Meson cloud effects in nucleon resonances at low and intermediate energies

Bojan Golli

Faculty of Education, University of Ljubljana and
J. Stefan Institute, Ljubljana, Slovenia

in collaboration with

Simon Širca (Ljubljana)

Manuel Fiolhais, Pedro Alberto (Coimbra)

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Outline

- Motivation
- Pion cloud effects in electro-excitation of the $\Delta(1232)$ resonance
- A short survey of the coupled channel approach incorporating quasi-bound quark-model states and meson clouds
- The underlying quark models used in our calculation
- Pion scattering and electro-production in the region of the Roper $(N(1440))$ resonance
- The role of the η and K mesons in the formation of the (two) low lying S11-wave resonances
- A short survey of the low-lying D-wave resonances
- Summary: an overview of the model-independent manifestation of the meson cloud

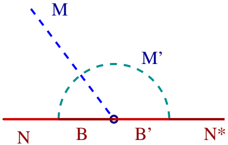
Motivation

- Which mechanism is responsible for the formation of a resonance:
  - João da Providência and João Urbano: 1978. Nucleon resonances arise as excitation of the pion cloud around the bare nucleon. Classical pion field interpreted as coherent state of pions; resonance with good J and T obtained by Peierls-Yoccoz projection.
  - Dynamical generation: a (quasi) bound state of the nucleon and a meson
  - Quark models: spin and radial excitations of the 3-valence quark core
  - "Unified models" of nucleon resonances include quark-core excitations + meson cloud around quark cores as an extent ion of the da Providência-Urbano approach.

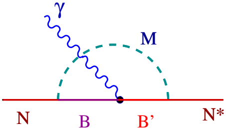
- How to disentangle different mechanisms?
  - In scattering it is possible to reproduce the elastic and inelastic amplitudes almost in any model by a suitable modification of the coupling constants and the interaction ranges
  - Meson electo-production amplitudes and the Q^2-dependence of different helicity amplitudes is a much more severe test to analyze the structure of various resonances

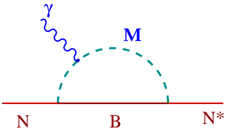
Meson-cloud effects, formally:

- MNN^* vertex renormalization

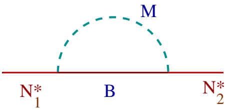


- γNN^* renormalization





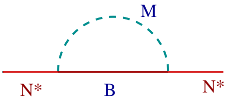
- resonance mixing



e.g. P11 $N^0(939) + N^0(1440)$,
S11 $N^0(1535) + N^0(1650)$,
D13 $N^0(1520) + N^0(1700)$

$$N^* = \cos\theta\, N_1^{(0)} + \sin\theta\, N_2^{(0)}, \quad \theta = \theta(W).$$

- self energy



The underlying quark models involving mesons

- σ-model with quarks
  (bosonized version of the NJL model) (early work, only for $\Delta(1232)$)
- Chromodielectric model
  dynamical confinement (early work, only for $\Delta(1232)$ and $N(1440)$)
- Cloudy Bag Model
  extended to pseudo-scalar SU(3) octet and the ρ meson:

$$\mathcal{L}_{\mathrm{CBM}}^{(\mathrm{quark-meson})} = -\frac{\mathrm{i}}{2f}\,\bar{q}\lambda_a(\gamma_5\phi_a + \boldsymbol{\gamma}\cdot\boldsymbol{A}_a)q\delta\left(r - R_{\mathrm{bag}}\right)\,,\; a = 1,2,\ldots,8$$

parameterizes the baryon-meson and baryon-photon coupling constants and form factors in terms of "f_π" and the bag radius R_{bag}.

Parameters:

$R_{\mathrm{bag}} = 0.83$ fm (from the ground state calculations)
$f_\pi = 76$ MeV (reproducing the experimental value of $g_{\pi NN}$)
$f_K = 1.2 f_\pi$, $f_\eta = f_\pi$ or $1.2 f_\pi$, $f_\pi < f_\rho < 2 f_\pi$
similar results for $0.75\ \mathrm{fm} < R_{\mathrm{bag}} < 1.0\ \mathrm{fm}$

Free parameters: bare masses of the resonances

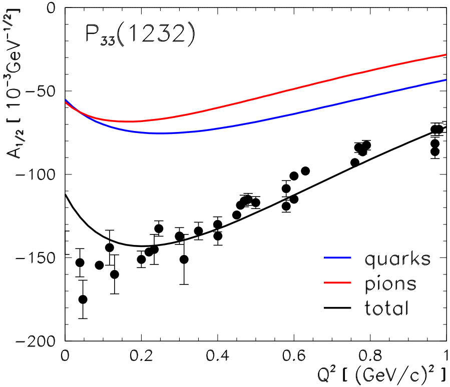
$\Delta(1232)$ helicity amplitude

$\Delta(1232)$ as the simplest system to investigate the interplay of quark and pion degrees of freedom.

Linear σ-model with quarks (LSM) and Chromodielectric model (CDM)

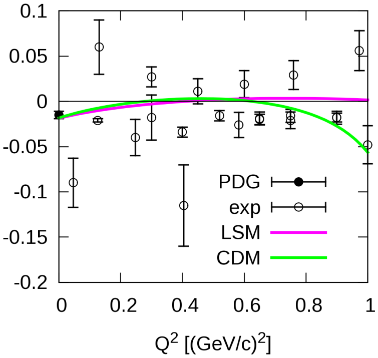
P.Alberto, M.Fiolhais, S.Širca, B.G. PLB **373** (1996) 229.

Strong contribution of the pion cloud to the leading M1 helicity amplitude:

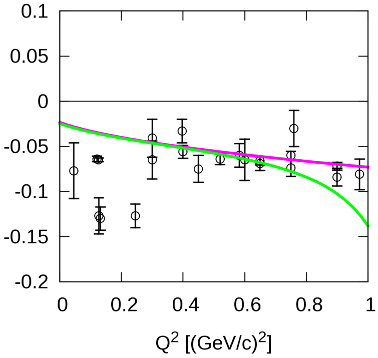


$\Delta(1232)$ EMR and CMR ratios

The pion cloud strongly dominates the E2 and C2 amplitudes: the different Q^2 behaviour of E2 and C2 amplitudes (equal for the point-like source) originates from the long-range part of N and Δ w.f.



$E2/M1$

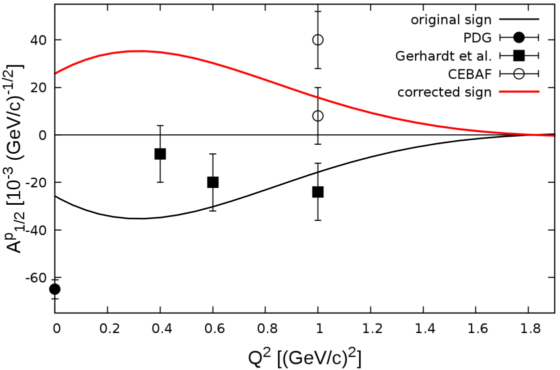


$C2/M1$

$N(1440)$ helicity amplitude

P. Alberto, M. Fiolhais, B. G., and J. Marques, Phys. Lett. B **523** (2001) 273.
Chromodielectric model: π and σ-meson clouds and a chromodielectric field which assures quark dynamical confinement. The Roper state includes beside the quark excitation to $2s$ state and the pion cloud, also the lowest vibrational mode of the σ-meson field around the nucleon:

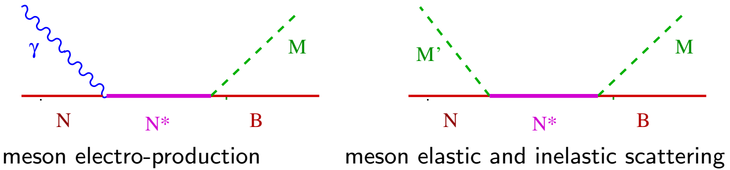
$$\Psi_R = c_1(1s)^3 + c_2(1s)^2(2s)^1 + c_\sigma a_\sigma^\dagger \Psi_N + c_\pi a_\pi^\dagger \Psi_N$$



Higher resonances

The helicity amplitudes are extracted from the measured meson electro-production amplitudes $A(W,Q^2)$ which provide more complete and less model-dependent informations about resonances.

Higher resonances are involved in processes like



The proper treatment of such processes requires a coupled channel approach. We have developed a formalism to compute the multi-channel K-matrix which includes many-body quasi-bound quark states in the scattering mechnism. The formalism is limitted to a class of chiral quark models in which the mesons are linearly coupled to the quark core.

Construction of the K-matrix

Aim: to include many-body states of quarks in the scattering formalism (Chew-Low type approach)

Construct K-matrix in the spin-isospin (JI) basis:

dressed states

$$K^{JI}_{M'B'\,MB} = -\pi\sqrt{\frac{\omega_M E_B}{k_M W}}\,\langle\Psi^{MB}_{JI}(W)||V_{M'}(k)||\Psi_{B'}\rangle$$

by using principal-value (PV) states

$$|\Psi^{MB}_{JI}(W)\rangle = \sqrt{\frac{\omega_M E_B}{k_M W}}\left\{\left[a^\dagger(k_M)|\Psi_B\rangle\right]^{JI} - \frac{\mathcal{P}}{H-W}\,[V(k_M)|\Psi_B\rangle]^{JI}\right\}$$

normalized as

$$\langle\Psi^{MB}(W)|\Psi^{M'B'}(W')\rangle = \delta(W-W')\delta_{MB,M'B'}(1+\mathbf{K}^2)_{MB,MB}$$

Ansatz for the channel PV states

free meson
(defines the channel)

$$|\Psi_{JI}^{MB}\rangle = \sqrt{\frac{\omega_M E_B}{k_M W}} \Big\{ [a^\dagger(k_M)|\Psi_B\rangle]^{JI}$$

bare (genuine) baryons (3q)

$$+ \sum_{\mathcal{R}} c_{\mathcal{R}}^{MB} |\Phi_{\mathcal{R}}\rangle$$

meson "clouds"
with amplitudes χ

$$+ \sum_{M'B'} \int \frac{\mathrm{d}k\, \chi^{M'B'MB}(k,k_M)}{\omega_k + E_{B'}(k) - W} [a^\dagger(k)|\Psi_{B'}\rangle]^{JI} \Big\}$$

Above the meson-baryon (MB) threshold:

$$K_{M'B'\,MB}(k,k_M) = \pi \sqrt{\frac{\omega_M E_B}{k_M W}} \sqrt{\frac{\omega_{M'} E_{B'}}{k_{M'} W}}\, \chi^{M'B'\,MB}(k,k_M)$$

T matrix for scattering and electroproduction

Solution for the K matrix

$$K_{MB,M'B'} = \pi\sqrt{\frac{\omega_M E_B}{k_M W}}\sqrt{\frac{\omega_{M'} E_{B'}}{k_{M'} W}}\left[\sum_{\mathcal{R}} \frac{\widetilde{\mathcal{V}}^{M}_{B\mathcal{R}}\widetilde{\mathcal{V}}^{M'}_{B'\mathcal{R}}}{(M_{\mathcal{R}} - W)} + \mathcal{D}_{MB,M'B'}\right]$$

$$|\widetilde{\Phi}_{\mathcal{R}}\rangle = \sum_{\mathcal{R}'} \mathcal{U}_{\mathcal{R}\mathcal{R}'}|\Phi_{\mathcal{R}}\rangle \qquad \widetilde{\mathcal{V}}_{B\mathcal{R}} = \frac{1}{\sqrt{Z_{\mathcal{R}}(W)}}\sum_{\mathcal{R}'} \mathcal{U}_{\mathcal{R}\mathcal{R}'}\mathcal{V}_{B\mathcal{R}'}$$

and for the T matrix

$$T_{MB,M'B'} = K_{MB,M'B'} + \mathrm{i}\sum_{M''K''} T_{MB,M''B''}K_{M''B'',M'B'}$$

Including the γN channel

$$T_{MB,\gamma N} = K_{MB,\gamma N} + \mathrm{i}\sum_{M'K'} T_{MB,M'B'}K_{M'B',\gamma N}$$

$$K_{M'B',\gamma N} = -\pi\sqrt{\frac{\omega_\gamma E_N}{k_\gamma W}}\langle \Psi^{M'B'}_{JI}||V_\gamma||\Psi_N\rangle$$

Separation of the electroproduction amplitudes

$$\mathcal{M}_{MB\,\gamma N} = \mathcal{M}^{(\mathrm{res})}_{MB\,\gamma N} + \mathcal{M}^{(\mathrm{bkg})}_{MB\,\gamma N}$$

The resonant part of the amplitude can then be written as:

$$\mathcal{M}^{(\mathrm{res})}_{MB\,\gamma N} = \sqrt{\frac{\omega_\gamma E_N^\gamma}{\omega_\pi E_N}}\,\frac{1}{\pi \mathcal{V}_{NN^*}}\,\underbrace{\langle \Psi^{(\mathrm{res})}_{N^*}(W)|\tilde{V}_\gamma|\Psi_N\rangle}_{A_{N^*}}\,T_{MB\,\pi N}$$

The helicity amplitude A_{N*} for electro-excitation:

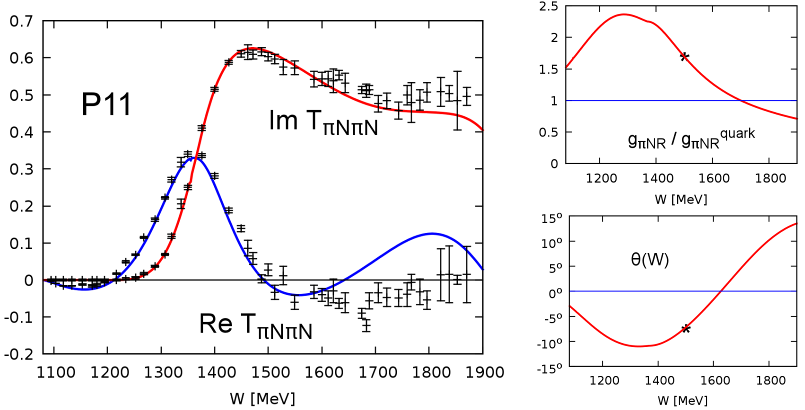
$$A_{N^*} \equiv \langle \Psi^{(\mathrm{res})}_{N^*}(W)|\tilde{V}_\gamma|\widetilde{\Psi}_N\rangle$$

The resonant state takes the form:

$$|\Psi^{(\mathrm{res})}_{N^*}(W)\rangle = \frac{1}{\sqrt{Z_{N^*}}}\left\{|\widetilde{\Phi}_{N^*}\rangle - \sum_{MB}\int \frac{\mathrm{d}k\quad \widetilde{\mathcal{V}}^{M}_{BN^*}(k)}{\omega_k + E_B - W}\,[a^\dagger(k)|\Psi_B\rangle]^{JI}\right\}$$

P11 scattering

Cloudy Bag Model, $R_{\text{bag}} = 0.83$ fm, $f_\pi = 76$ MeV

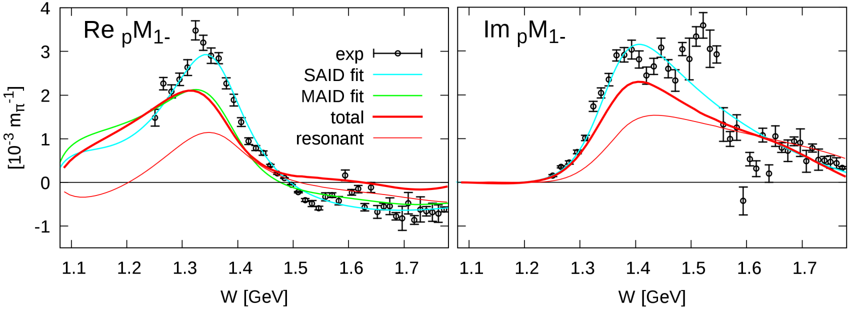


$$N(1440) = \cos\theta\ (1s)^3 + \sin\theta\ (1s)^2(2s)^1 = \cos\theta\ N^0(939) + \sin\theta\ N^0(1440)$$

Included $\pi\Delta$ and σN channels, $N(1710)$ resonance with $g_{\pi BN(1710)} \approx \frac{1}{5} g_{\pi BN^*}$

P11 photoproduction amplitudes on proton

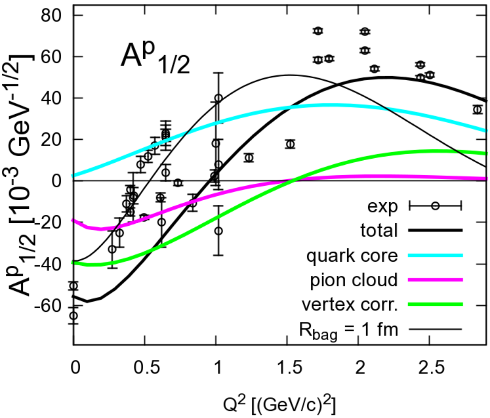
$$_pM_{1-}\,(\gamma + p \to \pi + N)$$



B. G. and S. Širca, Eur. Phys. J. A **38**, 271 (2008),
B. G., S. Širca, and M. Fiolhais, Eur. Phys. J. A **42**, 185 (2009)

P11 helicity amplitude on proton

$$A^{p}_{1/2}\left(\gamma + p \to N(1440)\right)$$

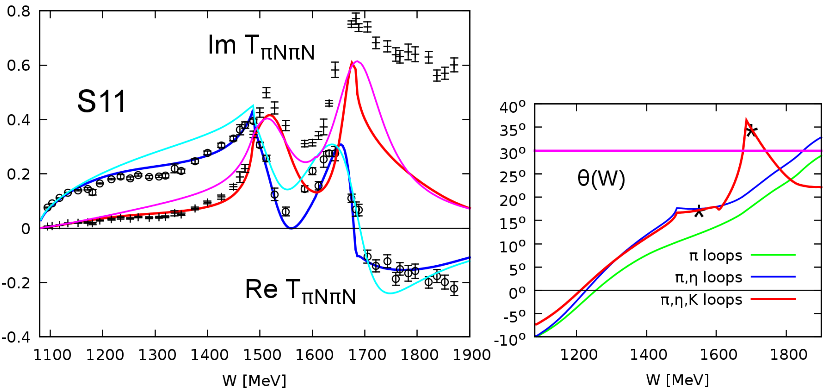


S11 scattering

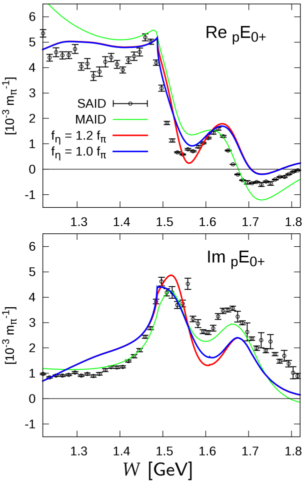
Single-quark excitations $1s \to 1p_{1/2}$ and $1s \to 1p_{3/2}$

$$N(1535) = \cos\theta|^2\mathbf{8}_{1/2}\rangle - \sin\theta|^4\mathbf{8}_{1/2}\rangle\,, \quad N(1650) = \sin\theta|^2\mathbf{8}_{1/2}\rangle + \cos\theta|^4\mathbf{8}_{1/2}\rangle$$

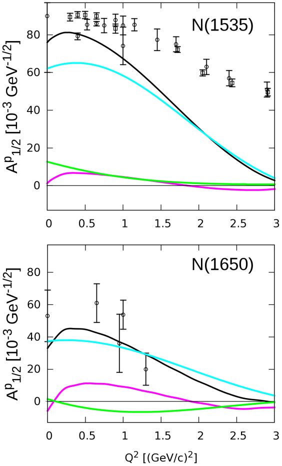
Included channels: πN, $\pi\Delta(1232)$, σN, ηN, $\pi N(1440)$, ρN, $K\Lambda$, $(K\Sigma)$.



S11 photoproduction and helicity amplitudes on proton



photoproduction



helicity amplitudes

B. G., S. Širca, Eur. Phys. J. A **47**, 61 (2011)

D-wave resonances in quark models

Single-quark excitations $1s \to 1p_{1/2}$ and $1s \to 1p_{3/2}$

D13 resonances: $$N(1520) = -\sin\theta|^4\mathbf{8}_{3/2}\rangle + \cos\theta|^2\mathbf{8}_{3/2}\rangle$$
$$N(1700) = \cos\theta|^4\mathbf{8}_{3/2}\rangle + \sin\theta|^2\mathbf{8}_{3/2}\rangle$$

$$^{2S+1}\mathbf{8}_{3/2} = c^S_{MS}|(1s)^2 1p_{3/2}\rangle_{MS} + c^S_{MA}|(1s)^2 1p_{3/2}\rangle_{MA} + c^S_{M}|(1s)^2 1p_{1/2}\rangle$$

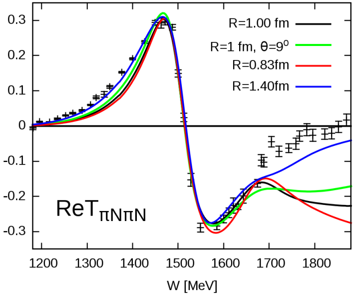
D33 resonance: $\Delta(1700) = |^2\mathbf{10}_{3/2}\rangle = \frac{\sqrt{5}}{3}|(1s)^2 1p_{3/2}\rangle - \frac{2}{3}|(1s)^2 1p_{1/2}\rangle\,,$

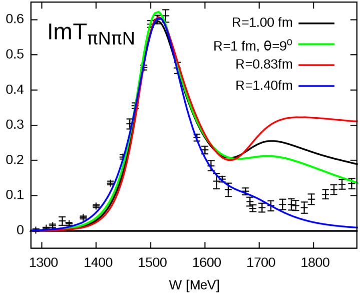
D15 resonance: $N(1675) = |^4\mathbf{8}_{5/2}\rangle = |(1s)^2 1p_{3/2}\rangle\,.$

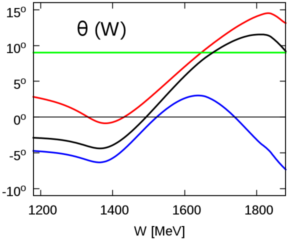
Considerable modification of quark-meson coupling constants with respect to the corresponding quark-model values:

B. G., S. Širca, Eur. Phys. J. A **49**, 111 (2013)

D13: elastic scattering amplitudes







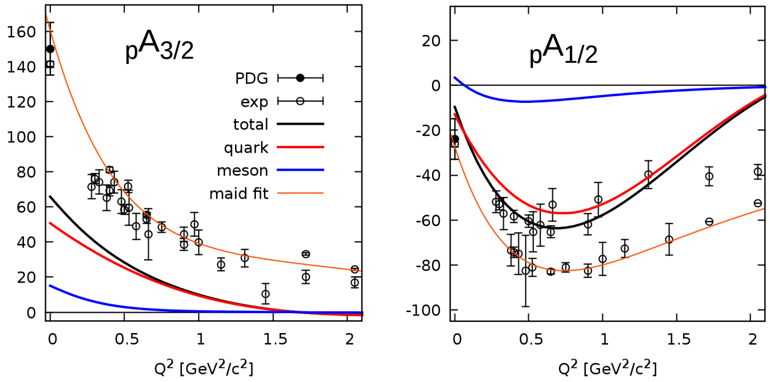
$R = 0.83$ fm: $g_{\pi NN^*} = 1.7\, g_{\pi NN^*}$ (QM)
$g^{s-\mathrm{wave}}_{\pi\Delta N^*} = 0.50\, g^{s-\mathrm{wave}}_{\pi\Delta N^*}$ (QM).

$R = 1.0$ fm: $g_{\pi NN^*} = 1.43\, g_{\pi NN^*}$ (QM),
$g^{s-\mathrm{wave}}_{\pi\Delta N^*} = 0.58\, g^{s-\mathrm{wave}}_{\pi\Delta N^*}$ (QM).

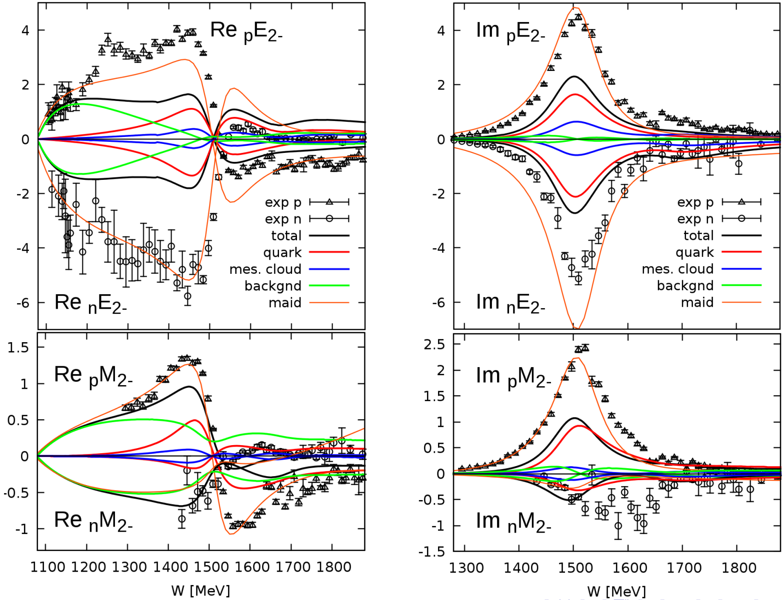
$R = 1.4$ fm: $g_{\pi NN^*} = 1.3\, g_{\pi NN^*}$ (QM),
$g^{s-\mathrm{wave}}_{\pi\Delta N^*} = 0.8\, g^{s-\mathrm{wave}}_{\pi\Delta N^*}$ (QM).

D13: helicity amplitudes ($R_{\rm bag} = 1.0$ fm)

$N(1520)\ 3/2^-$ resonance

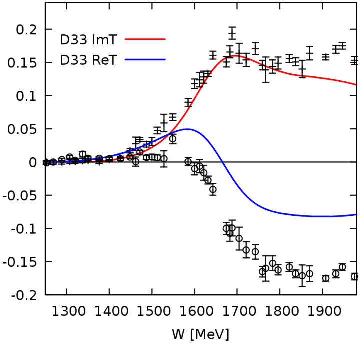


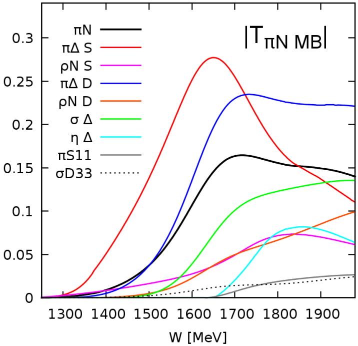
D13: photoproduction amplitudes



D33: scattering amplitudes

$$R_{\text{bag}} = 0.83 \text{ fm}, \quad g_{\pi NN^*} = 2.4\, g_{\pi NN^*} \text{ (QM)}, \; N^* \equiv \Delta(1700)\; 3/2^-$$

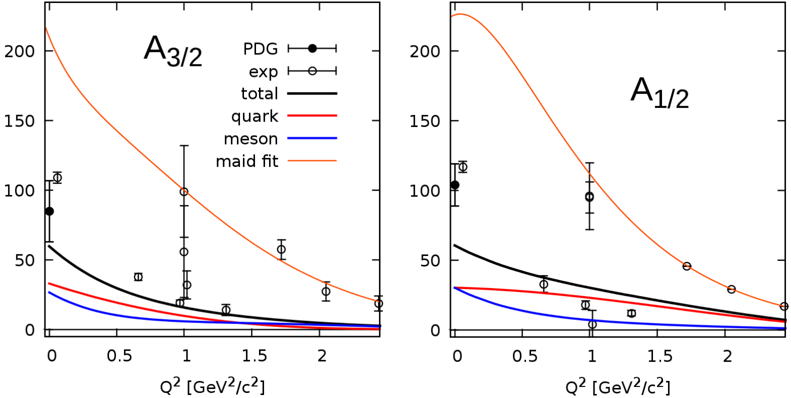




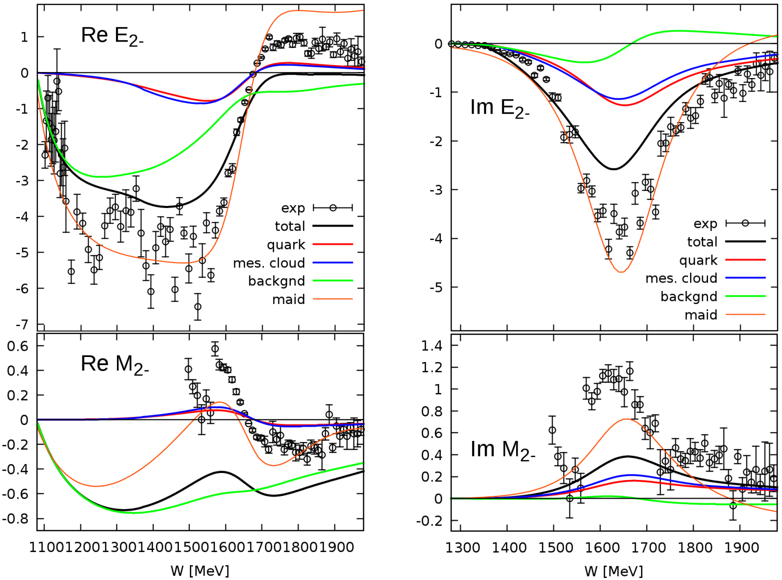
Res.	πN	$\pi\Delta\ (S)$	$\pi\Delta\ (D)$	ρN	σ
$\Delta(1700)$	15 %	50 %	29 %	4 % (S)	4 %
PDG	10–20 %	25–50 %	5–15 %	5–20 %	

D33: helicity amplitudes

$\Delta(1700)\ 3/2^-$ resonance



D33: photo-production amplitudes



Concluding remarks

- Pion cloud dominates (over other mesons)
  - p-wave pions: strong contribution for P11 and P33 resonances; similar results in different chiral quark models
  - s-wave and d-wave pions: comparable contribution for the S and D resonances; because of possible cancellations, the results may be model dependent
- η-meson and kaon clouds less important: sizable effects in the region of the upper S11 resonance.
- Bare masses: typically 200 MeV to 500 MeV higher than the "physical" masses; strange-meson loops contribute ~ 10 %

Where to look for the manifestation of the meson cloud?

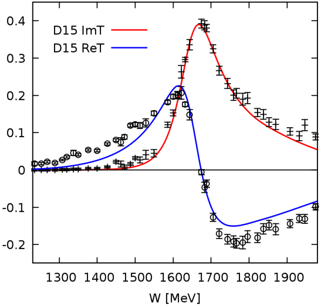
Manifestation of the meson cloud:

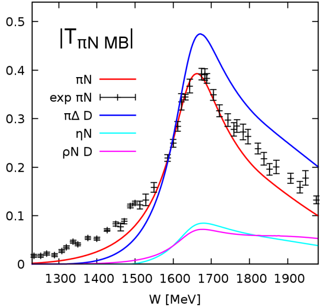
- Clear and "unmistakable" signal
  - E2/M1 and S2/M1 ratio of helicity amplitudes for $\Delta(1232)$
- Strong signal, commonly agreed upon
  - substantial enhancement of the $\pi N\Delta(1232)$ vertex
  - large enhancement of the $M1$ electro-production amplitude on the Δ
- Strong signal but still disputable (model dependent)
  - zero-crossing of the helicity amplitude for $N^*(1440)$
  - substantial enhancement of the $\pi NN^*(1440)$ vertex
- Moderate evidence for a sizable contribution
  - helicity amplitudes for D33

D15: scattering amplitudes

$R_{\text{bag}} = 1.0$ fm

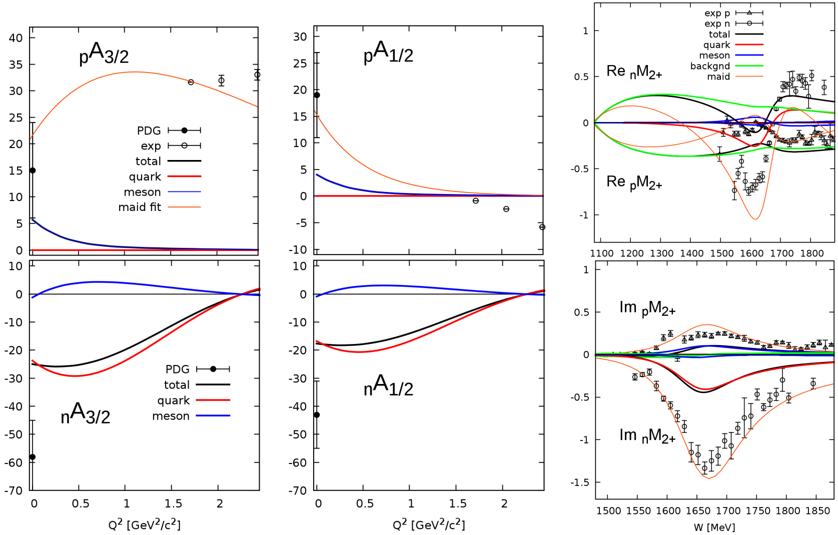
$g_{\pi NN^*} = 2.25\, g_{\pi NN^*}(\text{QM}), \quad g^{d-\text{wave}}_{MBN^*} = 1.45\, g^{d-\text{wave}}_{MBN^*}(\text{QM})\ MB \neq \pi N$





Res.	πN	$\pi\Delta\ (D)$	$\rho N\ (D)$	ηN(D)	$\sigma N\ (F)$
$N(1675)$	39 %	58 %	2 %	2 %	-
PDG	35–45 %	50 ± 15 %	1 ± 1 %	0 ± 1 %	7 ± 3 %

D15: helicity and photo-production amplitudes



Self-energy correction (in MeV)

Resonance	pion loops	$\eta\Lambda$ loops	$K\Lambda/K\Sigma$ loops	total
$\Delta(1232)P33$	201.5			201.5
$N(1440)P11$	66.8			66.8
$N(1535)S11$	60.9	24.5	25.2	110.6
$N(1650)S11$	133.0	-3.5	82.7	212.2
$N(1520)D13$	176.1	-	1.5	177.6
$N(1700)D13$	218.0	-	7.7	225.7
$\Delta(1700)D33$	484.3	-	9.3	493.6

Quark models with mesons

A natural environment: the model in which the mesons are linearly coupled to the quark core:

$$H = H_{\text{quark}} + \int \mathrm{d}k \sum_{lmt} \left\{ \omega_k\, a^\dagger_{lmt}(k) a_{lmt}(k) + [V_{lmt}(k) a_{lmt}(k) + \text{h. c.} \right.$$

$$T(k, k_0) = \pi \sqrt{\frac{\omega_k}{k}} \langle \Psi^-(k_0) || V(k) || \Phi_N \rangle$$

$$|\Psi(W)^\pm\rangle = a^\dagger(k_0)|\Phi_N\rangle + |\chi^\pm\rangle \,, \qquad |\chi^-\rangle = -\frac{1}{H - W \mp \mathrm{i}\epsilon} V(k)|\Phi_N\rangle$$

Generalized to

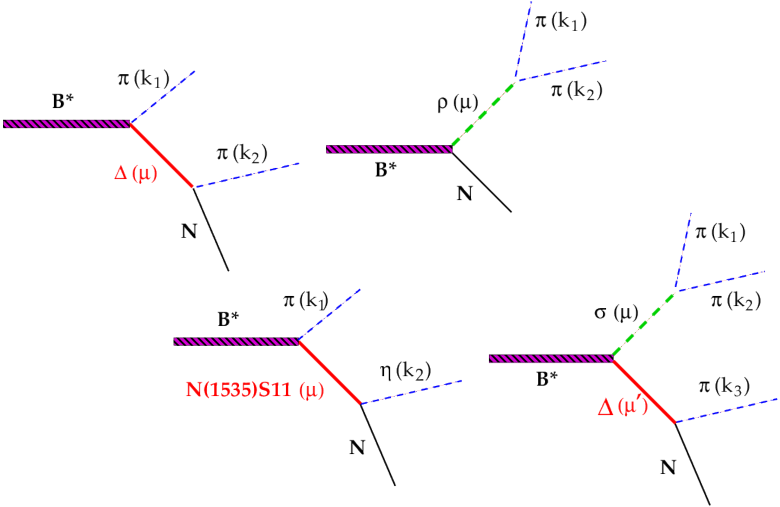
$$K^{JI}(k, k_0) = -\pi \sqrt{\frac{\omega_k E_N}{kW}} \langle \Psi^{\mathrm{P}}_{JI}(W) || V(k) || \Phi_N \rangle$$

The principle value (PV) states

$$|\Psi^{\mathrm{P}}_{JI}\rangle = \sqrt{\frac{\omega_0 E_N}{k_0 W}} \left\{ \left[a^\dagger(k_0)|\Phi_N\rangle \right]^{JI} - \frac{\mathcal{P}}{H - W} \left[V(k_0)|\Phi_N\rangle \right]^{JI} \right\}$$

Assumption about the two- and three-meson channels

2π, $\pi\eta$ and 3π decays through intermediate baryons ($\Delta(1232)$, $N(1535)$S11, ...) or mesons (σ, ρ, ...)



Intermediate baryon state

For $\mu \sim M_{\mathcal{R}}$, the intermediate baryon state can be written as

$$|\widetilde{\Psi}^{\alpha}(\mu)\rangle = \sum_{\beta} \left(\mathbf{1} + \mathbf{K}^2\right)^{-\frac{1}{2}}_{\beta\alpha} |\Psi^{\beta}(\mu)\rangle$$
$$\approx \frac{1}{\sqrt{2\pi}} \frac{\sqrt{\Gamma_{MB}(\mu)}}{\sqrt{(M_{\mathcal{R}} - \mu)^2 + \frac{1}{4}\Gamma^2(\mu)}} |\widehat{\Psi}_{\mathcal{R}}\rangle + \ldots ,$$

μ is the invariant mass of the baryon-meson system into which the intermediate baryon decays.

$\widehat{\Psi}_{\mathcal{R}}$ is the three-quark state surrounded by the meson cloud:

$$|\widehat{\Psi}_{\mathcal{R}}\rangle = Z_{\mathcal{R}}^{-\frac{1}{2}} \left[|\Phi_{\mathcal{R}}\rangle - \sum_{MB} \int \frac{dk \quad \mathcal{V}^{M}_{B\mathcal{R}}(k)}{\omega_k + E_B - W} [a^{\dagger}(k)|\widetilde{\Psi}_B\rangle]^{JI} \right]$$

Equations for meson amplitudes (Lippmann-Schwinger)

$$\chi^{M'B'\,MB}(k,k_M) = -\sum_{\mathcal{R}} c_{\mathcal{R}}^{MB}\, V_{B'\mathcal{R}}^{M'}(k) + \mathcal{K}^{M'B'\,MB}(k,k_M) + \sum_{M''B''}\int \mathrm{d}k'\, \frac{\mathcal{K}^{M'B'\,M''B''}(k,k')\chi^{M''B''\,MB}(k',k_M)}{\omega_k' + E_{B''}(k') - W}$$

with kernels

$$\mathcal{K}^{M'B'\,MB}(k,k') = \sum_{B''} f_{BB'}^{B''}\, \frac{\mathcal{V}_{B''B'}^{M'}(k')\, \mathcal{V}_{B''B}^{M}(k)}{\omega_k + \omega_k' + E_{B''}(\bar{k}) - W}$$

($f_{BB'}^{B''}$ are spin-isospin coefficients)

The solution assumes the form

$$\chi^{M'B'MB}(k,k_M) = -\sum_{\mathcal{R}} c_{\mathcal{R}}^{MB}\, \mathcal{V}_{B'\mathcal{R}}^{M'}(k) + \mathcal{D}^{M'B'MB}(k,k_M)$$

Solving the coupled equations

Dressed vertices then satisfy:

$$\mathcal{V}^{M}_{B\mathcal{R}}(k) = V^{M}_{B\mathcal{R}}(k) + \sum_{M'B'} \int \mathrm{d}k' \, \frac{\mathcal{K}^{MB\,M'B'}(k,k')\, \mathcal{V}^{M'}_{B'\mathcal{R}}(k')}{\omega'_k + E_{B'}(k') - W}$$

and similarly the background part of the amplitude:

$$\mathcal{D}^{M'B'\,MB}(k,k_M) = \mathcal{K}^{M'B'\,MB}(k,k_M) + \sum_{M''B''} \int \mathrm{d}k' \, \frac{\mathcal{K}^{M'B'\,M''B''}(k,k')\mathcal{D}^{M''B''\,MB}(k',k_M)}{\omega'_k + E_{B''}(k') - W}$$

The coefficients $c^{MB}_{\mathcal{R}'}$ in front of the quasi-bound states satisfy a set of equations:

$$\sum_{\mathcal{R}'} A_{\mathcal{R}\mathcal{R}'}(W)\, c^{MB}_{\mathcal{R}'}(W) = \mathcal{V}^{M}_{B\mathcal{R}}(k_M)$$

$$A_{\mathcal{R}\mathcal{R}'} = (W - M^0_{\mathcal{R}})\delta_{\mathcal{R}\mathcal{R}'} + \sum_{\mathcal{B}'} \int \mathrm{d}k \, \frac{\mathcal{V}^{M'}_{B'\mathcal{R}}(k)\, V^{M'}_{B'\mathcal{R}'}(k)}{\omega_k + E_{B'}(k) - W}$$

Approximation: separable kernels

$$\frac{1}{\omega_k + \omega'_k + E_{B''} - W} \approx \frac{(\omega_M + \omega_{M'} + E_{B''} - W)}{(\omega_k + E_{B''} - E_{B'})(\omega'_k + E_{B''} - E_B)}$$

$$W = E_B + \omega_M = E_{B'} + \omega_{M'}$$

The approximation has the property:

$$\mathcal{K}^{M'B'\,MB}(k, k_{M'})^{\text{approx}} = \mathcal{K}^{M'B'\,MB}(k, k_{M'})^{\text{exact}}$$

and preserves the symmetry of the K-matrix: $K_{MB,M'B'} = K_{M'B',MB}$ and, as a consequence, the unitarity of the S-matrix.

Further simplification; for $\mathcal{V}_{BB'}$ entering the kernel $\mathcal{K}^{M'B'\,MB}(k, k_M)$:

$$\mathcal{V}^M_{BB'}(\text{dressed}) = r^M_{BB'}\, V^M_{BB'}(\text{bare}) \qquad r^M_{BB'} = \text{const}$$

e.g. $r^{l=1\,\text{pions}}_{N\Delta} = 1.3$, as determined by solving the coupled equations in the $P33$ partial wave. Most $r^M_{BB'} = 1$.

Mixing of bare resonances

To solve the set of equations, diagonalize A to obtain U, along with the poles of the K matrix, and wave-function normalization Z:

$$UAU^T = \begin{bmatrix} Z_{\mathcal{R}}(W)(W - M_{\mathcal{R}}) & 0 & 0 \\ 0 & Z_{\mathcal{R}'}(W)(W - M_{\mathcal{R}'}) & 0 \\ 0 & 0 & Z_{\mathcal{R}''}(W)(W - M_{\mathcal{R}''}) \end{bmatrix}$$

As a consequence, $\Phi_{\mathcal{R}}$ mix:

$$|\widetilde{\Phi}_{\mathcal{R}}\rangle = \sum_{\mathcal{R}'} U_{\mathcal{R}\mathcal{R}'} |\Phi_{\mathcal{R}}\rangle \qquad \widetilde{\mathcal{V}}_{B\mathcal{R}} = \frac{1}{\sqrt{Z_{\mathcal{R}}(W)}} \sum_{\mathcal{R}'} U_{\mathcal{R}\mathcal{R}'} \mathcal{V}_{B\mathcal{R}'}$$

Solution for the K matrix

$$K_{MB,M'B'} = \pi \sqrt{\frac{\omega_M E_B}{k_M W}} \sqrt{\frac{\omega_{M'} E_{B'}}{k_{M'} W}} \left[\sum_{\mathcal{R}} \frac{\widetilde{\mathcal{V}}^{M}_{B\mathcal{R}} \widetilde{\mathcal{V}}^{M'}_{B'\mathcal{R}}}{(M_{\mathcal{R}} - W)} + \mathcal{D}_{MB,M'B'} \right]$$

and for the T matrix

$$T_{MB,M'B'} = K_{MB,M'B'} + \mathrm{i} \sum_{M''K''} T_{MB,M''B''} K_{M''B'',M'B'}$$

Meson-quark vertices

quark pion

$$V^{\pi}_{l=0,t}(k) = \frac{1}{2f_\pi}\sqrt{\frac{\omega_{p_{1/2}}\omega_s}{(\omega_{p_{1/2}}+1)(\omega_s-1)}}\,\frac{1}{2\pi}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_0(kR)}{kR}\sum_{i=1}^{3}\tau_t(i)\,\mathcal{P}_{sp}(i)\,,$$

$$V^{\pi}_{1mt}(k) = \frac{1}{2f_\pi}\,\frac{\omega_s}{(\omega_s-1)}\,\frac{1}{2\pi}\,\frac{1}{\sqrt{3}}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_1(kR)}{kR}\sum_{i=1}^{3}\tau_t(i)\left(\sigma_m(i) + r_{p_{1/2}}S^{[\frac{1}{2}]}_{1m}(i) + r_{p_{3/2}}S^{[\frac{3}{2}]}_{1m}(\right.$$

$$V^{\pi}_{2mt}(k) = \frac{1}{2f_\pi}\sqrt{\frac{\omega_{p_{3/2}}\omega_s}{(\omega_{p_{3/2}}-2)(\omega_s-1)}}\,\frac{\sqrt{2}}{2\pi}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_2(kR)}{kR}\sum_{i=1}^{3}\tau_t(i)\,\Sigma^{[\frac{1}{2}\,\frac{3}{2}]}_{2m}(i)\,.$$

$$\mathcal{P}_{sp} = \sum_{m_j}|sm_j\rangle\langle p_{1/2}m_j|\,,$$

$$S^{[\frac{1}{2}]}_{1m} = \sqrt{3}\sum_{m_j m_j'} C^{\frac{1}{2}m_j}_{\frac{1}{2}m_j'1m}|p_{1/2}m_j\rangle\langle p_{1/2}m_j'|\,,$$

$$\Sigma^{[\frac{1}{2}\,\frac{3}{2}]}_{2m} = \sum_{m_s m_j} C^{\frac{1}{2}m_s}_{\frac{3}{2}m_j2m}|sm_s\rangle\langle p_{3/2}m_j|\,,$$

$$S^{[\frac{3}{2}]}_{1m} = \frac{\sqrt{15}}{2}\sum_{m_j m_j'} C^{\frac{3}{2}m_j}_{\frac{3}{2}m_j'1m}|p_{3/2}m_j\rangle\langle p_{3/2}m_j'|\,,$$

$$r_{p_{1/2}} = \frac{\omega_{p_{1/2}}(\omega_s - 1)}{\omega_s(\omega_{p_{1/2}} + 1)}, \qquad r_{p_{3/2}} = \frac{2\omega_{p_{3/2}}(\omega_s - 1)}{5\omega_s(\omega_{p_{3/2}} - 2)}.$$

s-wave η and K mesons

$$V^\eta(k) = \frac{1}{2f_\pi}\sqrt{\frac{\omega_{p_{1/2}}\omega_s}{(\omega_{p_{1/2}}+1)(\omega_s-1)}}\,\frac{1}{2\pi}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_0(kR)}{kR}\sum_{i=1}^{3}\lambda_8(i)\,\mathcal{P}_{sp}(i),$$

$$V_t^K(k) = \frac{1}{2f_K}\sqrt{\frac{\omega_{p_{1/2}}\omega_s}{(\omega_{p_{1/2}}+1)(\omega_s-1)}}\,\frac{1}{2\pi}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_0(kR)}{kR}\sum_{i=1}^{3}(V_t(i)+U_t(i))\,\mathcal{P}_{sp}(i),$$

ρqq interaction involving the s-state and the $p_{j=1/2}$-state quarks

$$H^{SP}_{\rho\,Jl} = \int \mathrm{d}k\, V^{\rho SP}_{Jl}(k)\sum_{i=1}^{3}\sum_{Mt}\sigma_M(i)\tau_t(i)a_{1lMt}(k) + \mathrm{h.c},$$

with possible values $J = 1$ and $l = 0, 2$:

$$V^{\rho SP}_{10}(k) = \frac{1}{4\pi\tilde{f}_\rho}\sqrt{\frac{\omega_{p_{1/2}}\omega_s}{(\omega_{p_{1/2}}+1)(\omega_s-1)}}\,\frac{2}{3}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_0(kR)}{kR},$$

$$V^{\rho SP}_{12}(k) = \frac{1}{4\pi\tilde{f}_\rho}\sqrt{\frac{\omega_{p_{1/2}}\omega_s}{(\omega_{p_{1/2}}+1)(\omega_s-1)}}\,\frac{\sqrt{2}}{3}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_2(kR)}{kR}.$$

ρqq interaction between the s-state and the $p_{j=3/2}$-state quarks (SA):

$$H^{SA}_{\rho\, Jl} = V^{\rho SA}_{Jl}(k)\sum_{i=1}^{3}\sum_{Mt}\Sigma^{[\frac{3}{2}\,\frac{1}{2}]}_{JM}(i)\tau_t(i)a_{JMt}(k) + \text{h.c.}$$

with $J=1$, $l=0,2$ and $J=2$, $l=2$:

$$V^{\rho SA}_{10}(k) = \frac{1}{4\pi\tilde{f}_\rho}\sqrt{\frac{\omega_{p_{3/2}}\omega_s}{(\omega_{p_{3/2}}-2)(\omega_s-1)}}\,\frac{1}{\sqrt{3}}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_0(kR)}{kR}\,,$$

$$V^{\rho SA}_{12}(k) = -\frac{1}{4\pi\tilde{f}_\rho}\sqrt{\frac{\omega_{p_{3/2}}\omega_s}{(\omega_{p_{3/2}}-2)(\omega_s-1)}}\,\frac{1}{\sqrt{6}}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_2(kR)}{kR}\,,$$

$$V^{\rho SA}_{22}(k) = -\frac{1}{4\pi\tilde{f}_\rho}\sqrt{\frac{\omega_{p_{3/2}}\omega_s}{(\omega_{p_{3/2}}-2)(\omega_s-1)}}\,\sqrt{\frac{3}{2}}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_2(kR)}{kR}\,,$$

$$\langle \tfrac{3}{2}m_j|\Sigma^{[\frac{3}{2}\,\frac{1}{2}]}_{JM}|\tfrac{1}{2}m_s\rangle = C^{\frac{3}{2}m_j}_{\frac{1}{2}m_s JM}$$