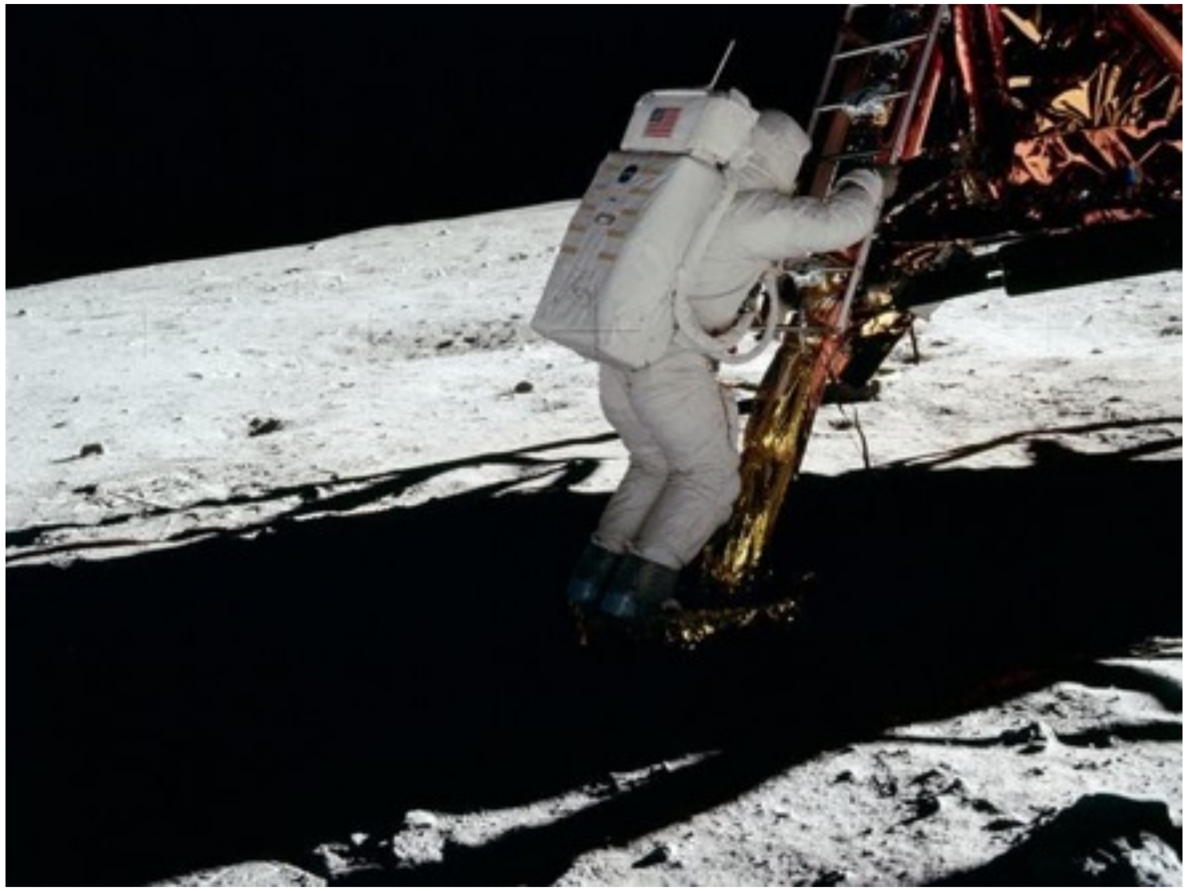
Quark matter subject to strong magnetic fields: phase diagram and applications

Parkes Telescope - Australia

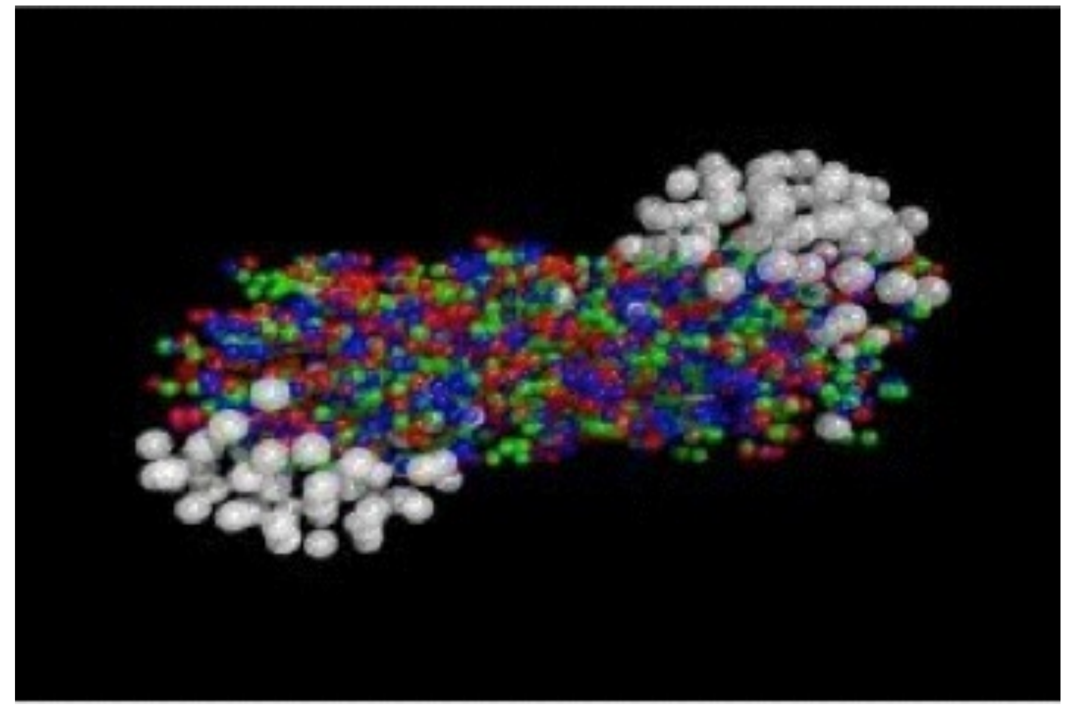


First Moon Walk

Débora Peres Menezes - Universidade Federal de Santa Catarina

Bled, Slovenia - 07/2014

Motivation: why magnetic fields?



Magnetars - $eB \simeq 0.5 m^2_\pi$

$m^2_\pi \simeq 3.5 \times 10^{18}$ G

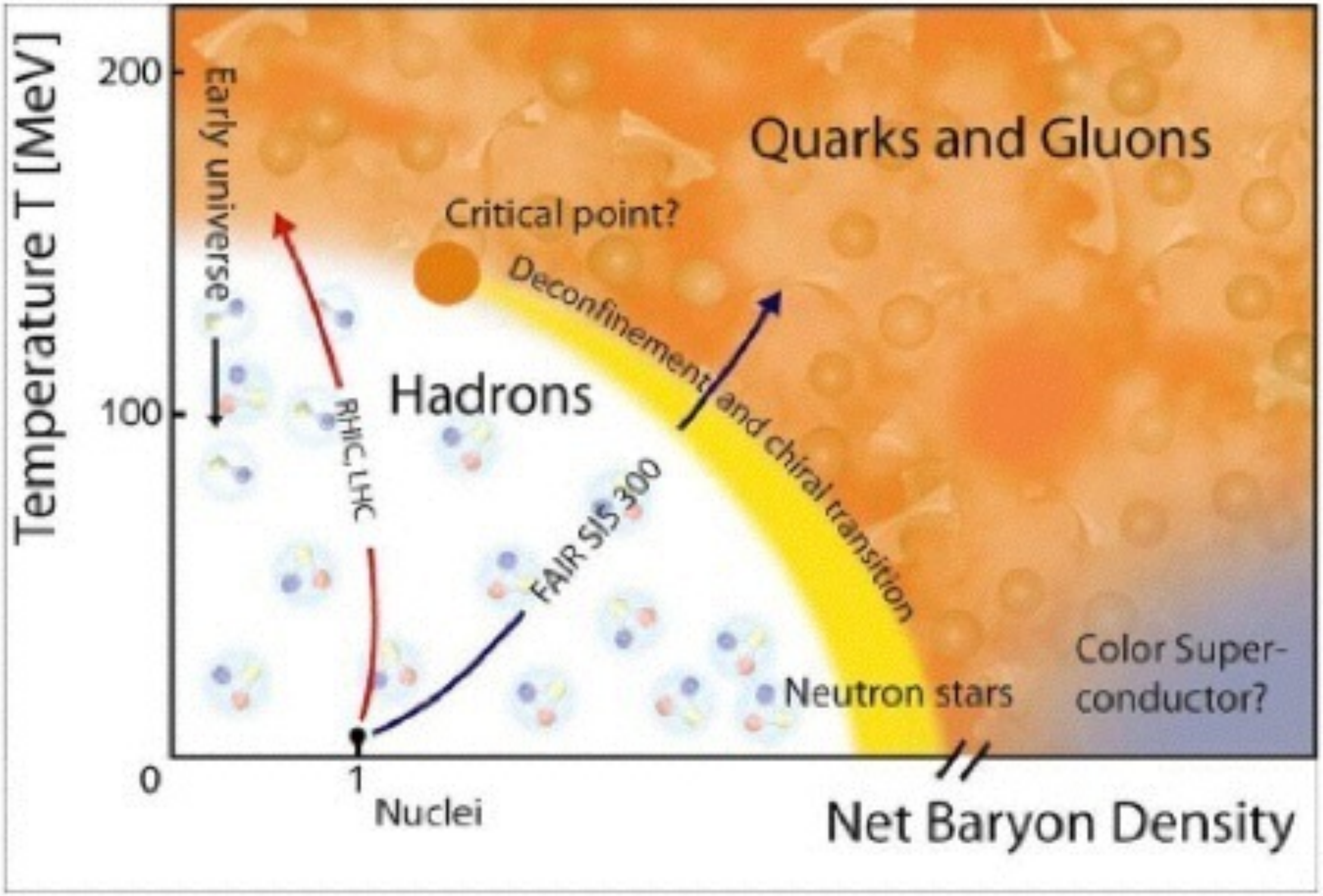
Non-central HIC - $eB \simeq 5 - 15 m^2_\pi$

Early Universe - $eB \simeq 30 m^2_\pi$

in natural units: $eB = 1\ \mathrm{GeV}^2 \quad B = 1.69 \times 10^{20} \mathrm{G}$

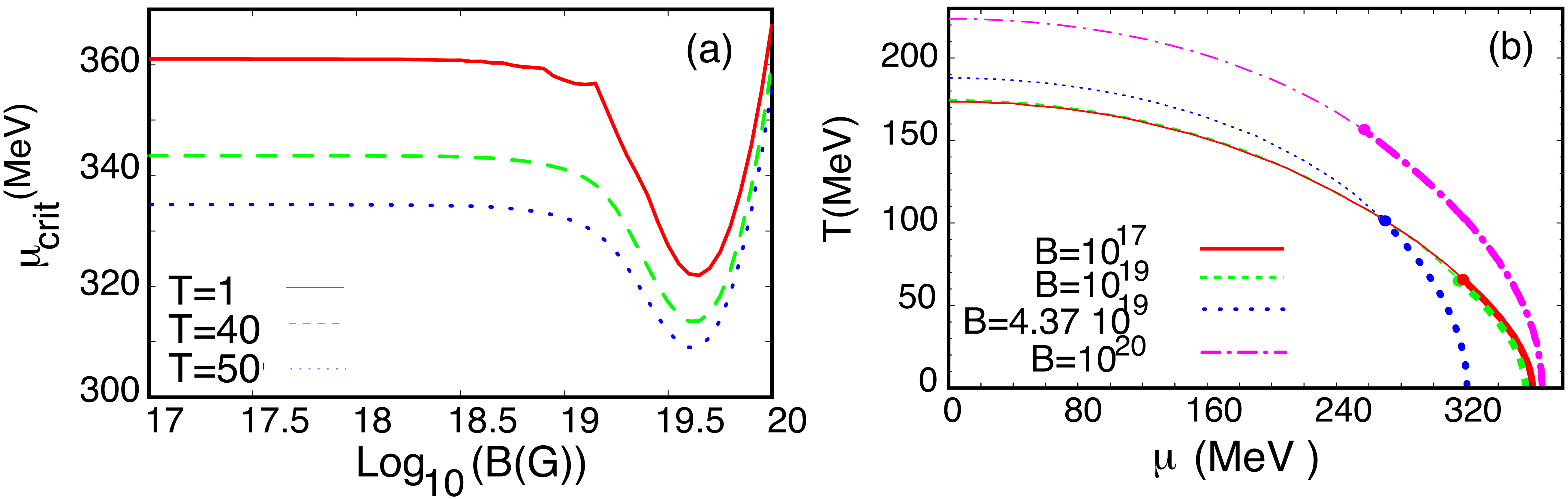
Heaviside-Lorentz, Gaussian and natural units lead to different conversions!

QCD Phase Diagram

- 

What would happen if matter were subject to strong magnetic fields ?

NJL model



S.S. Avancini, D.P. Menezes, M.B. Pinto and C. Providência - Phys. Rev. D 85, 091901(R) (2012)

The formalism

DPM, M. Benghi Pinto, S.S. Avancini, A. Perez Martinez and C. Providência, PRC 79, 035807 (2009).

$$\mathcal{L} = \bar{\psi}\left[\gamma_\mu\left(i\partial^\mu - qA^\mu\right) - \hat{m}_f\right]\psi \; + \; \mathcal{L}_{sym} + \; \mathcal{L}_{det} \; + \mathcal{L}_{vec} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\mathcal{L}_{sym} \; = \; G_S \sum_{a=0}^{8}\left[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}i\gamma_5\lambda_a\psi)^2\right]$$

$$\mathcal{L}_{det} \; = \; -K\left\{\det\left[\bar{\psi}(1+\gamma_5)\psi\right] + \det\left[\bar{\psi}(1-\gamma_5)\psi\right]\right\}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad\qquad A_\mu = \delta_{\mu 2} x_1 B$$

$$M_i = m_i - 4G_S\phi_i + 2K\phi_j\phi_k$$

$$P = \theta_u + \theta_d + \theta_s - 2G_S(\phi_u^2 + \phi_d^2 + \phi_s^2) + 4K\phi_u\phi_d\phi_s$$

$$\theta_f = \theta_f^{vac} + \theta_f^{mag} + \theta_f^{med} \qquad \phi_f = \phi_f^{vac} + \phi_f^{mag} + \phi_f^{med}$$

$$\theta_f^{vac} = -\frac{N_c}{8\pi^2}\left\{M_f^4 \ln\left[\frac{(\Lambda + \epsilon_\Lambda)}{M_f}\right] - \epsilon_\Lambda\,\Lambda\left(\Lambda^2 + \epsilon_\Lambda^2\right)\right\}$$

$$\epsilon_\Lambda = \sqrt{\Lambda^2 + M_f^2}$$

$$\theta_f^{mag} = \frac{N_c(|q_f|B)^2}{2\pi^2}\left[\zeta^{(1,0)}(-1, x_f) - \frac{1}{2}(x_f^2 - x_f)\ln x_f + \frac{x_f^2}{4}\right]$$

$$x_f = M_f^2/(2|q_f|B), \quad \zeta^{(1,0)}(-1, x) = d\zeta(z, x)/dz|_{z=-1}$$

$$\theta_f^{med} = \sum_{k=0}^{k_{f,max}} \alpha_k \frac{|q_f|BN_c}{4\pi^2} \int dp \left[\ln\left(1+\exp[-(E_f-\tilde{\mu}_f)/T]\right) + \ln\left(1+\exp[-(E_f+\tilde{\mu}_f)/T]\right)\right],$$

$$f_f^{\pm} = \frac{1}{\{1+\exp[(E_f \mp \tilde{\mu}_f)/T]\}}$$

$$\phi_f^{vac} = -\frac{M_f N_c}{2\pi^2}\left[\Lambda\epsilon_\Lambda - M_f^2 \ln\left(\frac{\Lambda+\epsilon_\Lambda}{M_f}\right)\right]$$

$$\phi_f^{mag} = -\frac{M|q_f|BN_c}{2\pi^2}\left[\ln\Gamma(x_f) - \frac{1}{2}\ln(2\pi) + x_f - \frac{1}{2}(2x_f-1)\ln(x_f)\right],$$

$$\phi_f^{med} = \sum_{k=0}^{k_{max}} \alpha_k \frac{M_f|q_f|BN_c}{2\pi^2} \int dp \frac{(f_f^+ + f_f^-)}{E_f}$$

$$\alpha_0 = 1,\ \alpha_{k>0} = 2, \quad E_f = \sqrt{p^2 + M_f^2 + 2|q_f|B}$$

Parameter set	Λ MeV	$G\Lambda^2$	$K\Lambda^5$	$m_{u,d}$ MeV	m_s MeV
Set 1 (HK)	631.4	1.835	9.29	5.5	135.7
Set 2 (RKH)	602.3	1.835	12.36	5.5	140.7

HK -T. Hatsuda and T. Kunihiro, Phys. Rep. 247, 221(1994)

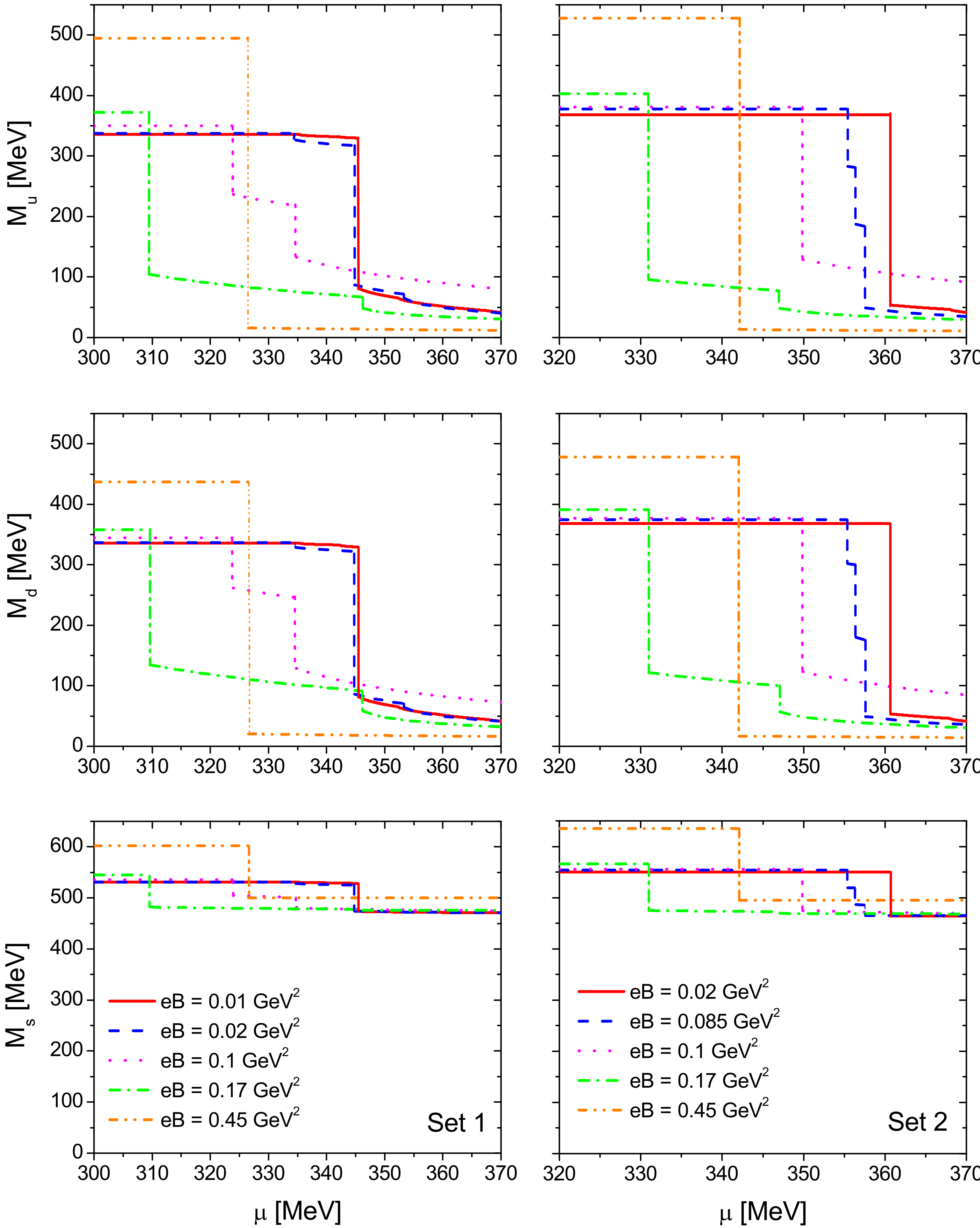
RKH - P. Rehberg, S. P. Klevansky and J. Hufner, Phys. Rev. C 53, 410 (1996)

T=0 / NJL
Phase structure of magnetized matter

Two cases:

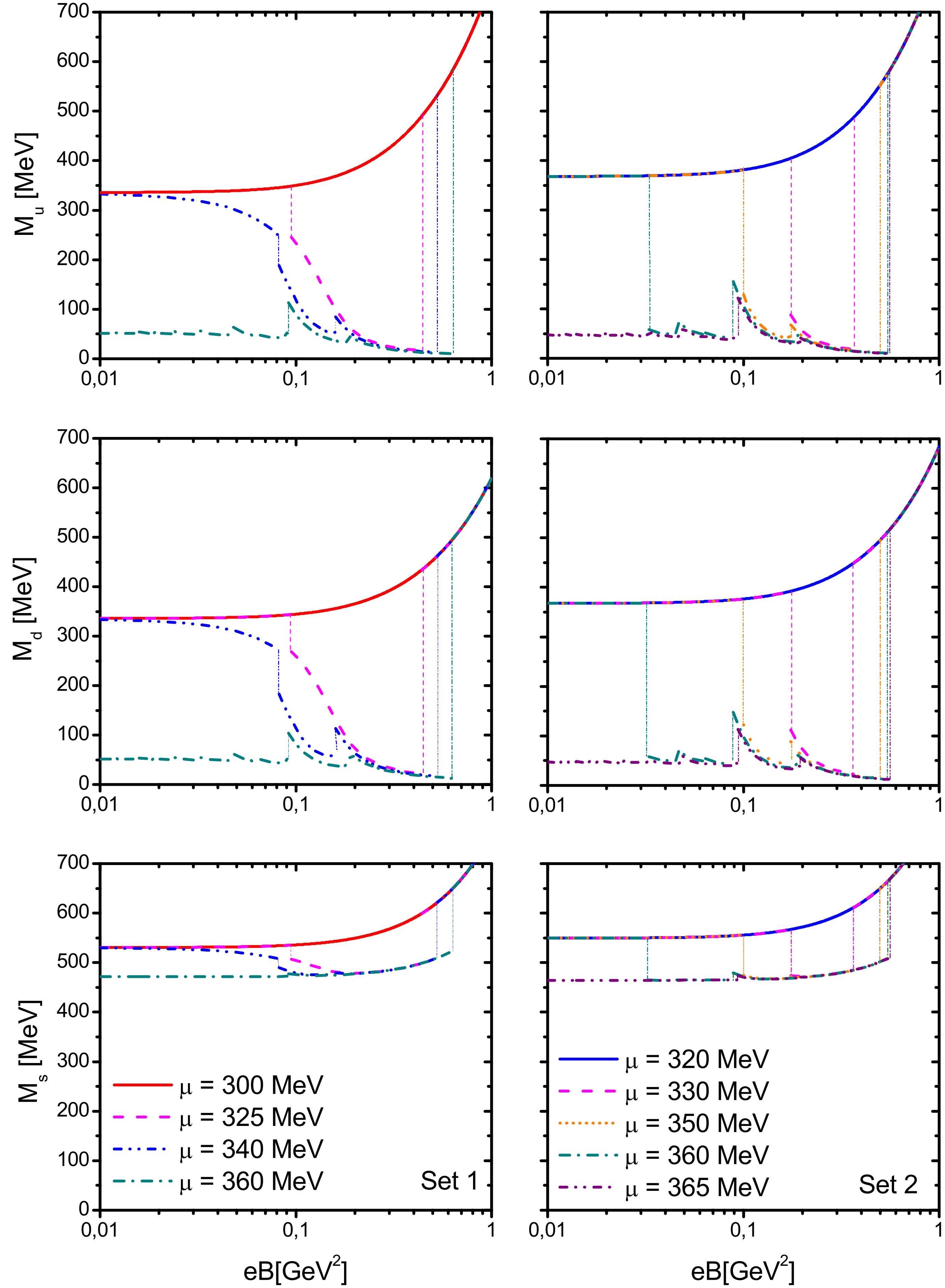
pure quark (*symmetric)* matter

matter in beta-equilibrium (with leptons)



The drop in the masses correspond to a 1st order phase transition, which goes from the fully chirally broken phase, where masses do not depend on the chemical potential, to a less massive phase, where they do.

The last jump indicates a state where the dressed u and d quarks are much smaller than their vacuum values, so that they are in the fully chirally restored phase.



M_u [MeV]
M_d [MeV]
M_s [MeV]
eB[GeV^2]
0,01
0,1
1
μ = 300 MeV
μ = 325 MeV
μ = 340 MeV
μ = 360 MeV
Set 1
μ = 320 MeV
μ = 330 MeV
μ = 350 MeV
μ = 360 MeV
μ = 365 MeV
Set 2

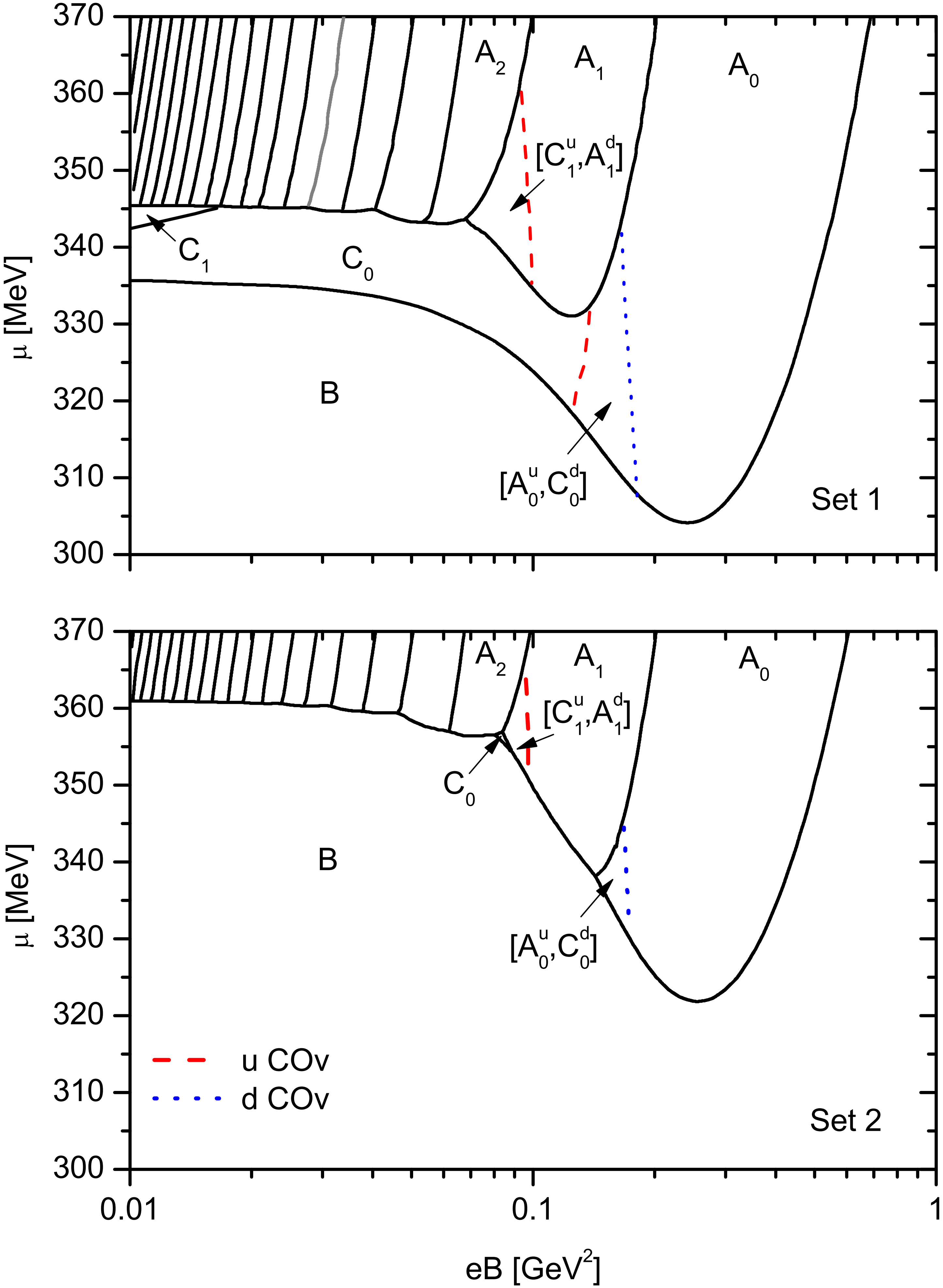
$$\chi_f = d\phi_f / dm_f$$

The peak of the susceptibilities are used to identify possible crossover transitions

Notation:
fully chirally broken phase = B
massive phases where M depends on mu = Ci
chirally restored phases = Ai

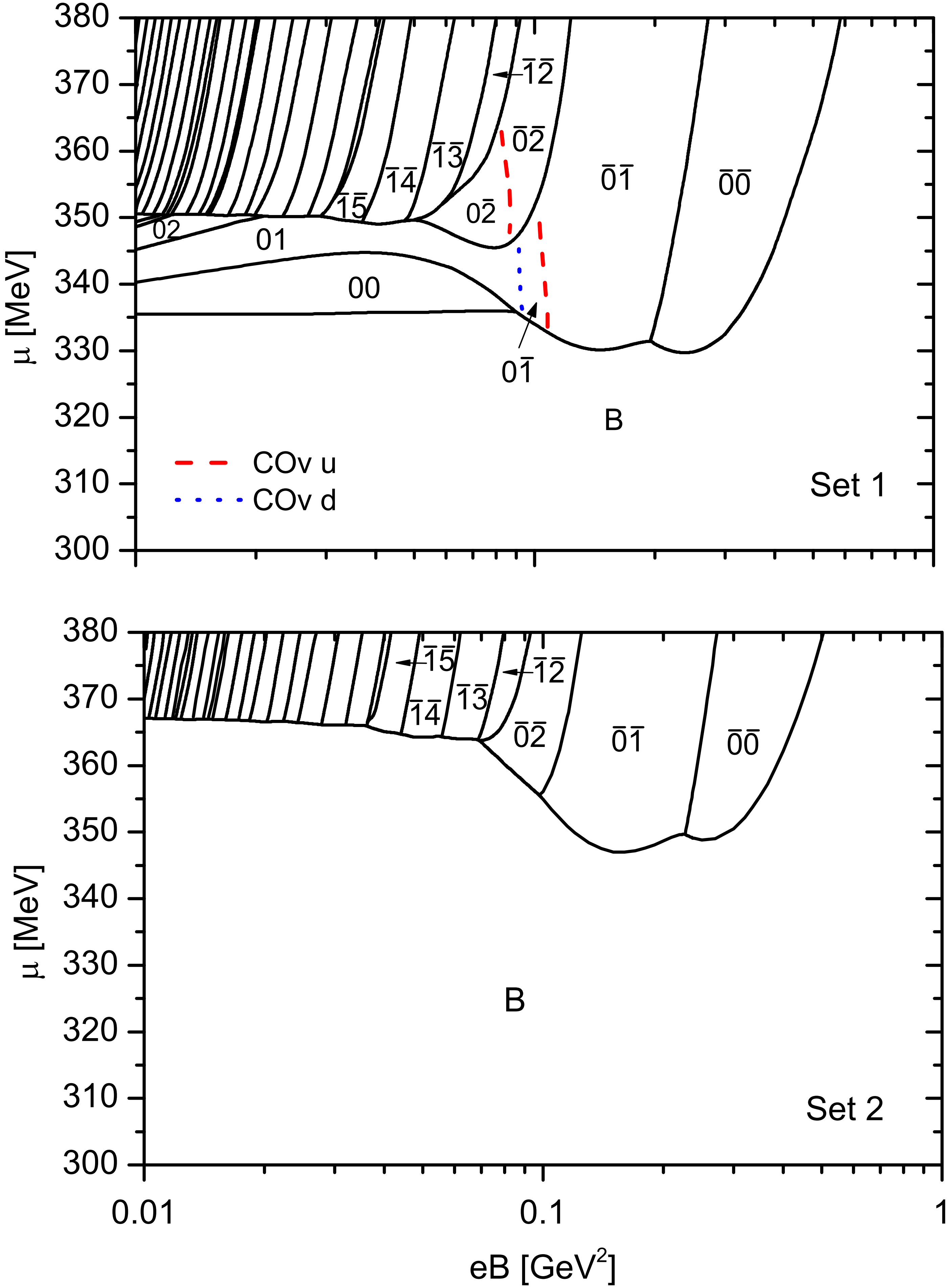
Ex: region where the u quark is in the A0 phase and the d quark in the C0 phase:

$$[\mathrm{A}_0^u, \mathrm{C}_0^d]$$



A2
A1
A0
[C1u,A1d]
C1
C0
B
[A0u,C0d]
Set 1
Set 2
u COv
d COv
μ [MeV]
eB [GeV2]
0.01
0.1
1
300
310
320
330
340
350
360
370

charge-neutral matter in beta equilibrium



$$\mu_s = \mu_d = \mu_u + \mu_e, \quad \mu_e = \mu_\mu$$

$$\rho_e + \rho_\mu = \frac{1}{3}(2\rho_u - \rho_d - \rho_s)$$

The pair of integers mn corresponds to the C_{mn} phase

The pair $\bar{m}\,\bar{n}$ to the A_{mn} phase

If one quark is in a C-type phase and the other in the A-type phase, a bar is placed on top of the integer associated with the A-type phase.

Conclusions -1

- Qualitative results are parameter dependent; set 1 gives a richer phase diagram than set 2
- Number of jumps in the quark masses are related to the number of filled LL
- Different susceptibilities imply different crossover transitions for each quark: there are some regions where the quarks of different flavors are in different phases
- Matter in beta-equilibrium exhibit 1st order phase transitions that take place at higher chemical potentials
- In matter subject to magnetic fields, spatially inhomogeneous condensate configurations can be favored

T=0 NJLv

$$\mathcal{L}_{vec} = -G_V \sum_{a=0}^{8} \left[(\bar{\psi}\gamma^\mu \lambda_a \psi)^2 + (\bar{\psi}\gamma^\mu \gamma_5 \lambda_a \psi)^2 \right]$$

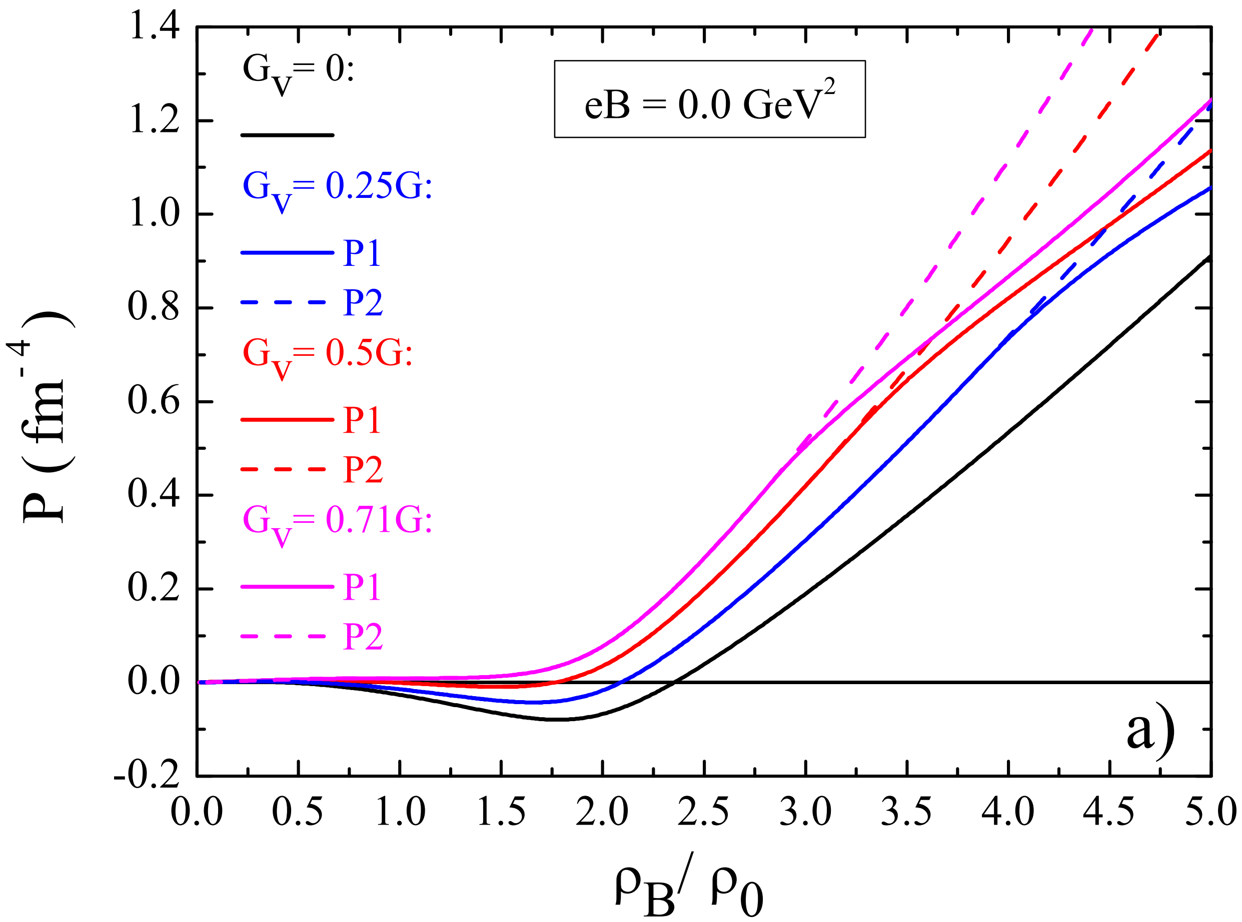
$$P_{P1} = P_{NJL} + 2G_V(\rho_u^2 + \rho_d^2 + \rho_s^2)$$

$$\tilde{\mu}_i = \mu_i - 4G_V\rho_i. \quad i = u, d, s$$

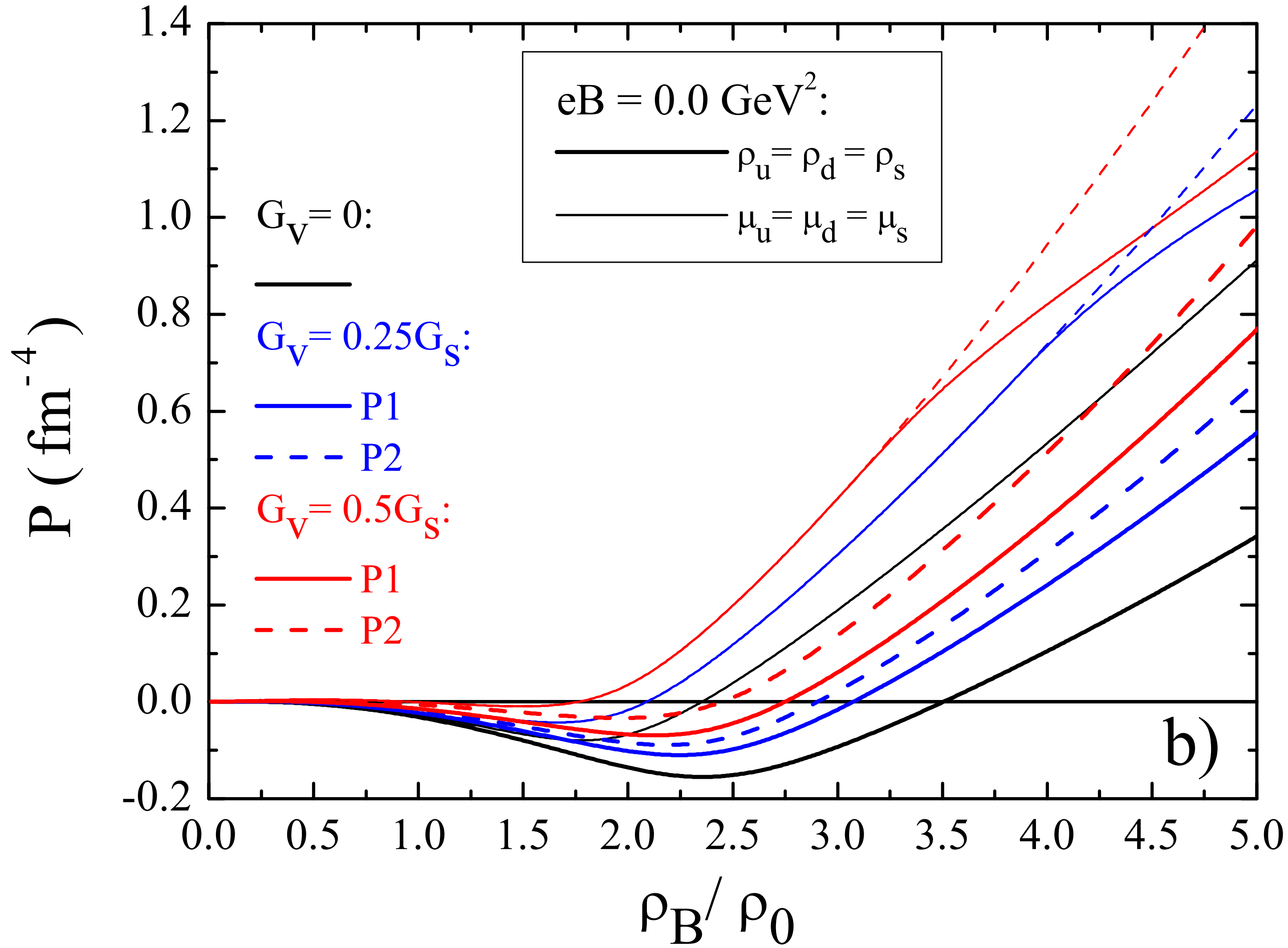
$$\mathcal{L}_{vec} = -G_V(\bar{\psi}\gamma^\mu\psi)^2$$

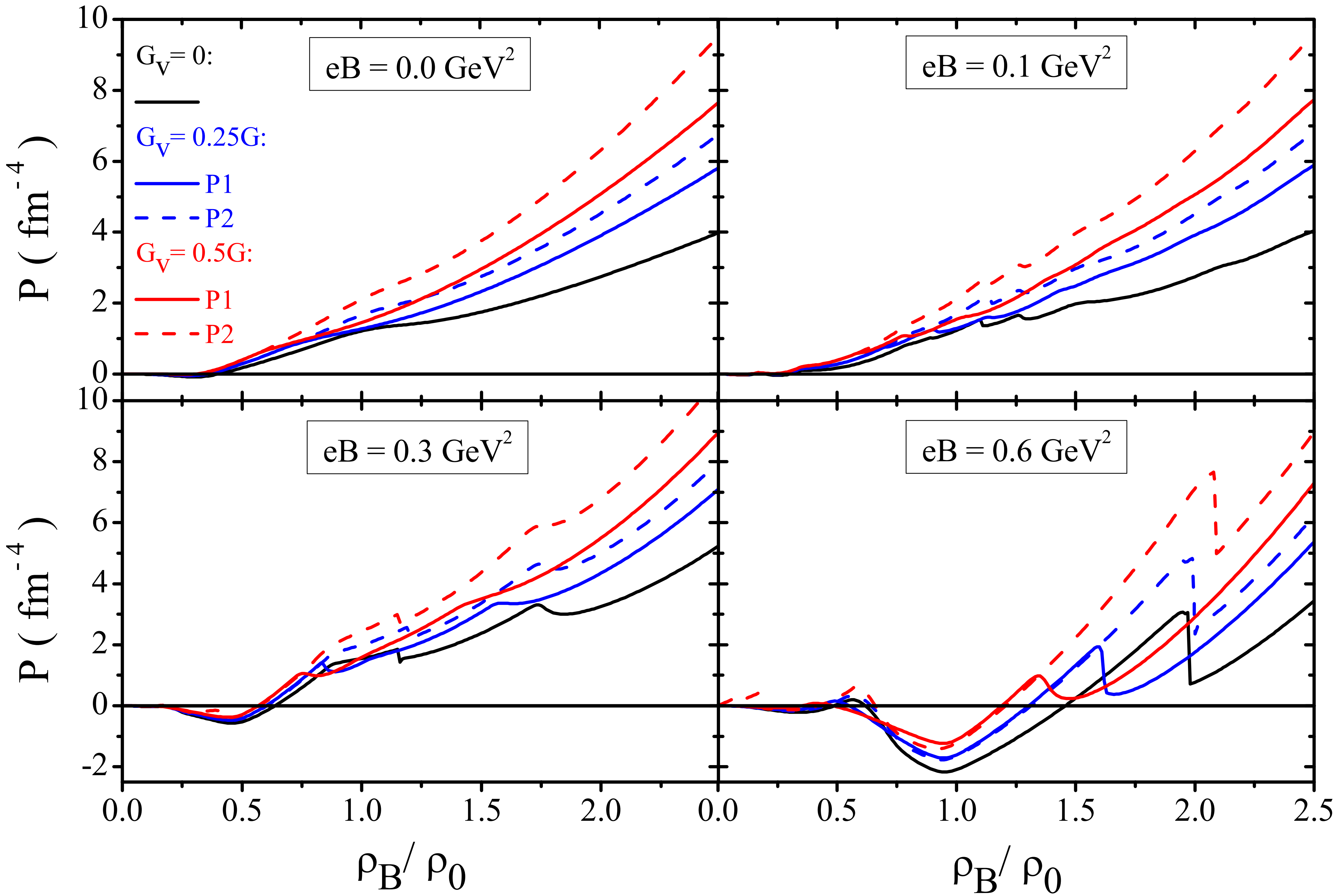
$$P_{P2} = P_{NJL} + G_V\rho^2 \qquad \tilde{\mu}_i = \mu_i - 2G_V\rho$$

D P M, Marcus B. Pinto, Luis R. B de Castro,Constança Providencia and Pedro Costa, Phys. Rev. C 89, 055207 (2014)

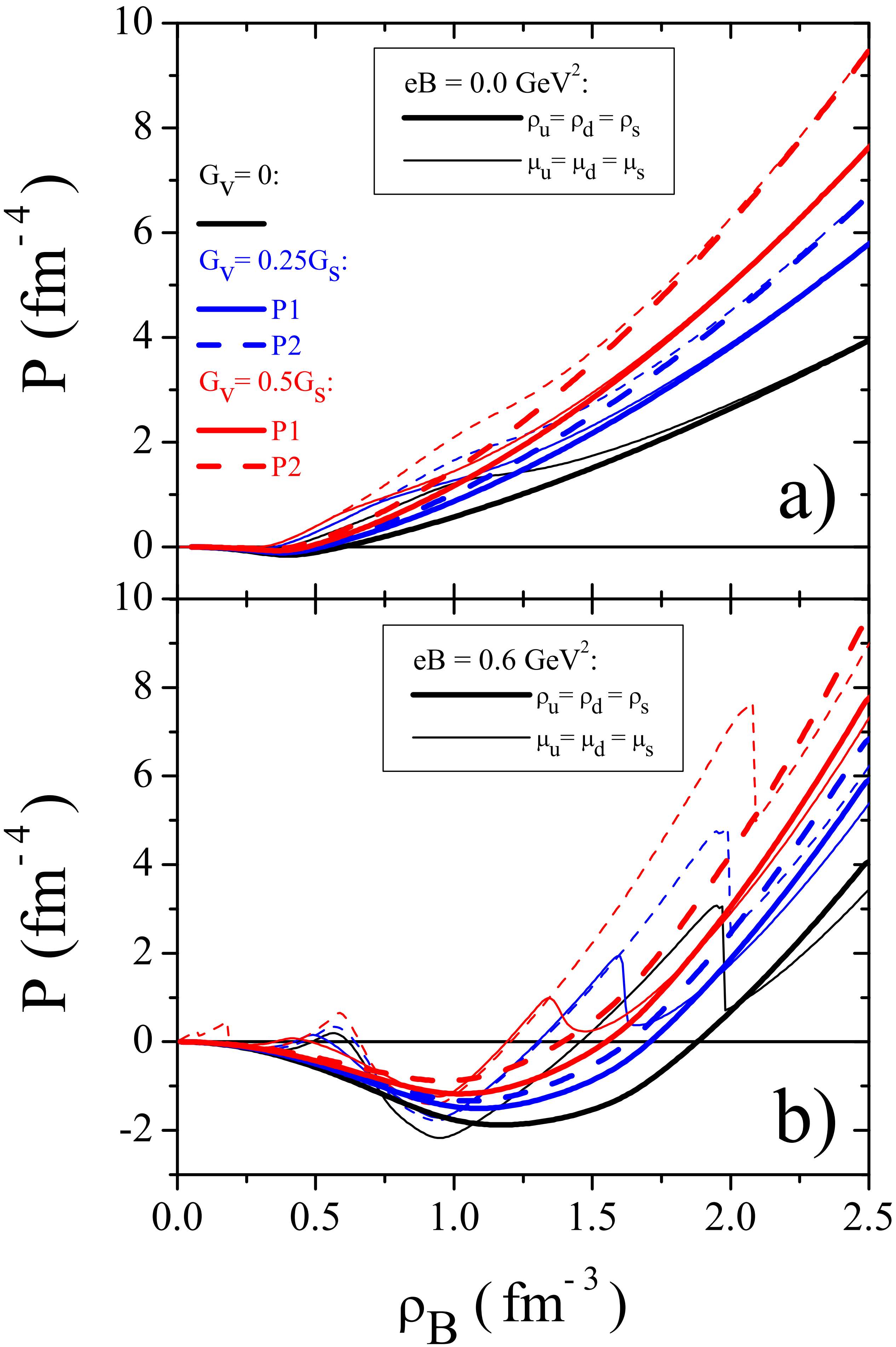


Pure quark matter
RKH

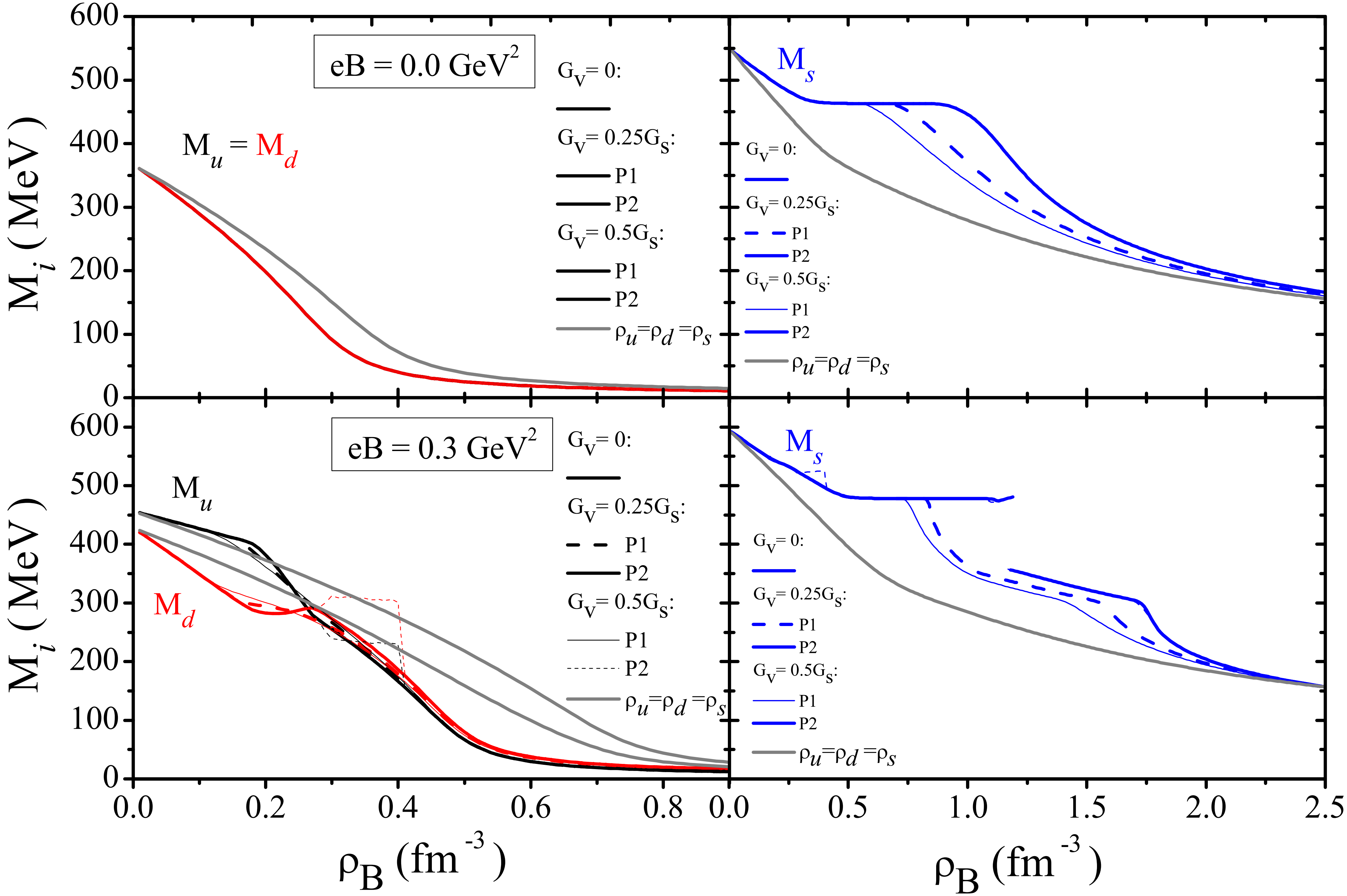




RKH - equal chemical potentials

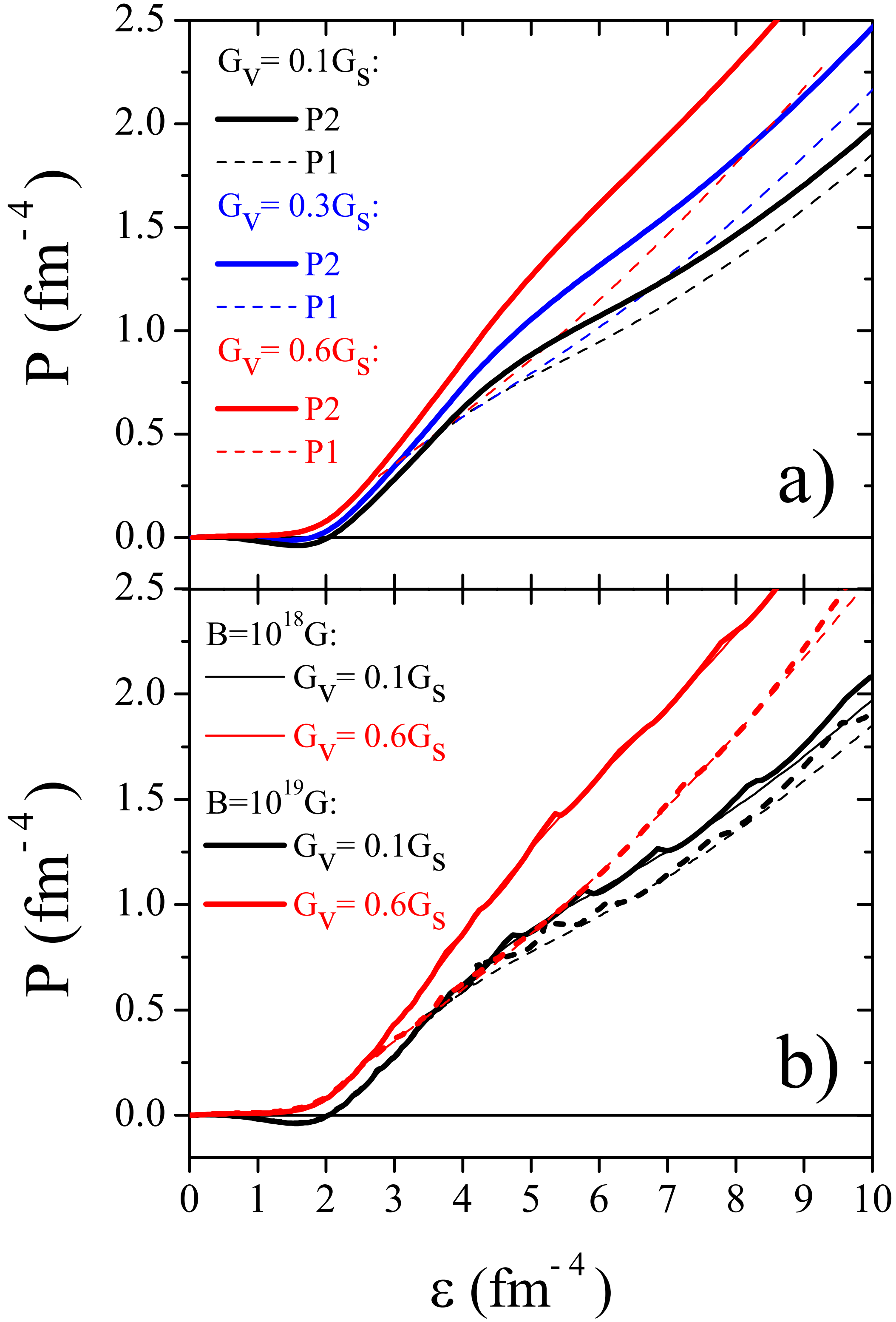


eB = 0.0 GeV^2:
$\rho_u = \rho_d = \rho_s$
$\mu_u = \mu_d = \mu_s$
$G_V = 0$:
$G_V = 0.25G_S$:
P1
P2
$G_V = 0.5G_S$:
P1
P2
a)
eB = 0.6 GeV^2:
$\rho_u = \rho_d = \rho_s$
$\mu_u = \mu_d = \mu_s$
b)
P (fm^{-4})
ρ_B (fm^{-3})

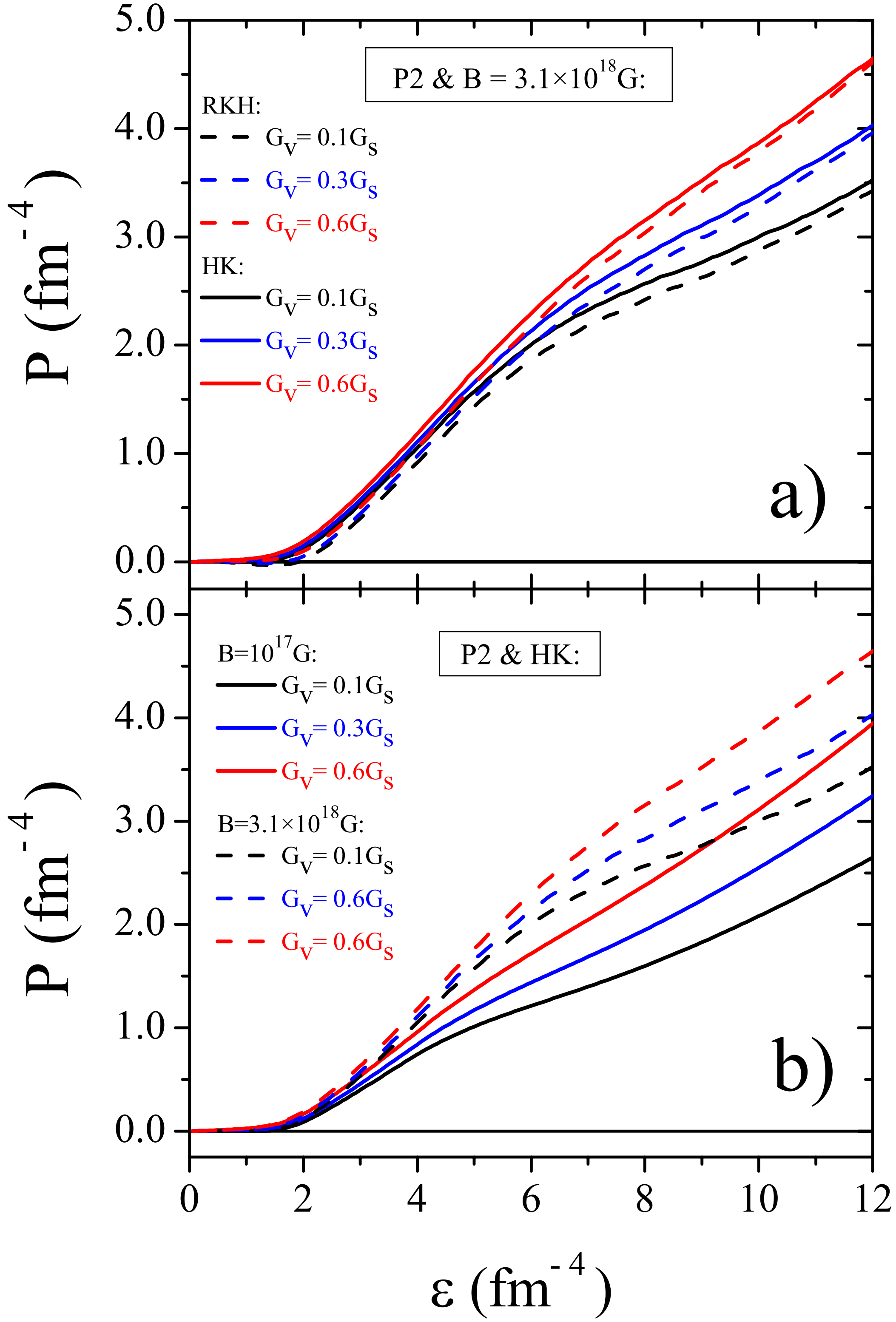


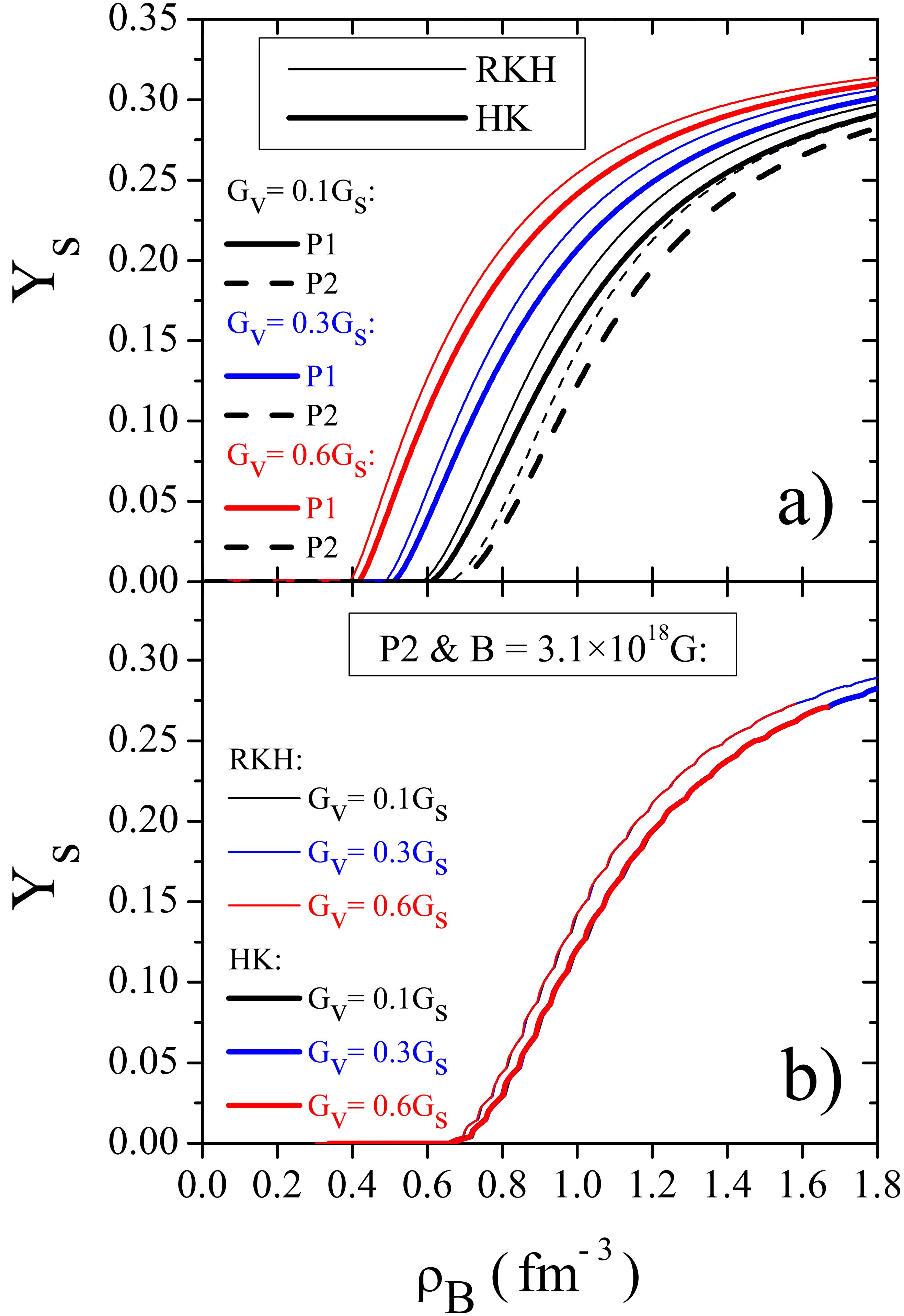
equal chemical potentials, except gray lines

Stellar matter - quark stars

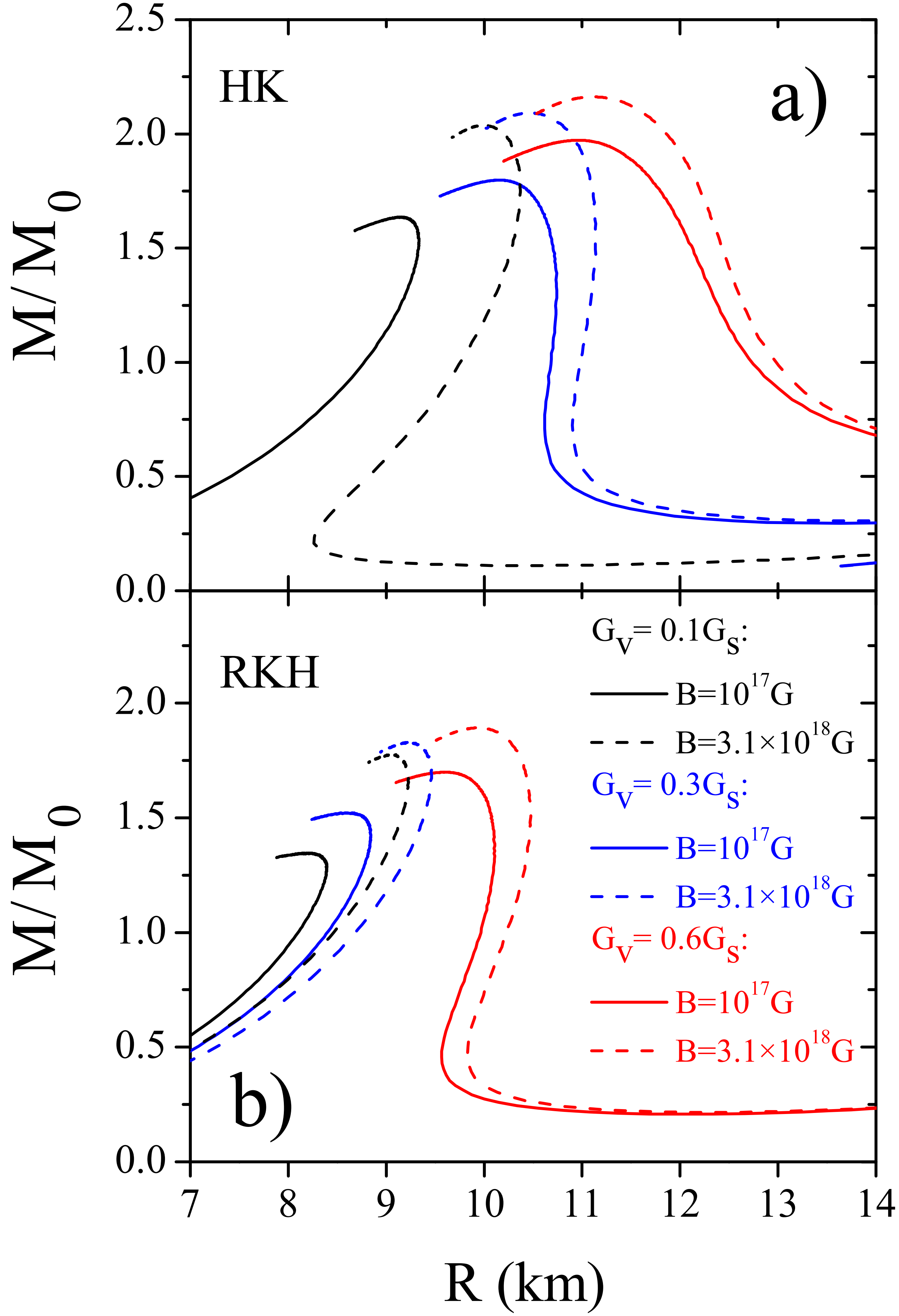


RKH - a) B=0





RKH
HK
$G_V = 0.1G_S$:
P1
P2
$G_V = 0.3G_S$:
P1
P2
$G_V = 0.6G_S$:
P1
P2
a)
P2 & B = 3.1×10^18 G:
RKH:
$G_V = 0.1G_S$
$G_V = 0.3G_S$
$G_V = 0.6G_S$
HK:
$G_V = 0.1G_S$
$G_V = 0.3G_S$
$G_V = 0.6G_S$
b)
Y_s
ρ_B (fm^{-3})

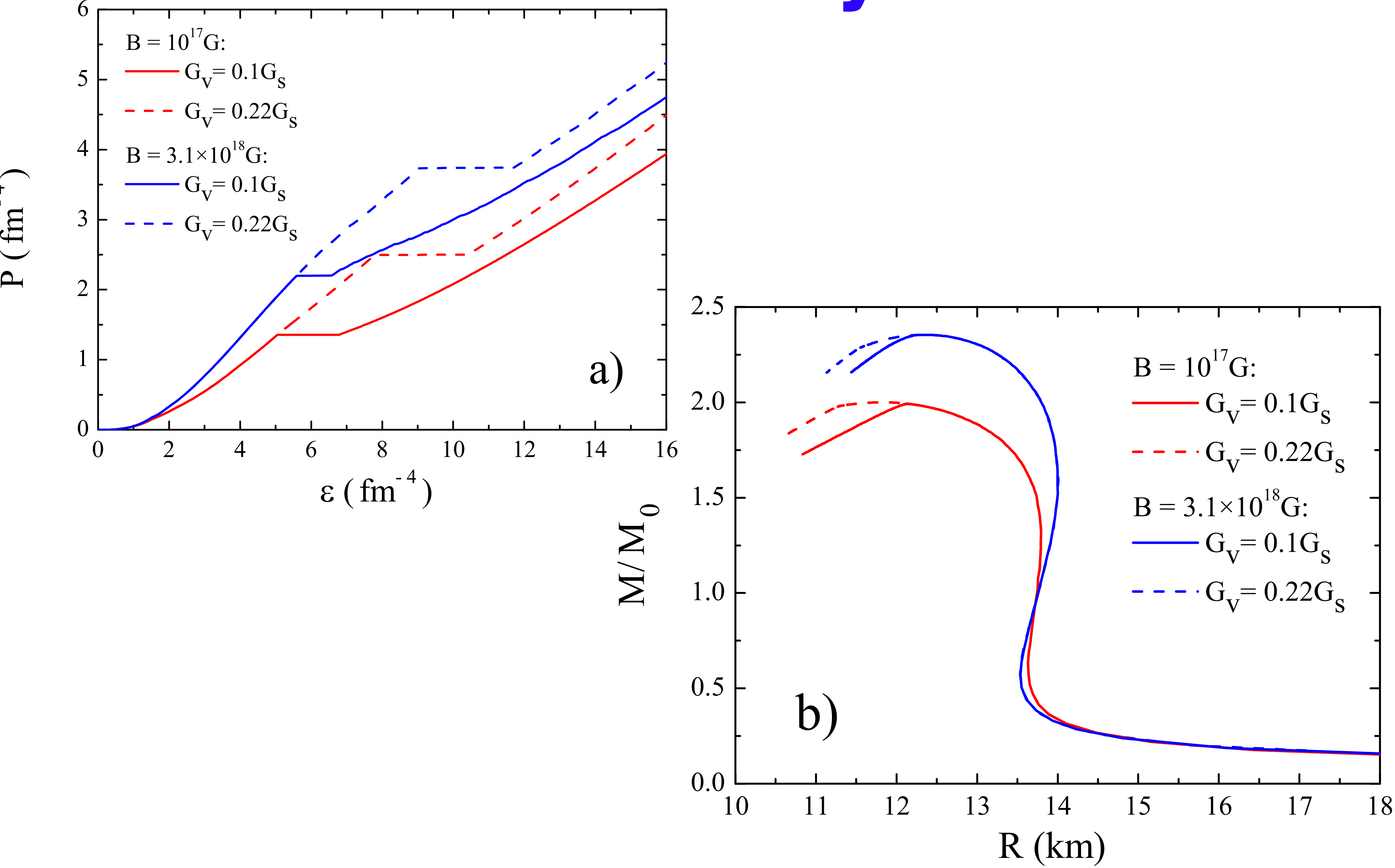


HK
a)
RKH
b)
M/M$_0$
R (km)
G$_V$= 0.1G$_S$:
B=10^{17}G
B=3.1×10^{18}G
G$_V$= 0.3G$_S$:
B=10^{17}G
B=3.1×10^{18}G
G$_V$= 0.6G$_S$:
B=10^{17}G
B=3.1×10^{18}G

Stellar macroscopic properties

			HK			RKH		
			$x = 0.1$	$x = 0.3$	$x = 0.6$	$x = 0.1$	$x = 0.3$	$x = 0.6$
$B = 0$ G	P1	M_{max} (M_0)	1.49	1.58	1.69	1.27	1.35	1.46
		R (km)	9.13	10.89	11.98	8.01	8.17	9.41
		ε_c (fm^{-4})	7.23	6.96	6.52	9.42	9.61	9.84
	P2	M_{max} (M_0)	1.56	1.72	1.91	1.35	1.54	1.74
		R (km)	9.15	10.61	11.47	8.22	8.60	9.91
		ε_c (fm^{-4})	7.35	7.37	6.92	8.71	8.58	8.09
$B = 10^{17}$ G P2		M_{max} (M_0)	1.56	1.72	1.91	1.35	1.54	1.74
		R (km)	9.16	10.16	10.95	8.21	8.58	9.60
		ε_c (fm^{-4})	7.41	7.36	6.98	8.80	8.94	8.11
$B = 3.1 \times 10^{18}$ G P2		M_{max} (M_0)	1.96	2.03	2.12	1.81	1.88	1.98
		R (km)	9.98	10.43	11.05	9.03	9.21	9.90
		ε_c (fm^{-4})	7.41	7.22	6.78	8.74	8.21	7.80

Stellar matter - hybrid stars



HK		M_{max} (M_0)	M_b (M_0)	R (km)	ε_c (fm^{-4})	ε(onset) (fm^{-4})	ρ_c	ρ	$\mu_B(\varepsilon_c)$ (MeV)	μ_B (MeV)
$B = 10^{17}$ G P2	$x = 0$	1.91	2.18	12.78	4.57	3.47	0.78	0.62	1360	1330
	$x = 0.10$	1.99	2.30	12.14	6.27	5.05	-	0.84	-	1503
	$x = 0.22$	2.00	2.31	11.82	5.93	7.79	0.95	1.18	1580	1726
$B = 3.1 \times 10^{18}$ G P2	$x = 0$	2.27	2.60	12.82	4.69	3.30	0.70	0.54	1324	1261
	$x = 0.10$	2.35	2.70	12.34	5.29	5.59	0.74	0.78	1427	1453
	$x = 0.22$	2.35	2.70	12.35	5.27	9.03	0.74	1.18	1426	1730

RKH		M_{max} (M_0)	M_b (M_0)	R (km)	ε_c (fm^{-4})	ε(onset) (fm^{-4})	ρ_c	ρ	$\mu_B(\varepsilon_c)$ (MeV)	μ_B (MeV)
$B = 10^{17}$ G P2	$x = 0$	1.97	2.26	12.48	4.29	4.28	-	0.74	-	1422
	$x = 0.10$	2.00	2.31	11.91	7.51	5.67	-	0.92	-	1557
	$x = 0.19$	2.00	2.31	11.83	5.91	7.83	0.95	1.18	1579	1728
$B = 3.1 \times 10^{18}$ G P2	$x = 0$	2.33	2.69	12.79	4.69	4.19	-	0.63	-	1335
	$x = 0.10$	2.35	2.70	12.34	5.30	6.52	0.74	0.88	1428	1531
	$x = 0.19$	2.35	2.70	12.34	5.30	9.05	0.74	1.18	1428	1731

Conclusions - 2

- The flavor independent vector interaction predicts smaller strangeness content and hence harder EOS

- At low densities, the magnetic field and the vector interaction have opposite effects (softens x hardens the EOS)

- Quarks stars - larger masses for the flavor independent vector interaction

- Hybrid stars may bear a core of deconfined quarks if neither B nor the vector interaction are too strong

Finite T - PNJL / EPNJL

$$\mathcal{L}_{(E)PNJL} = \mathcal{L}_{NJL} + \mathcal{U}(\Phi, \Phi^*; T)$$

$$\mathcal{U}(\Phi, \Phi^*; T) = T^4 \left\{ -\frac{a(T)}{2} \Phi^* \Phi + b(T) \ln \left[1 - 6\Phi^* \Phi + 4(\Phi^{*3} + \Phi^3) - 3(\Phi^* \Phi)^2 \right] \right\}$$

$$a(T) = a_0 + a_1 \left(\frac{T_0}{T} \right) + a_2 \left(\frac{T_0}{T} \right)^2 , \quad b(T) = b_3 \left(\frac{T_0}{T} \right)^3$$

$$a_0 = 3.51,\ a_1 = -2.47,\ a_3 = 15.2,\ b_3 = -1.75,\ T_0 = 210 \text{ MeV}$$

EPNJL effective vertex:

$$G(\Phi, \bar{\Phi}) = G \left[1 - \alpha_1 \Phi \bar{\Phi} - \alpha_2 (\Phi^3 + \bar{\Phi}^3) \right] ,\ \alpha_1 = 0.25,\ \alpha_2 = 0.10$$

Deconfinement and chiral restauration

$$\mu = 0$$

RKH parametrization

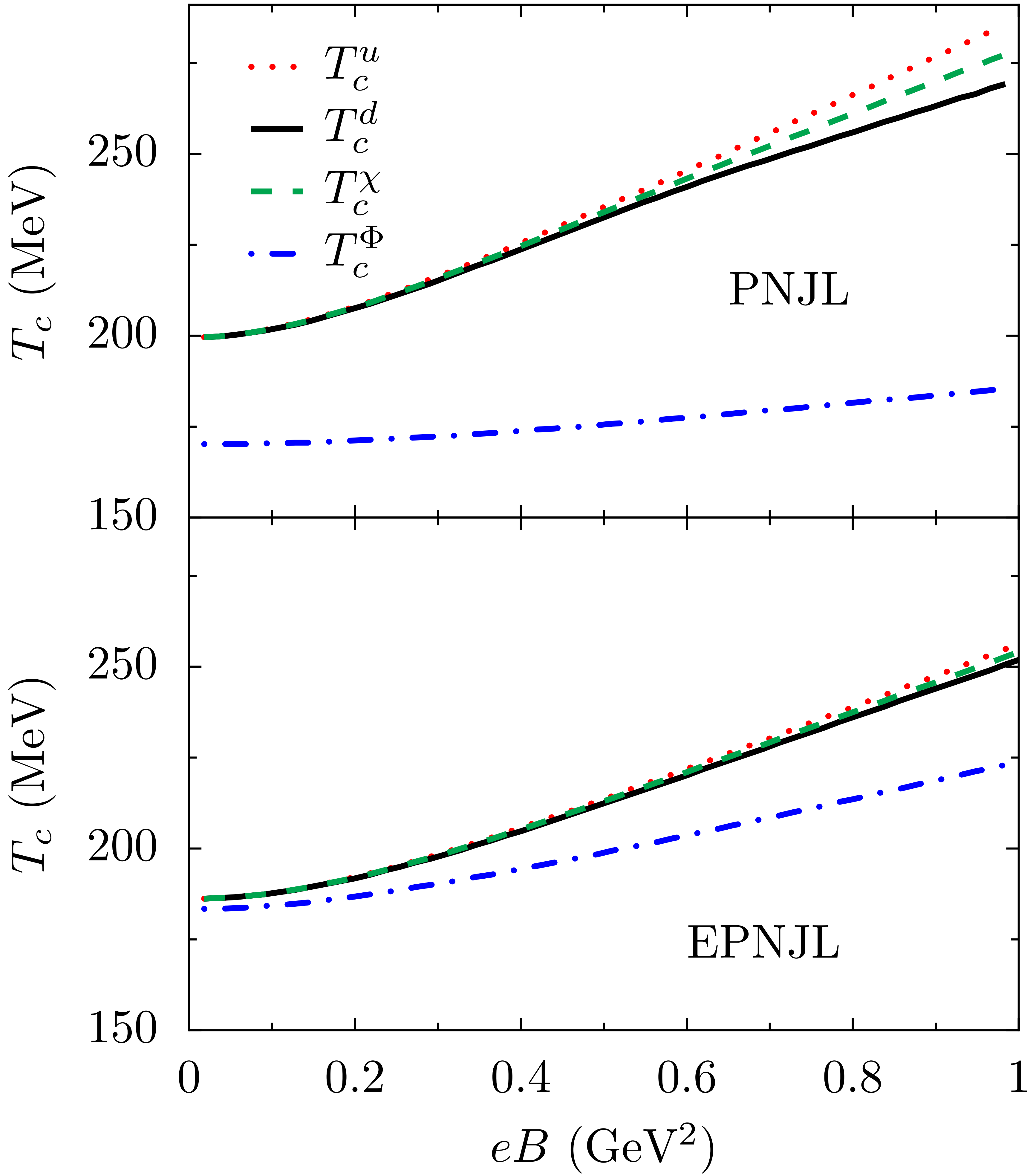
$$T_c^\chi = (T_u^\chi + T_d^\chi)/2,\ T_c^\Phi$$

pseudo-critical temperatures for the chiral and deconfinement transitions obtained from the peaks of the susceptibilities:

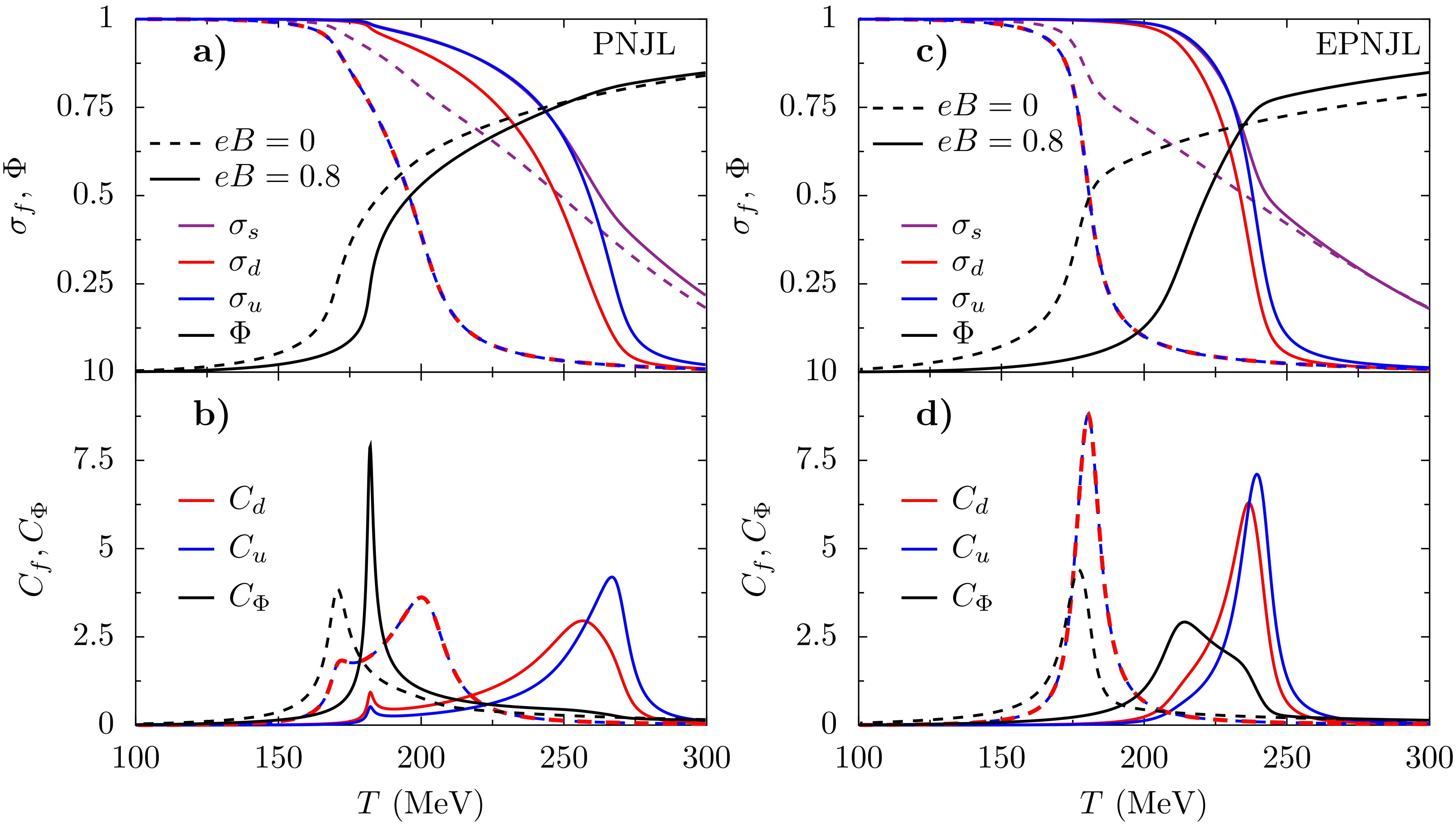
$$C_f = -m_\pi \partial\sigma_f/\partial T, \quad \sigma_f = \langle \bar{q_f} q_f \rangle (B,T)/\langle \bar{q_f} q_f \rangle (B,0)$$

$$C_\Phi = m_\pi \partial\Phi/\partial T$$

M. Ferreira,P. Costa, DPM, C. Providencia and N. Scoccola,
Phys. Rev. D 89, 016002 (2014)



T_c^u
T_c^d
T_c^χ
T_c^Φ
PNJL
EPNJL
T_c (MeV)
150
200
250
0
0.2
0.4
0.6
0.8
1
eB (GeV^2)



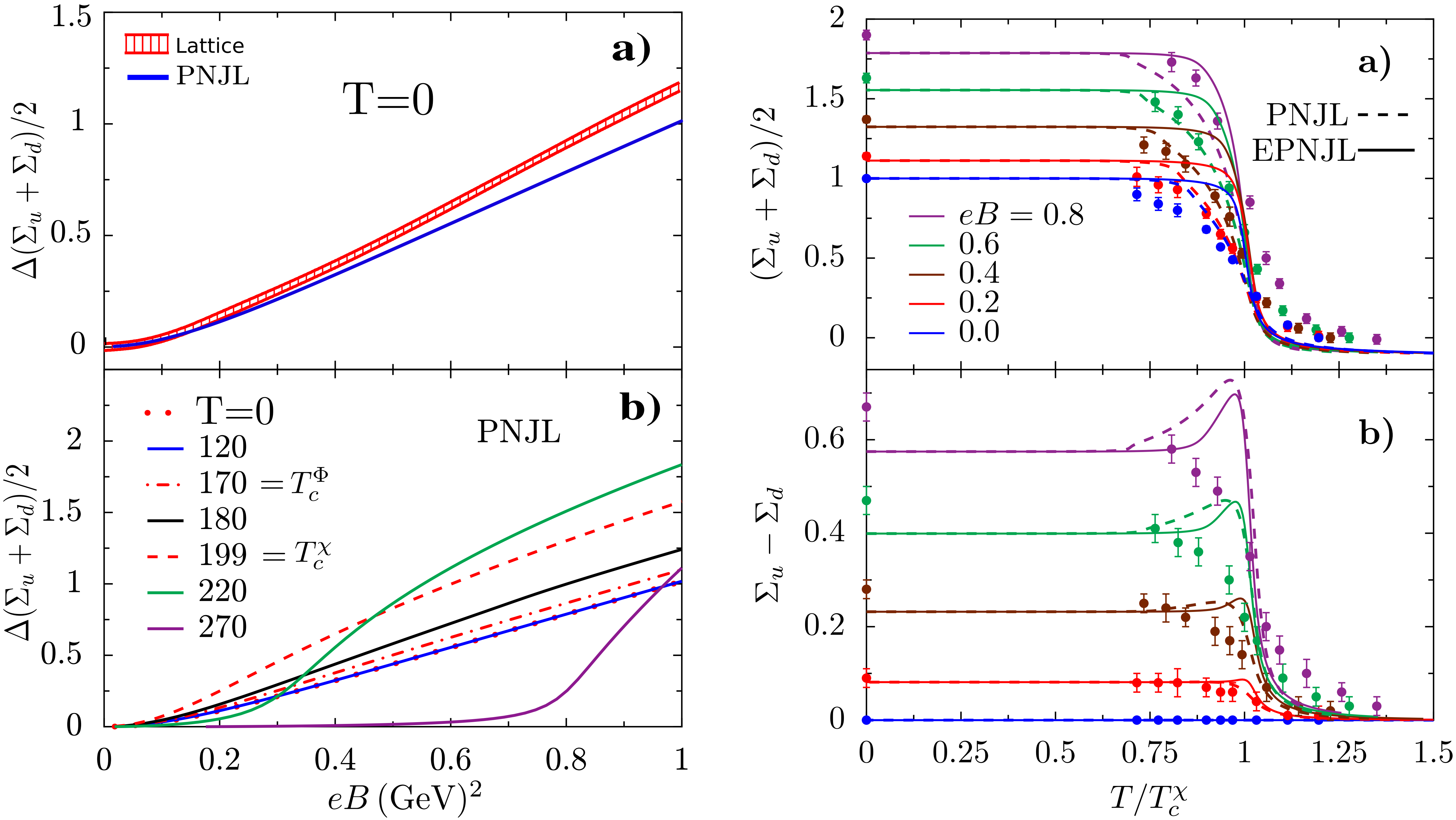
The existing coincidence at B=0 is destroyed by the magnetic field and *recovered* by the EPNJL

change of the light condensates due to B :

$$\Delta\Sigma_f(B,T) = \Sigma_f(B,T) - \Sigma_f(0,T)$$

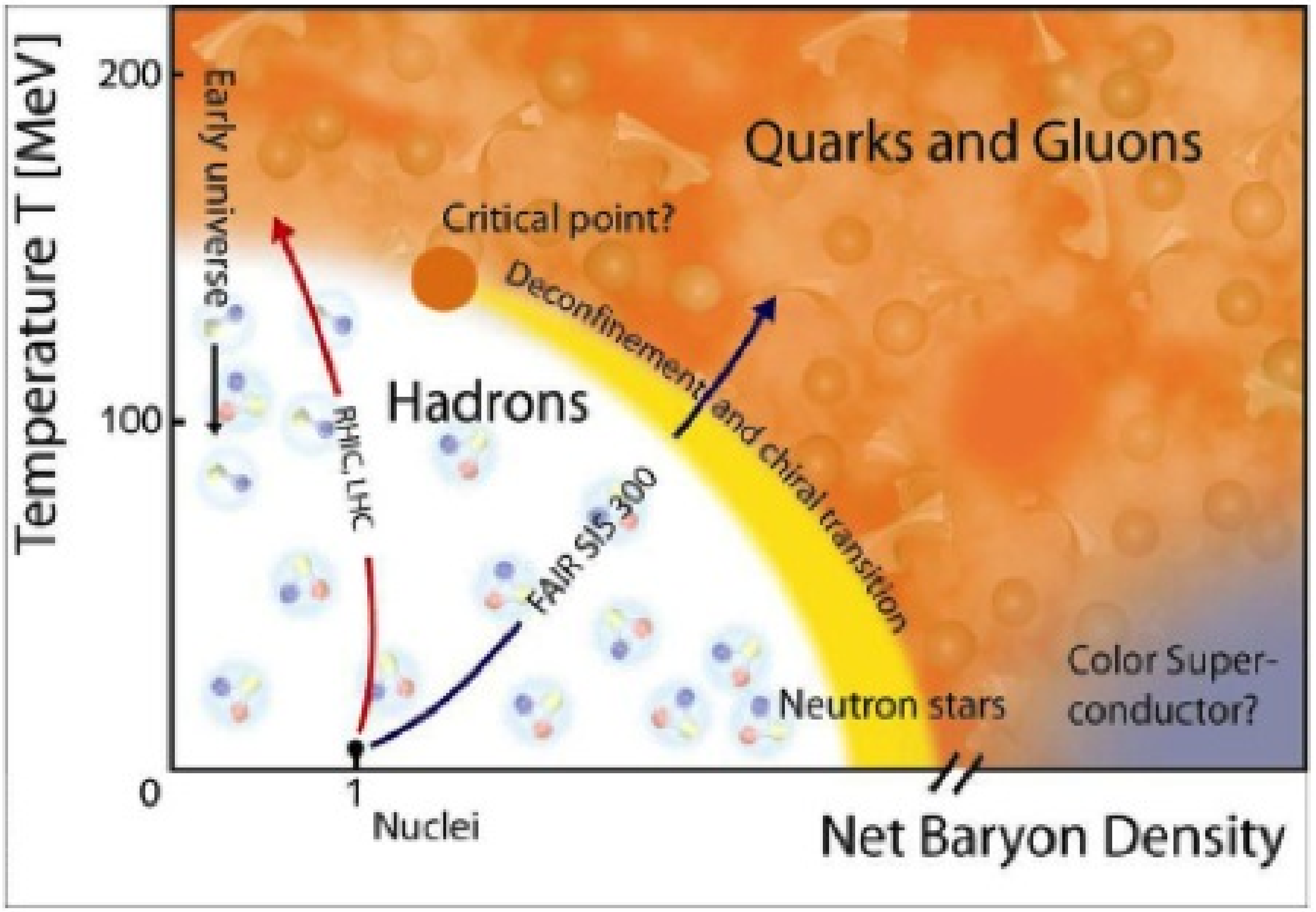
$$\Sigma_f(B,T) = \frac{2m_f}{m_\pi^2 f_\pi^2}\left[\langle \bar{q}_f q_f \rangle (B,T) - \langle \bar{q}_f q_f \rangle (0,0)\right] + 1$$

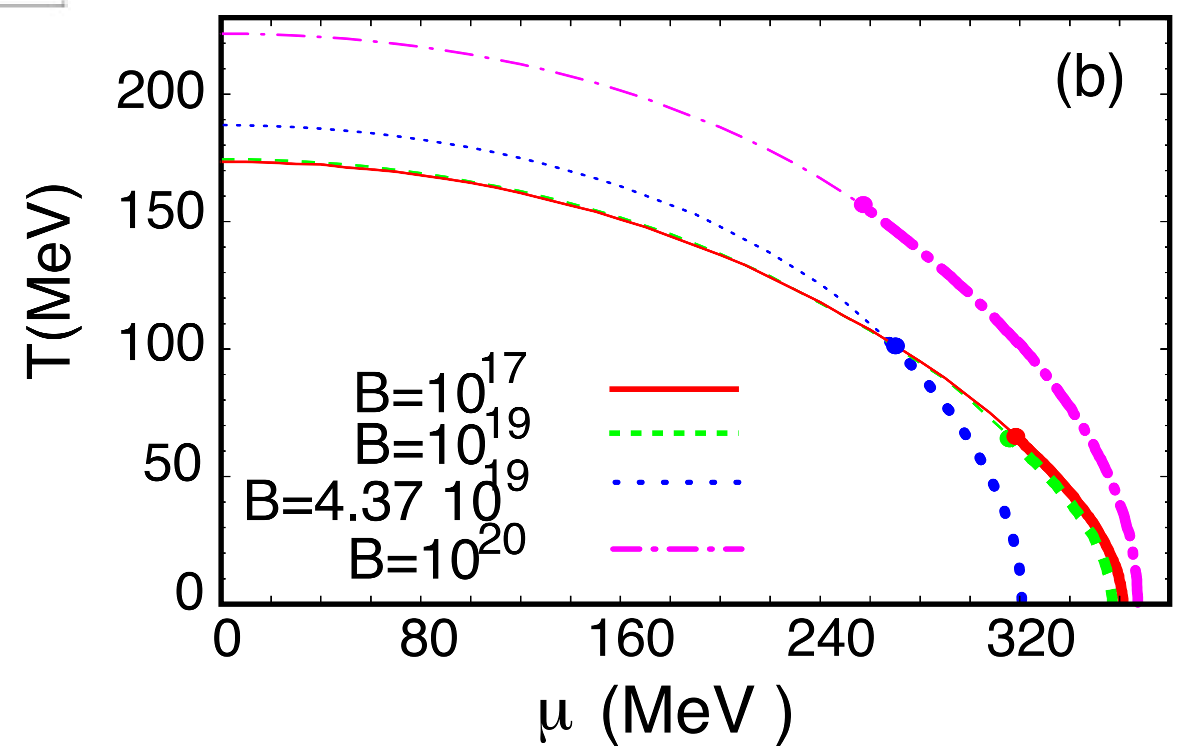
$$m_\pi = 135 \text{ MeV}, \quad f_\pi = 87.9 \text{ MeV}$$



Lattice results (courtesy): G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz, and A. Schafer, Phys. Rev. D 86, 071502 (2012)

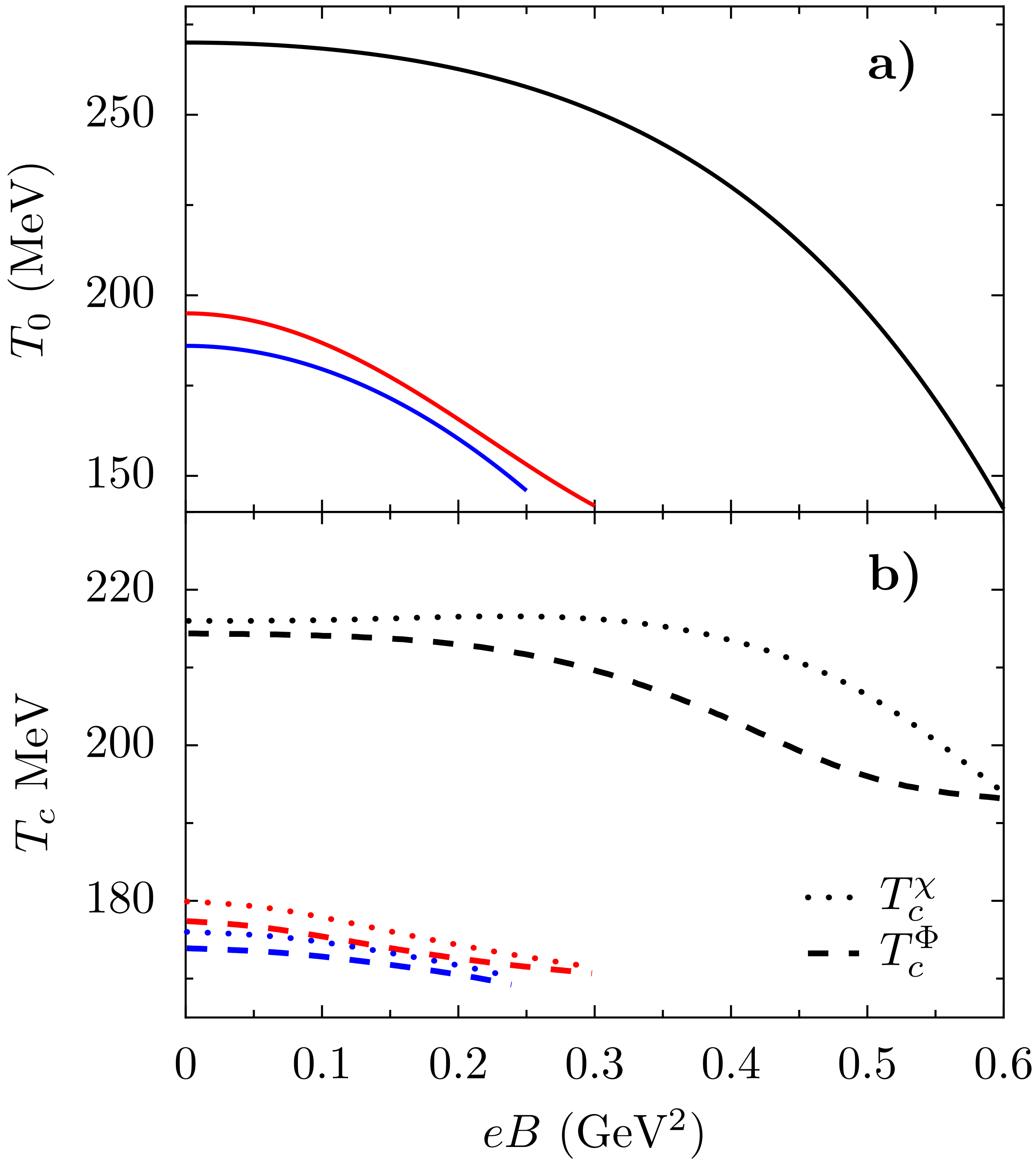
Inverse magnetic catalysis

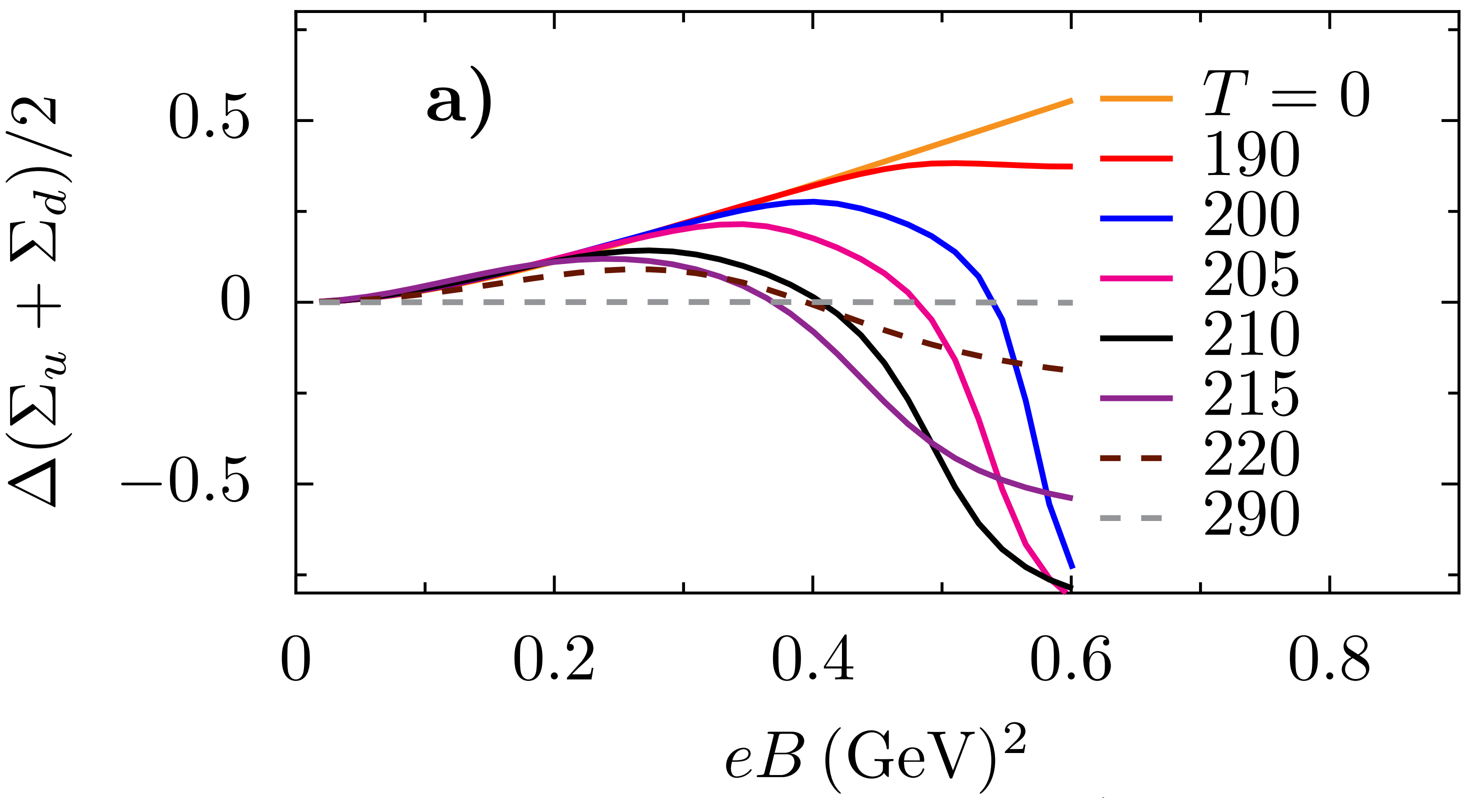




Inverse magnetic catalysis

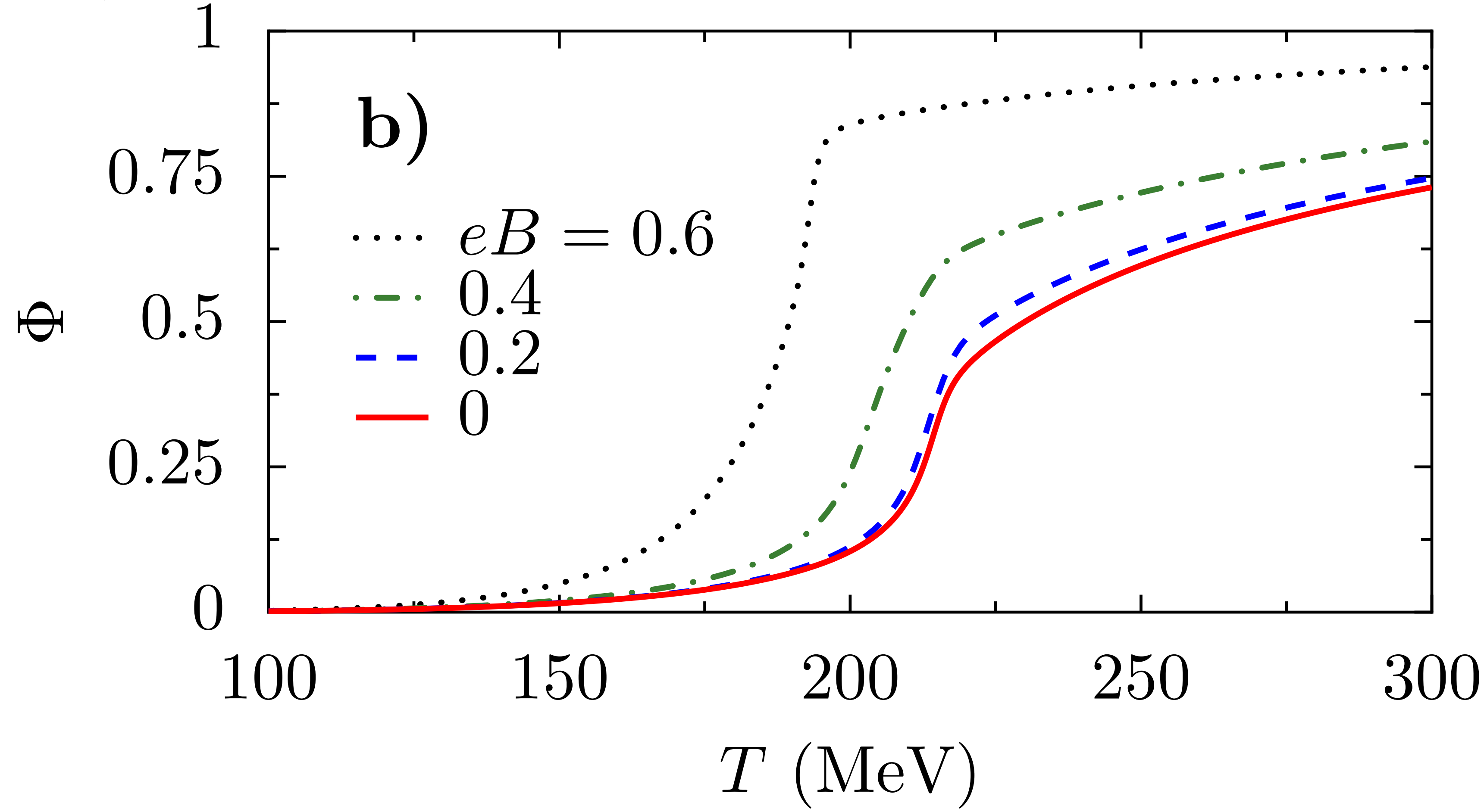
$$T_0(eB) = T_0(eB = 0) + \zeta(eB)^2 + \xi(eB)^4,\ T_0(eB = 0) = 186\text{MeV}$$





a)
$\Delta(\Sigma_u + \Sigma_d)/2$
0.5
0
−0.5
0
0.2
0.4
0.6
0.8
eB (GeV)^2
$T = 0$
190
200
205
210
215
220
290

EPNJL



b)
Φ
1
0.75
0.5
0.25
0
100
150
200
250
300
T (MeV)
$eB = 0.6$
0.4
0.2
0

Conclusions - 3

- **At T=0, the quark condensates agree with Lattice results**
- **At finite T, the effect due to electric charge quark difference is stronger in the Lattice**
- **Inverse magnetic catalysis can be reproduced within the EPNJL *if* the Polyakov loop scale parameter is made magnetic field dependent.**
- **However…with the EPNJL a first order phase transition (instead of a crossover) is obtained above 0.3 (0.61) GeV^2 with (without) the quark back-reaction of the Polyakov loop.**

CEP - NJL / PNJL

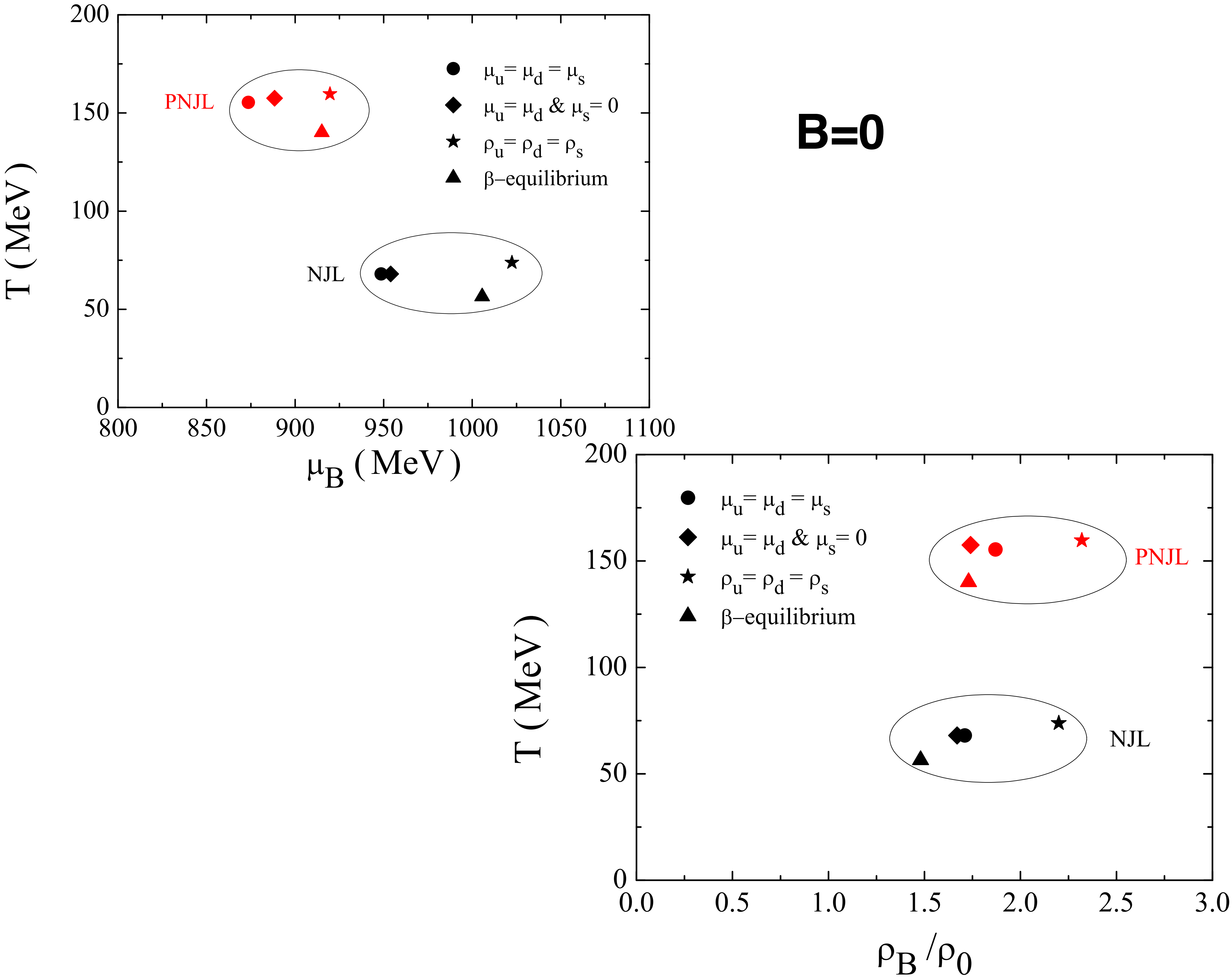
- **RKH parametrization / B=0 and strong B**
- **Different scenarios :**

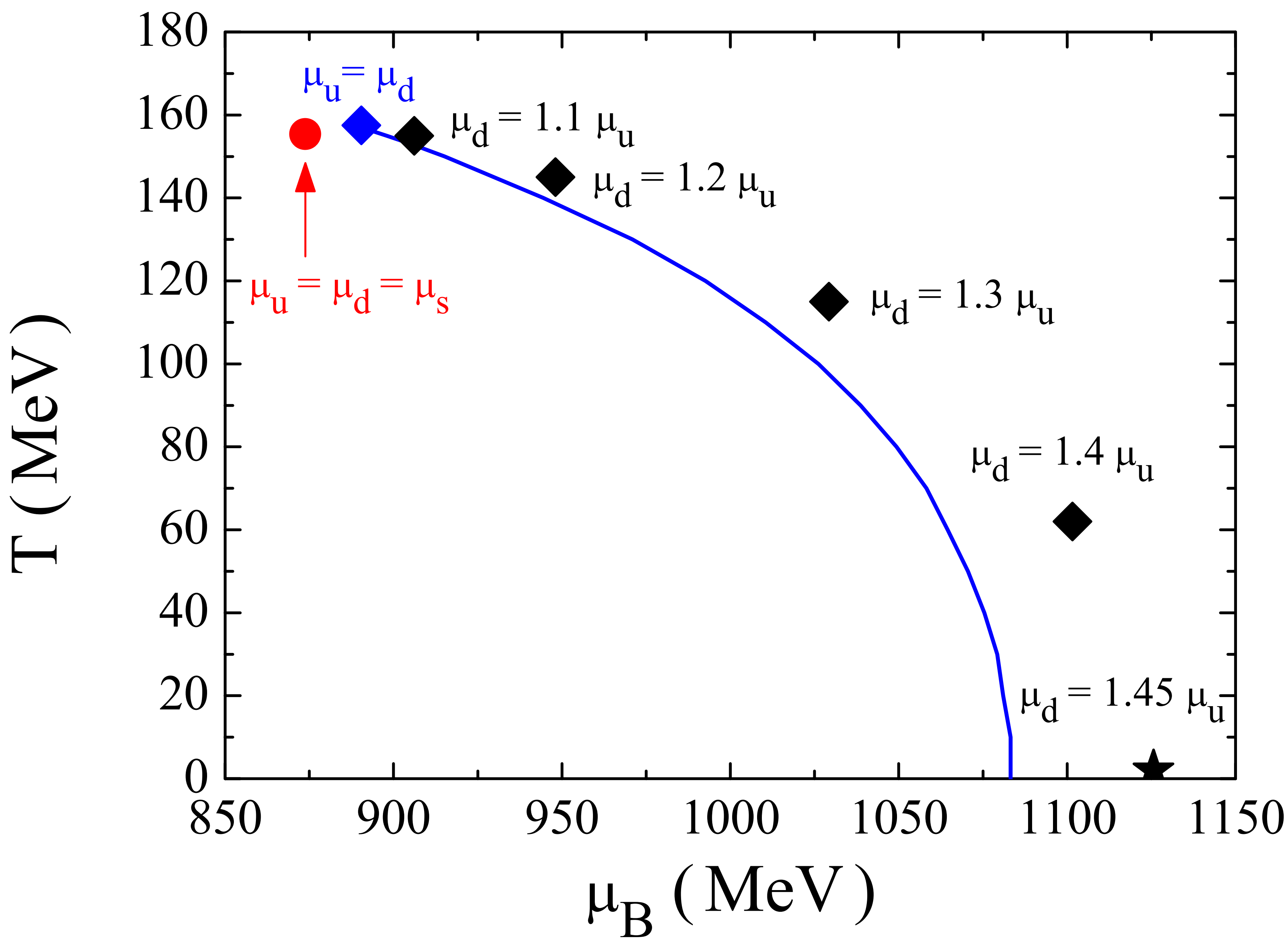
$$\mu_u = \mu_d = \mu_s$$

$$\mu_u = \mu_d, \quad \mu_s = 0$$

$$\rho_u = \rho_d = \rho_s$$

$$\beta - \text{equilibrium}$$


B=0
T (MeV)
μ_B (MeV)
ρ_B / ρ_0
PNJL
NJL
$\mu_u = \mu_d = \mu_s$
$\mu_u = \mu_d$ & $\mu_s = 0$
$\rho_u = \rho_d = \rho_s$
β–equilibrium
0
50
100
150
200
800
850
900
950
1000
1050
1100
0.0
0.5
1.0
1.5
2.0
2.5
3.0



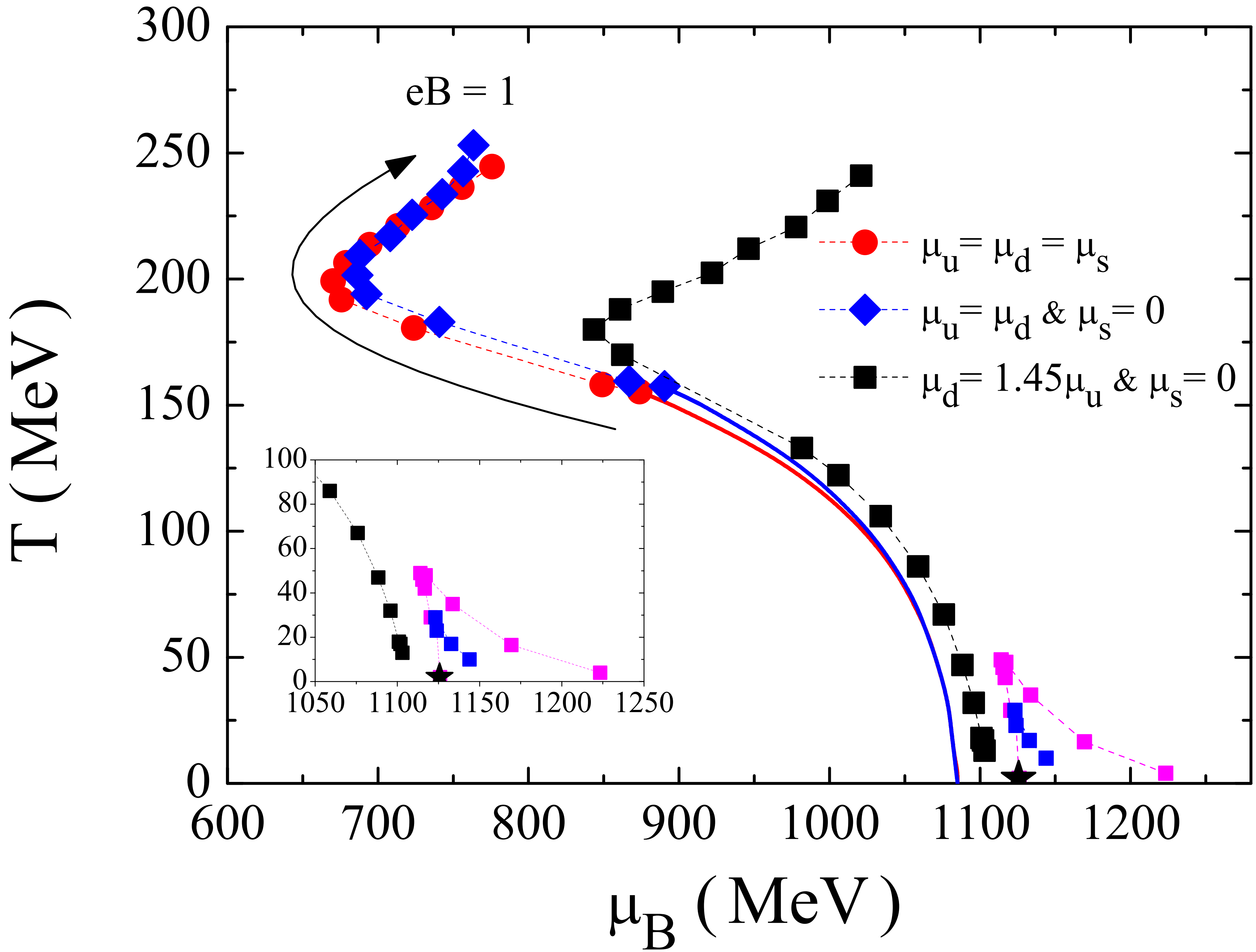
B=0

Effect of the isospin on the location of the CEP (PNJL)

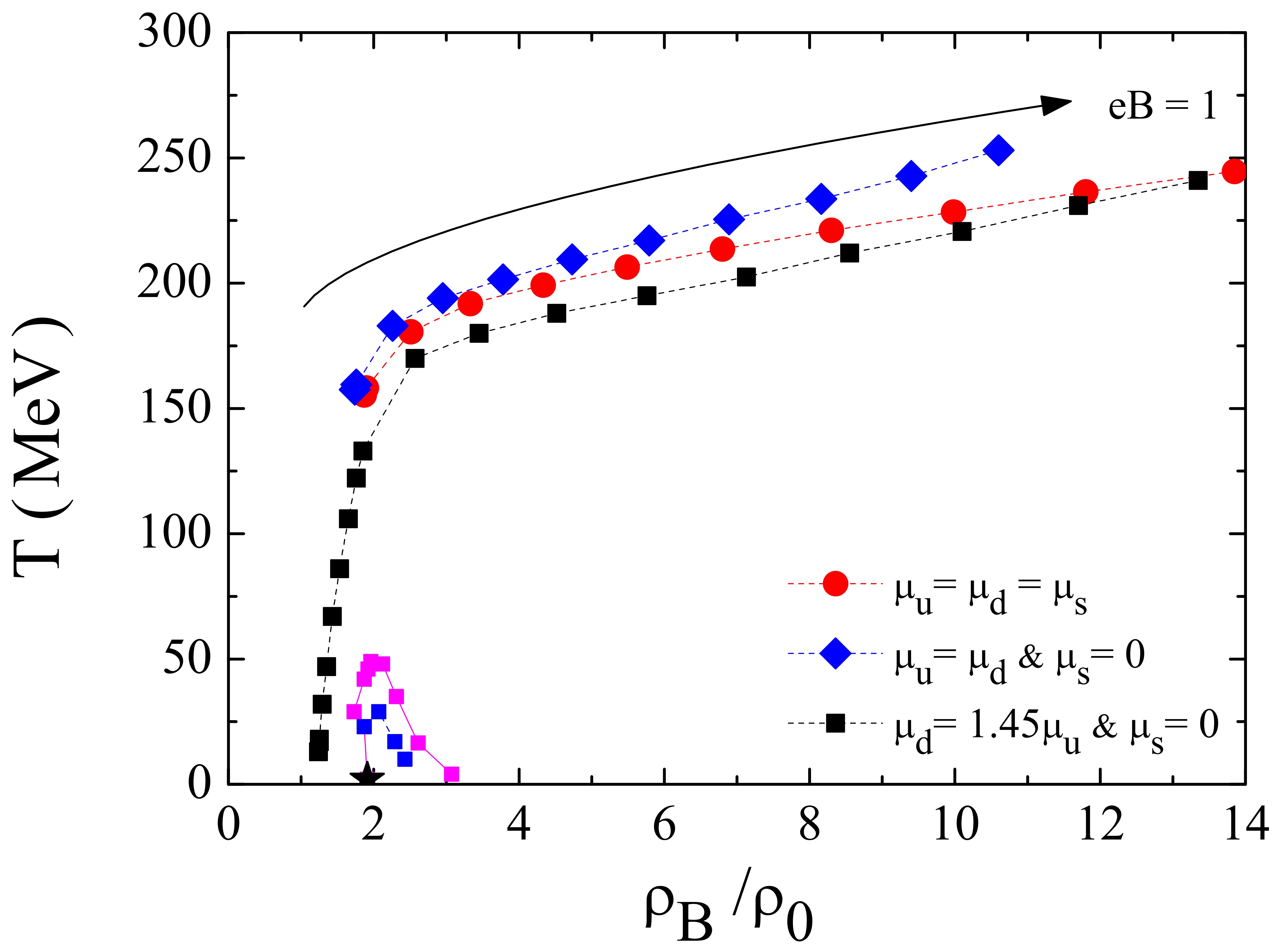
the line corresponds to zero isospin

$$\mu_u = \mu_d, \quad \mu_s = 0$$

	$\mu_u = \mu_d = \mu_s$			$\mu_u = \mu_d;\ \mu_s = 0$		
eB [GeV^2]	T [MeV]	μ_B [MeV]	ρ_B/ρ_0	T [MeV]	μ_B [MeV]	ρ_B/ρ_0
0	155.4	873.8	1.87	157.5	890.4	1.74
0.1	158.2	848.9	1.90	159.5	866.9	1.75
0.2	180.6	723.8	2.51	182.8	740.8	2.25
0.3	191.8	675.7	3.33	194.1	691.5	3.00
0.4	199.2	670.2	4.33	201.6	686.4	3.80
0.5	206.4	678.6	5.49	210.0	688.0	4.72
0.6	213.6	694.5	6.80	217.0	708.1	5.79
0.7	221.0	713.3	8.30	225.5	722.7	6.89
0.8	228.4	735.5	9.98	233.6	742.7	8.10
0.9	236.4	755.6	11.80	242.8	756.5	9.40
1	244.6	775.9	13.85	253.0	763.5	10.6



Full lines= 1st order transitions at eB=0; two CEPs at low T and strong B (pink and blue)



T (MeV)
ρ_B / ρ_0
eB = 1
$\mu_u = \mu_d = \mu_s$
$\mu_u = \mu_d$ & $\mu_s = 0$
$\mu_d = 1.45\mu_u$ & $\mu_s = 0$
0
50
100
150
200
250
300
0
2
4
6
8
10
12
14

Conclusions - 4

- For matter in beta-equilibrium, the CEP occurs at smaller Ts and densities (no B)
- For very asymmetric matter, no 1st order phase transition to a deconfined phase occurs (no B)
- CEP occurs at very small Ts if $eB < 0.1$ GeV^2 and a complicate structure appears, i.e., more than one CEP
- Strong Bs can drive the system without a CEP to a 1st order phase transition

- **Collaborators:**

- **Constança Providência (Coimbra / Portugal)**
- **Marcus Benghi Pinto (UFSC / Brazil)**
- **Norberto N. Scoccola (CONICET - CNEA / Argentina)**
- **Luis Benito Castro (UFSC / Brazil) - post-doc**
- **Pedro Costa (Coimbra / Portugal)**
- **Márcio Ferreira (Coimbra / Portugal) - Ph.D. student**
- **Ana Gabriela Grunfeld (CONICET - CNEA / Argentina)**

Thank you !

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