ANOMALOUS MAGNETIC MOMENT OF THE CONSTITUENT QUARK

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1 Units

Convenient units for magnetic moments are the nonrelativistic values

$$\mu_{\rm N} = \frac{e}{2m_{\rm N}} \text{.......nuclear magneton,}$$

$$\mu_{\rm q}^0 = \frac{e/3}{2m_{\rm q}} \text{.......quark magneton,}$$

$$\mu_\pi^0 = \frac{e}{2m_\pi} \text{.......pion magneton,}$$

Then for the u quark $\mu_{\mathbf{u}} = +2\mu_{\rm q}^0$ and for the d quark $\mu_{\mathbf{d}} = -1\mu_{\rm q}^0$. Usually we assume $m_{\rm q} = \frac{1}{3}m_{\rm N}$, then $\mu_{\rm q}^0 = \mu_N$.

The pion magneton is much larger due to the small pion mass, $\mu_\pi^0 = (m_{\rm N}/m_\pi)\,\mu_{\rm N} = 6.85\mu_{\rm N} = z\mu_{\rm N}$.

This is the magnetic moment of a free nonrelativistic pion. For a relativistic virtual pion we expect a smaller magnetic moment since, loosely speaking, istead of the mass, some energy (”effective mass”) appears in the denominator.

2 Relativistic derivation

The relativistic derivation for quarks starts from the Hamiltonian

$$H = \vec{\alpha} \cdot (\vec{p} - e_{\mathrm{q}} \vec{A}) + \ldots$$

By puting $\vec{A} = \frac{1}{2} \vec{r} \times \vec{B}$ we get

$$\mu_q = \langle \psi | \tfrac{1}{2} e_{\mathrm{q}} (\vec{r} \times \vec{\alpha})_z | \psi \rangle = \tfrac{2}{3} e_{\mathrm{q}} \int u(r) v(r) r^3 \, \mathrm{d}r$$

for the Dirac bispinor in s-state $\frac{1}{\sqrt{4\pi}}(u; \mathrm{i}\, \vec{\sigma} \cdot \hat{r}\, v)$

As a relativistic unit we then choose (defined for $e_{\mathrm{q}} = \frac{1}{3} e$)

$$\mu_{\mathrm{q}} = \tfrac{2}{9} e \int u(r) v(r) r^3 \, \mathrm{d}r \, .$$

Similarly, for pions it follows from the Hamiltonian

$$H = \tfrac{1}{2} [(\vec{p} - e\vec{A})\, \phi]^2 + \ldots, \qquad \mu_\pi = \tfrac{1}{4} \int \phi^2(r)\, r^2 \, \mathrm{d}r \, .$$

In the following we shall assume $\mu_{\mathrm{q}} \approx \mu_{\mathrm{q}}^0 \approx \mu_{\mathrm{N}}$ and $\mu_\pi = z\, \mu_{\mathrm{N}}$ with z probably between 1 and 6.85.

3 Magnetic moment of constituent quarks with pion cloud

$$|\mathrm{u}^{\uparrow}\rangle = \sqrt{(1-\frac{3}{2}a)}\,|\mathbf{u}^{\uparrow}\rangle - \sqrt{a}\,(\sqrt{\tfrac{1}{3}}|\mathbf{d}^{\uparrow}\pi_0^{+}\rangle - \sqrt{\tfrac{2}{3}}|\mathbf{d}^{\downarrow}\pi_1^{+}\rangle) + \sqrt{\frac{a}{2}}\,(\sqrt{\tfrac{1}{3}}|\mathbf{u}^{\uparrow}\pi_0^{0}\rangle - \sqrt{\tfrac{2}{3}}|\mathbf{u}^{\downarrow}\pi_1^{0}\rangle)$$

$$|\mathrm{d}^{\uparrow}\rangle = \sqrt{(1-\frac{3}{2}a)}\,|\mathbf{d}^{\uparrow}\rangle + \sqrt{a}\,(\sqrt{\tfrac{1}{3}}|\mathbf{u}^{\uparrow}\pi_0^{-}\rangle - \sqrt{\tfrac{2}{3}}|\mathbf{u}^{\downarrow}\pi_1^{-}\rangle) - \sqrt{\frac{a}{2}}\,(\sqrt{\tfrac{1}{3}}|\mathbf{d}^{\uparrow}\pi_0^{0}\rangle - \sqrt{\tfrac{2}{3}}|\mathbf{d}^{\downarrow}\pi_1^{0}\rangle)$$

We assume p-state pions and the subscript denotes the z-component of its angular momentum.

The basis of pure flavour quarks (and bare nucleons) is denoted by red **u**, **d**, **p** and **n**.

The magnetic moment of constituent quarks is defined as

$$\mu_u = \langle \mathrm{u}^{\uparrow}|\sigma_3(\tfrac{3}{2}\tau_3 + \tfrac{1}{2})\mu_{\mathrm{q}} + \Sigma_3 T_3\, \mu_{\pi}|\mathrm{u}^{\uparrow}\rangle$$

and similar for the d quark. The operators Σ and T act on pions.

The resulting magnetic moments are

$$\mu_{\mathbf{u}} = 2(1 - \tfrac{3}{2}a)\,\mu_{\mathrm{q}} + \tfrac{2}{3}a\mu_{\pi}\,,$$
$$\mu_{\mathbf{d}} = -(1 - a)\,\mu_{\mathrm{q}} - \tfrac{2}{3}a\mu_{\pi}\,.$$

For $a = 0.18$, $\mu_q = \mu_{\mathrm{N}}$ and $\mu_\pi = 6.85\,\mu_{\mathrm{N}}$ we get

$\mu_{\mathbf{u}} = 2.28\,\mu_{\mathrm{N}}$, $\mu_{\mathbf{d}} = -1.64\,\mu_{\mathrm{N}}$, $\mu_{\mathrm{p}} = 3.59\,\mu_{\mathrm{N}}$, $\mu_{\mathrm{n}} = -2.95\,\mu_{\mathrm{N}}$,

For $a = 0.18$, $\mu_q = \mu_{\mathrm{N}}$ and $\mu_\pi = 2\,\mu_{\mathrm{N}}$ or $\mu_\pi = \mu_{\mathrm{N}}$ we get

$\mu_{\mathbf{u}} = 1.70\,\mu_{\mathrm{N}}$, $\mu_{\mathbf{d}} = -1.06\,\mu_{\mathrm{N}}$, $\mu_{\mathrm{p}} = 2.62\,\mu_{\mathrm{N}}$, $\mu_{\mathrm{n}} = -1.98\,\mu_{\mathrm{N}}$,

$\mu_{\mathbf{u}} = 1.58\,\mu_{\mathrm{N}}$, $\mu_{\mathbf{d}} = -0.94\,\mu_{\mathrm{N}}$, $\mu_{\mathrm{p}} = 2.42\,\mu_{\mathrm{N}}$, $\mu_{\mathrm{n}} = -1.78\,\mu_{\mathrm{N}}$,

The ratio between proton and neutron magnetic moments is spoiled, it is no longer - 3/2 as in the simple quark model. In the first example above it is - 1.22, and in the second it is - 1.32 or 1.36. It is well known that any pion cloud brings the ratio from - 1.5 closer to - 1. Maybe the inclusion of ρ meson etc can correct this deficiency.

4 Pion cloud around bare nucleon

$$|\mathrm{p}^\uparrow\rangle = \sqrt{(1-\frac{3}{2}\,\alpha)}\,|\mathbf{p}^\uparrow\rangle - \sqrt{\alpha}\,(\sqrt{\tfrac{1}{3}}|\mathbf{n}^\uparrow\pi^+_0\rangle - \sqrt{\tfrac{2}{3}}|\mathbf{n}^\downarrow\pi^+_1\rangle) + \sqrt{\frac{a}{2}}\,(\sqrt{\tfrac{1}{3}}|\mathbf{p}^\uparrow\pi^0_0\rangle - \sqrt{\tfrac{2}{3}}|\mathbf{p}^\downarrow\pi^0_1\rangle$$

In the "oldfashioned' model with $\mu_{\mathbf{p}} = \mu_{\mathrm{N}}$, $\mu_{\mathbf{n}} = 0$ and $\mu_\pi = 6.85\,\mu_{\mathrm{N}}$ we get for $\alpha = 0.18$

$$\mu_{\mathrm{p}} = (1 - \tfrac{5}{3}\alpha + \tfrac{2}{3}z\alpha)\,\mu_{\mathrm{N}} = 1.52\mu_{\mathrm{N}}\,,$$

$$\mu_{\mathrm{n}} = -(\tfrac{1}{3}\alpha + \tfrac{2}{3}z\alpha)\,\mu_{\mathrm{N}} = -0.88\mu_{\mathrm{N}}\,.$$

We can fit the experimental values $\mu_{\mathrm{p}} = 2.79\mu_{\mathrm{N}},\ \mu_{\mathrm{n}} = -1.91\mu_{\mathrm{N}}$
with $\alpha = 0.06$, $z = 47$, or approximately
with $\alpha = 0.18$, $z = 17.47$ (correspondig to $\mu_{\mathrm{p}} = 2.79\mu_{\mathrm{N}},\ \mu_{\mathrm{n}} = -2.15\mu_{\mathrm{N}}$).

5 Comparison with other QUARK-PION MODELS

Example: L. Amoreira, M.Fiolhais, B. Golli, M.Rosina:
Int. J. of Modern Physics A **14**(1999)731-759.

$$H = H_{\rm q} + H_\pi + \int {\rm d}^3 r \vec{j}(\mathbf{r}) \cdot \vec{\pi}(\mathbf{r}) \,,$$

$$\vec{j}(\mathbf{r}) = \frac{1}{4\pi}\rho(r) \sum_{\nu=1}^{3} \hat{\mathbf{r}} \cdot \sigma(\nu)\vec{\tau}(\nu) \,,$$

$$H_{\rm q} = m_{0N} \sum_{N_i} |N_i\rangle\langle N_i| + m_{0\Delta} \sum_{\Delta_i} |\Delta_i\rangle\langle\Delta_i| \,,$$

$$H_\pi =: \int \tfrac{1}{2}[\vec{P}_\pi \cdot \vec{P}_\pi + \nabla\vec{\pi} \cdot \nabla\vec{\pi}{\rm d}^3 r : \;.$$

R[fm]	$\mu_{\rm p}/\mu_{\rm N}$	$\mu_{\rm n}/\mu_{\rm N}$
0.7	1.77	−1.40
0.9	2.04	−1.53
1.1	2.35	−1.69
1.3	2.67	−1.88
1.5	3.01	−2.08

Table 1: R (e.g. from Cloudy Bag Model) is a measure of pion amplitude

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I SHALL APPRECIATE YOUR CRITICISM AND SUGGESTIONS