

Tetraquarks

and Large N_c QCD



What does large N_c
have to tell us
about the
existence of
tetraquarks

TDC

R. F. Lebed

An Overview

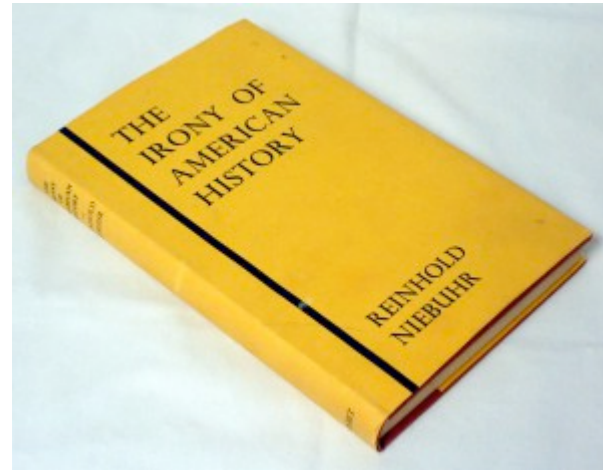


An overview

- Introduction
 - QCD vs the Quark Model
 - Exotics
 - Heavy quark vs light quark
 - Why Large N_c ?
 - Glueballs, Hybrids
- The Conventional Wisdom
 - Tetraquarks can't exist at large N_c
- Weinberg's critique
 - “Proof” that narrow tetraquarks don't exist is flawed
- QCD(AS) another variant of the the large N_c limit
 - Narrow tetraquarks must exist at large N_c in QCD(AS)
- QCD(F) (the usual large N_c limit) & Implications for the real world
 - Tetraquarks don't exist at large N_c in QCD(F)

Introduction

- History is full of Irony



- The history of QCD is no exception: the naïve quark model was an essential ingredient in the development of QCD, but given the existence of QCD it very hard to understand why the quark model works at all.

- The phrase “Quark Model” is composed of two words—“quark” and “model”

Both of these are ambiguous

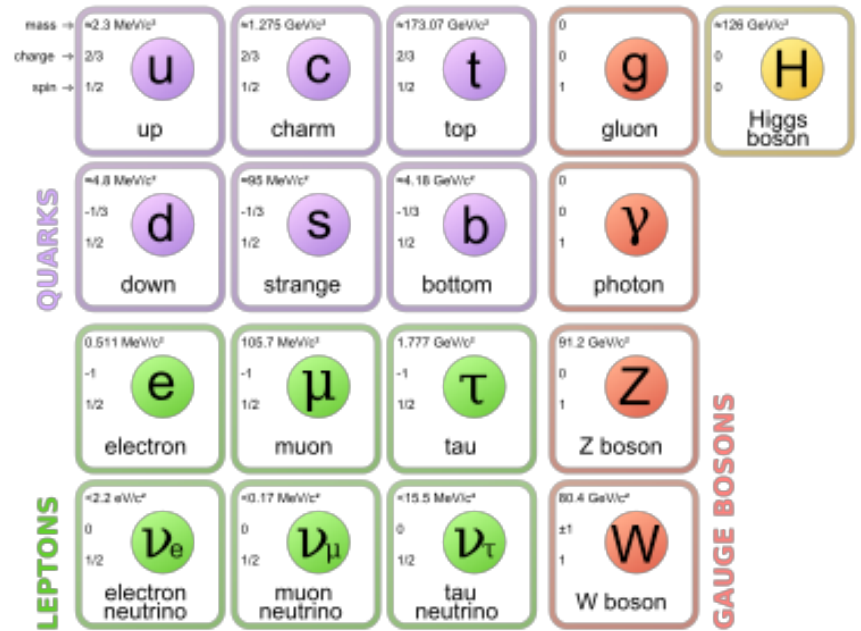
What does “quark” mean?

- It is a type of soft cheese product which exists (though hard to find) in the US but is really popular in Germany
- It is a nonsense word invented by James Joyce.
- It is an effective degree of freedom in the quark model.
- It is a fundamental degree of freedom in QCD.



All these meanings are fundamentally different

In what sense is the quark model a “model”?





- Note that from the modern perspective, the quark model is a “model” in the sense of a model ship rather than the standard model. It captures *some* aspects of the real thing but misses others.
- It is not a complete theory which directly predicts experimental observables like the standard model.
- It is definitely not beautiful like a supermodel.

- However, the simple quark model still strongly influences the language that we use to describe hadrons and remains a basic way most hadronic physicists think about states.
- Exotic hadrons are ones which do not fit into a quark model description and are important in that they help clarify what QCD has and the quark model does not.
 - There are two types
 - Quantum number exotics. States which by their quantum numbers **cannot** be made in the quark model. (eg. an isospin 2 meson)
 - Cryptoexotics. States which by their quantum numbers can be made in the simple quark model but which dynamically are dominated by components which are not of the quark model type.

- There is strong evidence for heavy tetraquarks
 - During the past decade there has been a zoo of heavy quark states, the X,Y,Z.
 - Several of these appear to have tetraquark quantum numbers (in this discussion I am not distinguishing between “tetraquarks” and “molecules” but focusing on quantum numbers. It is not clear that the distinction between “tetraquarks” and “molecules” is meaningful outside the context of models)
 - Example X(3872) which appears to be $c u \bar{c} \bar{d}$
 - However these probably are special to heavy quark physics and need not indicate the generic existence of tetraquarks.
- Why is the case of heavy quarks special?
 - Generically, two heavy particles are much more likely to bind with the same force as two lighter particles.
 - In a molecule picture in which two heavy-light mesons interact at long distance via pion exchange, one can rigorously prove that in the *extreme* heavy quark limit, these must bind.

- In the case $ll\bar{H}\bar{H}$ type channels, the *extreme* heavy quark limit not only must yield tetraquarks but their properties are fixed. (TDC & P. Holhler, Phys. Rev D M.J. Savage and M.B. Wise, Phys. D74 094003 (2006). Lett. B 248, 177(1990)).
 - The two heavies can form a tightly bound state into 3 representation of color due to color-coulomb. Since it is very very heavy and compact it acts like a static point-like color source in exactly the same way as a heavy quark does (note in the heavy quark limit the spin of the heavy is irrelevant) so the excitation spectrum will look like that of a baryon with one heavy quark.
 - Unfortunately, the mass must be VERY large (greater than M_B) to be in this regime.
- Still the general, tendency of heavy quarks to bind into tetraquark states is generic as the heavy quarks get heavy.

- What about light quarks:
 - What is the empirical situation for exotics made of light quarks?
 - There does seem to be strong evidence for least one hybrid state, the $\pi_1(1400)$ but there is no compelling evidence for a quantum number exotic tetraquark.
 - There is a long history of identifying scalars as crypto-tetraquarks (Jaffe 1977) but it remains an open question. The key point is that “there are too many scalars”.
 - The $f_0(980)$ is often thought to be a crypto-tetraquark with large amount of hidden strangeness. Evidence: despite having virtually no phase space the $f_0(980)$ decays into 2 kaons. How compelling is this?

Guidance from Large N_c ?

- In many cases the large N_c limit provides a crude cartoon of the real world of $N_c=3$ and this enables one to get insights into the real world. For example:
 - The OZI rule becomes exact at large N_c
 - Explains why baryons are heavier than mesons
 - Explains why decays with the smallest number of mesons typically dominate decays
- In some cases the large N_c limit provides a semi-quantitative understanding in a particular for baryons
 - An contracted $SU(2N_f)$ spin-flavor symmetry emerges at large N_c and makes quantitative predictions with corrections of order $1/N_c$ or $1/N_c^2$ Gervais and Sakita 1983; Dashen and Manhar 1993

In many cases the large N_c limit provides a crude cartoon of the real world of $N_c=3$ and this enables one to get insights into the real world.

For example:

- The OZI rule becomes exact at large N_c
- Explains why baryons are heavier than mesons
- Explains why decays with the smallest number of mesons typically dominate decays

– In some cases the large N_c limit provides a semi-quantitative understanding in a particular for baryons

- An contracted $SU(2N_f)$ spin-flavor symmetry emerges at large N_c and makes quantitative predictions with corrections of order $1/N_c$ or $1/N_c^2$ Gervais and Sakita 1983; Dashen and Manhar 1993

The Conventional Wisdom

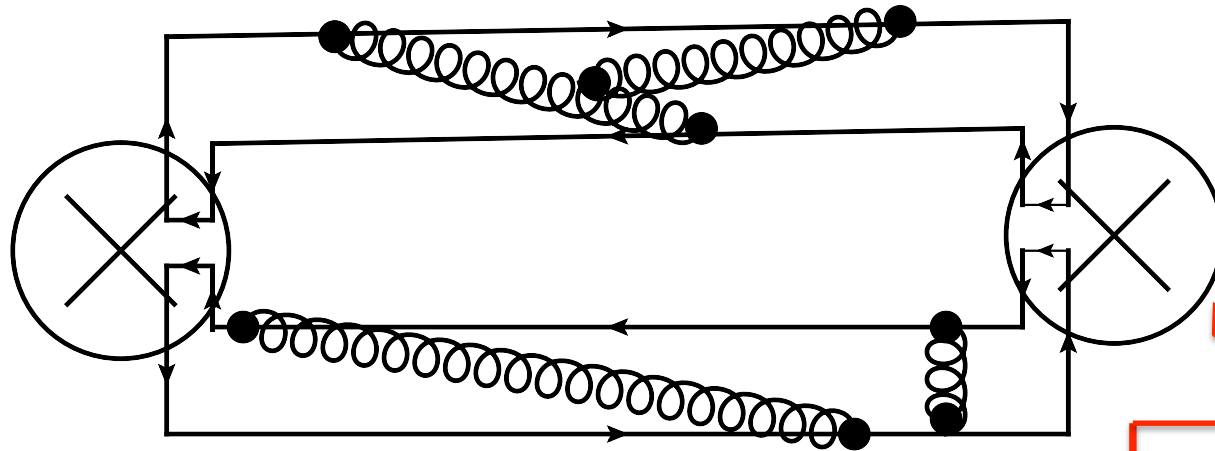
- Tetraquarks do not exist at large N_c . (Witten 1979; Coleman 1985)

Basic argument:

The standard method to study hadrons at large N_c is via a study of the correlation functions for sources with the appropriate quantum numbers. It is easy to show that with a minimal tetraquark source of two bilinears at the same point, the leading order diagram ($\mathcal{O}(N_c^2)$) is just a disconnected diagram which behaves like two non-interacting mesons. It does not act like a tetraquark.

Disconnected graphs $\mathcal{O}(N_c^2)$

a typical diagram at
quark/gluon level:
dominated by loops
with planar gluons
inside



Source

$$J = \bar{q}^a(x)q_a(x)\bar{q}^a(x)q_b(x)$$

a,b are color indices

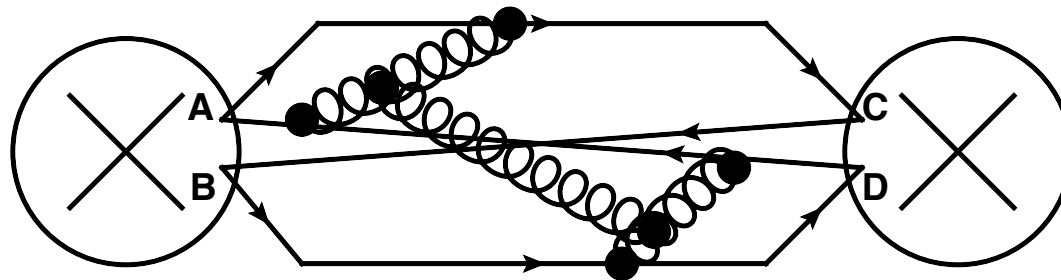


hadronic level
two mesons

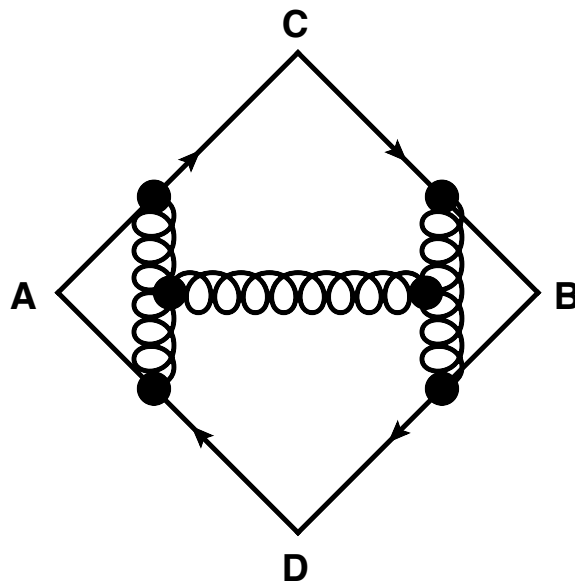
Weinberg's Critique

- Recently this has been called into question. Weinberg recently pointed out (PRL 110,24130 (2013)) that this standard argument is not valid.
- The argument is wrong for a very simple reason: the fact that leading order correlator implies that that the tetraquark operator “makes two meson and nothing else” is irrelevant. One needs to look at the leading diagrams in which the four quarks all interact---i.e. the leading connected diagram---to see states which look like two interacting mesons. *Whether or not these resonate into tetraquarks is separate question from whether the leading diagrams only make noninteracting mesons.*

Connected graphs $\mathcal{O}(N_c)$



A typical diagram at quark/gluon level : dominated by a single loop with planar gluons inside. Written as a sensible looking space-time type diagram, it does not seem to be by a single loop with planar gluons inside.



But topologically it is, and the N_c counting only depends on the topology

- If tetraquarks do exist they can be found in the dynamics of these connected diagrams.
- Note that the logic by which tertraquarks must be absent since they do not appear in the $\mathcal{O}(N_c^2)$ leading order contribution to the correlator must be wrong
 - The same argument could be applied to a 4-quark source with the nonexotic quantum number of two-pions combined to a vector-isovector. The $\mathcal{O}(N_c^2)$ leading order contribution indeed just makes two non-interacting pions. However one cannot deduce from that a ρ meson does not exist. They do, and can be seen in the leading order connected contribitons $\mathcal{O}(N_c)$

- Witten (1979) had another argument against tetraquarks at large N_c : Since the meson-meson interaction is weak at large N_c , they cannot bind into tetraquarks.
- However, this argument is also specious:
 - The same argument can be made in nonexotic channels, eg. a source composed of two pseudoscalars-isovectors (π channels) coupled together into a vector-isovector (ρ channel). That fact that π - π scattering is generically weak at large N_c does not mean that the pions do not resonate into a ρ . They do.

- The reason is that if there is a ρ at large N_c (which can be shown to be true) and is weakly coupled to the 2-pion channel (which can also be shown to be true—they have a coupling which scales as $N_c^{-1/2}$) then the π - π scattering is weak but the ρ exists.
- In the same way, there is no logical reason that a tetraquark cannot exist as a narrow resonance weakly coupled to two-meson channels as $N_c^{-1/2}$ while being fully consistent with weak meson-meson scattering,
- The question is whether they do, in fact, exist

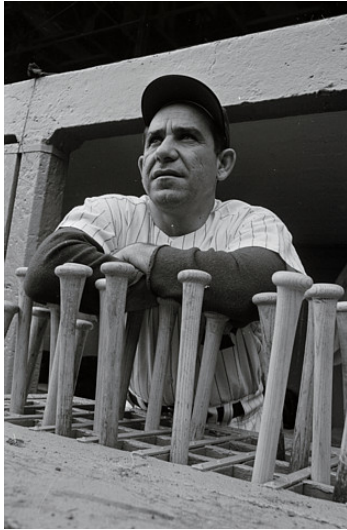
- It is important to note, however, that Weinberg has **NOT** shown that tetraquarks do exist as narrow resonances at large N_c . Merely that the argument to disprove the existence of tetraquark is wrong.
- At this stage there is no known way to show that tetraquarks exist with the standard version of large N_c QCD. Indeed, I will argue at the end that they do not!!
- However there is a variant of large N_c QCD in which it is possible to show that quantum number exotics must exist and become narrow as $N_c \rightarrow \infty$.

QCD (AS)

- The large N_c limit of QCD is not unique
 - For gluons there is a unique prescription $SU(3) \rightarrow SU(N_c)$
 - However for quarks, we can choose different representations of the gauge group
 - Asymptotic freedom restricts the possibilities to the fundamental (F), adjoint (Adj), two index symmetric (S), two index anti-symmetric (AS).
 - Adj transforms like gluons (traceless fundamental color-anticolor); dimension $N_c^2 - 1$; 8 for $N_c = 3$ (unlike our world).
 - S transforms like two colors (eg fundamental quarks) with indices symmetrized; dimension $\frac{1}{2}N_c(N_c + 1)$; 6 for $N_c = 3$ (unlike our world).
 - AS transforms like two colors (eg fundamental quarks) with indices antisymmetrized; dimension $\frac{1}{2}N_c(N_c - 1)$; 3 for $N_c = 3$ (just like our world).

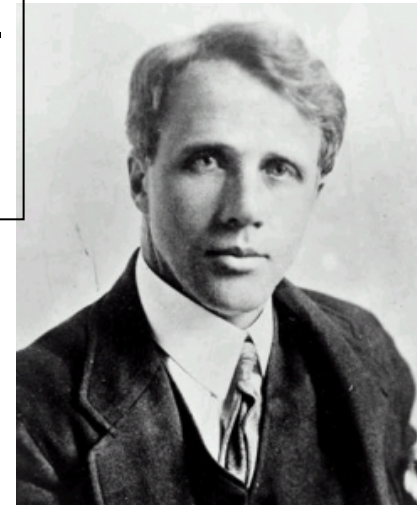
- Note that $N_c=3$ quarks in the AS representation are indistinguishable from the (anti-)fundamental. (In essence antisymmetric $r\ b$ is the same as \bar{g} .)
- However quarks in the AS and F extrapolate to large N_c in **different ways**.
 - The large N_c limits are physically different
 - The $1/N_c$ expansions are different.
 - A priori it is not obvious which expansion is better
 - It may well depend on the observable in question
- The idea of using QCD (AS) at large N_c is old
 - Corrigan & Ramond (1979)
 - Idea was revived in early 2000's by Armoni, Shifman and Veneziano who discovered a remarkable duality that emerges at large N_c .

Two Roads to Large N_c QCD



Quarks in
Fundamental

Quarks in 2-
index anti-
symmetric



“When you come to a
fork in the road, take it.”

---Yogi Berra,
American baseball
player, coach and part-
time philosopher

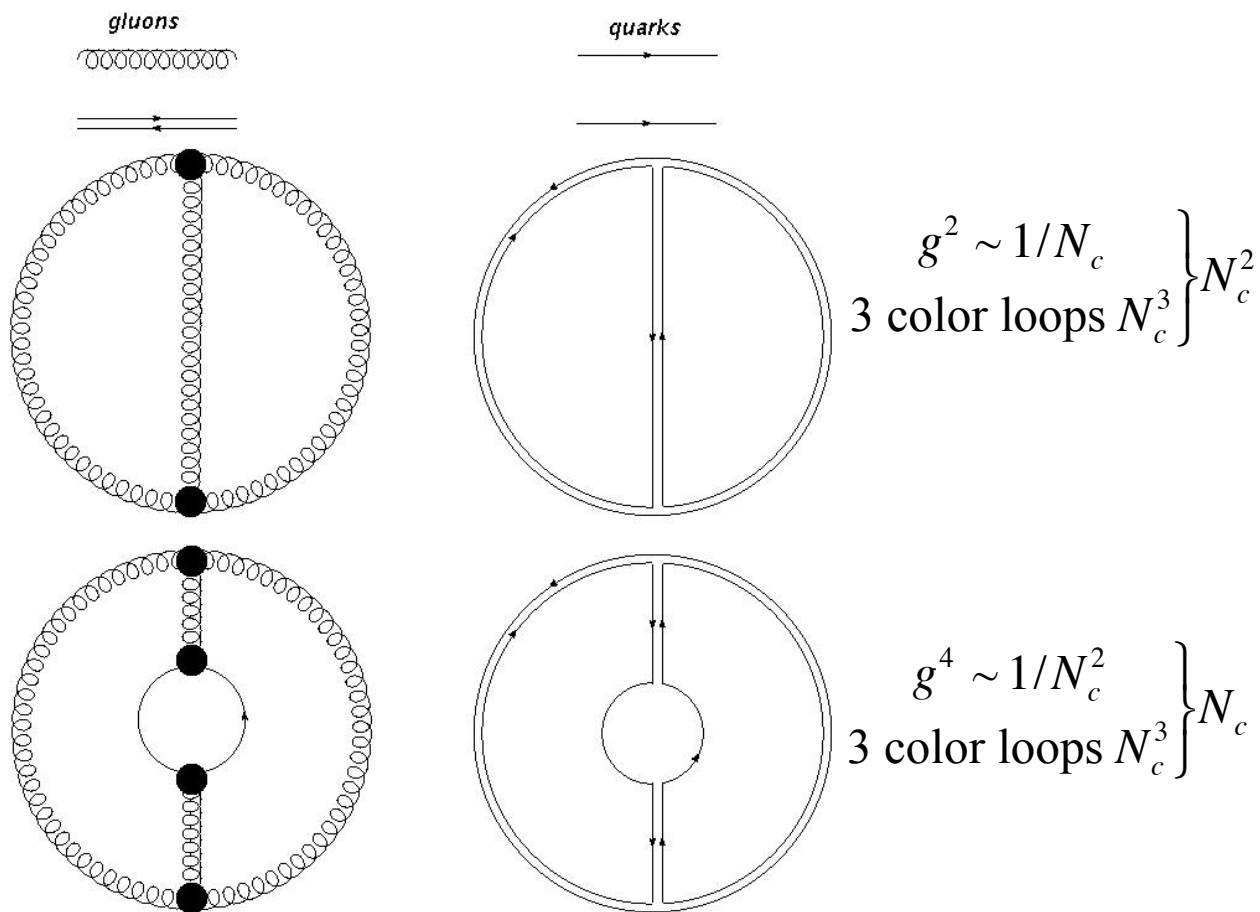
“Two roads diverged in a
wood, and I—
I took the one less traveled
by And that has made all the
difference.”

---Robert Frost,
American poet

Large N_c QCD

Principal difference between QCD(AS) and QCD(F) at large N_c is in the role of quarks loops

Easy to see this using 't Hooft color flow diagrams

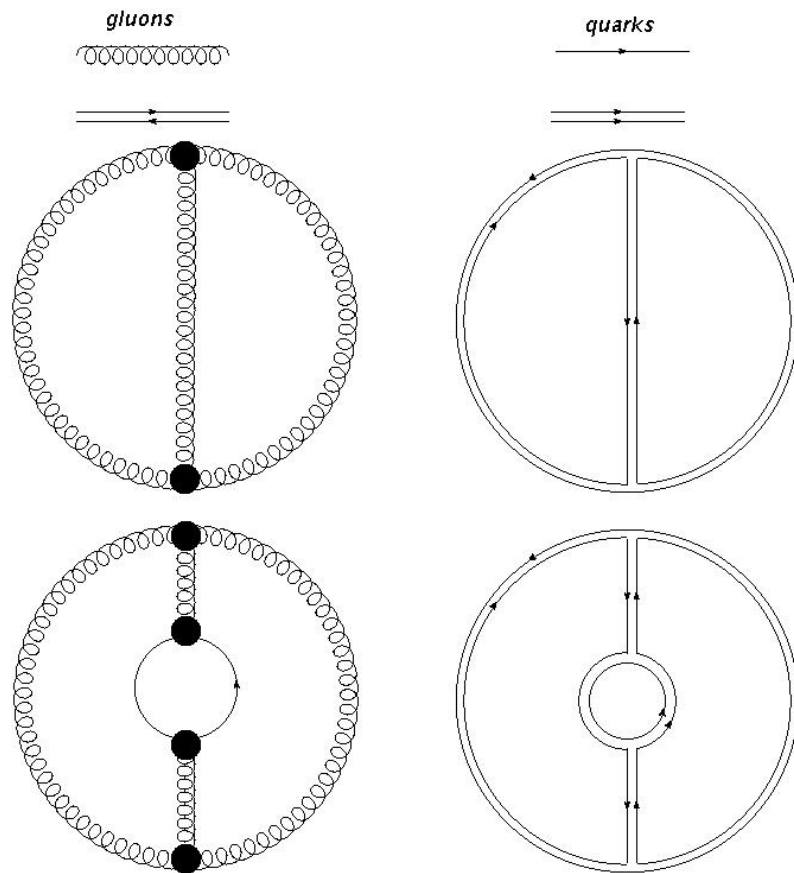


Recall the 't Hooft large N_c limit keeps $g^2 N_c$ fixed
So $g^2 \sim 1/N_c$

QCD(F)

Insertion of a planar quark loops yields a $1/N_c$ suppression.

Leading order graphs are made of planar gluons



$$\left. \begin{array}{l} g^2 \sim 1/N_c \\ 3 \text{ color loops } N_c^3 \end{array} \right\} N_c^2$$

QCD(AS)

Insertion of a planar quark loops does not lead to a $1/N_c$ suppression.

$$\left. \begin{array}{l} g^4 \sim 1/N_c^2 \\ 4 \text{ color loops } N_c^4 \end{array} \right\} N_c^2$$

Leading order graphs are made of planar gluons and quarks

Principal phenomenological difference between the two is the inclusion of quark loop effects at leading order in QCD(AS). Whether this is a bug or a feature depends upon the observable. In baryon spectroscopy based on emergent symmetry, both QCD(AS) and QCD(F) appear to have predictive power (Cherman, Cohen & Lebed 2009, 2012).

- QCD(AS) naturally includes quark loops at leading order. Thus one might expect that in non-quantum number exotic channels tetraquarks will mix with ordinary mesons at leading order.
 - This can be shown to be correct.
- More interestingly, in quantum number exotic channels, QCD(AS) **MUST** have narrow tetraquarks at large N_c (i.e. narrow states which have at least 2 quarks and 2 antiquarks)Cohen&Lebed PRD 89, 054018 (2014).

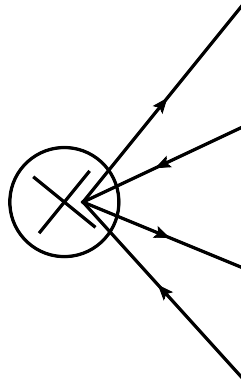
Key ingredient: there are single color trace tetraquark sources in QCD(AS). That is the source cannot be broken up into two separate color singlets (except for N_c^{-2} contributions). This cannot be done in QCD(F)

$$J(x) = \sum_{\substack{A,B \\ a,b,c,d}} C_{AB} \bar{q}^{ab}(x) \Gamma_A q_{bc}(x) \bar{q}^{cd}(x) \Gamma_B q_{da}(x)$$

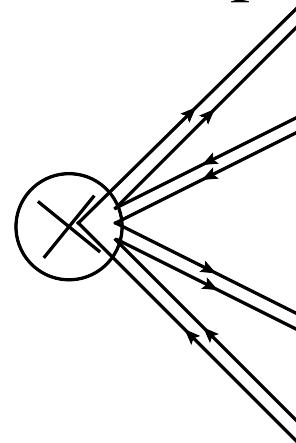
Γ_A, Γ_B are matrices in Dirac-flavor space.

a, b, c, d are fundamental color indices

choice of C_{AB} fixes quantum #s; for simplicity chose an exotic

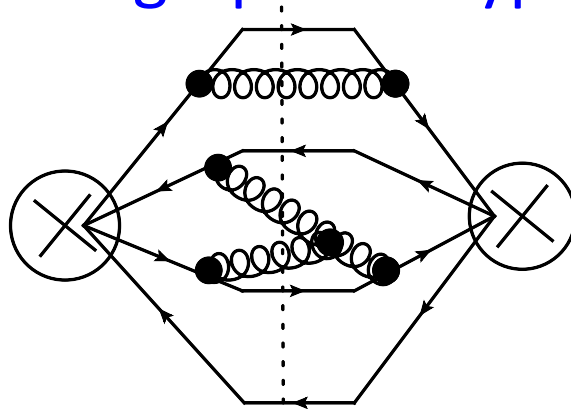


Source as a Feynman diagram

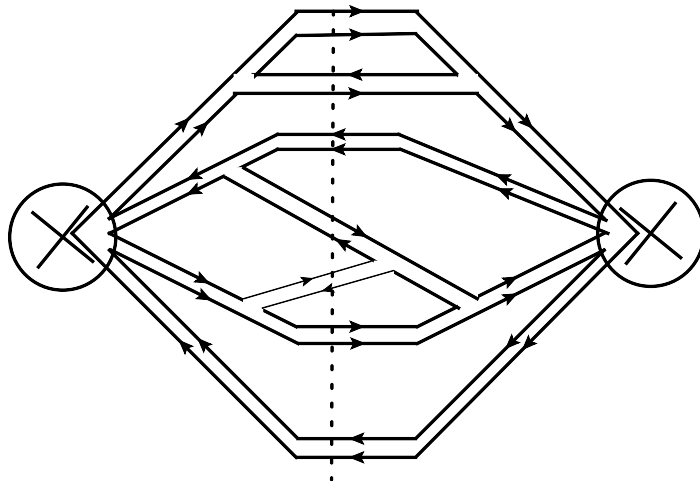


Source as a color-flow diagram

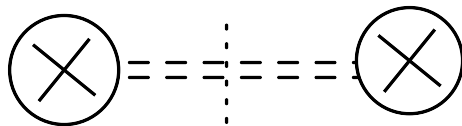
Look at the JJ correlation function. It is dominated by planar graphs. A typical diagram scales as N_c^4



Feynman diagram

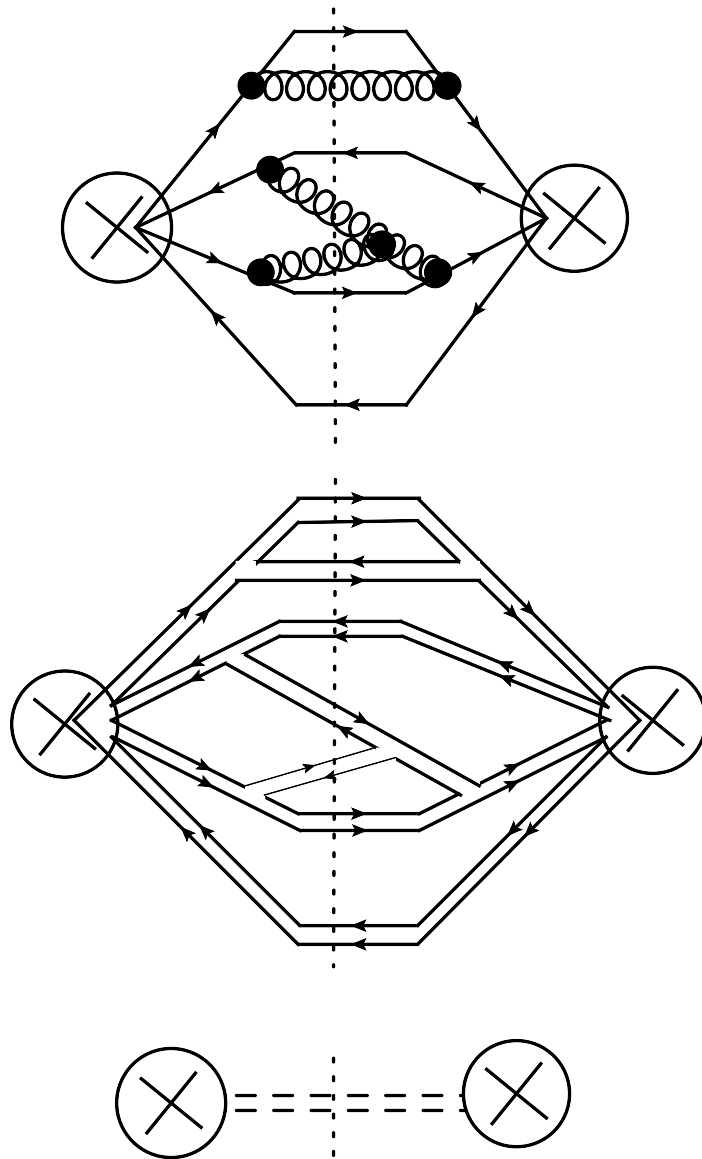


Color-flow diagram; 7 color loops
 $\sim N_c^7$; 6 factors of $g \sim N_c^{-3}$; overall
 scaling $\sim N_c^4$



Hadronic level diagram:
 propagation of a single
 tetraquark

The reason this corresponds to a single tetra hadrons can be understood in terms of a cut of the diagram.



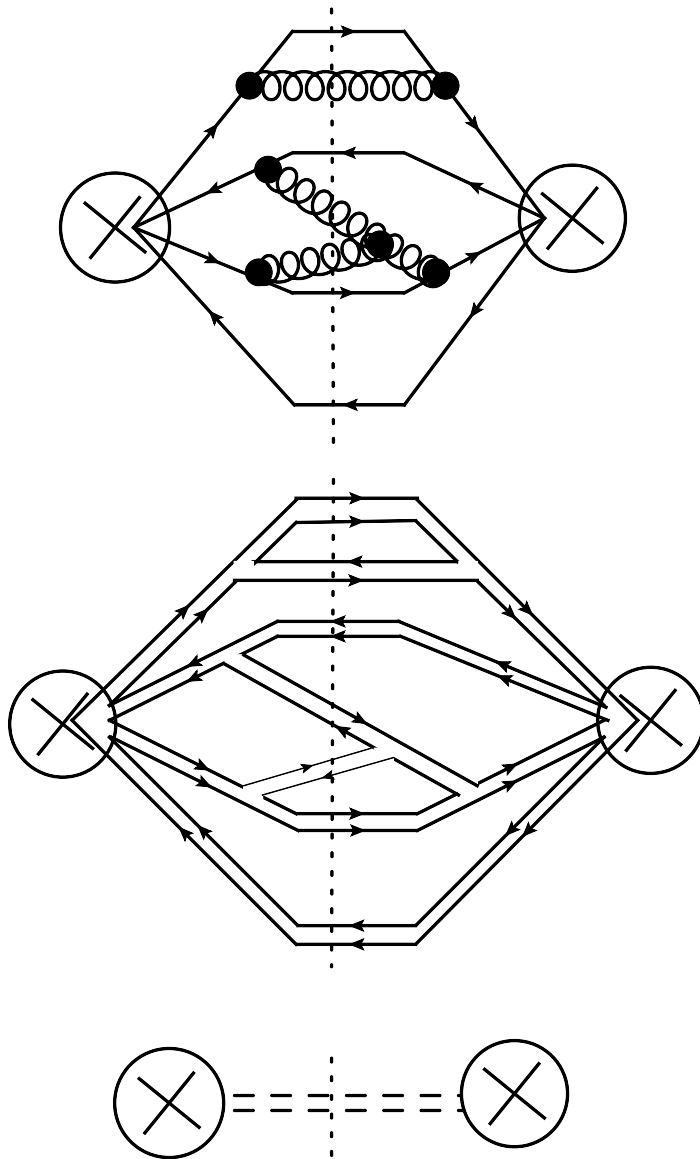
Short dashed line indicates a cut which reveals the intermediate state structure of the diagram.

The cut shown here corresponds to a state of the form

$$\bar{q}^{ab} q_{bc} A_d^c A_e^d \bar{q}^{ef} A_f^g q_{ga}$$

This is a single color-trace object. It can not be divided into two separate color singlets (except by a $1/N_c^2$ contribution)

This is generic: all cuts yield single-color trace objects

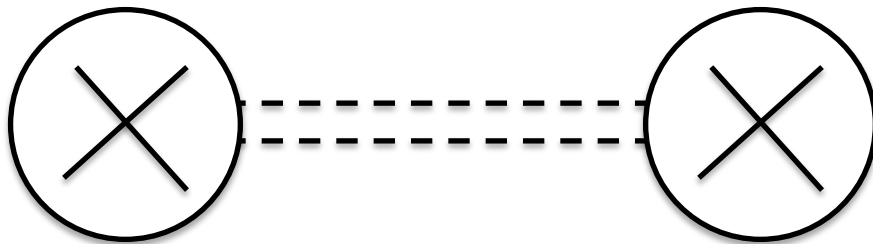


If one includes confinement, this implies that the state must be a single hadron at leading order. It cannot break up into two color singlet hadrons since all intermediate states consist of a single indivisible color singlet.

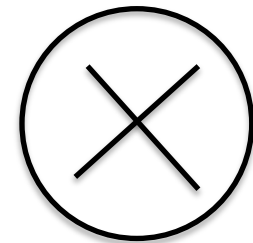
It must be narrow as components with more than one hadron are suppressed in the $1/N_c$ expansion.

- This can be seen to be true self-consistently
 - One can use standard kind of large N_c analysis for correlators with appropriate changes to account for QCD(AS) to deduce that a generic multi-hadron vertex scales as N_c^{2-n} where n is the number of hadrons (mesons, glueballs, hybrids & tetraquarks). This is true whether or not one has exotic channels. The only constraint is that quantum numbers do not exclude the vertex
 - Thus, tetraquark width $\sim N_c^{-2}$
as advertised the tetraquark is narrow

An example: the tetraquark-2 meson vertex.
Proceed by studying the scaling of the appropriate correlator.



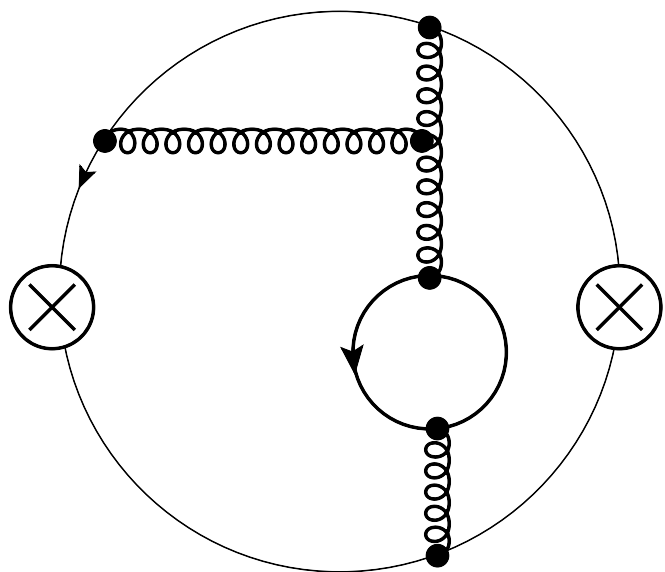
Recall at quark-gluon level we deduced that the tetraquark two point function scaled as N_c^4



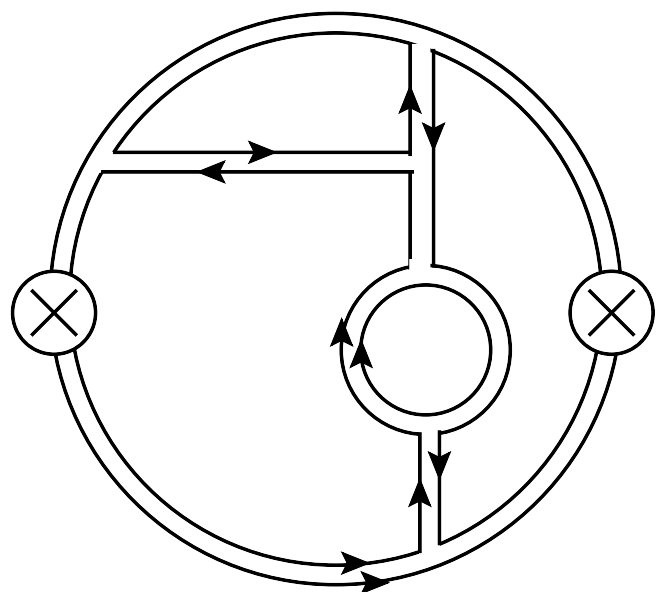
Thus the source scales as N_c^2



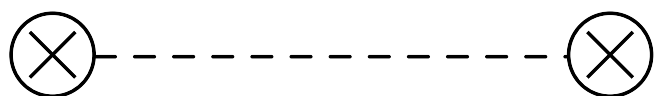
The meson source scales as N_c^1 by analogous reasoning



Feynman diagram

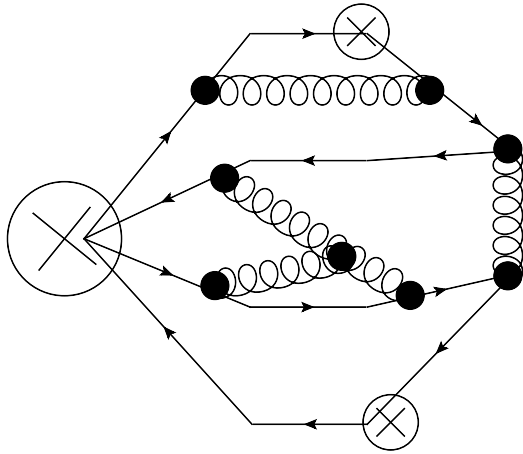


Color-flow diagram; 5 color loops
 $\sim N_c^5$; 6 factors of $g \sim N_c^{-3}$; overall
 scaling $\sim N_c^2$



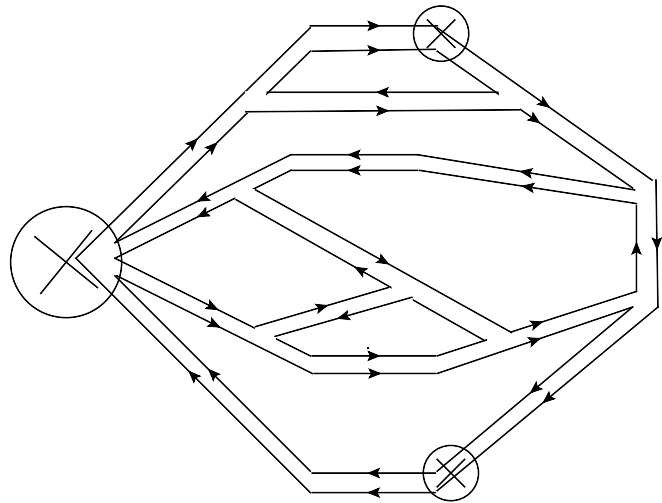
Hadronic level diagram:
 propagation of a single meson;
 source scales as N_c^1

A typical graph

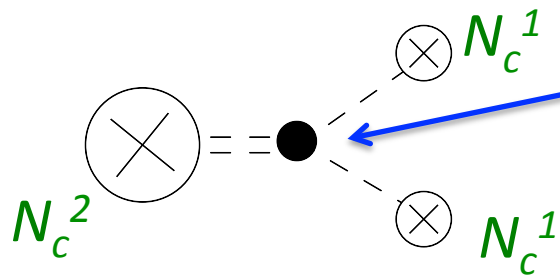


Feynman
diagram: 8
factors of g

Overall N_c
scaling $N_c^3 =$
 $(N_c^{-4})(N_c^7)$



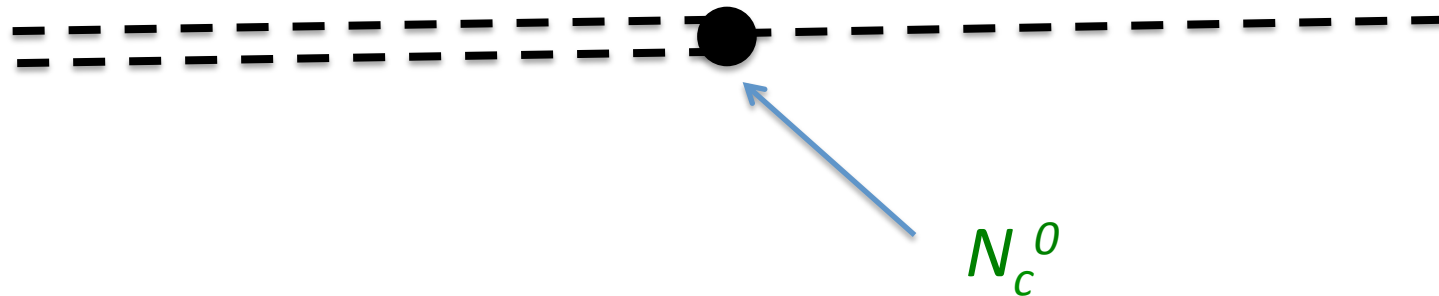
Color flow
diagram: 7
color loops



Hadronic
level diagram

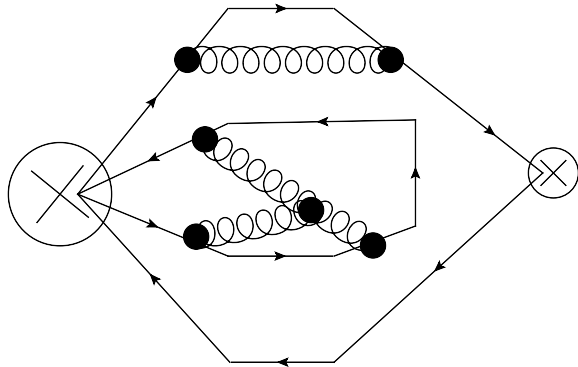
To match scaling at QCD
level the tetraquark-2
meson vertex must scale
as N_c^{-1} yielding a width of
 N_c^{-2} .

- What about non-exotic channels?
 - Tetraquarks and ordinary mesons mix at leading order. Thus follows from our claimed scaling of a generic multi-hadron vertex scales as N_c^{2-n} where n is the number of hadrons (mesons, glueballs, hybrids & tetraquarks) since this is a 2 point function.

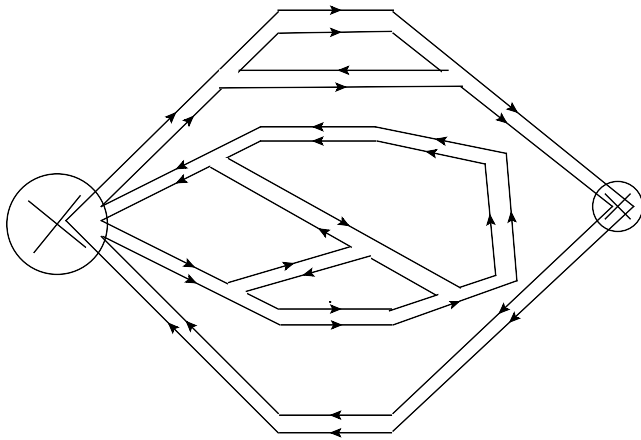


- To see that this scaling hold in this case consider a correlation function with one tetraquark sources and one meson source

A typical diagram



Feynman diagram



Color-flow diagram; 6 color loops
 $\sim N_c^6$; 6 factors of $g \sim N_c^{-3}$; overall
 scaling $\sim N_c^3$

$$N_c^2 \quad \text{---} \bigcirc \text{---} \text{---} \bullet \text{---} \text{---} \bigcirc \text{---} N_c^1$$

The diagram shows a matching rule for the hadronic level. On the left, a vertex (a circle with a cross) is connected to a black dot by a dashed line. This is equated to a vertex connected to a black dot by a dashed line, which is then connected to another vertex by a dashed line. The left vertex is labeled N_c^2 and the right vertex is labeled N_c^1 .

Hadronic level diagram:
 matching requires mixing
 vertex to scale as N_c^0 .

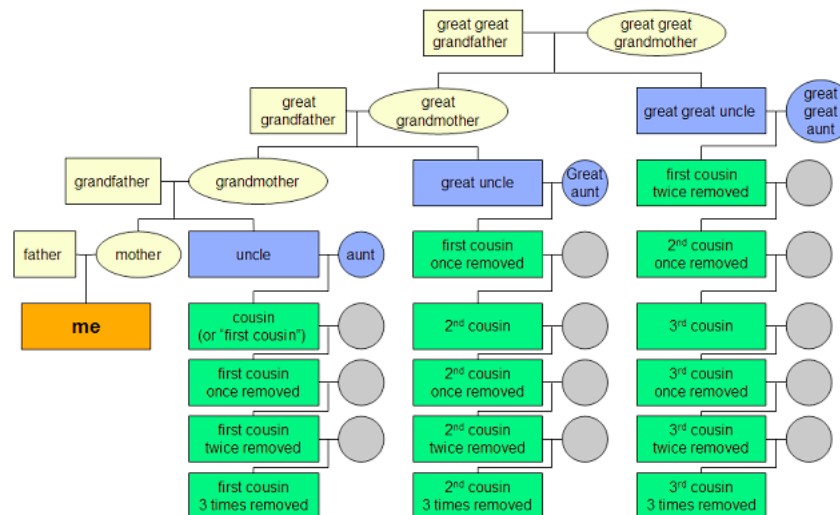
- This can be generalized.
 - The same kind of analysis will yield narrow hexaquarks, octaquarks etc.
 - Again, for all types of hadrons an n point hadronic vertex will (if allowed by quantum numbers) scale as N_c^{2-n} .

Summary for QCD(AS)

- Tetraquarks (as well as higher multi-quark hadrons) exist as narrow resonances in the large N_c limit of QCD(AS).
- Non-exotic tetraquarks exist and mix with ordinary mesons.
- The generic n -hadronic vertex will (if allowed by quantum numbers) scale as N_c^{2-n} .
- The width of all hadrons with phase space to decay will scale as N_c^{-2} .

Implications For the Real World

- Minimally, this result shows that nothing in the structure of gauge theories such as QCD excludes exotic tetraquark states made from light quarks as narrow resonances---a cousin of QCD has been shown to have them.
- One question is “how close a cousin”?



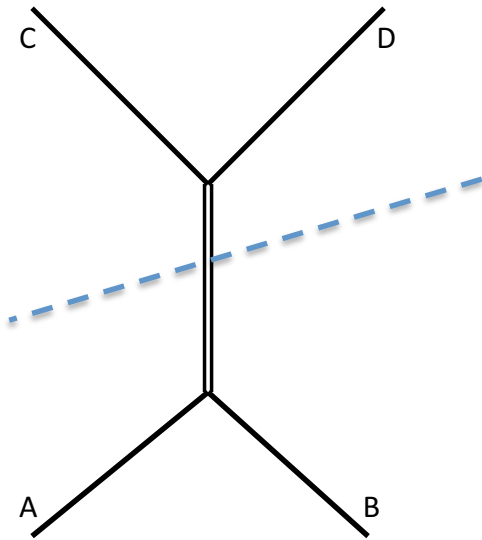
- If it is a close cousin and the real world resembles QCD(AS) at large N_c then one expects narrow exotic tetraquarks to exist.
- This does not mean that one can necessarily find nonexotic tetraquarks such as the f_0 ; parametrically tetraquarks and ordinary mesons will mix at leading order. However, nothing in principle prevents such a state from being *dynamically* dominated by tetraquark components numerically. It is just not *parametrically* isolated at large N_c

QCD(F)

- **Key question: Is QCD(AS) at large N_c a close cousin of the real world so far as these observables?** One hint on this would be the behavior of QCD(F) (the more standard type of large N_c QCD); if **both** large N_c limits behave the same way it is more compelling
- There is a somewhat subtle argument that despite Weinberg's critique of Coleman and Witten tetraquarks in fact, do not exist at large N_c in QCD(F) cohen & Lebed ArXiv 1403.0890
 - Recall that Weinberg merely showed that the Witten/Coleman argument against tetraquarks is flawed, not that tetraquarks themselves exist.

A sketch of the argument why there are no tetraquarks in QCD(F) at Large N_c

If exotic tetraquarks exist they will couple to ordinary meson with a coupling strength $\sim N_c^{-1/2}$. Thus it **must** appear as a singularity in the s-channel of scattering for incident mesons.



This is based on standard Mandelstam type dispersion analysis. The scattering amplitude can depend on two of the Mandelstam variables (say t , and s). At fixed t , the dispersion relation is

$$T(s, t) = \text{pole terms} + \frac{1}{\pi^2} \int_{\text{threshold}}^{\infty} ds' \frac{\rho(s', t)}{s - s' + i\epsilon}$$

A tetraquark must appear as a sharp structure in ρ ; it will become a δ function at large N_c .

- To proceed use standard assumptions
 - Scaling with N_c of physical observables will match the N_c scaling of the leading order family of diagrams.
 - A cut in the diagram corresponds to intermediate particles going on-shell
- Focus on the the scattering amplitude and in particular the spectral function
 - A key point is that the LSZ reduction relates the scattering amplitude to the **amputated** 4-point function.
 - That is it multiplies by inverse propagators to eliminate singularities associated with the incident and final particles

$$T = Z_A^{-1/2} Z_B^{-1/2} Z_C^{-1/2} Z_D^{-1/2} (q_A^2 - m_A^2)(q_B^2 - m_B^2)(q_C^2 - m_C^2)(q_D^2 - m_D^2) \Pi_4^{ABCD}(q_A, q_B, q_C, q_D)$$



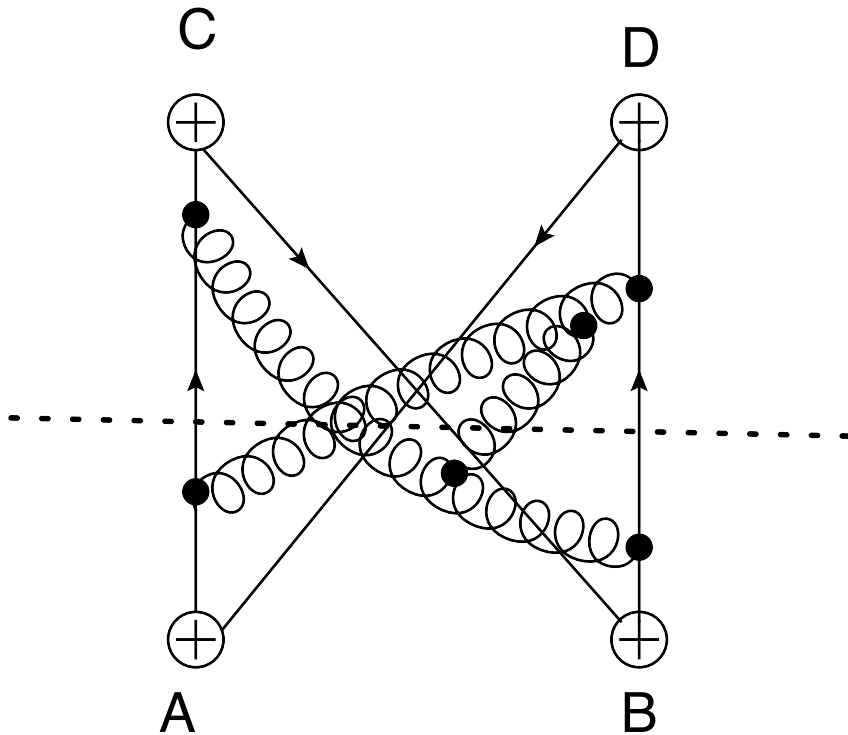
4 point function in momentum space for
currents A, B,,C, D

$$Z_A = |(q_A^2 - m_A^2) \Pi_2^{AA}(q_A)|$$

There is a topological argument that amputated 4-point function at leading order **every diagram** in an exotic channel only has singularities in the s-channel associated with the asymptotic mesons (either initial or final) in the sense that the cut has two color singlets carry the initial four momenta of each; thus there are no singularities associated with intermediate object.

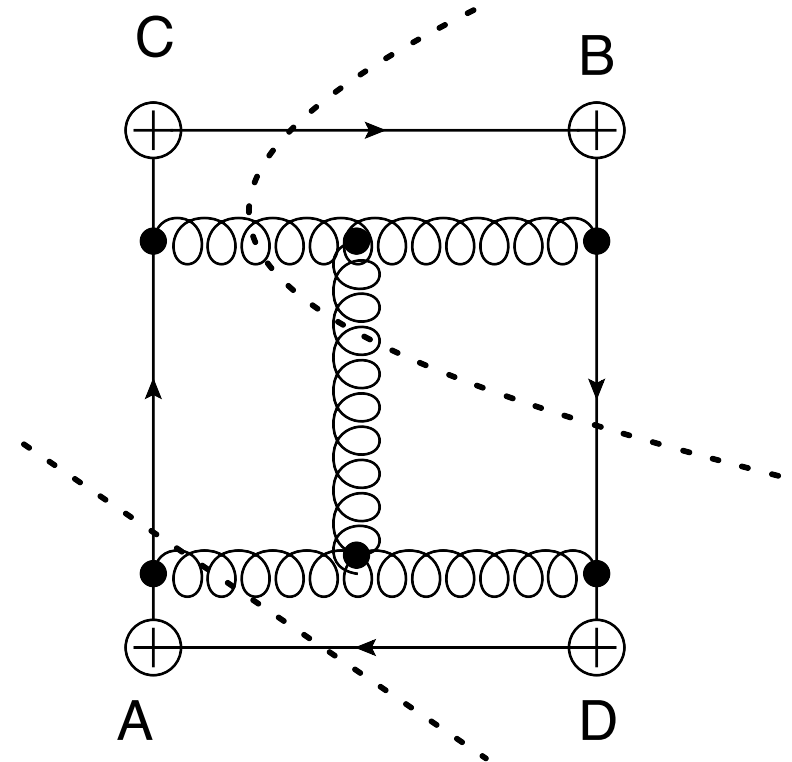
Key point is to distinguish between a space-time description of the process $A+B \rightarrow C+D$ from the topology of the color flow

A typical contribution to the full 4-pt function



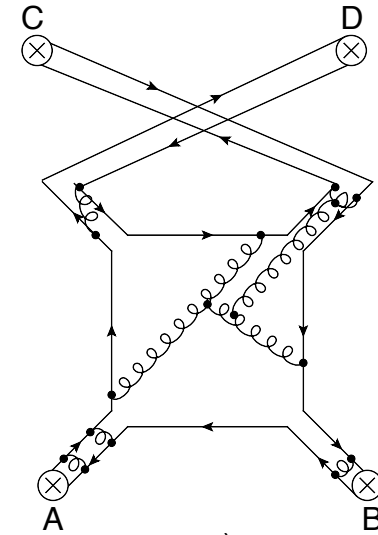
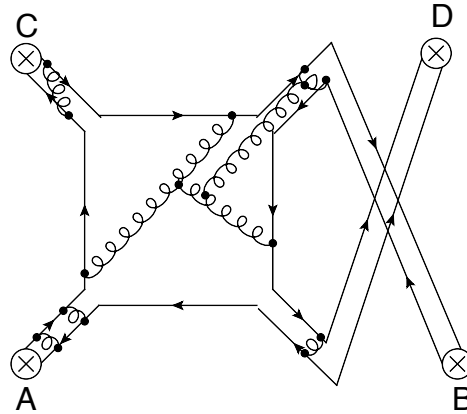
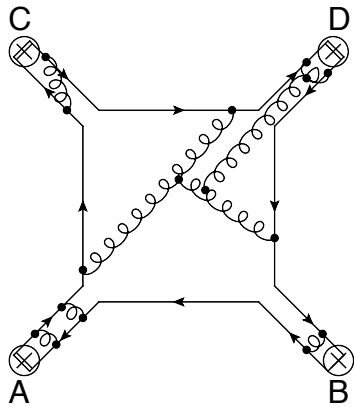
Space-time diagram

This behavior is generic and occurs in all diagrams associated with exotic channels

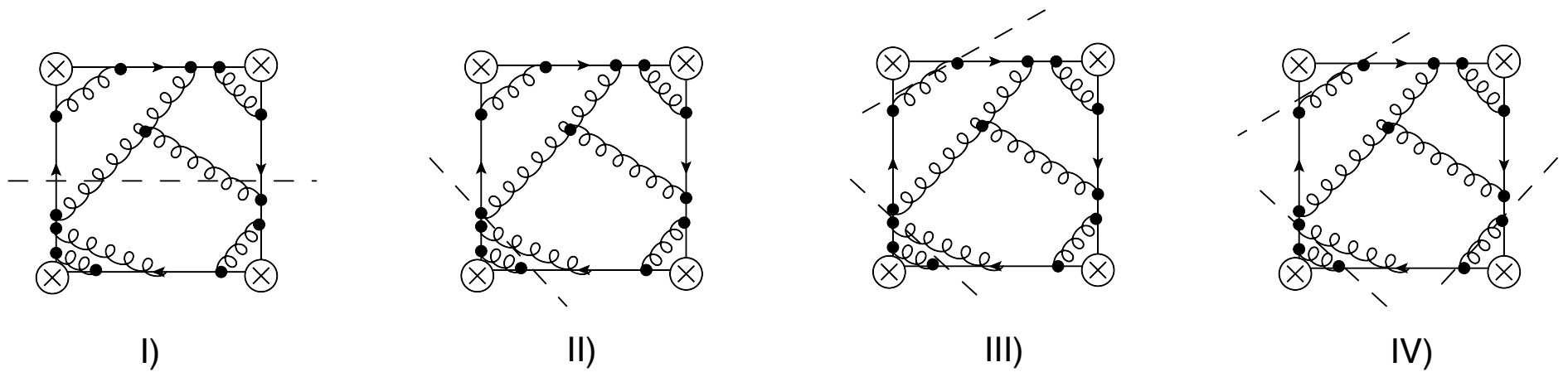


Topologically equivalent planar graph. Note that the cut has broken in two and each part carries the 4-momenta inserted at A or B

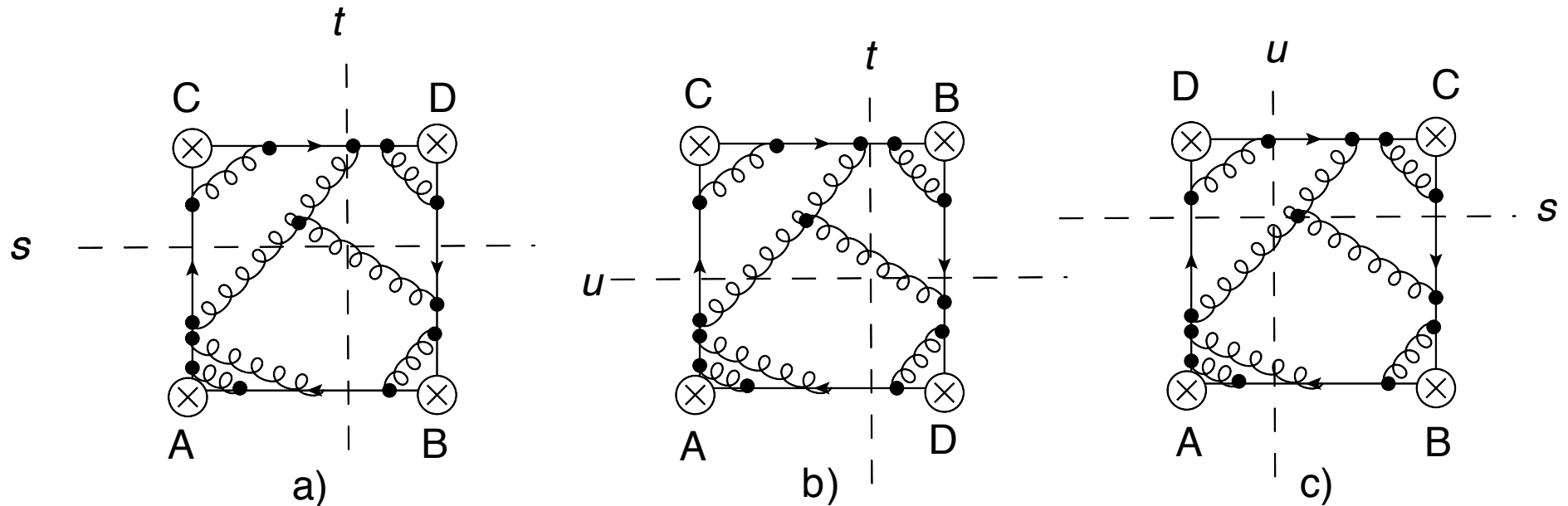
- Note that the nature of the cut when drawn as a space-time diagram is such that you might think it is associated with a tetraquark.
- However from the diagram drawn in planar form, it is clear that the cut merely cuts corners carrying exactly the momentum brought in at A & B and thus correspond to on-shell incident or mesons.
 - Hence when going to the scattering amplitude from the 4-point function (i.e. amputating the external legs) this diagram will vanish.
- Claim: this is generic and exotic channels do not have tetraquark cuts.



Relating leading space-time diagrams to topological ones. Topologically all that matters is order of the four corners. Moreover, time-reversal invariance means **ABCD** is identical to **ADCB**. Thus, there are only three classes of diagrams **ABCD**, **ADBC** and **ABDC**.

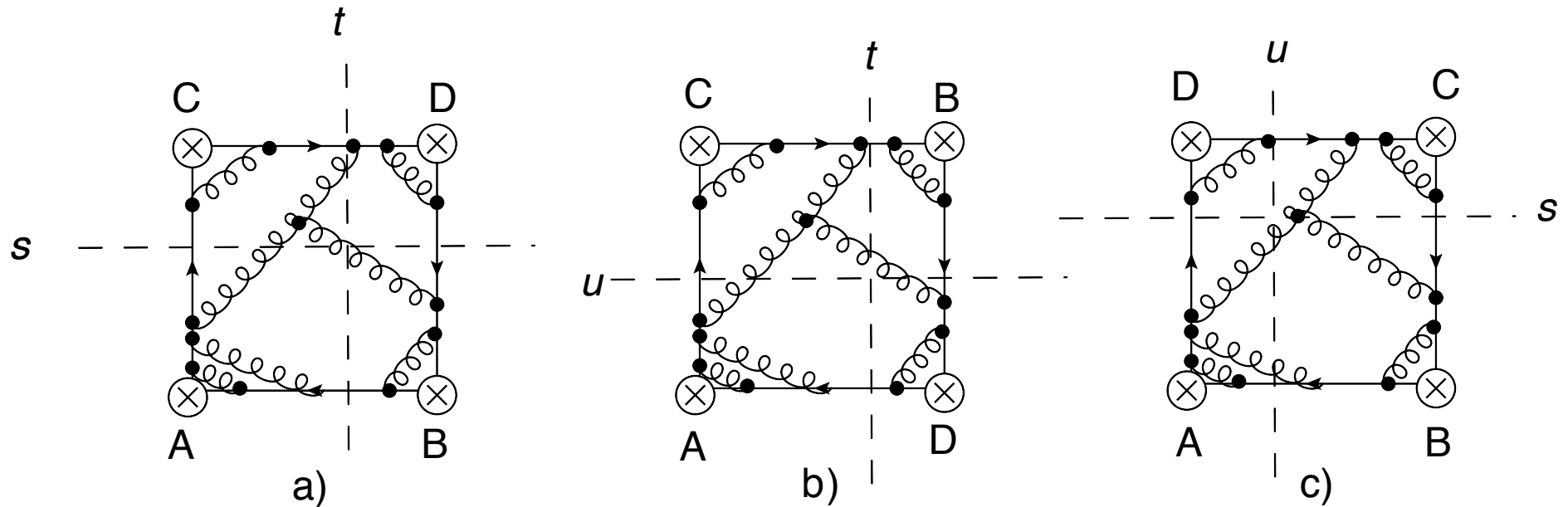


Broad categories of cuts. Note that except for category I) these all cut through a corner. These corner cuts all are associated with the momentum carried in at the corner and are eliminated when looking at the amputated diagram, AKA the scattering amplitude. Thus the only singularities in category I that could be associated with scattering going through a tetraquark. We will show below that this is not possible for exotic channels by looking at theses in detail



Type I cuts. The three topological classes in terms of ordering are given here. Note that for each there are cuts in only two of the tree Mandelstam variables.

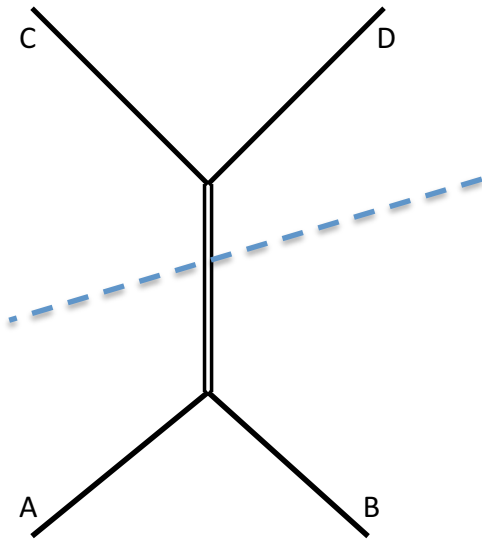
S-channel cuts exist in type a) and c) but not b). If we can show that in exotic channels only have topology b) then there is no s-channel cut at leading order in $1/N_c$ expansion and hence no tetra quark



Note that in a) and c) type diagrams A & B are adjacent to each other. If the channel is a flavor exotic (say isospin 2), then a quark line (isospin $\frac{1}{2}$) cannot run past an adjacent A&B since doing so must change its isospin to $\frac{3}{2}$ or $\frac{5}{2}$ but cannot keep it as $\frac{1}{2}$.

Thus, as advertised only b) is possible and it has no s-channel cut.

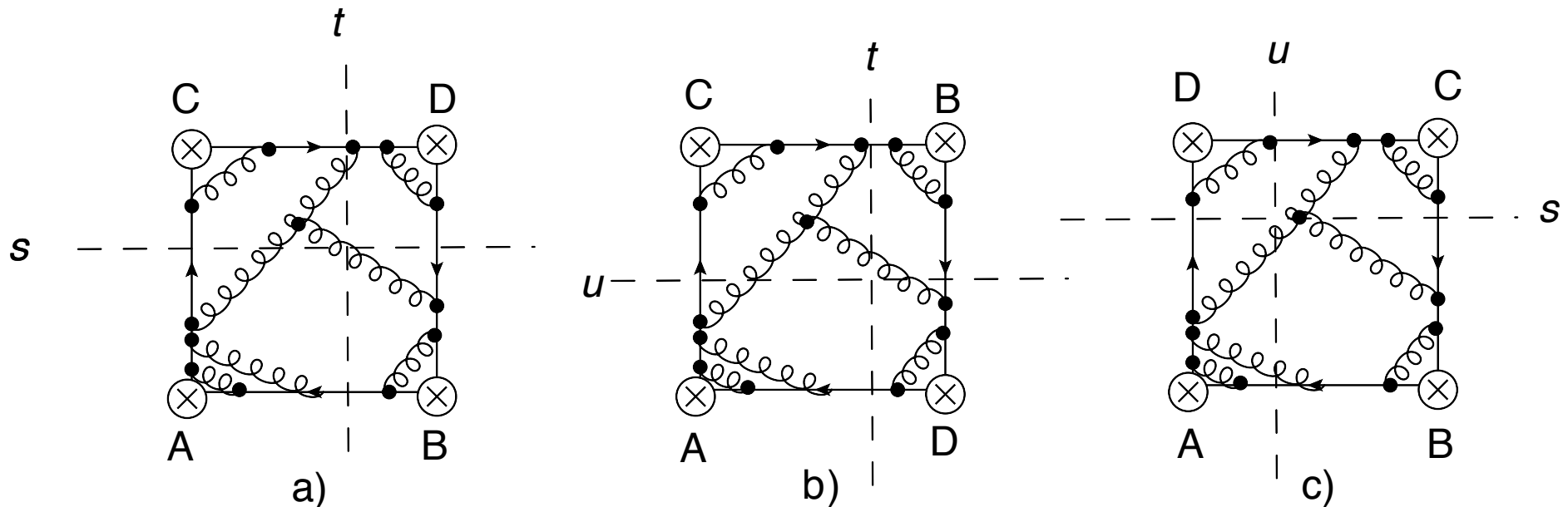
Recall that, if exotic tetraquarks exist they will couple to ordinary meson with a coupling strength $\sim N_c^{-1/2}$. Thus it **must** appear as a singularity in the s-channel of scattering for incident mesons.



Since no diagrams have s-channel cuts we conclude that quantum number exotic tetraquarks do not appear at large N_c in QCD(F).

Thus, QCD(F) and QCD(AS) behave in fundamentally different ways for this observable.

What about non-exotic channels in QCD(F ?)



The non-exotic channels **do** have s -channel singularities a) and c) are not forbidden by quantum numbers. But note they cut exactly one quark-antiquark pair. Thus they are associated with ordinary mesons. They are NOT tetraquarks.

- Thus, in QCD(F) at large N_c , there are no tetraquarks in either exotic or nonexotic channels.
 - This depends on standard assumptions used in large N_c analysis. In particular, it depends on perturbative graphs capturing the correct leading N_c counting.
 - There is a loophole; if tetraquarks were to exist for some unknown reason and couple to mesons weaker than $N_c^{-1/2}$ there is no inconsistency but there is no reason to expect that this scenario occurs; it seems very unlikely.
 - It would mean that tetraquarks exist only due to subleading connected graphs.

- Thus whether tetraquarks exist in the real world depends on whether the real world is closer (in this aspect) to QCD(F) at large N_c or to QCD(AS) at large N_c . This is a dynamical question.

The bottom line



Whether narrow tetraquarks made of light quarks exist in the real world is a matter of dynamical detail which generic large N_c arguments cannot answer. **Large N_c arguments show they cannot be excluded as incompatible with QCDlike theories**