + \frac{\bar{K}_1}{\Lambda N_c} + \frac{\bar{g}_{9,10}}{\Lambda^2 N_c} \right)$$

$=$ 

$$\propto \hat{m}\, \mathcal{O}(\Lambda^2 \times N_c)$$

KAPLAN-MANOHAR AMBIGUITY

There is definite freedom in the definition of the external source χ. In fact, the sources

$$\chi^{(c_i)} = \chi + \frac{c_1}{\Lambda}(\det\chi^+)\chi(\chi^+\chi)^{-1} + \frac{c_2}{\Lambda^2}\chi\chi^+\chi + \frac{c_3}{\Lambda^2} tr(\chi^+\chi)\chi,$$

with three independent constants c_i have the same symmetry transformation properties as χ. Therefore, we could have used $\chi^{(c_i)}$ everywhere that we used χ. As a result we would come to the same Lagrangian with the following redefinitions of couplings:

$$\bar{\kappa}_1 \to \bar{\kappa}_1' = \bar{\kappa}_1 + c_1/2; \quad g_5 \to g_5' = g_5 - \bar{\kappa}_2 c_1;$$
$$\bar{g}_7 \to \bar{g}_7' = \bar{g}_7 + \bar{\kappa}_2 c_1/2; \quad \bar{g}_8 \to \bar{g}_8' = \bar{g}_8 + \bar{\kappa}_2 c_1/2;$$
$$\bar{g}_9 \to \bar{g}_9' = \bar{g}_9 + c_2 - 2\bar{\kappa}_1 c_1; \quad \bar{g}_{10} \to \bar{g}_{10}' = \bar{g}_{10} + c_3 + 2\bar{\kappa}_1 c_1.$$

$$\boxed{\bar{\kappa}_1' = \bar{g}_9' = \bar{g}_{10}' = 0}$$

HOW CAN CURRENT QUARK MASSES BE PROPERLY DEFINED IN THE MODEL?

$$m_i = \mu_i \left(1 + \frac{\overline{g}_9}{4\Lambda^2} \mu_i + \frac{\overline{g}_{10}}{4\Lambda^2} \mu^2 \right) + \frac{\overline{\kappa}_1}{2\Lambda} t_{ijk} \mu_j \mu_k$$

$$m_i = \mu_i$$

This procedure is more restricted here than in the chiral perturbation theory.

BOSONIZATION

Let us introduce in the vacuum functional

$$Z = \int dq\, d\bar{q} \exp\left(i \int d^4 x L\right)$$

the functional unity (Alkofer, Reinhardt, 1988)

$$1 = \int \prod_a ds_a dp_a \delta(s_a - \bar{q}\lambda_a q)\delta(p_a - \bar{q} i\gamma_5 \lambda_a q)$$

$$= \int \prod_a ds_a dp_a d\sigma_a d\phi_a \exp\left\{i \int d^4 x \left[\sigma_a (s_a - \bar{q}\lambda_a q) + \phi_a (p_a - \bar{q} i\gamma_5 \lambda_a q)\right]\right\}$$

thus obtaining

$$Z = \int \prod_a d\sigma_a d\phi_a \underbrace{dq d\bar{q} \exp\left(i \int d^4 x L_{q\bar{q}}\right)}_{\text{Gaussian integral, heat kernel expansion}} \underbrace{\int \prod_a ds_a dp_a \exp\left(i \int d^4 x L_{aux}\right)}_{\text{stationary phase approx.}}.$$

Gaussian integral
heat kernel expansion

stationary phase approx.

Here

$$L_{q\bar{q}} = \bar{q}\left[i\gamma^{\mu}\partial_{\mu} - \left(\sigma + i\gamma_5\phi\right)\right]q \equiv \bar{q}Dq$$

$$L_{aux} = s_a\left(\sigma_a - m_a\right) + p_a\phi_a + L_{\text{int}}(s,p) + \sum_{i=2}^{8} L_i'\left(s,p,\mu\right)$$

quartic polynomial in auxiliary fields

cubic polynomial in auxiliary fields

$$L_{\text{int}}\left(s,p\right) = L_{4q} + L_{6q} + L_{8q}$$

INTEGRATION OVER AUXILIARY FIELDS

The stationary phase trajectory are given by the extremum conditions

$$\frac{\partial L_{aux}}{\partial s_a} = 0, \quad \frac{\partial L_{aux}}{\partial p_a} = 0.$$

which must be fulfilled in the neighborhood of the uniform vacuum state of the theory, i.e. $\sigma \to \sigma + M$, $\langle \sigma \rangle = 0$. We seek solutions in the form

$$s_a^{st} = h_a + h_{ab}^{(1)} \sigma_b + h_{abc}^{(1)} \sigma_b \sigma_c + h_{abc}^{(2)} \phi_b \phi_c + \ldots$$

$$p_a^{st} = h_{ab}^{(2)} \phi_b + h_{abc}^{(3)} \sigma_b \phi_c + \ldots$$

We are led to the result:

$$L_{aux} = h_a \sigma_a + \frac{1}{2} h_{ab}^{(1)} \sigma_a \sigma_b + \frac{1}{2} h_{ab}^{(2)} \phi_a \phi_b + \ldots$$

STATIONARY PHASE EQUATIONS

$$M_i - m_i + \frac{\kappa}{4} t_{ijk} h_j h_k + \frac{h_i}{2}\left(2G + g_1 h^2 + g_4 \mu h\right) + \frac{g_2}{2} h_i^3$$

$$+ \frac{\mu_i}{4}\left[3 g_3 h_i^2 + g_4 h^2 + 2\left(g_5 + g_6\right)\mu_i h_i + 4 g_7 \mu h\right]$$

$$+ \kappa_2 t_{ijk} \mu_j h_k = 0.$$

INTEGRATION OVER QUARKS

After integrating out the fermions, we obtain

$$\int dq d\overline{q} \exp\left(i \int d^4 x \overline{q} D q\right) \propto \det D$$

The modulus of the quark determinant is chirally invariant and may be defined in the proper-time regularization scheme as

$$S_E = -\ln\left|\det D_E\right| = \frac{1}{2} \int_0^\infty \frac{dt}{t} \rho\left(t\Lambda^2\right) \underbrace{Tr\left[\exp\left(-t D_E^+ D_E\right)\right]}_{\text{heat kernel}}$$

where

$$D_E^+ D_E = M^2 - \partial_\alpha^2 + Y, \qquad \rho\left(t\Lambda^2\right) = 1 - \left(1 + t\Lambda^2\right) \exp\left(-t\Lambda^2\right),$$

$$Y = i\gamma_\alpha\left(\partial_\alpha \sigma + i\gamma_5 \partial_\alpha \phi\right) + \sigma^2 + \phi^2 + \left\{M, \sigma\right\} + i\gamma_5\left[\sigma + M, \phi\right].$$

HEAT KERNEL EXPANSION

The trace of the heat kernel operator can be presented as

$$Tr\left[\exp\left(-tD_E^+D_E\right)\right] = \int d^4x_E\, tr\langle x|\exp\left(-tD_E^+D_E\right)|x\rangle$$

correspondingly

$$S_E = \int \frac{d^4x_E d^4p}{2(2\pi)^4}\int_0^\infty \frac{dt}{t^3}\rho\left(t\Lambda^2\right)e^{-p^2}tr\left(e^{-t\left(M^2+A\right)}\right)\times 1$$

where

$$A = -\partial^2 + Y - \frac{2i}{\sqrt{t}}\,p\partial$$

and

$$\left[M^2, A\right] = \left[M^2, Y\right] \neq 0.$$

SCHWINGER'S PROPER-TIME TECHNIQUE

If $\left[M^2, A\right] = 0$, i.e. $M^2 = diag\left(m^2, m^2, m^2\right) \sim m^2 \times 1$, we obtain

$$tr\left(e^{-t\left(M^2+A\right)}\right) = e^{-t\,m^2} tr\left(e^{-tA}\right) = e^{-t\,m^2} tr\left(\sum_0^\infty t^n a_n\right),$$

where a_n are the Seeley-DeWitt coefficients. The subsequent integrations over p and proper-time t give the known result:

$$S_E = \int \frac{d^4 x_E}{32\pi^2} \sum_{n=0}^{\infty} J_{n-1}\left(m^2\right) tr\left(a_n\right)$$

with

$$J_n\left(m^2\right) = \int_0^\infty \frac{dt}{t^{2-n}} e^{-t\,m^2} \rho\left(t\Lambda^2\right).$$

$$\delta_\omega a_n = i\left[\omega, a_n\right],$$
$$\omega = \alpha + \gamma_5 \beta.$$

GENERALIZATION

Since the explicit chiral symmetry breaking leads us to the inequality:

$$\left[M^2, A\right] \neq 0.$$

the classical Schwinger-DeWitt technique must be modified. This modification, however, should not destroy the covariant grouping for the background fields. The successful algorithm has been found in 2001.

$$J_l\left(m_j^2\right) - J_l\left(m_i^2\right) = \sum_{n=1}^{\infty} \frac{\Delta_{ij}^n}{2^n n!}\left[J_{l+n}\left(m_i^2\right) - (-1)^n J_{l+n}\left(m_j^2\right)\right],$$

and the mass dependent factors are

$$I_n = \frac{1}{N_f} \sum_{i=1}^{N_f} J_n\left(m_i^2\right), \quad \Delta_{ij} = m_i^2 - m_j^2 .$$

THE MODIFIED SCHWINGER-DEWITT SERIES

$$S_E = \int \frac{d^4 x_E}{32\pi^2} \sum_{n=0}^{\infty} I_{n-1}\, tr(b_n)$$

for new covariant coefficients we have found:

$$b_0 = 1, \quad b_1 = -Y, \qquad b_2 = \frac{Y^2}{2} + \frac{\lambda_3}{2}\Delta_{12} Y + \frac{\lambda_8}{2\sqrt{3}}\left(\Delta_{13} + \Delta_{23}\right) Y,$$

$$b_3 = -\frac{Y^3}{6} - \frac{1}{12}\left(\partial Y\right)^2 - \frac{\lambda_3}{12}\Delta_{12}\left(\Delta_{31} + \Delta_{32}\right) Y + \frac{\lambda_3 Y^2}{4}\Delta_{21}$$

$$+ \frac{\lambda_8 Y}{12\sqrt{3}}\left[\Delta_{13}\left(\Delta_{21} + \Delta_{23}\right) + \Delta_{23}\left(\Delta_{12} + \Delta_{13}\right)\right] + \frac{\lambda_8 Y^2}{4\sqrt{3}}\left(\Delta_{31} + \Delta_{32}\right),$$

and one can check that $\delta_\omega b_n = i\left[\omega, b_n\right].$

THE TOTAL LAGRANGIAN OF THE BOSONIZED THEORY

a). The gap equation

$$h_i + \frac{N_c}{6\pi^2} M_i \left[3I_0 - \left(3M_i^2 - M^2 \right) I_1 \right] = 0.$$

where

$$M^2 = M_u^2 + M_d^2 + M_s^2 .$$

From now on I will consider the case with an exact SU(2) isospin symmetry: $M_u = M_d = \hat{M} \neq M_s .$

b). The small perturbations

$$L = \frac{N_c I_1}{16\pi^2} tr\left[(\partial\phi)^2 + (\partial\sigma)^2\right] + \frac{N_c I_0}{4\pi^2}\left(\phi_a^2 + \sigma_a^2\right)$$
$$- \frac{N_c I_1}{12\pi^2}\Big\{\Delta_{ns}\left[2\sqrt{2}\left(3\sigma_0\sigma_8 + \phi_0\phi_8\right) - \phi_8^2 + \phi_i^2\right]$$
$$+2\left(2\hat{M}^2 + M_s^2\right)\sigma_0^2 + \left(\hat{M}^2 + 5M_s^2\right)\sigma_8^2 + \left(7\hat{M}^2 - M_s^2\right)\sigma_i^2$$
$$+\left(\hat{M} + M_s\right)\left(\hat{M} + 2M_s\right)\sigma_f^2 + \left(M_s - \hat{M}\right)\left(2M_s - \hat{M}\right)\phi_f^2\Big\}$$
$$+\frac{1}{2} h_{ab}^{(1)}\sigma_a\sigma_b + \frac{1}{2} h_{ab}^{(2)}\phi_a\phi_b + \ldots$$

The kinetic term requires a redefinition of meson fields

$$\sigma_a = g\sigma_a^R, \quad \phi_a = g\phi_a^R, \quad g^2 = \frac{4\pi^2}{N_c I_1} = \frac{\hat{M}^2}{f_\pi^2}.$$

The interaction terms

for 2-body decays at meson tree level

$$L_{int} = L_{int}^{(hk)} + L_{int}^{SPA}. \qquad (1)$$

$$L_{int}^{SPA} = \sigma_a \left(\frac{1}{3} h_{abc}^{(1)} \sigma_b \sigma_c + h_{abc}^{(2)} \phi_b \phi_c \right). \qquad (2)$$

$$\begin{aligned} L_{int}^{(hk)} &= -\frac{N_c}{2\pi^2} I_1 M_a \left[d_{ab\rho} d_{ce\rho} \sigma_b \left(\sigma_c \sigma_e + \phi_c \phi_e \right) \right. \\ &+ \left. 2 f_{ac\rho} f_{be\rho} \sigma_b \phi_c \phi_e \right] \qquad (3) \end{aligned}$$

.

- L_{int}^{SPA}: All dependence on the explicit χ_{SB} parameters absorbed in $h_{ab}^{(1,2)}$ and $h_{abc}^{(1,2,3)}$ $\longrightarrow$ Same formal structure as in case without these interactions.

- $L_{int}^{(hk)}$: difference in constituent quark masses leads to a resummation of the heat kernel series for the modified Seeley-DeWitt coefficients b_i.

.

Strong decays of scalars S into pseudoscalars P_1, P_2

$$\Gamma_\beta = \frac{|\vec{p_\beta}|}{8\pi m_S^2} |g_\beta|^2 \equiv \bar{g}_\beta |\vec{p}_\beta| \tag{4}$$

$$|\vec{p}_\beta| = \sqrt{\frac{[m_S^2 - (m_1 + m_2)^2]\,[m_S^2 - (m_1 - m_2)^2]}{4m_S^2}}$$

β specifies kinematic characteristics of the channel $S \to P_1 P_2$, and the masses m_S, m_1, m_2 of the states.

$\bar{g}_\beta$ in eq.(4) include all flavor and symmetry factors associated with the final state.

.

• **Flatté distributions** for $a_0(980)$ and $f_0(980)$ decays: to accomodate threshold effects associated with two kaon production, on grounds of analyticity and unitarity.

Close to threshold the elastic scattering cross section is parametrized by a two-channel resonance

$$\begin{aligned} \sigma_{el} &= 4\pi|f_{el}|^2, \\ f_{el}^{\beta} &= \frac{1}{|\vec{p}_\beta|}\frac{m_R\Gamma_\beta}{m_R^2 - s - im_R(\Gamma_\beta + \Gamma^S_{K\bar{K}})} \end{aligned} \quad (5)$$

index β: $a_0\pi\eta$ or $f_0\pi\pi$ channels

$$\Gamma^S_{K\bar{K}} = \begin{cases} \bar{g}^S_K\sqrt{\frac{s}{4} - m_K^2} & \text{above threshold} \\ i\bar{g}^S_K\sqrt{m_K^2 - \frac{s}{4}} & \text{below threshold.} \end{cases} \quad (6)$$

$\bar{g}^S_K$: coupling of S to the two kaons, case $S = a_0$ or f_0.

.

Radiative decays

• Additional information on the structure of the mesons is obtained through the study of their radiative decays.

• We consider two photon decays at the quark one-loop order $S \to \gamma\gamma$ and $P \to \gamma\gamma$. The corresponding integrals are finite.

• The anomalous $P \to \gamma\gamma$ decays belong to the imaginary part of the action. By the Adler-Bardeen theorem they are fully determined by the 3-point function Feynman amplitudes involving one quark loop; higher orders only redefine the couplings.

• Source of uncertainty: model dependent determination of the coupling of the η and η' mesons to the quarks. In our approach they are calculated within the heat kernel technique.

.

$$S(s) \to \gamma(p_1, \epsilon^*_\mu) + \gamma(p_2, \epsilon^*_\nu)$$

Minimal coupling: $\mathcal{L}_\gamma = -e\bar{q}\gamma^\mu Q q A_\mu$, $Q = \frac{1}{2}(\lambda_3 + \frac{1}{\sqrt{3}}\lambda_8)$

$\mathcal{L}_{\mu\nu} = (p_2^\mu p_1^\nu - \frac{1}{2} s g^{\mu\nu})$; $\Gamma_{S\gamma\gamma} = \frac{m_S^3}{64\pi}|A_{S\gamma\gamma}|^2$

$$\begin{aligned}
A^{\mu\nu}_{S\gamma\gamma} &= \mathcal{L}^{\mu\nu} A_{S\gamma\gamma}; \qquad S = \sigma, f_0(980), a_0(980) \\
A_{\sigma\gamma\gamma} &= \frac{5}{9}\, T_u \cos\bar{\psi} - \frac{\sqrt{2}}{9}\, T_s \sin\bar{\psi} \\
A_{f_0\gamma\gamma} &= -\frac{5}{9}\, T_u \sin\bar{\psi} - \frac{\sqrt{2}}{9}\, T_s \cos\bar{\psi} \\
A_{a_0\gamma\gamma} &= \frac{1}{3}\, T_u
\end{aligned} \tag{7}$$

T_i: 3-point Feynman amplitudes, keeping only the contribution corresponding to the first non-vanishing order in the heat kernel action, the Seeley-DeWitt coefficient b_3.

.

The anomalous decay of $P = (\pi^0, \eta, \eta')$ $P(p) \to \gamma(p_1, \epsilon^*_\mu) + \gamma(p_2, \epsilon^*_\nu)$

$$
\begin{aligned}
A^{\mu\nu}_{P\gamma\gamma} &= \epsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta} A_{P\gamma\gamma} \\
A_{\eta\gamma\gamma} &= -\frac{5}{9}\, T^P_u \sin\bar{\psi}_P - \frac{\sqrt{2}}{9}\, T^P_s \cos\bar{\psi}_P \\
A_{\eta'\gamma\gamma} &= \frac{5}{9}\, T^P_u \cos\bar{\psi}_P - \frac{\sqrt{2}}{9}\, T^P_s \sin\bar{\psi}_P \\
A_{\pi^0\gamma\gamma} &= \frac{1}{3}\, T^P_u \qquad (8)
\end{aligned}
$$

Widths are calculated as

$$
\Gamma_{P\gamma\gamma} = \frac{|\vec{p}|^3}{8\pi} |A_{P\gamma\gamma}|^2 \qquad (9)
$$

with $|\vec{p}| = \sqrt{m_P^2/4}$ and m_P the pseudoscalar mass.

• The only parameter dependence in the radiative decays of S, P enters through the wave function normalization g, common to all decays, and through M.

• There is also an explicit dependence on the scale Λ in the case of the scalar decays.

• The PCAC hypothesis establishes a relation between g, the weak decay couplings and M

$$f_\pi = \frac{\hat{M}}{g}; \qquad f_K = \frac{\hat{M} + M_s}{2g}. \tag{10}$$

• **They allow to eliminate all dependence on the constituent quark masses** from the pseudoscalar radiative decays

$$T^P(\hat{M}) = \frac{N_c \alpha}{\pi f_\pi}, \quad T^P(M_s) = \frac{N_c \alpha}{\pi(2f_K - f_\pi)}. \tag{11}$$

.

- **The Adler-Bardeen theorem allows to infer:**

(i) that the study and measurement of the anomalous decays are a reliable means of determination of the mixing angle of the η and η' mesons,

(ii) which must comply with the mixing angle determination extracted from the mass spectrum.

- **With the present model Lagrangian one is able to:**

(I) account properly for the $SU(3)$ breaking effects in the description of f_π and f_K,

(II) obtain the correct empirical η and η' meson masses

which has been an open problem until now. This is important for the numerical consistency in the amplitudes.

.

Fixing parameters, numerical results and discussion

Table: Parameter sets (a),(b),(c),(d) differ by varying the mixing angles and m_σ: sets (a), (b) and (d) with $m_\sigma = 550$ MeV versus set (c) with $m_\sigma = 600$ MeV,sets (a),(c) and (d) with $\theta_P = -12^\circ$ versus set (b) with $\theta_P = -15^\circ$. The scalar mixing angle is kept constant, $\theta_S = 25^\circ$, in (a),(b),(c) and increased to $\theta_S = 27.5^\circ$ in set (d). Units: MeV. Input (marked with *).

m_π	m_K	m_η	$m_{\eta'}$	f_π	f_K	m_κ	m_{a_0}	m_{f_0}
138*	**494***	**547***	**958***	**92***	**113***	**850***	**980***	**980***

.

Table: Parameter sets of the model: $\hat{m}, m_s$, and Λ are given in MeV. The couplings have the following units: $[G] = \text{GeV}^{-2}$, $[\kappa] = \text{GeV}^{-5}$, $[g_1] = [g_2] = \text{GeV}^{-8}$. We also show here the values of constituent quark masses $\hat{M}$ and M_s in MeV. See also caption of Table 1.

Sets	$\hat{m}$	m_s	$\hat{M}$	M_s	Λ	G	$-\kappa$	g_1	g_2
a	4.0*	100*	372	541	830	9.74	121.1	3136	133
b	4.0*	100*	372	542	829	9.83	118.5	3305	-158
c	4.0*	100*	370	539	830	10.45	120.3	2081	102
d	4.0*	100*	373	544	828	10.48	122.0	3284	173

.

Table: **Explicit symmetry breaking interaction couplings**. The couplings have the following units: $[\kappa_1] = \text{GeV}^{-1}$, $[\kappa_2] = \text{GeV}^{-3}$, $[g_3] = [g_4] = \text{GeV}^{-6}$, $[g_5] = [g_6] = [g_7] = [g_8] = \text{GeV}^{-4}$, $[g_9] = [g_{10}] = \text{GeV}^{-2}$.

Sets	κ_1	κ_2	$-g_3$	g_4	g_5	$-g_6$	$-g_7$	g_8	g_9	g_{10}
a	0*	6.14	6338	657	210	1618	105	-65	0*	0*
b	0*	5.61	6472	702	210	1668	100	-38	0*	0*
c	0*	6.12	6214	464	207	1598	133	-66	0*	0*
d	0*	6.17	6497	1235	213	1642	13.3	-64	0*	0*

Strong Decays of Scalars

m_R: resonance mass in MeV,

Γ^{BW} and Γ^{Fl} are the Breit-Wigner width and the Flatté distribution width in MeV,

$R^S = \frac{\bar{g}_K^S}{\bar{g}_\beta}$

.

The experimental status for strong decays of scalars

σ meson until recently had a large uncertainty

$m_\sigma = (400 \div 1200)$ MeV and a full width $\Gamma_\sigma = (600 \div 1000)$ MeV.

Presently PDT:2011 narrowed to

$m_\sigma = (400 \div 550)$ MeV and $\Gamma_\sigma = (400 \div 700)$ MeV.

The result based on the average over the dispersion analysis of Colangelo:2001,Caprini:2006,Kaminski:2011,Moussalam:2011: a very sharp value for the pole position
$M - i\Gamma/2 = (446 \pm 6) - (276 \pm 5)$ MeV.

.

$f_0(980)$ **meson**:

$m_{f_0(980)} = 990 \pm 20$ MeV and $\Gamma_{f_0(980)} = 40 \div 100$ MeV

$a_0(980)$ **meson**:

$a_0(980)$ meson: $m_{a_0(980)} = 980 \pm 20$ MeV and $\Gamma_{a_0(980)} = 50 \div 100$ MeV.

$\kappa(800)$ **meson**:

$\kappa(800)$ quoted in the PDG table from a Breit-Wigner fit: pole at $(764 \pm 63^{+71}_{-54}) - i(306 \pm 149^{+143}_{-82})$ MeV.

Set	Decays	m_R	Γ^{BW}	Γ^{FI}	$\bar{g}_\beta$	$\bar{g}^S_K$	R^S	θ_P	θ_S
a	$\sigma \to \pi\pi$	550	465		1.95	0.97	0.497	-12	25
	$f_0 \to \pi\pi$	980	108	60	0.23	0.32	1.397		
	$\kappa \to K\pi$	850	310		1.2	0			
	$a_0 \to \eta\pi$	980	419	45	1.32	2.69	2.05		

Set	Decays	m_R	Γ^{BW}	Γ^{FI}	$\bar{g}_\beta$	$\bar{g}^S_K$	R^S	θ_P	θ_S
b	$\sigma \to \pi\pi$	550	465		1.955	0.986	0.504	-15	25
	$f_0 \to \pi\pi$	980	108	60	0.230	0.312	1.356		
	$\kappa \to K\pi$	850	310		1.2	0			
	$a_0 \to \eta\pi$	980	459	50	1.44	2.805	1.944		

Set	Decays	m_R	Γ^{BW}	Γ^{FI}	$\bar{g}_\beta$	$\bar{g}^S_K$	R^S	θ_P	θ_S
c	$\sigma \to \pi\pi$	600	635		2.39	1.52	0.61	-12	25
	$f_0 \to \pi\pi$	980	108	61	0.23	0.30	1.32		
	$\kappa \to K\pi$	850	310		1.2	0			
	$a_0 \to \eta\pi$	980	419	46	1.31	2.67	2.03		

Set	Decays	m_R	Γ^{BW}	Γ^{FI}	$\bar{g}_\beta$	$\bar{g}^S_K$	R^S	θ_P	θ_S
d	$\sigma \to \pi\pi$	550	461		1.94	0.63	0.33	-12	27.5
	$f_0 \to \pi\pi$	980	62	30	0.23	0.30	3.90		
	$\kappa \to K\pi$	850	310		1.2	0			
	$a_0 \to \eta\pi$	980	420	46	1.32	2.73	2.07		

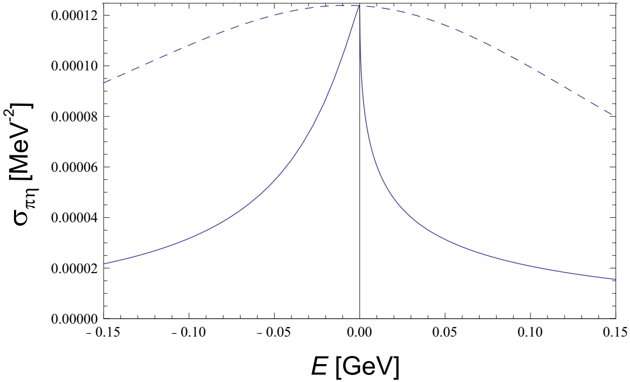


Figure: The $\pi\eta$ cross section as function $E = \sqrt{s} - 2m_K$ for the a_0 resonance channel from the Flatté distribution (solid line) with parameters of set (b), $\bar{g}_{a_0\pi\eta} = 1.44$, $\bar{g}_K^{a_0} = 2.8$, $R^{a_0} = 1.944$. The width read at half peak value is $\Gamma^{Fl} = 50$ MeV. Dashed line corresponds to the single $\pi\eta$ channel.

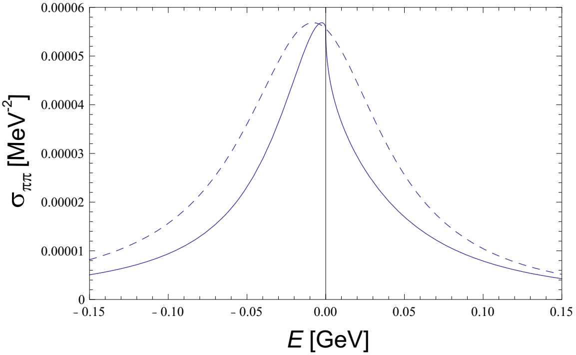


Figure: The $\pi\pi$ cross section as function $E = \sqrt{s} - 2m_K$ for the f_0 resonance channel from the Flatté distribution (solid line) with parameters of set (b), $\bar{g}_{f_0\pi\pi} = 0.23$, $\bar{g}_K^{f_0} = 0.31$, $R^{f_0} = 1.36$. The width read at half peak value is $\Gamma^{Fl} = 60$ MeV. Dashed line corresponds to just the two pion channel.

.

We obtain that the σ mass and $\sigma \to \pi\pi$ decay are within the recent limits for sets (a-b) and (d) while set (c) has a mass larger than the upper limit by ~ 50 MeV.

While in set (a-b) and (d) the calculated width is smaller than the nominal mass of the resonance, the opposite behavior is seen in set (c).

$\bar{g}_{\sigma\pi\pi}$ increases comparing set(a) to (c) explaining the larger width,

however the ratio $R^\sigma = \frac{\bar{g}^\sigma_K}{\bar{g}_{\sigma\pi\pi}}$ of the σ to kaon and to the pion couplings also increases by 20%.

- The obtained ratios for R^σ are in agreement with the experimental value $R^\sigma_{exp} = 0.5 \pm 0.1$ in Bugg:2006 for sets (a-c) and slightly below for set (d).

We expect some effect on the width if these channels were taken into account, but only a moderate one since the coupling to pions dominates, $R^{\sigma} \sim 0.3 \div 0.5$.

The decay width for $\kappa(800) \to K\pi \sim 310$ MeV is smaller roughly by a factor two than the quoted central value but lies still within the limits.

- The ratio of the couplings $\frac{\bar{g}_{\kappa K\pi}}{\bar{g}_{\sigma\pi\pi}} \frac{m_\kappa^2}{m_\sigma^2} = 1.5$ (the ratio of meson masses corrects for the different definitions of the couplings in Bugg:2006) is within the experimental values in Bugg:2006, as opposed to the $q\bar{q}$ and $q^2\bar{q}^2$ model approaches considered in the same paper.

- The widths of the $a_0(980) \to \pi\eta$ and $f_0(980) \to \pi\pi$ decays are well accomodated within a Flatté description.

- Ratio $R^{a_0} \sim 2$ shows that the dominant component is in the coupling to the kaons.

- As demonstrated in Baru:2005 the ratio $R^S = \frac{\bar{g}_K^S}{\bar{g}^\beta}$ is a relatively stable quantity in despite of the large fluctuations in the experimental values extracted for the individual couplings. Our R^S are compatible with the indicated values in Baru:2005.

- The ratio $R^{f_0} = \frac{\bar{g}_K^{f_0}}{\bar{g}_{f_0 \pi\pi}}$ is strongly dependent on the mixing angle θ_S of the scalar sector. The larger ratio $R^{f_0} = 3.9$ in set (d), which agrees well with the experimental value $R^{f_0}_{exp} = 4.21 \pm 0.46$ of BES

- The crossed ratio $r = \frac{R^{f_0}}{R^{a_0}}$ is usually assumed to be larger than unity. The $a_0(980)$ does not depend on the θ_S mixing angle (an eventual correlation with the $f_0(980)$ meson through isospin mixing is discarded here), but does depend on the pseudoscalar θ_P angle through its decay into the $\pi\eta$.

- The θ_P is fixed in the pseudoscalar sector to yield the correct η and η' masses, as well as their radiative two photon decay widths. Therefore the ratio R^{a_0} of the a_0 couplings to kaons and to the $\pi\eta$ channels remains approximately constant for all parameter sets $(R^{a_0})^{-1} \sim 0.5$.

This value is not too bad in comparison with the experimental quoted ratio $(R^{a_0}_{exp})^{-1} = 0.75 \pm 0.11$ Hyams:1973. Requiring the ratio $r > 1$ constrains further the angle to be larger than $\theta_S \sim 26^\circ$.

- On the other hand the ratio R^{f_0} increases until θ_S reaches ideal mixing. In the interval $\theta_{id} < \theta_S \leq \frac{\pi}{4}$ it decreases but stays much larger than the experimental accepted ratio, e.g. at $\theta_S = 44^\circ$ one has $R^{f_0} \sim 11$.

- The combined requirement $r > 1$ and $R^{f_0}_{exp}$ confines the mixing angle to the narrow window $27^\circ < \theta_S < 28^\circ$.

.

- From the point of view of the calculated strong decay widths the smaller angle $\theta_S = 25^\circ$ is also acceptable.

- Our interval of values for the mixing angle $25^\circ < \theta_S < 28^\circ$, corresponding to $-10.3^\circ < \bar{\psi} < -7.3^\circ$ are within the values $-14^\circ < \bar{\psi} < -3^\circ$ estimated in Escribano:2002, more specifically $\bar{\psi} \sim -9^\circ$ if a Flatté distribution is used in a complementarity approach of CHPT and the Linear Sigma Model.

.

Radiative decays

Table: Radiative decays of the scalar mesons $\Gamma_{S\gamma\gamma}$ in KeV , resonance masses in MeV $m_\sigma = 550$ for sets (a,b,d), $m_\sigma = 600$ for set (c), $m_{a_0} = m_{f_0} = 980$ in all sets.

Set a	$\Gamma_{S\gamma\gamma}$	Set b	$\Gamma_{S\gamma\gamma}$	Set c	$\Gamma_{S\gamma\gamma}$	Set d	$\Gamma_{S\gamma\gamma}$
$\sigma \to \gamma\gamma$	0.212	$\sigma \to \gamma\gamma$	0.212	$\sigma \to \gamma\gamma$	0.277	$\sigma \to \gamma\gamma$	0.210
$f_0 \to \gamma\gamma$	0.055	$f_0 \to \gamma\gamma$	0.055	$f_0 \to \gamma\gamma$	0.055	$f_0 \to \gamma\gamma$	0.080
$a_0 \to \gamma\gamma$	0.389	$a_0 \to \gamma\gamma$	0.386	$a_0 \to \gamma\gamma$	0.392	$a_0 \to \gamma\gamma$	0.383

Table: Anomalous decays $\Gamma_{P\gamma\gamma}$ for sets (a) and (c) in KeV, corresponding to $\theta_P = -12^\circ$, m_R is the particle mass in MeV. [For set (b), corresponding to $\theta_P = -15^\circ$, we have $\Gamma_{\eta\gamma\gamma} = 0.6$ KeV, $\Gamma_{\eta'\gamma\gamma} = 4.8$ KeV.]

Decays	m_R	$\Gamma_{P\gamma\gamma}$	$\Gamma^{exp}_{P\gamma\gamma}$ (PDT:2011)
$\pi^0 \to \gamma\gamma$	136	0.00798	$0.00774637 \div 0.00810933$
$\eta \to \gamma\gamma$	547	0.5239	$(39.31 \pm 0.2)\%\,\Gamma_{tot} = 0.508 \div 0.569$
$\eta' \to \gamma\gamma$	958	5.225	$(2.18 \pm 0.08)\%\,\Gamma_{tot} = 3.99 \div 4.70$

- The two photon decays of the pseudoscalars are in very good agreement with data.

- For the radiative widths of the σ, there is a large spread in the experimental data from different facilities.

- Our results for $\sigma \to \gamma\gamma$ only account for about 20% of the value (1.2 ± 0.4) KeV Bernabeu:2008 obtained from the nucleon electromagnetic polarizabilities, which is one of the lowest estimates for this width.

- For the $f_0(980) \to \gamma\gamma$ the PDG average is quoted as $(0.29^{+0.07}_{-0.06})$ KeV. Sets (a-c) yield approximately 20% and set (e) 30% of this value.

- These results meet the current expectations that a direct coupling to the photons via a quark loop are not sufficient to account for the observed radiative widths of these mesons.

.

• **Question: why are the strong widths described reasonably well in all channels and the radiative ones fall short of the empirical values for the σ, f_0 decays?**

• **Answer: only the strong decays probe directly the multi-quark couplings g_i contained in the stationary phase piece $\mathcal{L}_{int}^{SPA}$ of the total interaction Lagrangian (1).**

.

- Since $\mathcal{L}_{int}^{SPA}$ has no derivative terms only the heat kernel Lagrangian $\mathcal{L}_{int}^{HK}$ involves the electromagnetic interaction, after minimal coupling.

- The information of the SPA conditions which leaks through the gap equations to the electromagnetic sector is rather weak; it is contained only in

(i) the wave function normalization which is the same for all mesons,

(ii) in the quark constituent masses

(iii) and scale Λ

which remain approximately constant in all parameter sets.

.

• **Thus, effectively, the two photon decays of the scalars yield a clean signature whether the electromagnetic decay of the mesons proceeds dominantly through a $q\bar{q}$ channel or not.**

• **This in turn ties up with the strength distribution in the HK and SPA contributions to the coupling g_{SPP} The HK piece relates directly to the meson-$q\bar{q}$ channel, the SPA part to the higher order multiquark interactions.**

Table: The coefficients coef^{HK} and coef^{SPA} of the heat kernel and of the SPA contributions to the total value of the coupling $g_{SP_1P_2}$ resulting from the interaction Lagrangian for the open decay channels. Values are for the neutral channels. Units are in GeV.

$g_{SP_1P_2}$	**coef^{HK}/g^3**	**coef^{SPA}/g^3**	**total/g^3**
$\sigma\pi^0\pi^0$	**-0.0450**	**0.0215**	**-0.0235**
$f_0\pi^0\pi^0$	**-0.0061**	**-0.0047**	**-0.0109**
$\kappa^0\bar{K}^0\pi^0$	**0.0660**	**-0.0257**	**0.0403**
$a_0^0\eta\pi^0$	**-0.0666**	**-0.0178**	**-0.0844**

Table: The coefficients coef^{HK} and coef^{SPA} of the heat kernel and of the SPA contributions to the total value of the coupling $g_{SK\bar{K}}$ resulting from the interaction Lagrangian. Values are for the neutral channels. Units are in GeV.

$g_{SK\bar{K}P_2}$	**coef^{HK}/g^3**	**coef^{SPA}/g^3**	**total/g^3**
$\sigma K\bar{K}$	**-0.041**	**0.0178**	**-0.0232**
$f_0K\bar{K}$	**0.118**	**-0.081**	**0.0372**
$a_0^0K\bar{K}$	**0.0246**	**0.0968**	**0.121**

.

a_0 meson: the calculated $a_0(980) \to \gamma\gamma \sim 0.39$ KeV overestimates the average PDG value $0.21^{+0.08}_{-0.04}$ and points to the dominance of the direct one quark loop coupling to photons of a_0.

(i) The large bare width that we obtain for the $a_0 \to \pi\eta$ decay stems mainly from the HK coefficient represented with 80% of the total strength.

(ii) The a_0 meson in the $q\bar{q}$ picture is composed only of u and d quarks, thus its coupling to the $K\bar{K}$ mesons requires a flavor change at the kaon vertices, as opposed to the $\eta\pi$ case. It is more favorable to couple to kaons through the multiquark vertices (SPA), 80% total strength.

(iv) It is thus imperative to take this mode into account through the two-channel Flatté distribution.

.

- From the point of view of the two photon decay of a_0, a $\pi\eta$ loop does not couple directly to two photons, the decay proceeds through the quark loop of u or d quarks with the large strength of the HK component.

- To access the dominant SPA component the two photon decay would have to proceed through coupling to the $K\bar{K}$ loop, a sub-leading process in N_c counting as compared to the direct $q\bar{q}$ loop. Furthermore, due to the relatively large mass of the kaons, this loop is not expected to contribute significantly.

.

σ, f_0 channels:

- **Lack of a pronounced dominance of the HK:**

the $q\bar{q}$ coupling of these mesons to the photons represents only a fraction of the total width.

The remaining strength must derive from the multiquark channels which should be included in an extra step, taking into account explicitly meson loop contributions.

- For the strong decay of the f_0:

stronger participation of the multi-quark interactions and cancellations in the kaon channel (as opposed to the pion channel) $\longrightarrow$ a coupling to the kaon channel through the Flatté approach is not imperative.

.

- Rescattering effects have been shown in several approaches to yield the main contribution, e.g. for the $\sigma \to \gamma\gamma$ extracted from the dispersion analysis of $\gamma\gamma \to \pi^0\pi^0$ Oller:2008.

- Claims for a tetraquark structure Jaffe:1977 of the σ meson were forwarded e.g. in Giacosa:2006, and in Achasov:2008 interpreted as pion and kaon loop contributions. Our approach sheds light on these phenomena from a different angle.

.

● Radiative decays of the scalar mesons have been calculated a long time ago in a variant of the NJL model, with and without meson loop contributions, Ebert:1997.

The amplitudes differ from ours in two key aspects:

we use an unified description for all non-anomalous decays based on the generalized heat kernel approach which leads

(i) to a common wave function normalization for all mesons that implies the reduction factor of $\sim \frac{2}{3}$ in the amplitude

(ii)and in the case of the radiative decays to the regularized one loop integrals carrying the factors $(\frac{\Lambda^2}{\Lambda^2+M_i^2})^2$. Tis reduces the amplitude by 1/2. The combined effect is a dramatic reduction by a factor ~ 10 in the decay widths, as compared to Ebert:1997 for the quark loop contribution.

.

Concluding remarks

• We have generalized the effective multi-quark Lagrangians of the NJL type including higher order terms in the current quark-mass expansion.

• The procedure is based on the very general assumption that the scale of spontaneous chiral symmetry breaking determines the hierarchy of local multi-quark interactions. There is a finite subset of vertices which are responsible for the explicit χ_{SB} at each order considered.

• We have classified these vertices at NLO, the same order as the 't Hooft determinant and eight quark terms previously analyzed in the literature.

.

- **These new terms carry either signatures of violation of the Zweig-rule or of admixtures of $q^2\bar{q}^2$ states to the quark-antiquark ones and are thus potentially interesting candidates in the quest of analyzing the structure and interaction dynamics of the low lying mesons.**

CONCLUSIONS:

1) WE FIT THE LOW LYING PSEUDOSCALAR SPECTRUM, THE WEAK PION AND KAON DECAY CONSTANTS, AND THE SINGLET-OCTET MIXING ANGLE TO PERFECT ACCURACY

$$\kappa_2; \quad g_8 \rightarrow \eta - \eta'$$

$$L_2 = \frac{\overline{\kappa}_2}{\Lambda^3} e_{ijk} e_{mnl} \chi_{im} \Sigma_{jn} \Sigma_{kl} + h.c. \quad L_8 = \frac{\overline{g}_8}{\Lambda^4} \left(Tr\, \Sigma^+ \chi - h.c. \right)^2$$

2) WE FIT THE LOW LYING SCALAR SPECTRUM WHICH TYPICALLY (IN NJL-MODEL) IS

$$m_\sigma < m_{a_0} < m_\kappa < m_{f_0}$$

IN ACCORDANCE WITH EMPIRICAL EVIDENCE

$$m_\kappa < m_{a_0} \approx m_{f_0}$$

THE g_3 INTERACTIONS IS THE MAIN REASON FOR THE REVERSE ORDERING, $m_{a_0} > m_\kappa$ THE COUPLING g_6 BEING RESPONSIBLE FOR THE FINE TUNING OF THE RESULT.

$$L_3 = \frac{\bar{g}_3}{\Lambda^6} Tr\left(\Sigma^+ \Sigma \Sigma^+ \chi\right) + h.c. \quad L_6 = \frac{\bar{g}_6}{\Lambda^4} Tr\left(\Sigma \Sigma^+ \chi \chi^+ + \Sigma^+ \Sigma \chi^+ \chi\right)$$

.

- The fitting of the $\eta - \eta'$ mass splitting together with the overall successful description of the whole set of low-energy pseudoscalar characteristics is actually a solution for a long standing problem of NJL-type models.

- The scalar meson strong decay widths are within current expectations.

- The radiative decays of the scalar mesons into two photons show that the main channel for the $a_0(980)$ decay proceeds through coupling to a quark-antiquark state, while the radiative decays of singlet-octet states σ, f_0 must proceed through more complex strutures.

- Finally, the radiative decays of the pseudoscalars are in very good agreement with data.

PERSPECTIVES

The low-energy meson physics, e.g. pi-pi scattering lengths.

The extension to the vector and axial-vector mesons

The model in hot and dense environment.

The electromagnetic interactions. The model in the strong magnetic field.