Structure and transitions of nucleon excitations from lattice QCD

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Baryon structure via variational analysis

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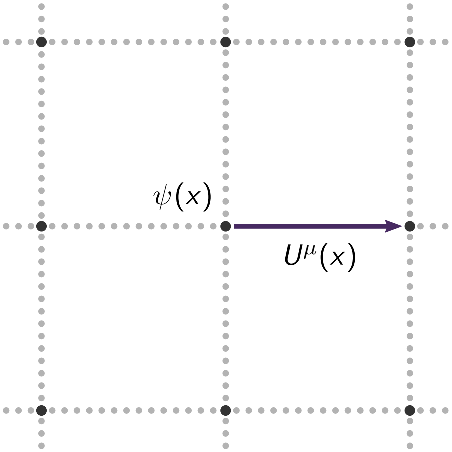
- Discretise quantum chromodynamics on finite 4D lattice



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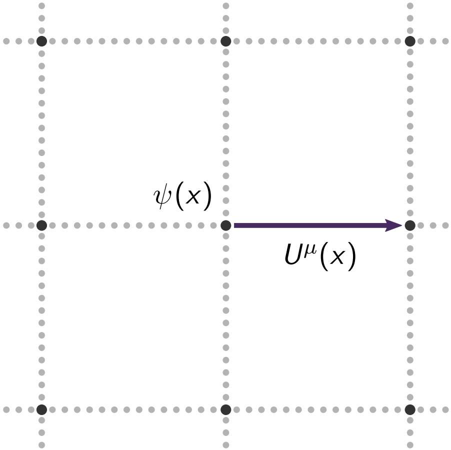

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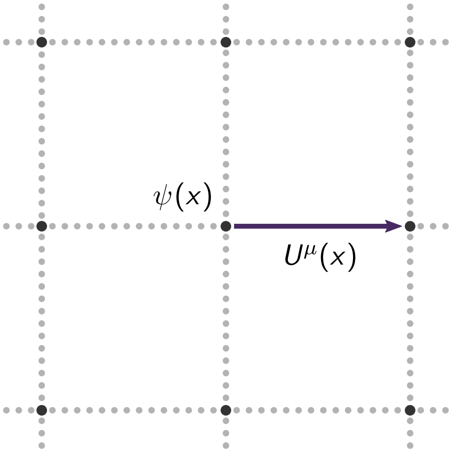

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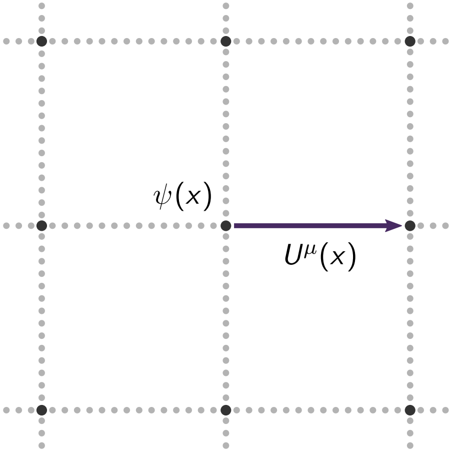

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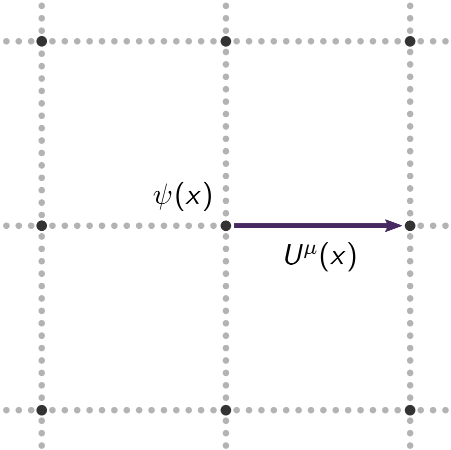

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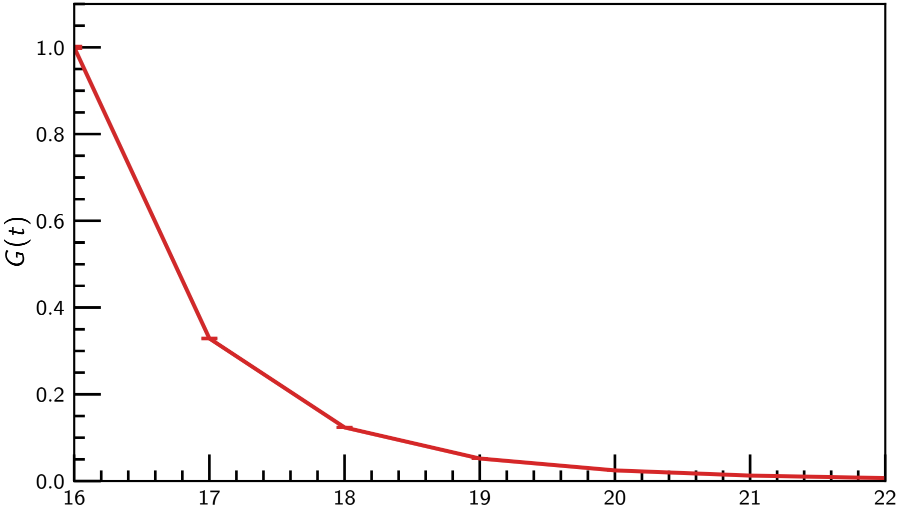
Correlation function

$G_{11}(\Gamma_+; \mathbf{0};\ t)$



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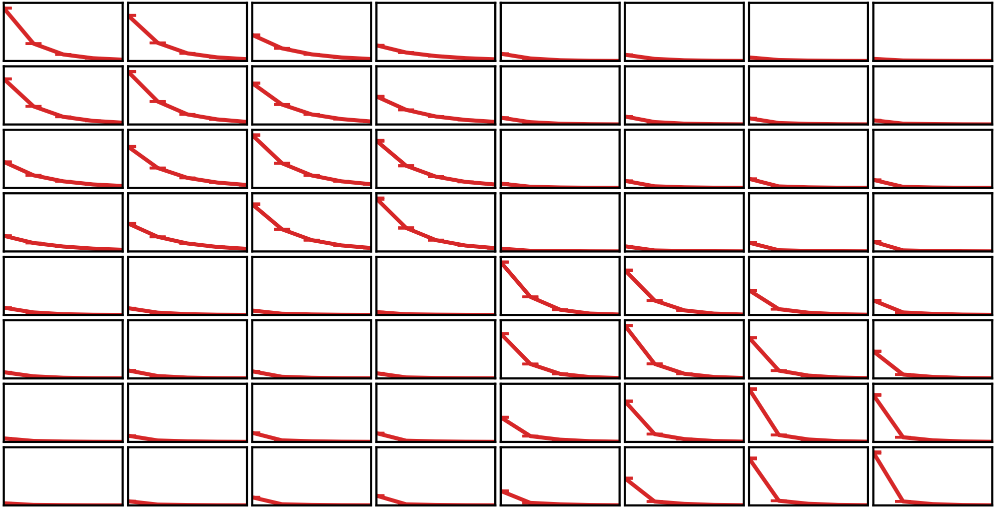

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- At non-zero momentum, opposite-parity contaminations come in at $O(|\boldsymbol{p}|)$

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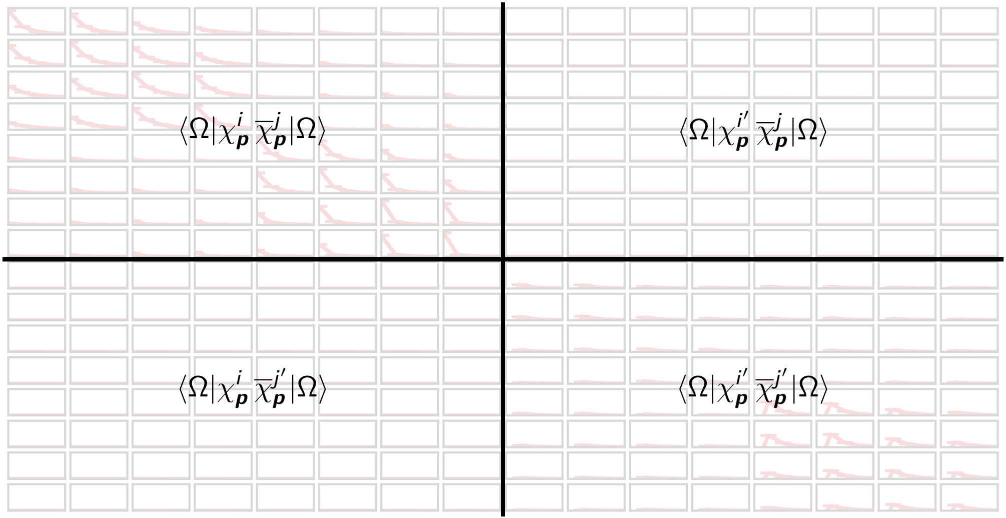
PEVA correlation matrix

$\boldsymbol{p}^2 = 0\,\mathrm{GeV}^2$



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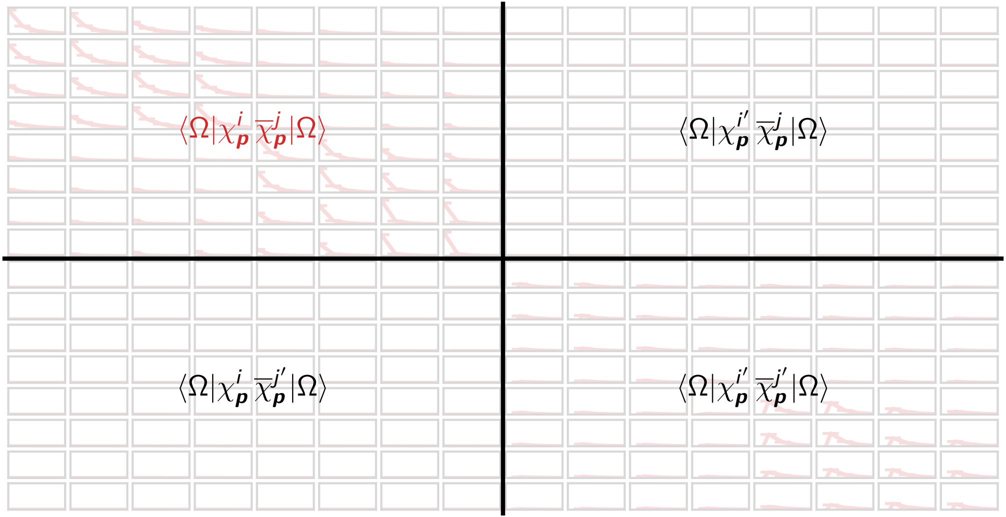

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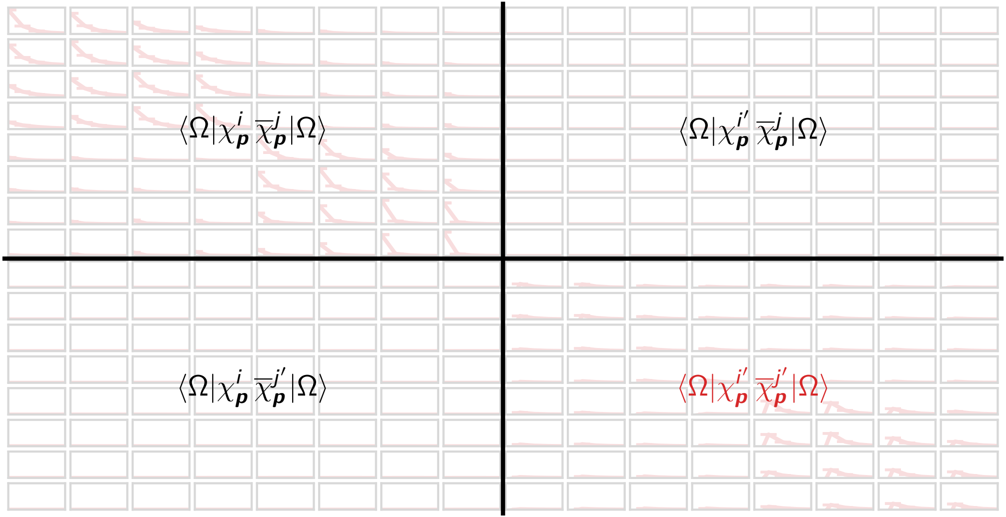

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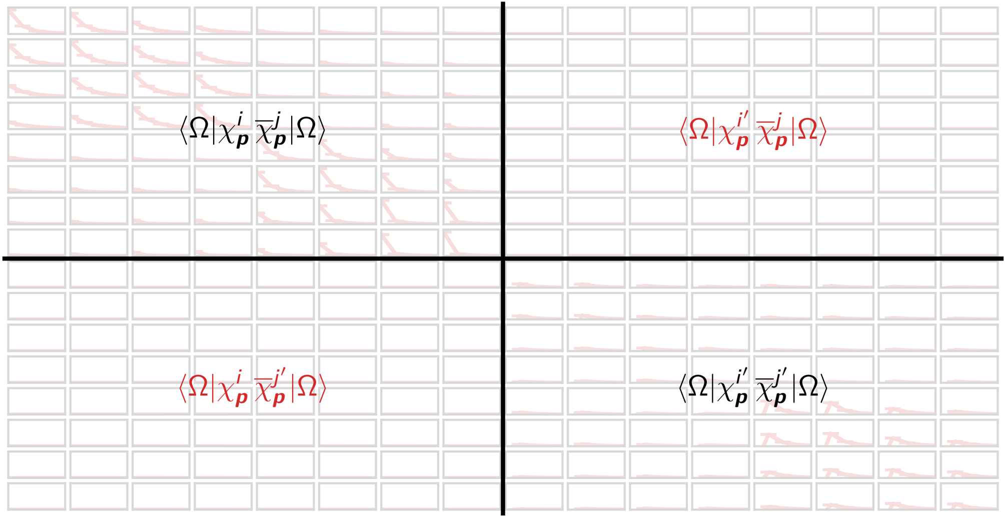

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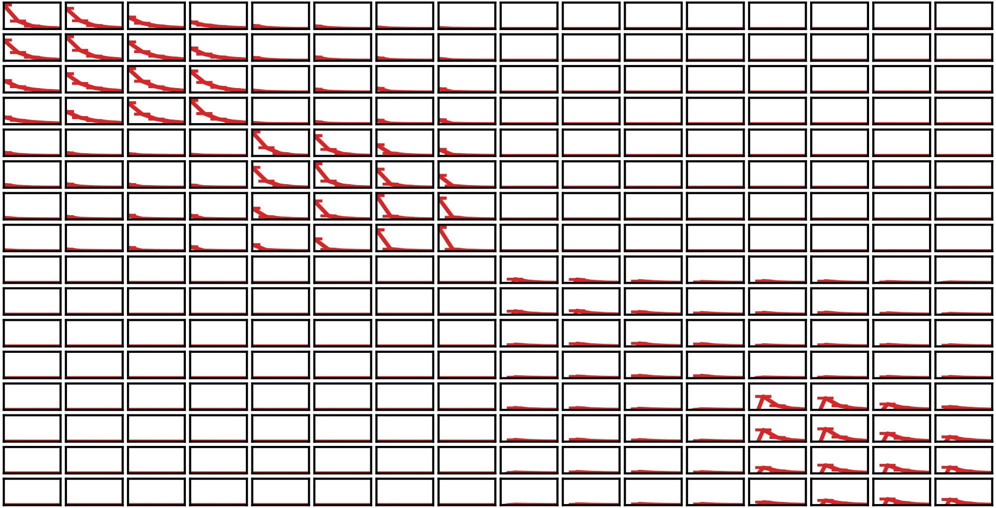

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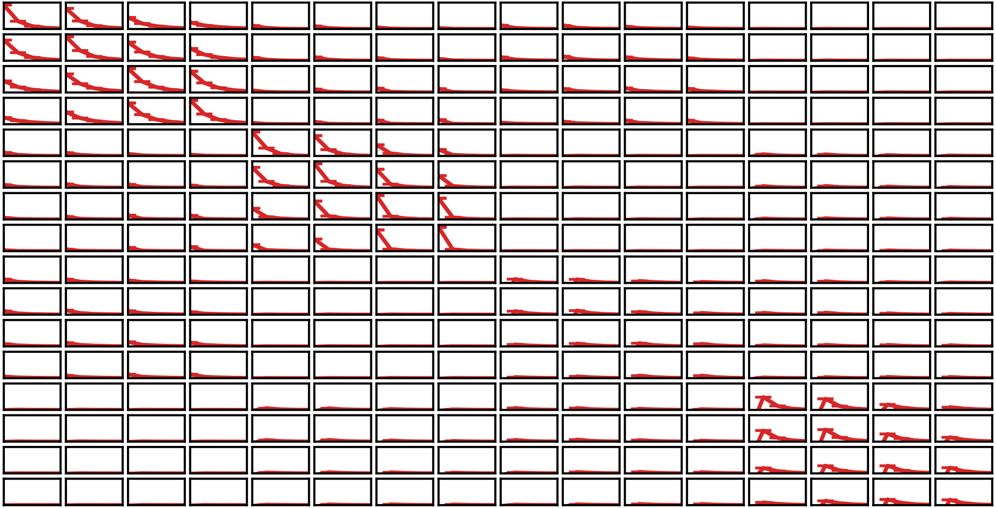
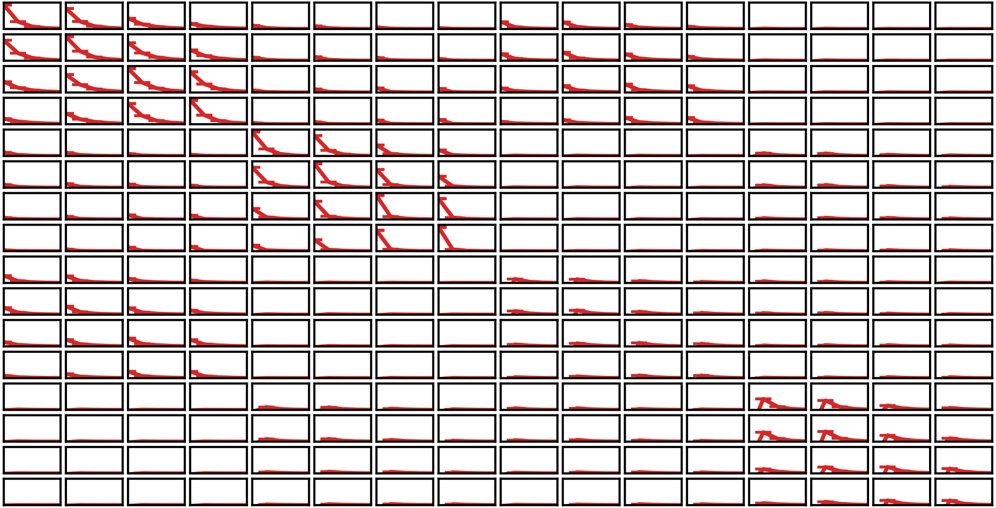


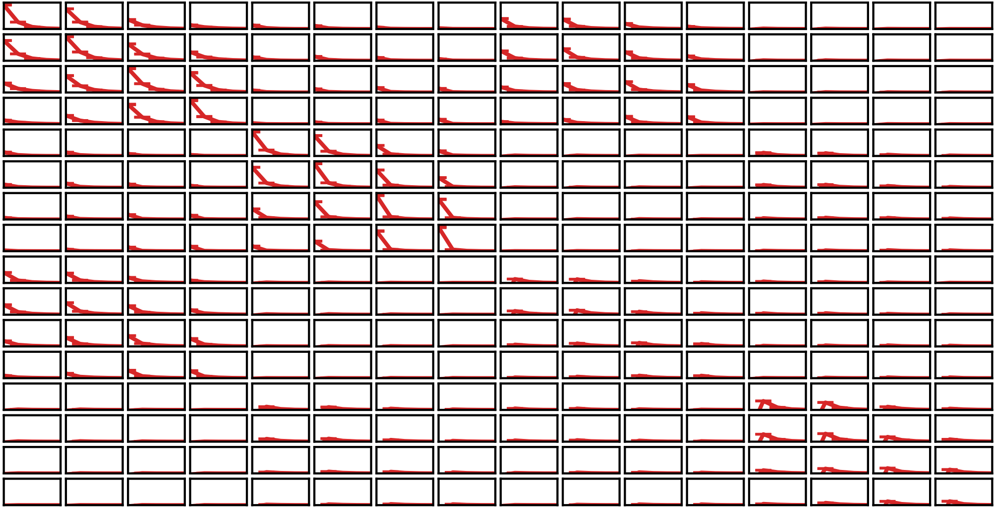
PEVA correlation matrix

$p^2 = 0.332\,\mathrm{GeV}^2$



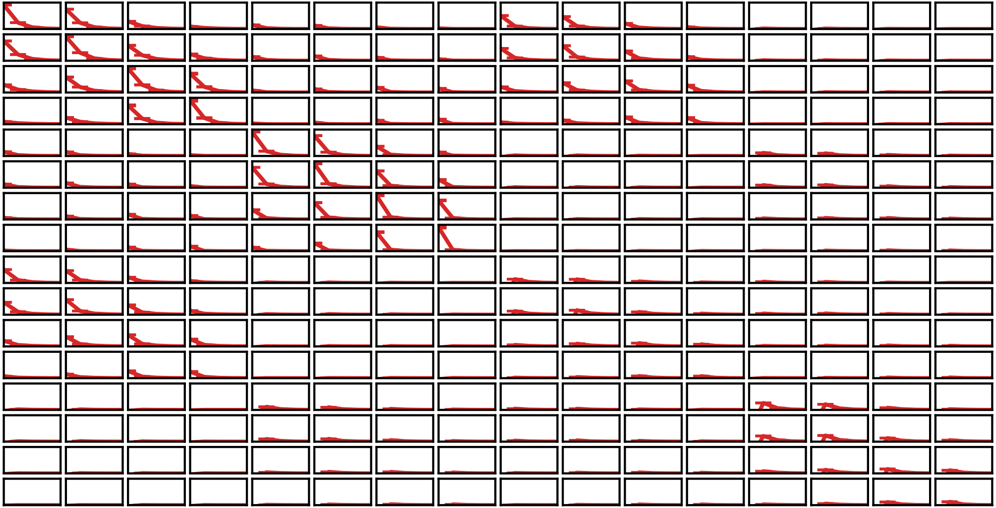
PEVA correlation matrix

$\boldsymbol{p}^2 = 0.498\,\mathrm{GeV}^2$



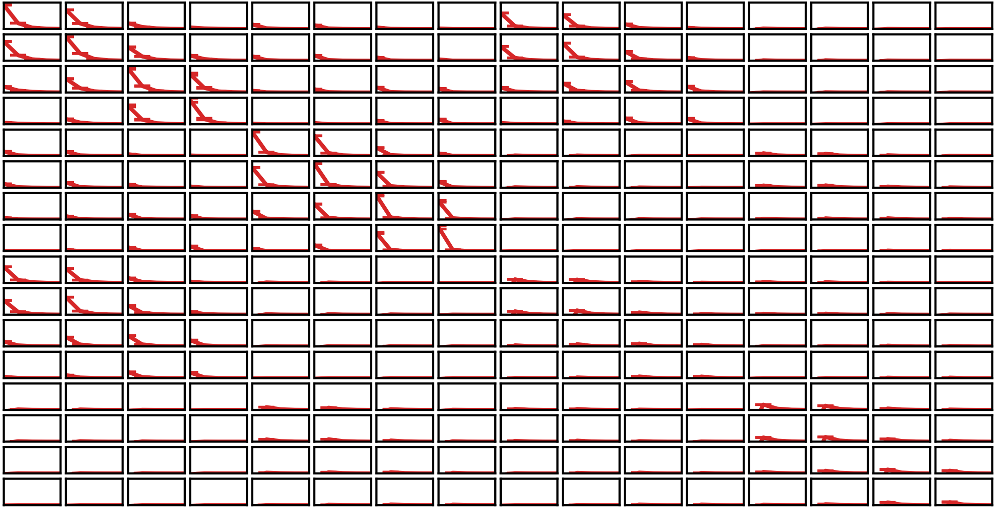
PEVA correlation matrix

$\boldsymbol{p}^2 = 0.664\,\mathrm{GeV}^2$



PEVA correlation matrix

$\boldsymbol{p}^2 = 0.830\,\mathrm{GeV}^2$



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- Seek operators $\{\phi^\alpha_{\boldsymbol{p}}\}$ that couple strongly to a single energy eigenstate

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- Coefficients can be found by solving generalised eigenvalue problem

$$\boldsymbol{v}^{\alpha}(\boldsymbol{p})\ G(\boldsymbol{p};\ t_0) = \exp(-E^{\alpha}(\boldsymbol{p})\,\Delta t)\ \boldsymbol{v}^{\alpha}(\boldsymbol{p})\ G(\boldsymbol{p};\ t_0 + \Delta t)$$
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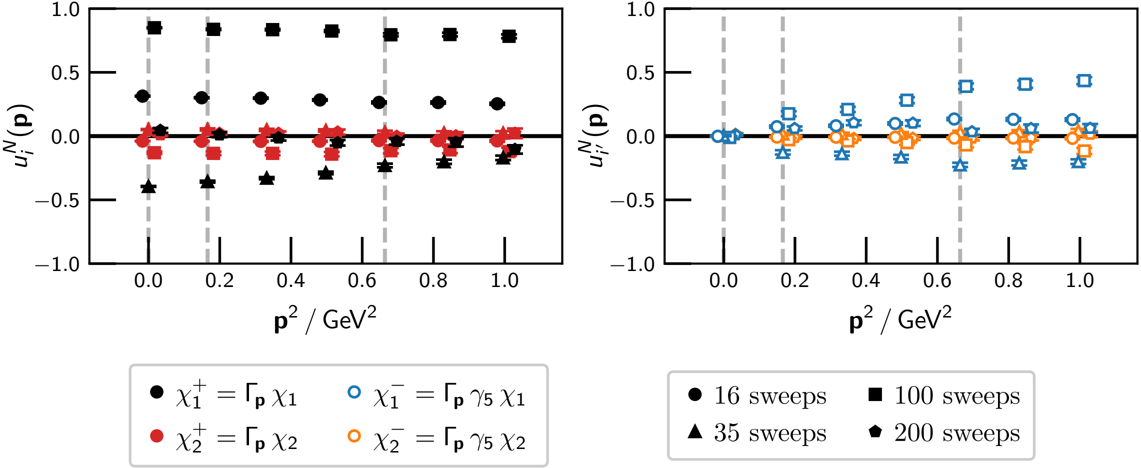
Expect E_α^{eff} to approximately obey dispersion relation

$$E_\alpha^{\mathrm{eff}}(\boldsymbol{p}, t) \approx \sqrt{{m_\alpha}^2 + \boldsymbol{p}^2}$$

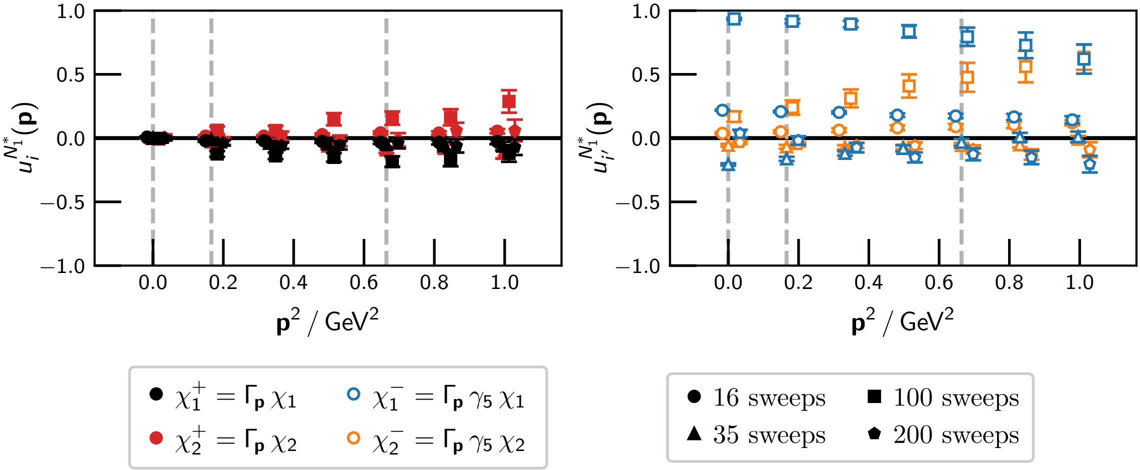
Eigenvector components

Ground state



Eigenvector components

First negative parity excitation



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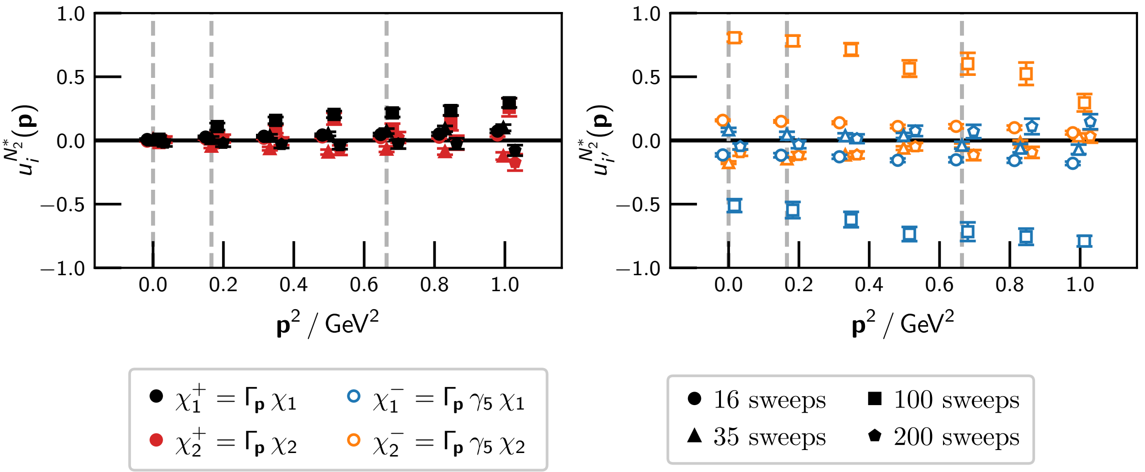

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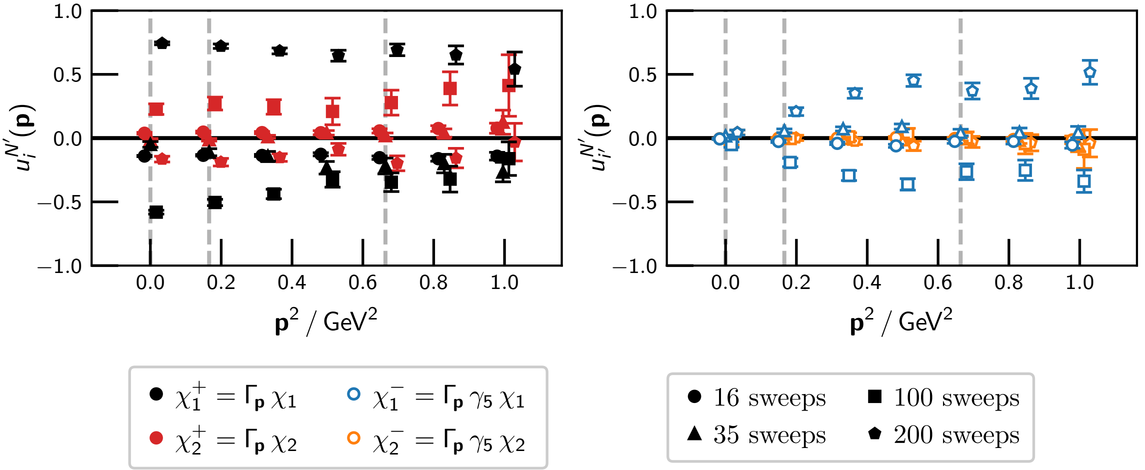
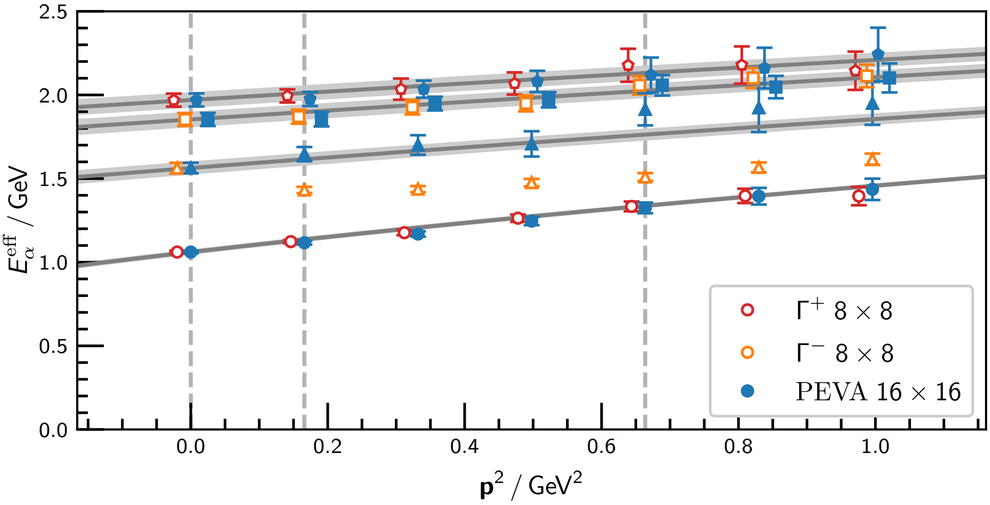


Effective energy

Nucleon spectrum



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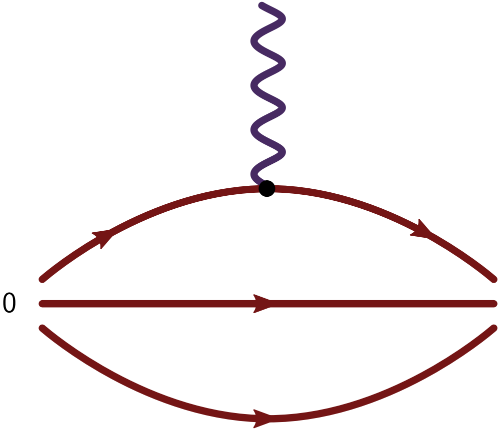

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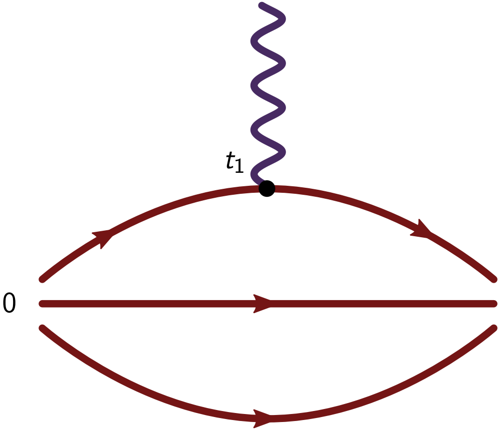

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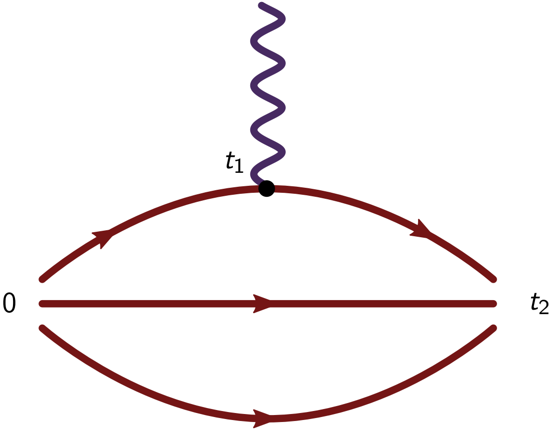

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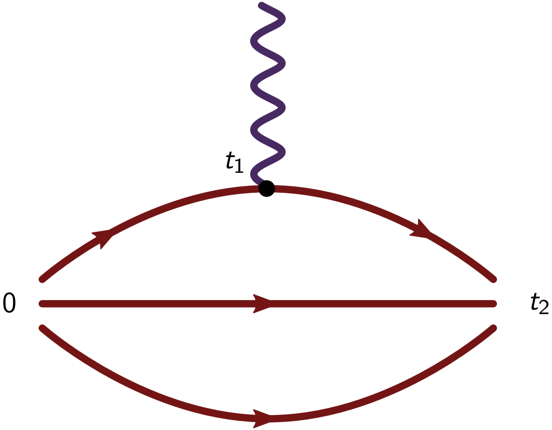
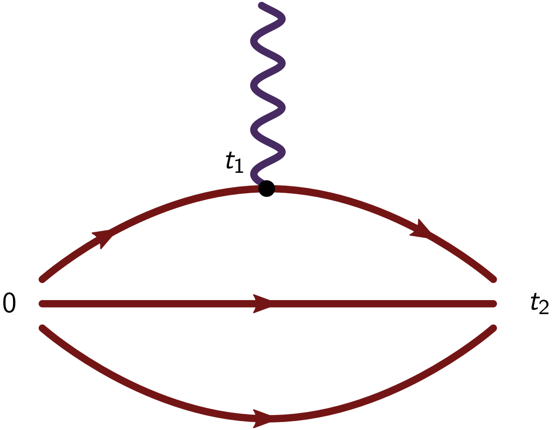


Matrix element

$$\langle B\,;p'\,;s'|\,j^\mu|\,B\,;p\,;s\rangle \propto$$
$$\overline{u}_B\left(\gamma^\mu F_1(Q^2) - \frac{\sigma^{\mu\nu}q_\nu}{2m_B}F_2(Q^2)\right)u_B$$

Step 3: Compute three point correlation function



Matrix element

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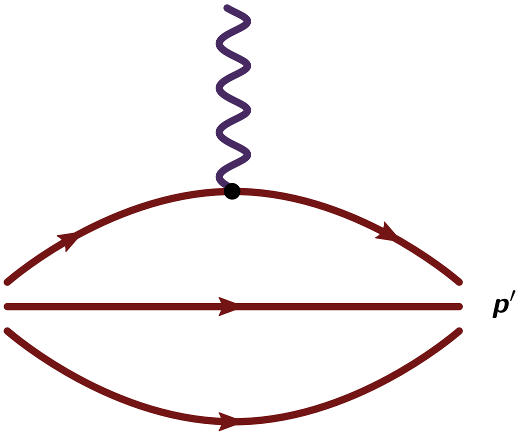

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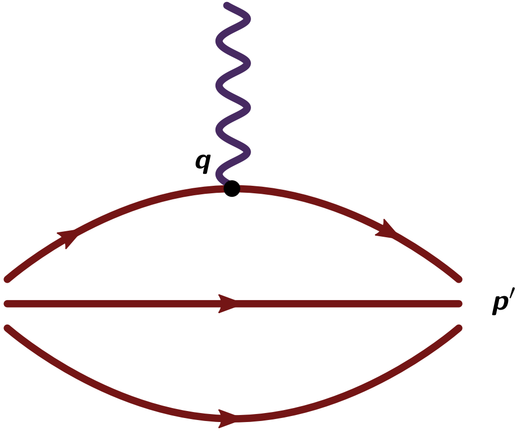

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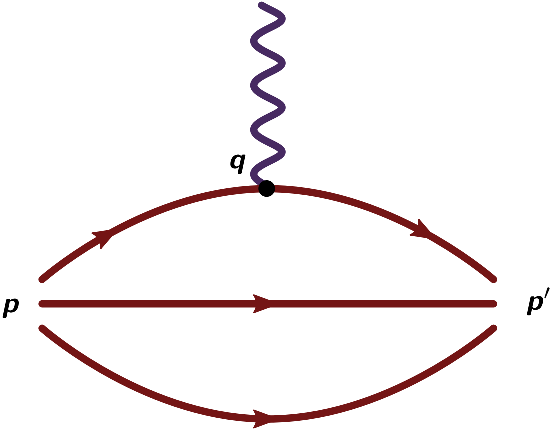

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Baryon structure via variational analysis

Step 1: Construct correlation matrix

Start with basis of operators and construct a matrix of correlation functions $G_{ij}(\boldsymbol{p}; \Gamma; t)$

Step 2: Perform variational analysis

Perform a variational analysis to construct optimised operators

Step 3: Compute three point correlation function

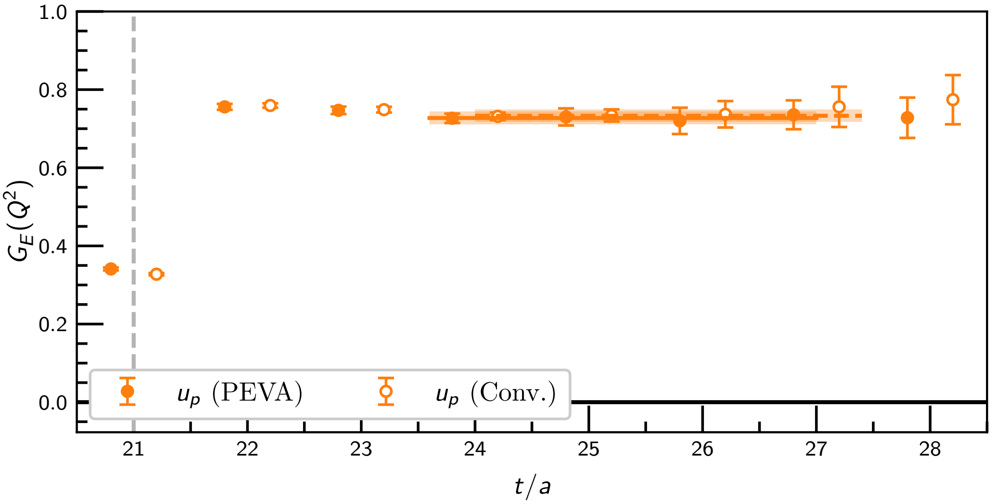
Use these optimised operators to construct relevant three point correlation functions

Step 4: Extract form factors

Take ratios of three point to two point functions and extract e.g. $G_E(Q^2)$ and $G_M(Q^2)$

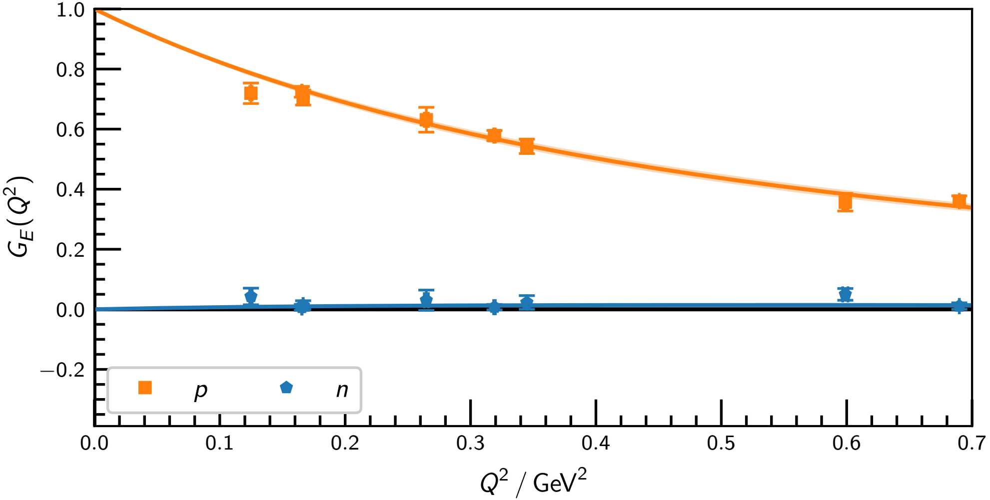
Ground state

Fits to $G_E(Q^2 = 0.166(4))$ ($m_\pi = 156\,\mathrm{MeV}$)



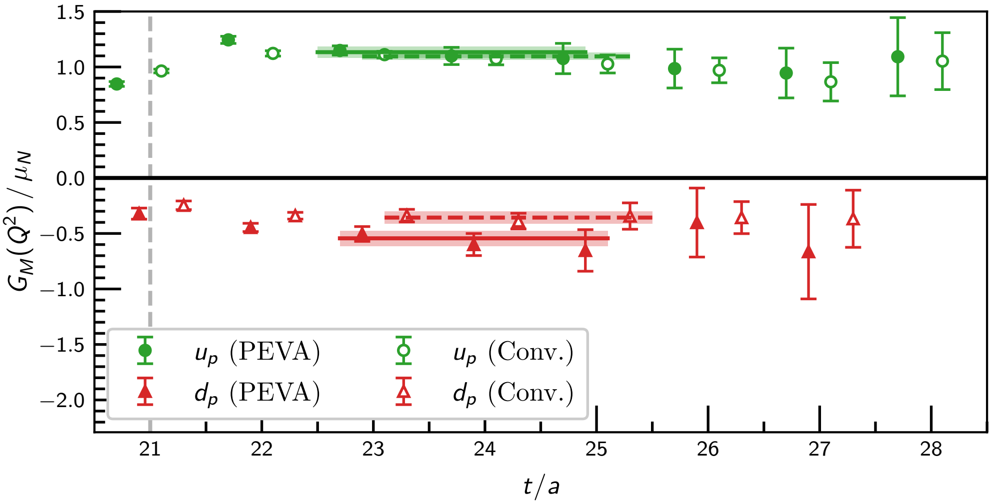
Ground state

Momentum-dependence of $G_E(Q^2)$ ($m_\pi = 156\,\mathrm{MeV}$)



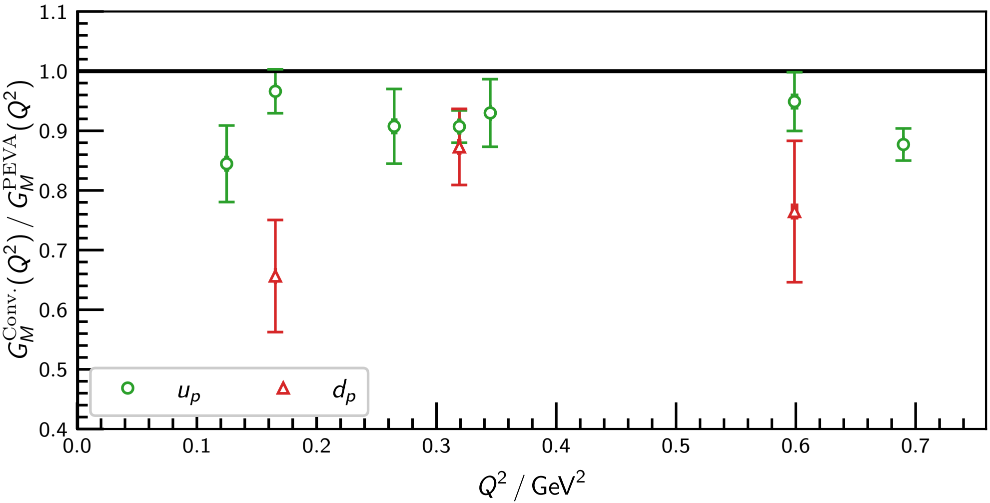
Ground state

Fits to $G_M(Q^2 = 0.166(4))$ ($m_\pi = 156\,\mathrm{MeV}$)



Ground state

Ratios of conventional $G_M(Q^2)$ plateaus to PEVA ($m_\pi = 156\,\text{MeV}$)



Ground state

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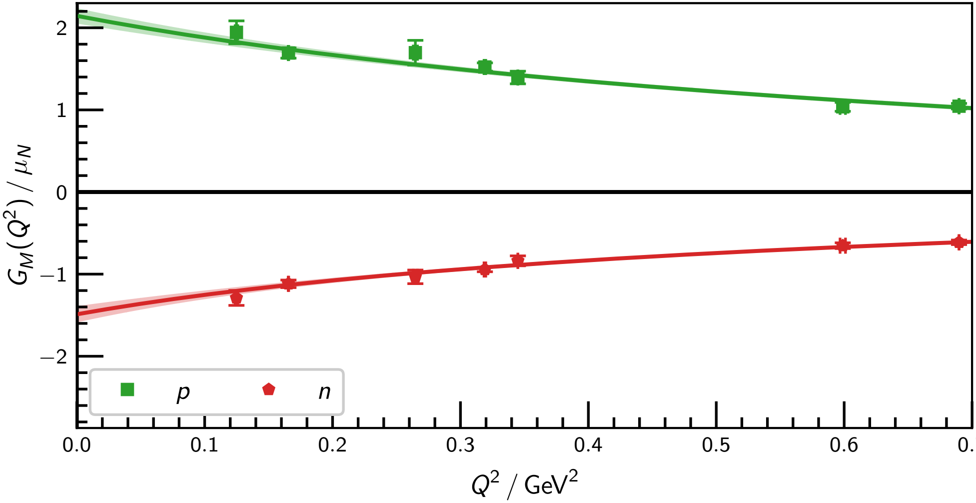

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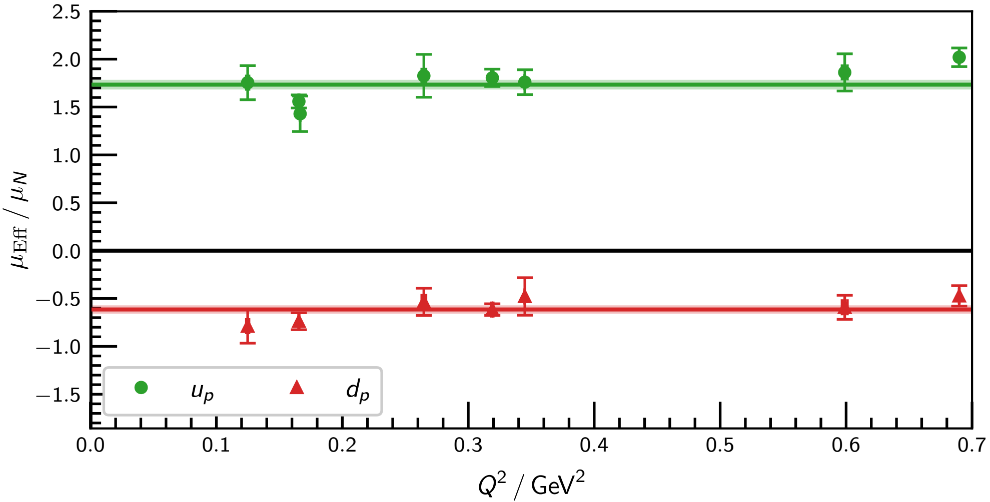
Effective magnetic moment

$$\mu_{\mathrm{Eff}}(Q^2) = \frac{G_M(Q^2)}{G_E(Q^2)}$$

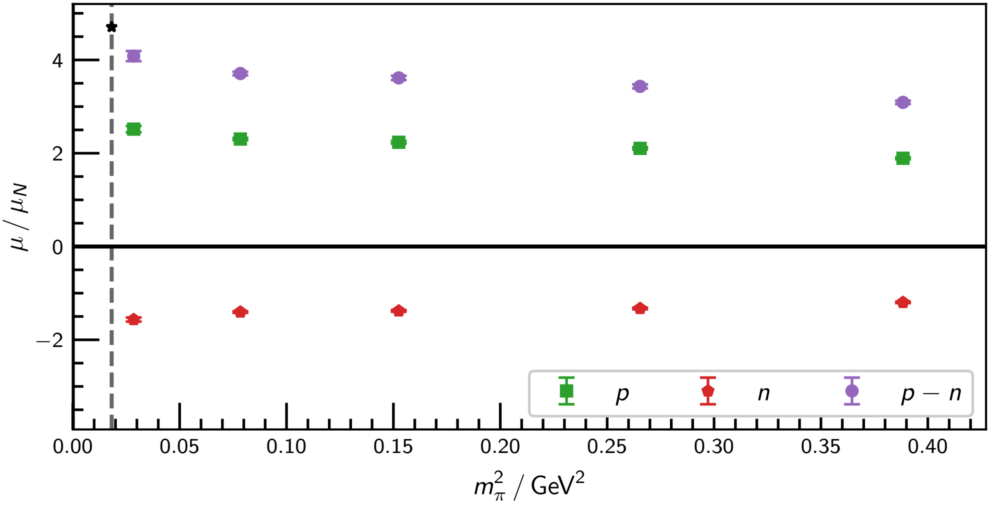
Ground state

Magnetic moment estimate ($m_\pi = 156\,\text{MeV}$)



Ground state

Pion-mass dependence of magnetic moment



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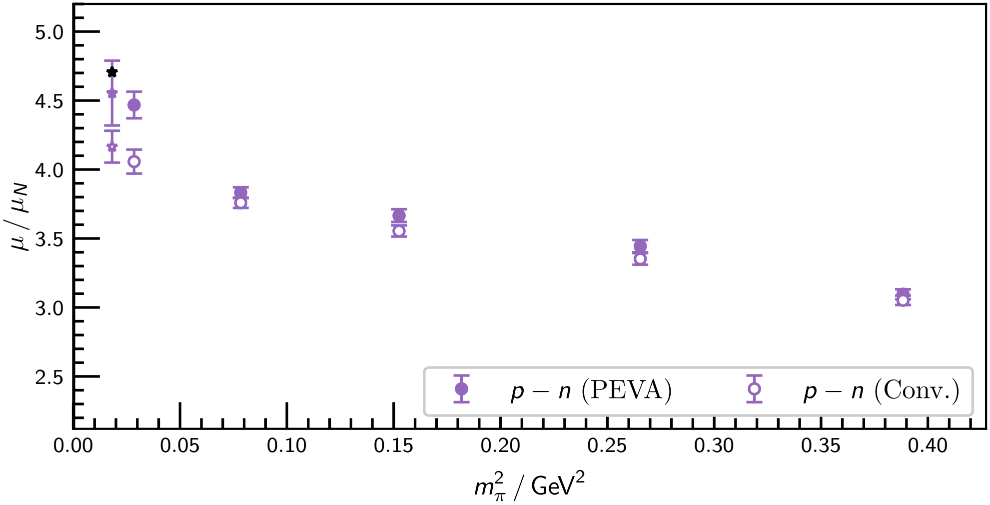
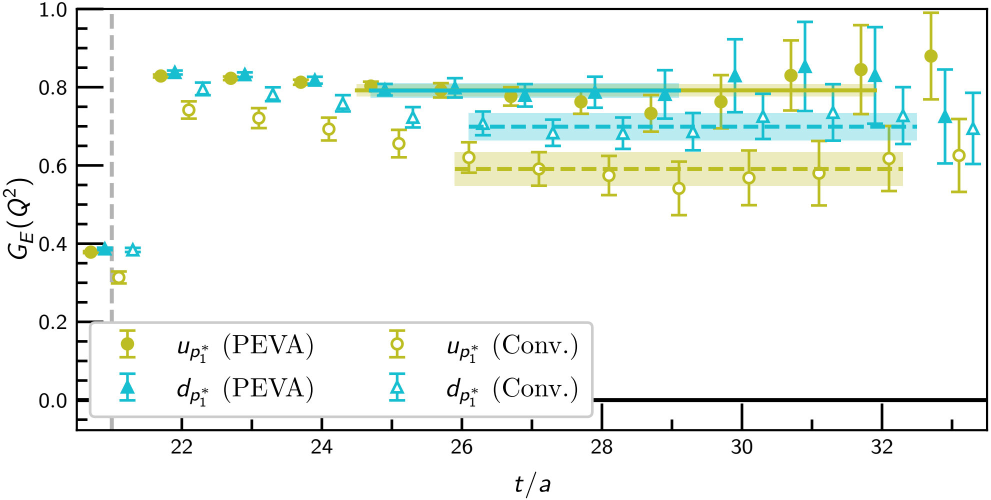


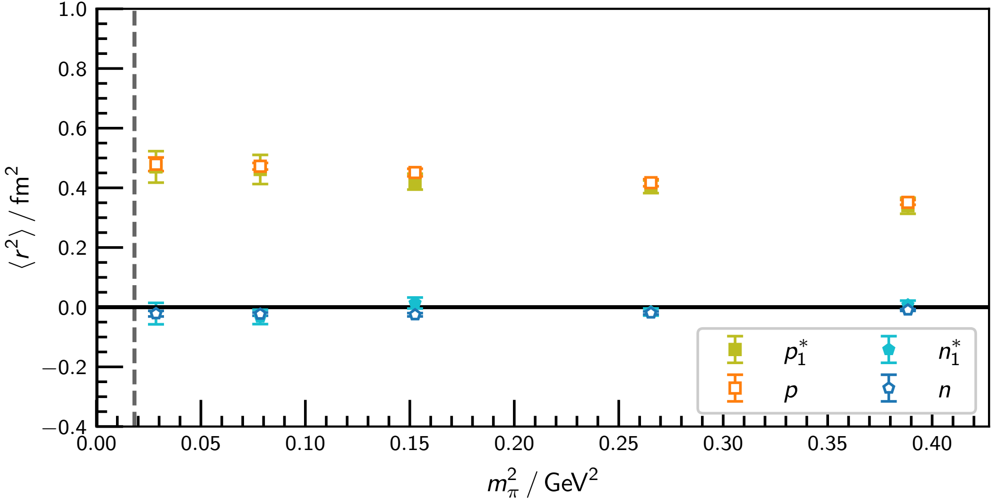
First negative-parity excitation

Fits to $G_E(Q^2 = 0.142(4))$ $(m_\pi = 702\,\mathrm{MeV})$



First negative-parity excitation

Pion-mass dependence of charge radius



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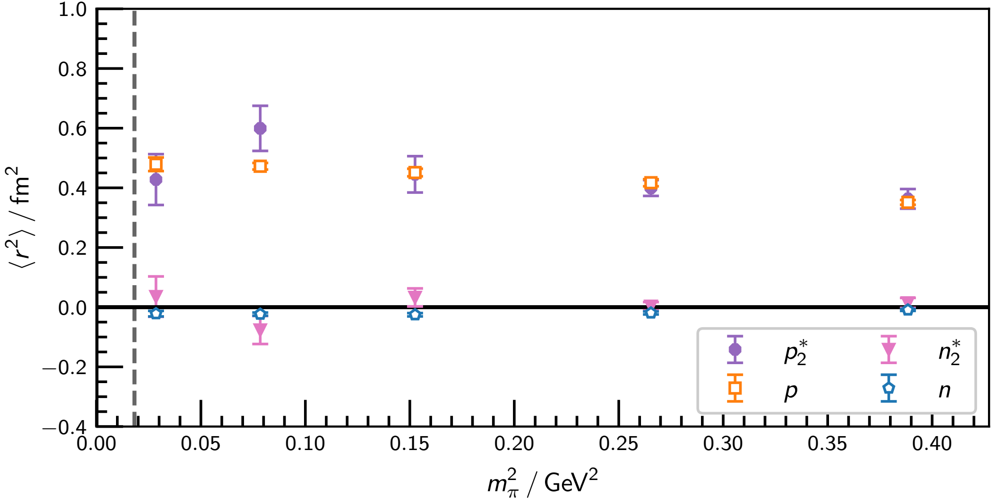

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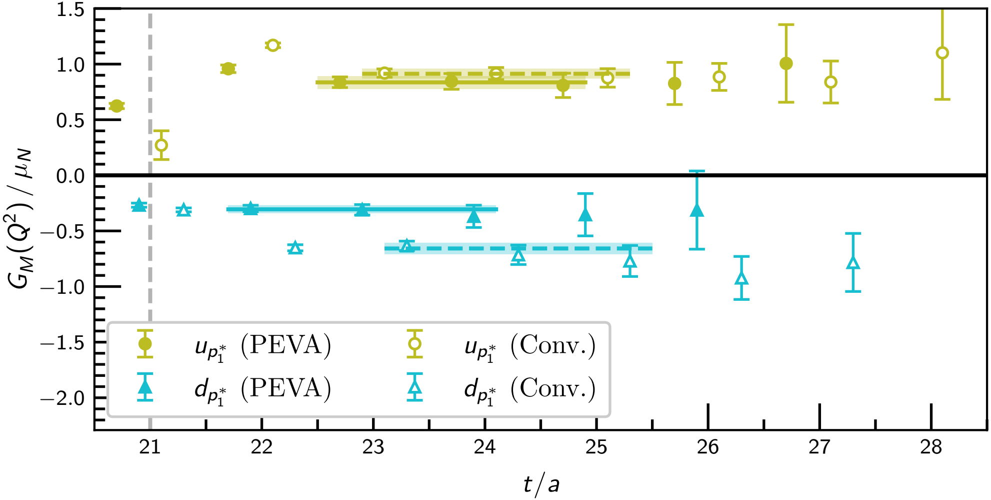
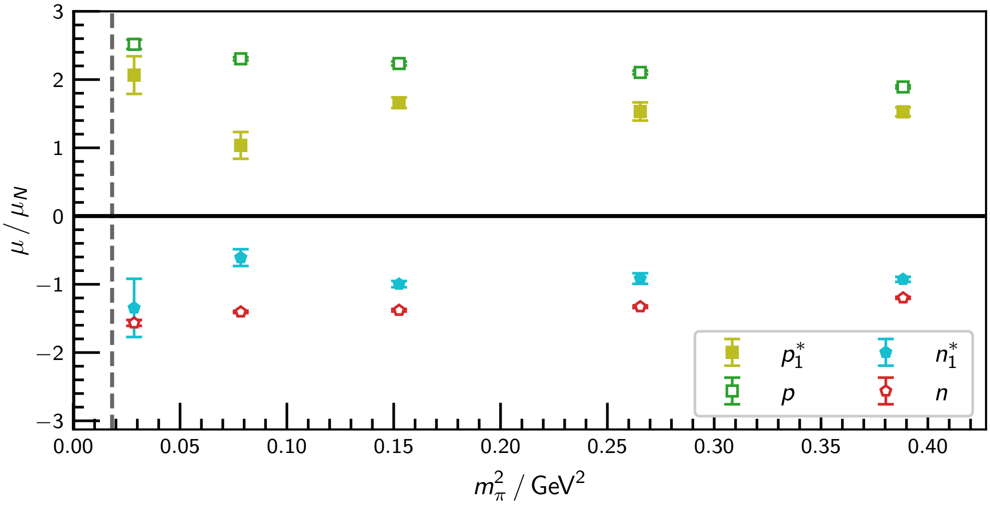


First negative-parity excitation

Pion-mass dependence of magnetic moment



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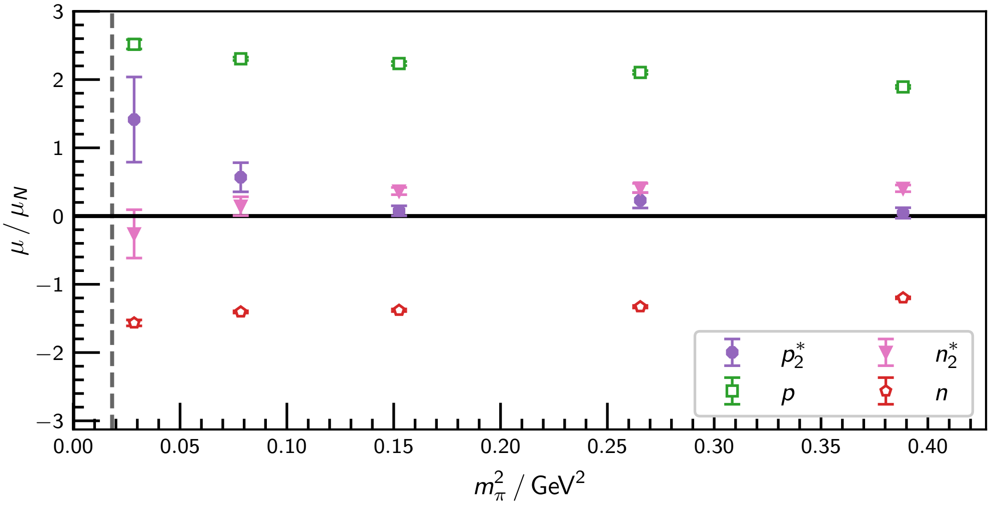
Pion-mass dependence of magnetic moment



Comparison to constituent quark model

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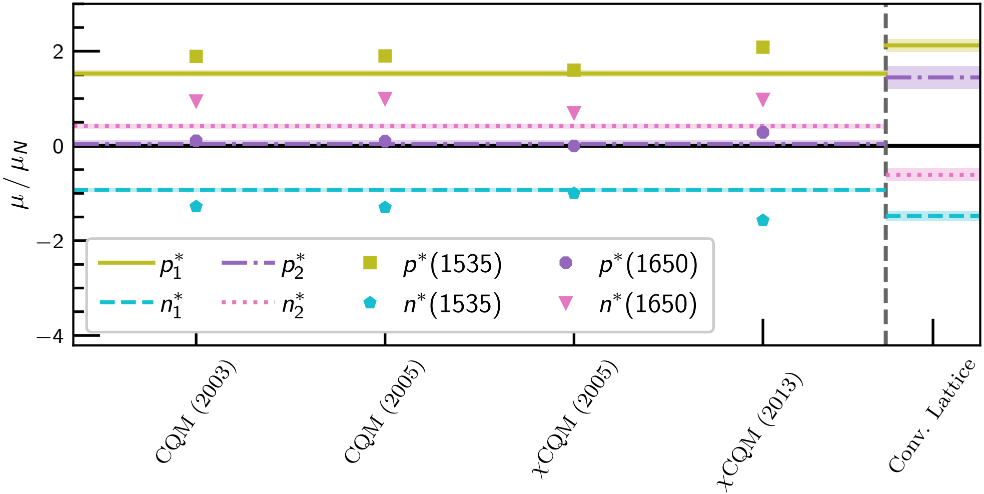

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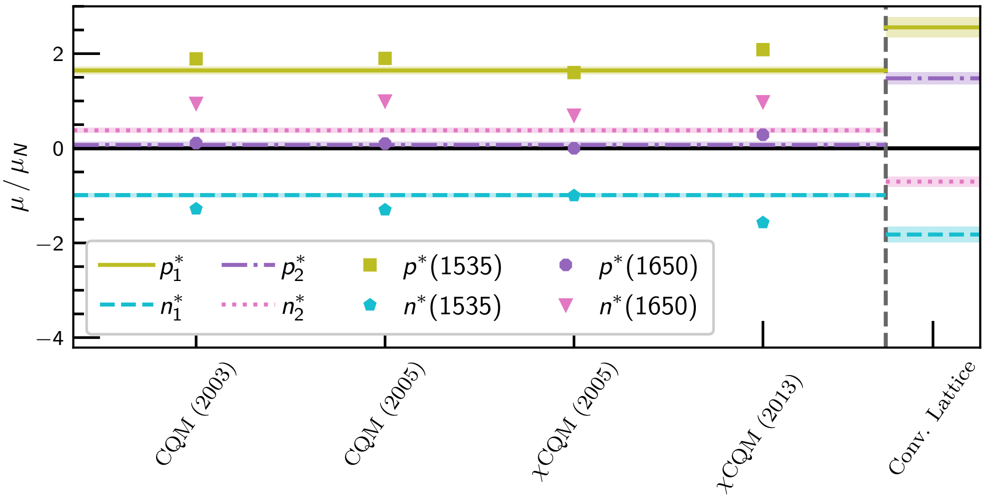

Comparison to constituent quark model

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- W.-T. Chiang, S. N. Yang, M. Vanderhaeghen and D. Drechsel, Nucl. Phys. A **723** (2003), doi:10.1016/S0375-9474(03)01160-6.
- J. Liu, J. He and Y. B. Dong, Phys. Rev. D **71** (2005), doi:10.1103/PhysRevD.71.094004.
- N. Sharma, A. Martinez Torres, K. P. Khemchandani and H. Dahiya, Eur. Phys. J. A **49** (2013), doi:10.1140/epja/i2013-13011-2.

Step 3: Compute three point correlation function

Matrix element

$$\langle \beta^- ; p' ; s' | j^\mu | \alpha^+ ; p ; s \rangle \propto$$
$$\overline{u}_\beta \left(\left(\delta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \gamma^\nu \gamma^5 F_1^*(Q^2) - \frac{\sigma^{\mu\nu} q_\nu}{m_\beta - m_\alpha} \gamma^5 F_2^*(Q^2) \right) u_\alpha$$

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Helicity amplitudes

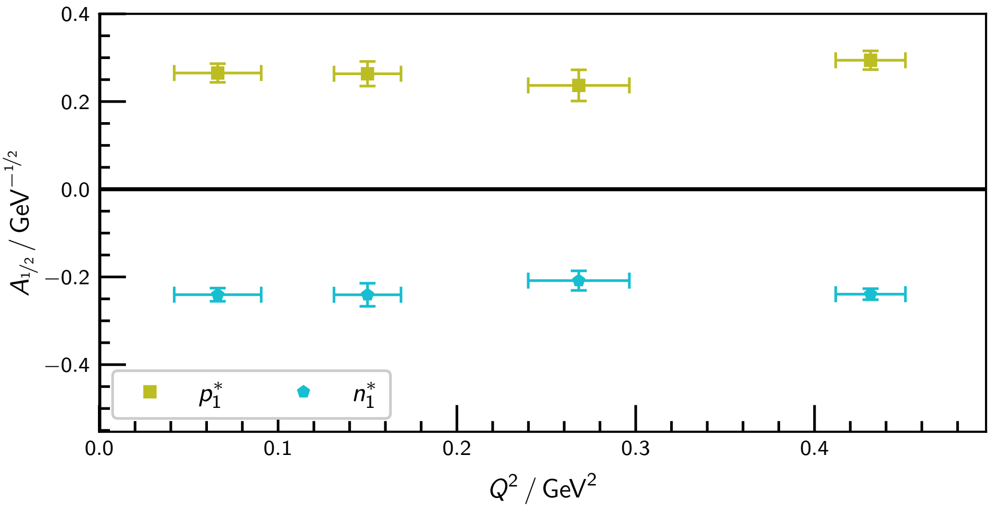
$$A_{1/2}(Q^2) = 2b_- \left(F_1^*(Q^2) + F_2^*(Q^2) \right)$$

$$S_{1/2}(Q^2) = -\sqrt{2} b_- \frac{(m_\beta - m_\alpha) |\vec{q}|}{Q^2} \left(F_1^*(Q^2) - \frac{Q^2}{m_\beta - m_\alpha} F_2^*(Q^2) \right)$$

$$b_- = \sqrt{\frac{Q^2 + (m_\beta + m_\alpha)^2}{8 m_\alpha (m_\beta^2 - m_\alpha^2)}}$$

Transition to first negative parity excitation

Transverse helicity amplitude at $m_\pi = 702\,\mathrm{MeV}$



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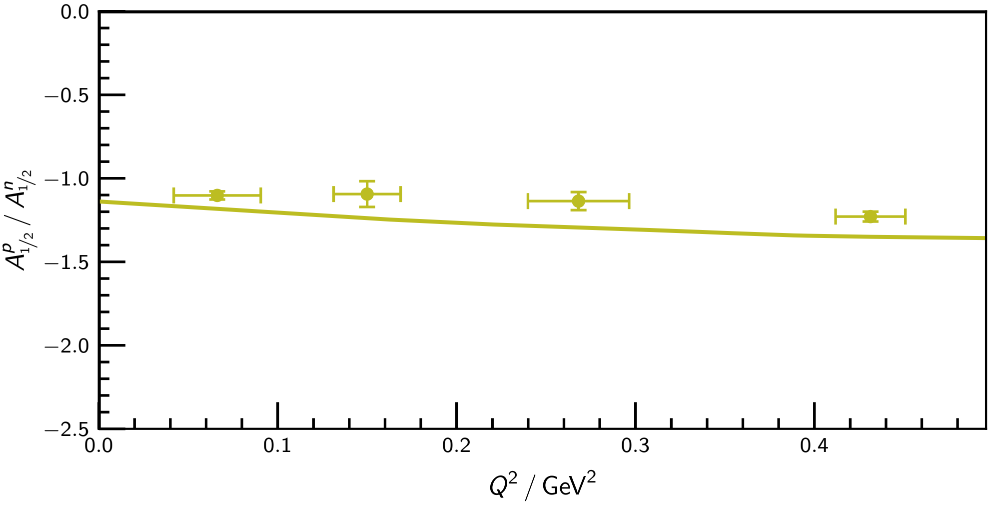
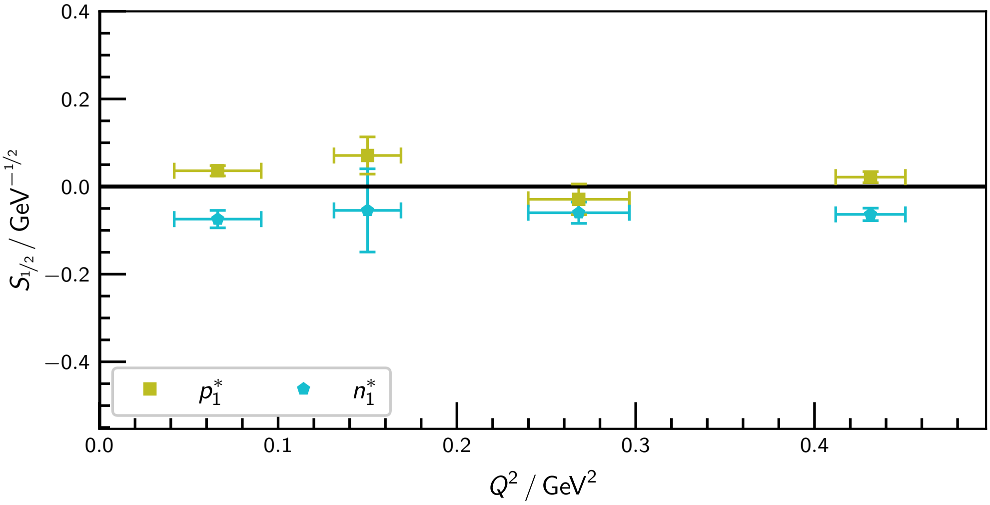
Transverse helicity amplitude ratio at $m_\pi = 702\,\mathrm{MeV}$



Transition to first negative parity excitation

Longitudinal helicity amplitude at $m_\pi = 702\,\mathrm{MeV}$



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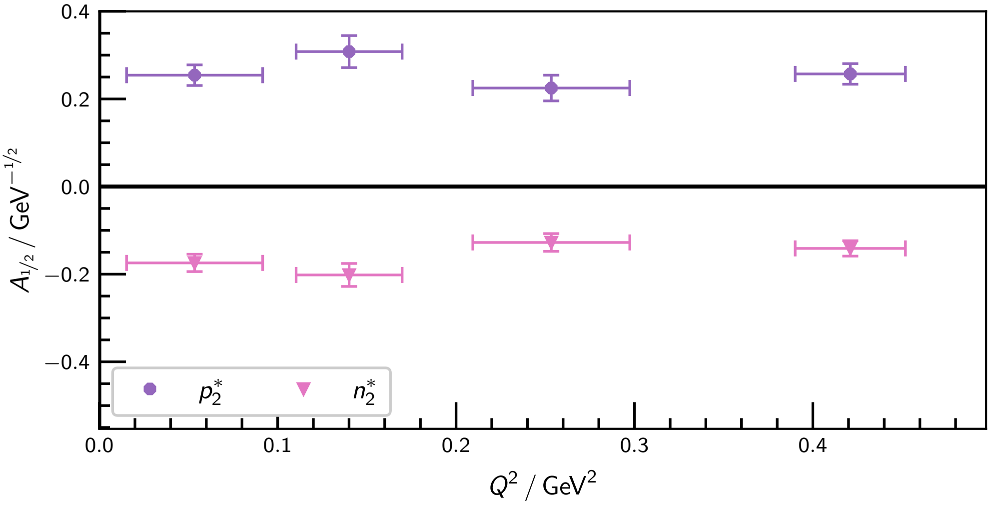

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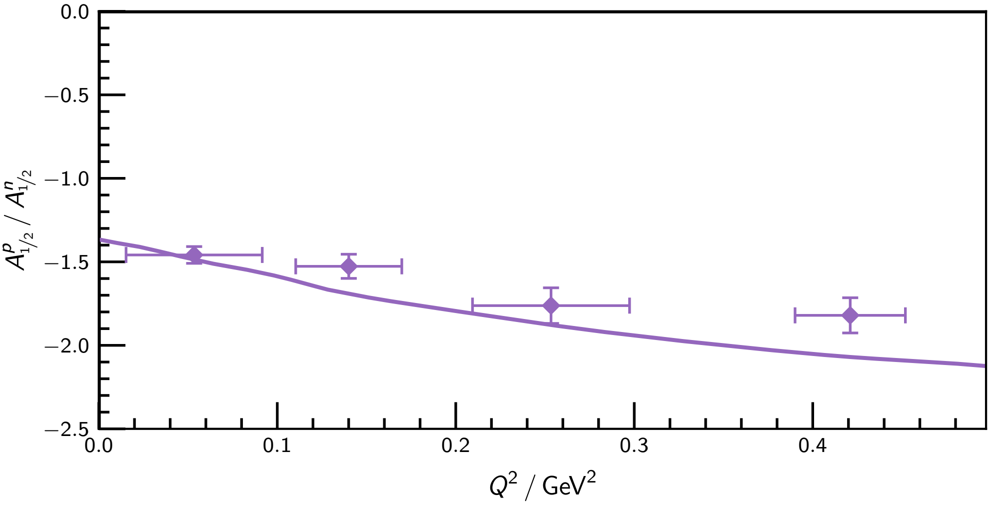

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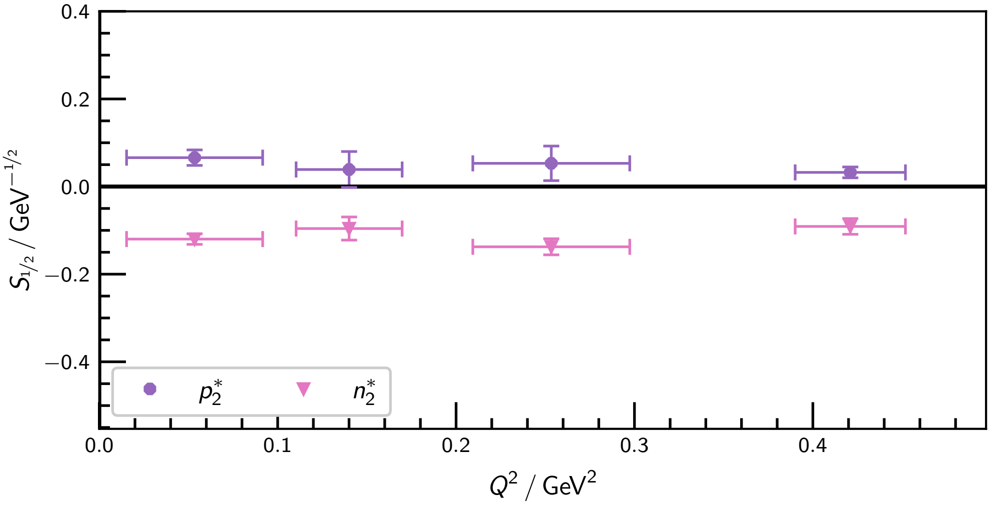

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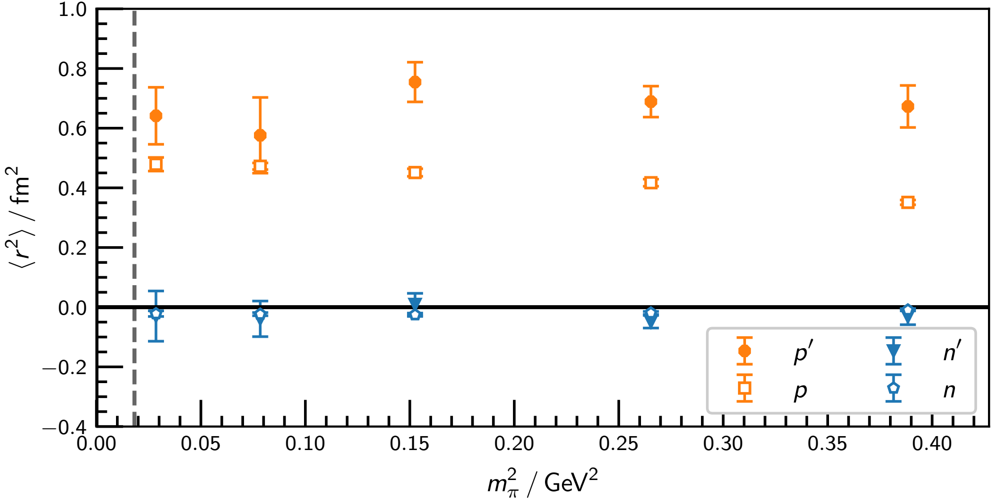
Pion-mass dependence of charge radius



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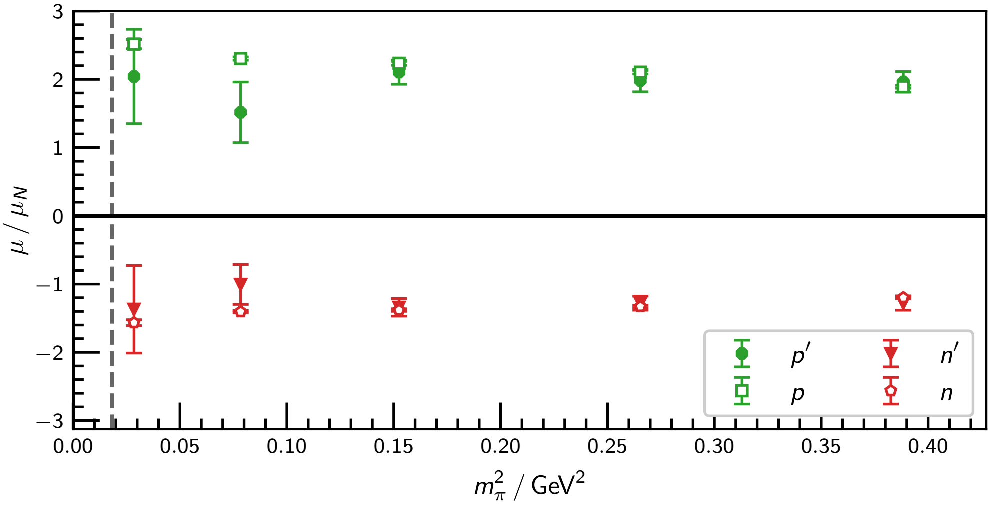
Pion-mass dependence of magnetic moment



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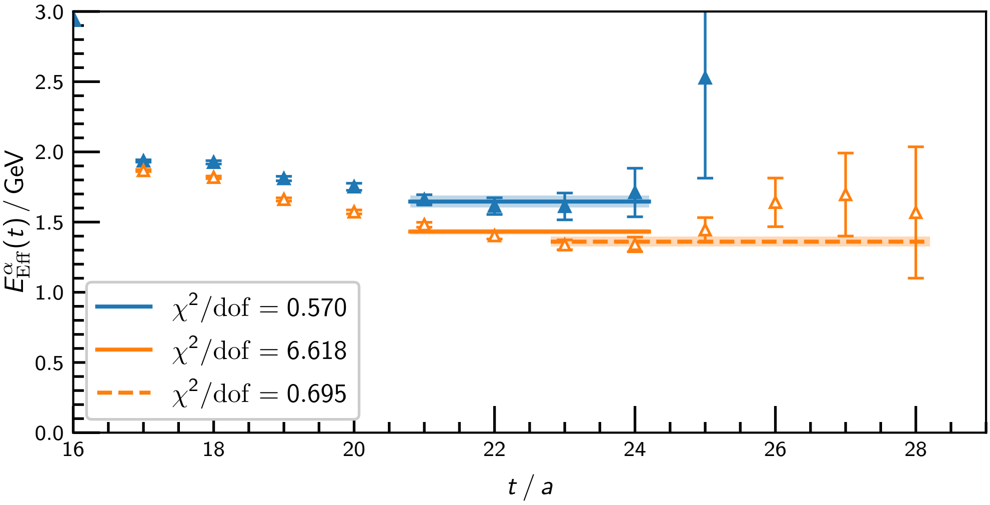
- Inclusion of multi-particle scattering operators is important

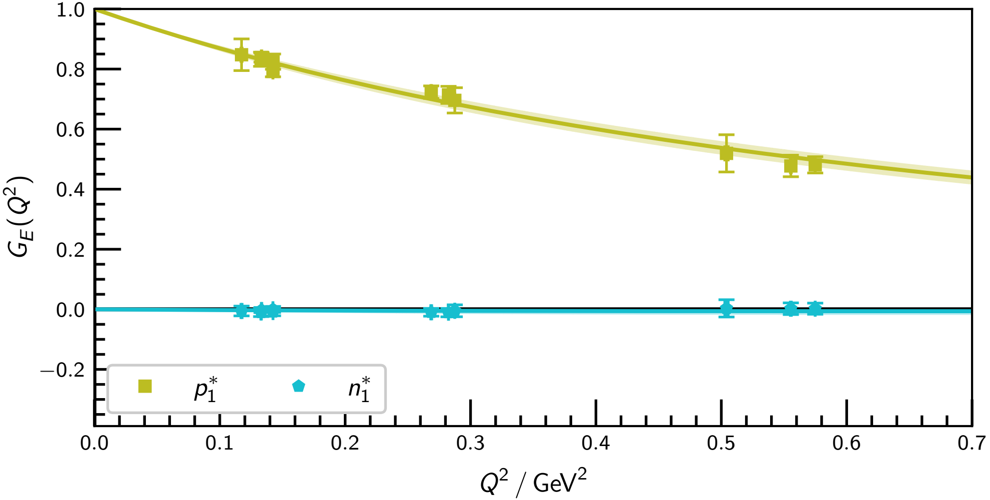
Effective energy

First negative parity excitation - $\boldsymbol{p}^2 \simeq 0.166\,\mathrm{GeV}^2$



First negative-parity excitation

Momentum-dependence of $G_E(Q^2)$ ($m_\pi = 702\,\mathrm{MeV}$)



Second negative-parity excitation

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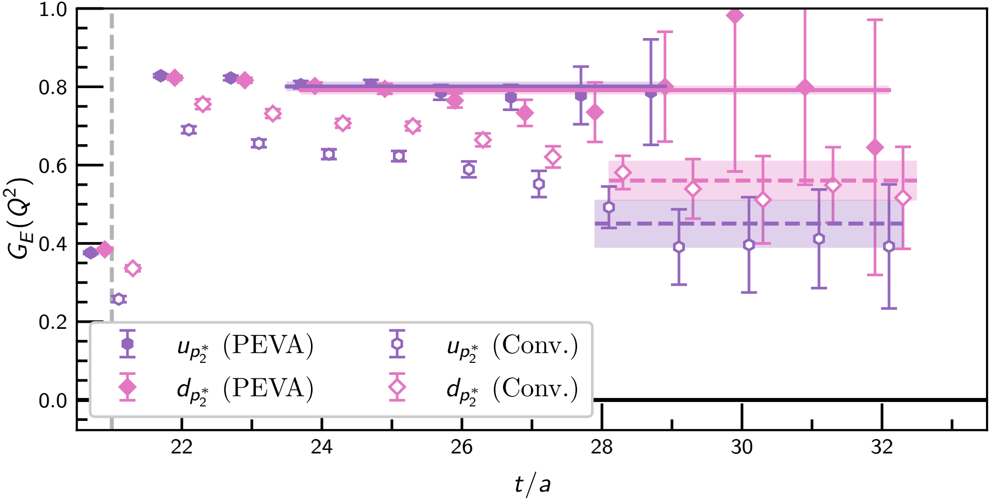

Second negative-parity excitation

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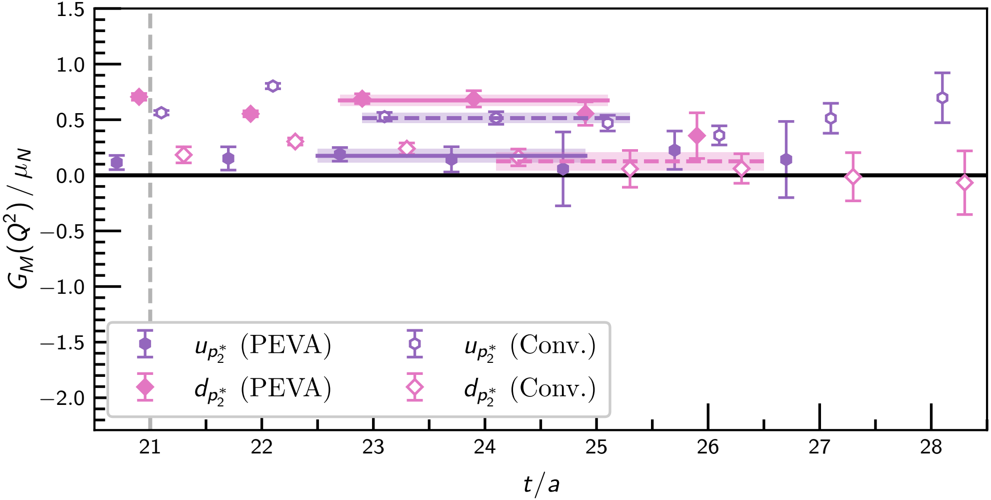
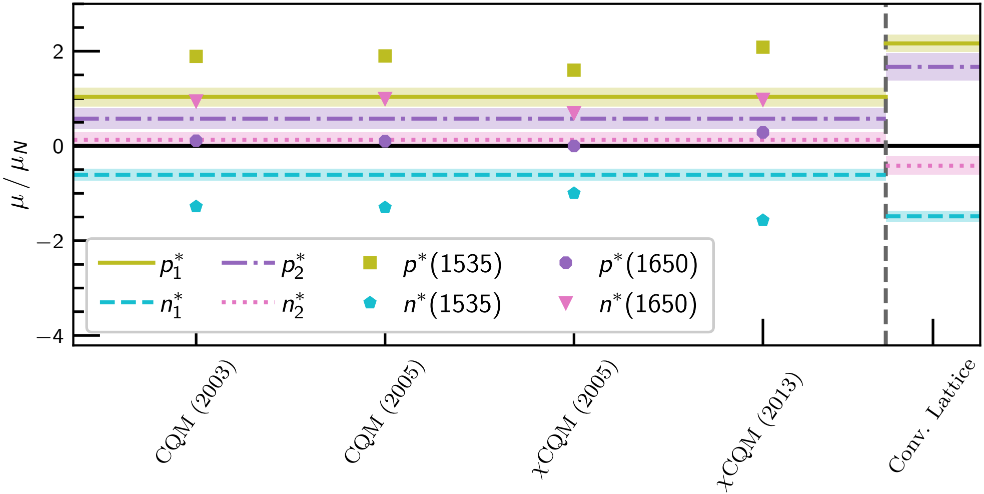


Comparison to constituent quark model

$m_\pi = 296\,\mathrm{MeV}$



Parity projection

$$\mathcal{G}_{ij}(\boldsymbol{p};\ t) = \sum_{B^\pm} \mathrm{e}^{-E_{B^\pm}(\boldsymbol{p})\,t}\, \lambda_i^{B^\pm}\, \overline{\lambda}_j^{B^\pm}\, \frac{-i\,\gamma\cdot p \pm m_{B^\pm}}{2E_{B^\pm}(\boldsymbol{p})}$$

- Introduce $\Gamma_\pm = (\gamma_4 \pm \mathbb{I})/2$ and define $G_{ij}(\Gamma_\pm; \boldsymbol{p};\ t) \equiv \mathrm{Tr}\left(\Gamma_\pm\, \mathcal{G}_{ij}(\boldsymbol{p};\ t)\right)$

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$$G_{ij}(\Gamma_+;\boldsymbol{p};\ t) = \sum_{B^+} \mathrm{e}^{-E_{B^+}(\boldsymbol{p})\,t}\, \lambda_i^{B^+}\, \overline{\lambda}_j^{B^+}\, \frac{E_{B^+}(\boldsymbol{p}) + m_{B^+}}{2E_{B^+}(\boldsymbol{p})} + \sum_{B^-} \mathrm{e}^{-E_{B^-}(\boldsymbol{p})\,t}\, \lambda_i^{B^-}\, \overline{\lambda}_j^{B^-}\, \frac{E_{B^-}(\boldsymbol{p}) - m_{B^-}}{2E_{B^-}(\boldsymbol{p})}$$

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Parity projection

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Zero momentum

- At zero momentum, $E_B(\mathbf{0}) = m_B$, so

$$\frac{E_B(\mathbf{0}) + m_B}{2E_B(\mathbf{0})} = 1$$

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- So projected correlators only contain terms for states of a single parity

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- Can analyse states of each parity independently