COMPARISON OF QUARK MASSES

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1 Difference between current and constituent masses

	PDG	$\overline{\text{PDG}}$	Bha	ΔBha	AL1	ΔAL1	Rel	ΔRel
u	2.3±0.6	2.15±0.15						
d	4.8±0.4	4.70±0.20						
$\frac{1}{2}$(u+d)	3.5±0.5	3.40±0.25	337	334	315	312	340	337
s	95±5	93.5±2.5	600	506	577	483	480	386
c	1275±25	1275±25	1870	595	1836	561	1675	400
b($\overline{\text{MS}}$)	4180±30	4180±30	5260	1080	5227	1047	5055	875
b(1S)	4660±30	4650±30	5260	610	5227	577	5055	405

Table 1: Bha = Bhaduri parameters, AL1 = Grenoble AL1 parameters, Rel = Relativistic CQM, Δ = difference with respect to $\overline{\text{PDG}}$.

Reference for current quark masses (PDG 2012):
J. Beringer et al. (Particle Data Group), PRD86, 010001 (2012)
and 2013 partial update for the 2014 edition
u, d, s masses ($\overline{\text{PDG}}$) are given in the $\overline{\text{MS}}$ scheme at $\mu = 2$ GeV.
c, b, t masses ($\overline{\text{PDG}}$) are given in the $\overline{\text{MS}}$ scheme at $m_q = m_q(m_q)$.
(1S) is a certain different scheme.

Reference for nonrelativistic constituent quark model masses:
D.Janc and M.Rosina, FBS 35 (2004) 175 (pages 193-194),
R.K.Bhaduri, L.E.Cohler, Y.Nogami, Nuovo Cim. A65 (1981) 376,
B.Silvestre-Brac, FBS 20 (1996) 1.

Reference for relativistic constituent quark model masses:
J. P. Day, W. Plessas, Ki-Seok Choi, Bled Workshops in Phys.13,1(2012)5,
J. P. Day, W. Plessas, arXiv 1205.6918v1 (hep-ph).

CONCLUSION:

The dressing of quarks (∆model= model mass-bare mass) amounts to 330 MeV for u and d quarks and grows to 400-600 MeV for heavier quarks

(and even to 1000 MeV for the b quark for one of the two popular schemes.)

2 Determination of current quark masses

Reference: A.V. Manohar and C.T. Sachrajda, Phys.Rev.D86(2012)010001.

For light quarks, one can use the techniques of chiral perturbation theory to extract quark mass ratios. To first order in M one finds that

$$\begin{aligned}
m^2(\pi^0) &= B(m_u + m_d), \\
m^2(\pi^\pm) &= B(m_u + m_d) + D_{em}, \\
m^2(K^0) &= m^2(\bar{K^0}) = B(m_d + m_s), \\
m^2(K^\pm) &= B(m_u + m_s) + D_{em}, \\
m^2(\eta) &= (1/3)B(m_u + m_d + 4m_s),
\end{aligned}$$

with two unknown constants B and D_{em}, the electromagnetic mass difference. One obtains the ratioes.

$$\frac{m_u}{m_d} = \frac{(2m^2(\pi^0) - m^2(\pi^+) + m^2(K^+) - m^2(K^0))}{(m^2(K^0) - m^2(K^+) + m^2(\pi^+))} = 0.56\,,$$

$$\frac{m_s}{m_d} = \frac{(m^2(K^0) + m^2(K^+) - m^2(\pi^+))}{(m^2(K^0) + m^2(\pi^+) - m^2(K^+))} = 20.2\,.$$

For light quarks the authors emphasize the trust in Lattice QCD derivations, the corresponding values are given in the table in the column $\overline{\text{PDG}}$.

For heavy quarks one inputs experimental values of masses and decay parameters of selected heavy hadrons and applies the "heavy quark effective theory"(HQET) in conjunction with the chiral perturbation theory to obtain running masses at a convenient scale.

3 Determination of constituent quark masses – nonrelativistic constituent quark model with a two-body OGE potential

Bhaduri potential:

$$V_{ij}^{B} = -\frac{\lambda_i^C}{2} \cdot \frac{\lambda_j^C}{2} \Big(U_0 + \frac{\alpha}{r_{ij}} + \beta r_{ij} + \alpha \frac{\hbar^2}{m_i m_j c^2} \frac{e^{-r_{ij}/r_0}}{r_0^2 r_{ij}} \sigma_i \cdot \sigma_j \Big),$$

$$r_{ij} = |\vec{r}_i - \vec{r}_j|;$$

$m_{\mathrm{b}} = 5259$ MeV, $m_{\mathrm{c}} = 1870$ MeV,
$m_{\mathrm{s}} = 600$ MeV, $m_{\mathrm{u}} = m_d = 337$ MeV,
$U_0 = 685$ MeV, $\alpha = 77$ MeVfm,
$\beta = 706.95$ MeV/fm, $r_0 = 0.4545$ fm.

Grenoble AL1 potential:

$$V_{ij}^{AL1} = -\frac{\lambda_i^C}{2} \cdot \frac{\lambda_j^C}{2} \Big(U_0 + \frac{\alpha}{r_{ij}} + \beta r_{ij} + \tilde{\alpha} \frac{2\pi\hbar^2}{3m_i m_j c^2} \frac{e^{-r_{ij}^2/r_0^2}}{\pi^{3/2} r_0^3} \sigma_i \cdot \sigma_j \Big),$$

$$r_0(m_i, m_j) = A \Big(\frac{m_i + m_j}{2m_i m_j} \Big)^B, \qquad r_{ij} = |\vec{r}_i - \vec{r}_j|;$$

$m_{\mathrm{b}} = 5227$ MeV,	$m_{\mathrm{c}} = 1836$ MeV,	$A = 1.6553$ GeV^{B-1},
$m_{\mathrm{s}} = 577$ MeV,	$m_{\mathrm{u}} = m_d = 315$ MeV,	$B = 0.2204$,
$U_0 = 624.075$ MeV,	$\alpha = 74.895$ MeVfm,	
$\beta = 629.315$ MeV/fm,	$\tilde{\alpha} = 274.948$ MeVfm.	

4 The chromodielectric model

A simple example is given in:
M. Rosina, H.J. Pirner, Fizika (Zagreb) 19 (1987) 2, 43-46,
(Proc.Workshop on Mesonic Degrees of Freedom in Hadrons,
Bled/Ljubljana 1987)

$$\mathcal{L} = -\frac{1}{4}\chi^4 B_a^{\mu\nu} B_{\mu\nu}^a + \bar{\psi}\left(\gamma^\mu(i\partial_\mu - gB_\mu\hat{C}) - \frac{m}{\chi}\right)\psi + \frac{1}{2}w^2(\partial_\mu\chi)^2 - \frac{1}{8}M^2w^2\chi^2(2-\chi)^2\,.$$

the "current mass" $m = 20$ Mev is a model parameter,
and the "constituent mass" $m/\chi = 160$ MeV comes out by minimization of energy and fitting model parameters to energy and density of nuclear matter.

5 The constituent quark as a soliton in a linear σ model

B.Golli and M.Rosina, Phys.Lett B393 (1997) 161-166

$$\mathcal{L} = \mathrm{i}\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi + g\bar{\psi}(\hat{\sigma} + \mathrm{i}\vec{\tau}\cdot\hat{\vec{\pi}}\gamma_5)\psi + \tfrac{1}{2}\partial_{\mu}\hat{\sigma}\partial^{\mu}\hat{\sigma} + \tfrac{1}{2}\partial_{\mu}\hat{\vec{\pi}}\cdot\partial^{\mu}\hat{\vec{\pi}} - \mathcal{U}(\hat{\sigma},\hat{\vec{\pi}})$$

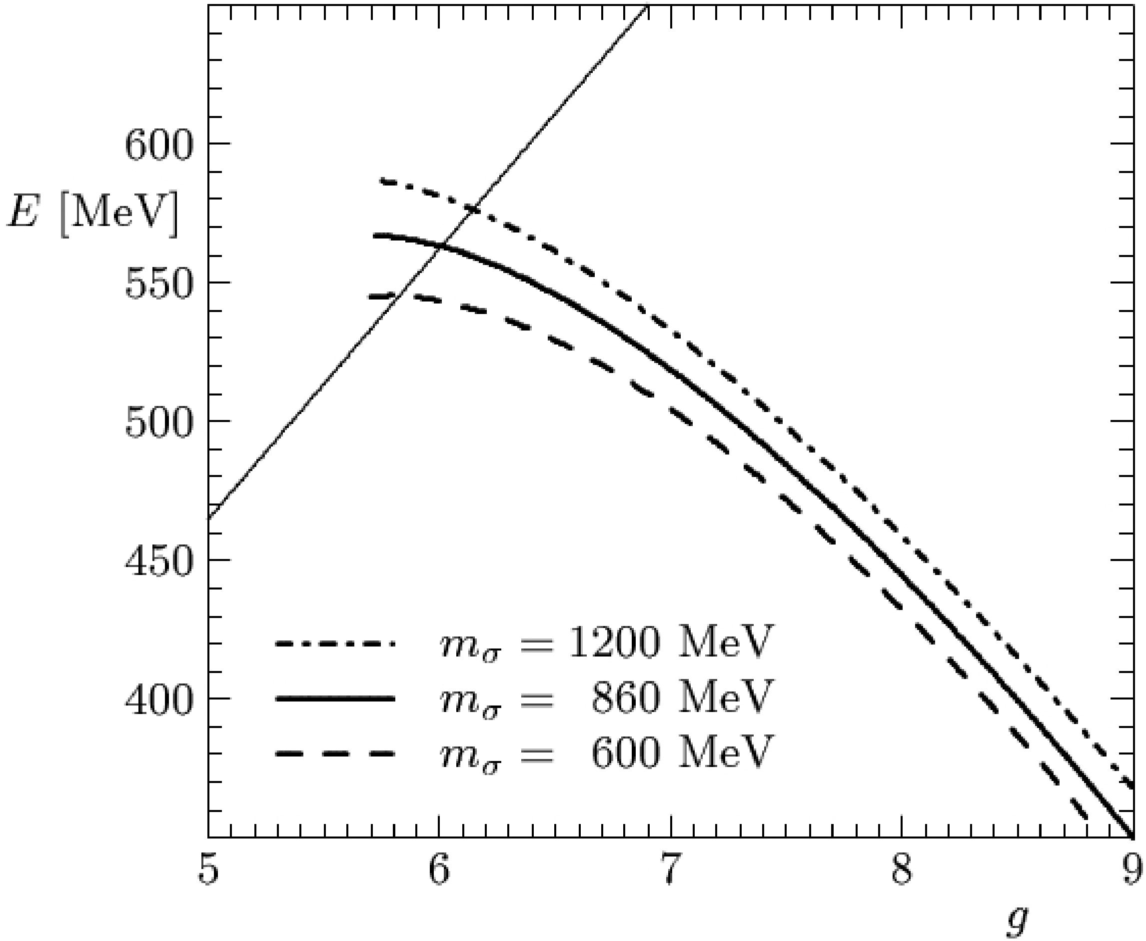
For nucleon soliton, nucleon observables are well reproduced with $g = 6$.

Vacuum expectation value of σ is assumed to be f_π.
Then, $gf_\pi \approx 550$ MeV acts as a mass term for the quark.

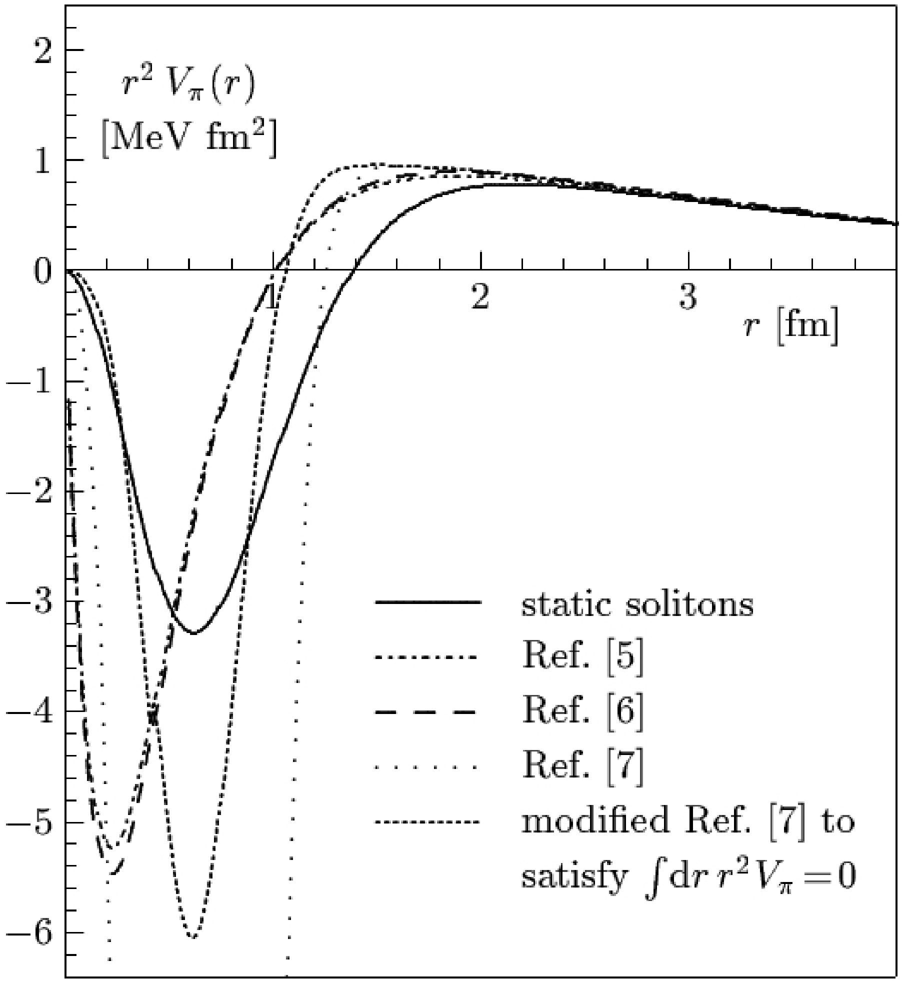
For $g > 6$, the quark soliton gets dressed in pions and its energy decreases. It reaches 350 MeV at $g = 9$ which is much too large. System of three separate quark solitons would become unstable and change into a nucleon soliton.

A description including quark soliton – quark soliton interaction would aleviate the situation since tha σ-exchange potential contributes -320 MeV at the origin.

The pion-exchange potential also gives some credit to the quark soliton model.



600
550
500
450
400
E [MeV]
5
6
7
8
9
g
m_σ = 1200 MeV
m_σ = 860 MeV
m_σ = 600 MeV



$r^2 V_\pi(r)$
[MeV fm^2]
2
1
0
−1
−2
−3
−4
−5
−6
1
2
3
r [fm]
static solitons
Ref. [5]
Ref. [6]
Ref. [7]
modified Ref. [7] to
satisfy $\int dr\, r^2 V_\pi = 0$

6 The Nambu–Jona-Lasinio Model

1. We assume a sharp 3-momentum cutoff $0 \leq |\vec{p_i}| \leq \Lambda$;
2. The space is restricted to a box of volume $\mathcal{V}$ with periodic boundary conditions. This gives a finite number of discrete momentum states, $\mathcal{N} = N_c N_f \mathcal{V} \Lambda^3 / 3\pi^2$ occupied by N quarks.
3. $|\vec{p_i}| \to P = \frac{3}{4}\Lambda$.

$$\begin{aligned} H_{NJL} &= \sum_{i=1}^{N} \Big(\gamma_5(i) h(i) P + m_0 \beta(i) \Big) \\ &- \frac{2G}{\mathcal{V}} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j\neq i}}^{N} \Big(\beta(i)\beta(j) + \big(\mathrm{i}\beta(i)\gamma_5(i) \big) \big(\mathrm{i}\beta(j)\gamma_5(j) \big) \Big) \, \mathcal{P} \, , \end{aligned}$$

$$\mathcal{P} = \sum_{\vec{p_i}'}^{\Lambda} \sum_{\vec{p_j}'}^{\Lambda} \sum_{\vec{p_i}}^{\Lambda} \sum_{\vec{p_j}}^{\Lambda} \delta_{\vec{p_i}' + \vec{p_j}', \, \vec{p_i} + \vec{p_j}} \, |\vec{p_i}', \, \vec{p_j}' \rangle \, \langle \vec{p_i} \, , \, \vec{p_j} | \tag{1}$$

This projector restricts momenta to a sharp cutoff Λ
and conserves momentum.

$$
\begin{aligned}
M &= \sqrt{\Big(E_g(N) - E_g(N-1)\Big)^2 - P^2} \\
Q &= \langle\, g|\bar{\psi}\psi|g\rangle = \frac{1}{\mathcal{V}}\langle\, g|\sum_i \beta(i)|g\,\rangle \\
m_\pi &= E_1(N) - E_g(N)\,.
\end{aligned}
$$

Hartree-Fock + RPA gives

$$
\begin{aligned}
M - m_0 &= \sqrt{(\frac{4}{\pi^2}\,G\Lambda^3)^2 - \frac{(M-m_0)^2}{M^2}P^2} \\
Q &= \frac{\Lambda^3}{\pi^2}\frac{M}{\sqrt{M^2+P^2}} \\
m_\pi &\approx \sqrt{\sqrt{\frac{M^2+P^2}{M^2}}G\Lambda^3 m_0}\,.
\end{aligned}
$$

$$
\Lambda = 648\,\mathrm{MeV}, \quad G = 40.6\,\mathrm{MeV\ fm}, \quad m_0 = 4.58\,\mathrm{MeV}.
$$

These values compare favourably with those of full Nambu-Jona Lasinio

$$
\mathrm{Coimbra}\;:\;\Lambda = 631\,\mathrm{MeV}, \quad G = 40\,\mathrm{MeV\ fm}, \quad m_0 \approx 5\,\mathrm{MeV},
$$

$$
\mathrm{Buballa}\;:\;\Lambda = 664\,\mathrm{MeV}, \quad G = 37.8\,\mathrm{MeV\ fm}, \quad m_0 = 5.0\,\mathrm{MeV}.
$$