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Proceedings of the Mini-Workshop

Selected Few-Body Problems in Hadronic and Atomic Physics

Bled, Slovenia, July 7–14, 2001

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Preface

The series of mini-workshops at Bled, which started in 1987 with the workshop on *Mesonic Degrees of Freedom in Hadrons*, has established its own character of friendly but productive confrontation of ideas, such that we intend to organize similar workshops every year.

It is now for the seventh time that a small group of enthusiasts met in this renowned holiday resort to clarify several open problems of common interest. The topics of this meeting ranged from few-quark problems, schematic models for hadronic and nuclear physics, effective interactions between constituent quarks, baryon-baryon and meson-meson effective interaction, semirelativistic few-body calculations, accurate few-body calculations, and quasilinearization method. The participants enjoyed a focused, intense discussion and critical confrontation of their results and ideas in a friendly atmosphere. Every participant had up to one hour time for his exposition which could be interrupted by questions and remarks, plus half an hour of general discussion. The advantage of such mini-workshops is the ease with which the participants sincerely acknowledge not only the successes, but also the weak points and open problems in their research.

The rather diverse set of topics covered by a relatively small group of people did not, as one would perhaps expect, cause a breakup into smaller groups. On the contrary, the spirit of interdisciplinarity was abundant, criticism was relaxed and direct, and old friends retraced their paths through physics.

The mini-workshop took place in Villa Plemelj, bequeathed to the Society of Mathematicians, Physicists and Astronomers by the renowned Slovenia mathematician Josip Plemelj. The beautiful environment of Lake Bled helped a lot to the cheerful atmosphere and optimism in the presentations; and the occasional inclement weather contributed to the patience for long afternoon discussions.

Ljubljana, December 2001

M. Rosina
R. Krivec
B. Golli

Workshops organized at Bled

- What Comes beyond the Standard Model (June 29–July 9, 1998)
- Hadrons as Solitons (July 6-17, 1999)
- What Comes beyond the Standard Model (July 22–31, 1999)
- Few-Quark Problems (July 8-15, 2000)
- What Comes beyond the Standard Model (July 17–31, 2000)
- Statistical Mechanics of Complex Systems (August 27–September 2, 2000)
- What Comes beyond the Standard Model (July 17–27, 2001)
- Studies of Elementary Steps of Radical Reactions in Atmospheric Chemistry (August 25–28, 2001)

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Effective interaction for the hyperspherical formalism*

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Abstract. The effective interaction method, traditionally used in the framework of an harmonic oscillator basis, is generalized to the hyperspherical formalism. It is shown that with the present method one obtains an enormous improvement of the convergence of the hyperspherical harmonics series in calculating ground state properties, excitation energies and transitions to continuum states.

In general we would like to use the hyperspherical harmonics (HH) basis functions to solve the A–body Hamiltonian

$$H = \sum_{i=1}^{A} \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j}^{A} V_{ij} \ , \tag{1}$$

where m is the nucleon mass and V_{ij} is the NN interaction. In practice, looking for the eigenvectors of H in terms of the HH expansion turns out to be a notoriously difficult task. Therefore, one usually has to introduce correlation functions in order to accelerate the convergence of the calculation [1–4]. In this work, however, we shall explore another possibility and instead of using correlation functions we shall use the method of effective interactions [5]. This approach is largely used in shell–model calculations [6,7], where the harmonic oscillator basis is used in a truncated model space. Instead of the bare NN interaction one uses effective interactions inside the model space. Defining P as the projection operator onto the model space and $Q = 1 - P$ as the projection onto the complementary space, the model space Hamiltonian can be written as

$$H_P = P \left[\sum_{i=1}^{A} \frac{\mathbf{p}_i^2}{2m} \right] P + P \left[\sum_{i<j}^{A} V_{ij} \right]_{eff} P \ . \tag{2}$$

In general the effective interaction appearing in Eq. (2) is an A–body interaction. If it is determined without any approximation, then the model–space Hamiltonian provides a set of eigenvalues which coincide with a subset of the eigenvalues

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of the original full–space Hamiltonian, Eq. (1). However, calculation of the exact A–body effective interaction is as difficult as finding the full–space solution. It is therefore customary to approximate V_{eff} by a sum of two–body effective interactions determined from a 2–body problem.

In the effective interaction hyperspherical harmonic (EIHH) approach [9] the division of the total HH space in P and Q spaces is realized via the HH quantum number K (P(Q) space: $K \leq (>) K_{max}$). The total Hamiltonian is written in hyperspherical coordinates,

$$H = \frac{1}{2m}\left(-\Delta_\rho + \frac{\hat{K}^2}{\rho^2}\right) + \sum_{i<j} V_{ij} \ , \tag{3}$$

where ρ is the hyperradius and Δ_ρ contains derivatives with respect to ρ only. The grand-angular momentum operator $\hat{K}^2$ is a function of the variables of particles A and $(A-1)$ and of $\hat{K}_{A-2}$ the grand angular momentum operator of the $(A-2)$ residual system [8]. Then from the total Hamiltonian one can extract a "two-body" Hamiltonian of particles A and $(A-1)$

$$H_2(\rho) = \frac{1}{2m}\frac{\hat{K}^2}{\rho^2} + V_{A(A-1)} \ , \tag{4}$$

which, however, contains the hyperspherical part of the total kinetic energy. Since the HH functions of the $(A-2)$ system are eigenfunctions of $\hat{K}^2_{A-2}$ one has an explicit dependence of H_2 on the quantum number K_{A-2} of the residual system, i.e. $H_2 \rightarrow H_2^{K_{A-2}}$. Applying the hermitian version of the Lee-Suzuki method [5] to H_2 one gets an effective Hamiltonian H_{2eff}. The effective interaction V_{eff} is obtained from

$$V_{eff}^{K_{A-2}}(\rho) = H_{2eff}^{K_{A-2}}(\rho) - \frac{1}{2m}\frac{\hat{K}^2}{\rho^2} \tag{5}$$

This V_{eff} replaces V_{ij} in Eq. (3) when we project the solution on the P-space. This effective potential has the following property: $V_{eff} \rightarrow V_{ij}$ for $P \rightarrow 1$. Due to the "effectiveness" of the operator the solution of the Schrödinger equation converges faster to the true one. The HH formulation leads to various advantages: (i) V_{eff} itself is ρ dependent, therefore it contains some information on the "medium", (ii) because of the above mentioned K_{A-2} dependence the (A-2) residual system is not a pure spectator, and (iii) an additional confining potential is not needed, since the presence of ρ in Eq. (4) automatically confines the two-body system to the range $0 \leq r_{A-(A-1)} < \sqrt{2}\rho$. We would like to point out that $V_{eff}(K_{max})$ can be viewed as a kind of momentum expansion, since the short range resolution is increased with growing K_{max}. The convergence for the calculation of mean values can be improved introducing the corresponding effective operators. Of course for the calculation of the mean value of the Hamiltonian, i. e. E_b, one already makes use of an effective operator, namely $H_{2eff}^{K_{A-2}}$.

To this end let us present a simple example for the rate of convergence of the HH series with the effective interaction for ground state energy, and rms matter radii. In Fig. 1 we illustrate the convergence patterns with bare and effective interactions for binding energy and radius of ^{4}He with the MTV potential. It is

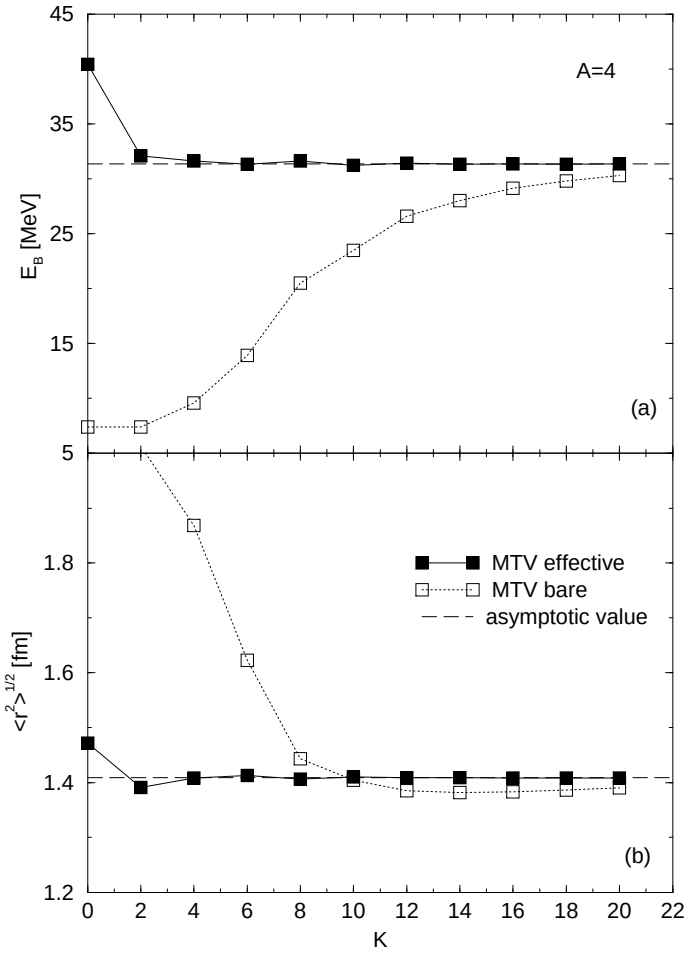

Fig. 1. Binding energy (a) and root mean square radius (b) of the A=4 system for the Malfliet–Tjon potential MTV as a function of the hyperangular quantum number K. The asymptotic value has been indicated by a dashed line.

readily seen that the effective interaction improves the convergence drastically. For the effective interaction one nearly obtains the correct values for energy and radius with a rather low K of 4. The convergence with the bare interaction is considerably worse since even with $K = 20$ one does not have completely converged results.

Summing up, in this talk we have presented the recently developed hyperspherical effective interaction method. In this approach the two-body effective interaction depends on the A-body hyperradius and on the state of the A-2 rest system explicitly. The EIHH method has been so far applied to few–body nuclei in the mass range $A = 3 \div 6$, interacting through central and noncentral potential models including the realistic NN force AV14. Our results for these systems

show that the method leads to an excellent convergence of the hyperspherical expansion.

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Do we see the chiral symmetry restoration in baryon spectrum?

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One feature of QCD that is well understood is that the theory possess an approximate $SU(2)_L \times SU(2)_R$ symmetry and that this symmetry is spontaneously broken. As a consequence the properties of the low-lying hadrons are strongly affected by spontaneous breaking of chiral symmetry (SBCS). One striking feature of hadronic physics is the appearance of parity doublets for highly excited baryons (baryons with a mass of ~ 2 GeV and above). Recently it has been suggested that these parity doublets can be explained by an effective restoration of chiral symmetry for these highly excited states [1].

One of the experimentally established concepts, the quark-hadron duality, states that the experimentally observed spectral density $\rho(s)$ at $s \to \infty$ is dual to the contribution of the free quark loop plus perturbative corrections (e.g. in $e^+e^- \to hadrons$; in this case the "asymptotic" regime sets up at $s \sim 2-3$ GeV). However in perturbation theory there is no SBCS. Thus one sees immediately that $SU(2)_L \times SU(2)_R$ symmetry must be manifest in the spectrum at asymptotically high mass. Assuming that the process of chiral restoration is smooth (once one goes up in excitation energy from the ground state), as it indeed follows from the operator product expansion, one can understand the appearance of the systematical parity doublets in the upper part of baryon spectrum as a manifestation of chiral symmetry restoration [2]. One of the immediate implications is that the concept of constituent quarks, which may be adequate in the SBCS regime, becomes irrelevant high in the spectrum [1].

Effective chiral restoration implies that the physical baryon states high in the spectrum fall into multiplets of $SU(2)_L \times SU(2)_R$. Constraint of parity conservation implies that such multiplets must be a direct sum of two irreducible representations: $(1/2,0) \oplus (0,1/2)$, $(3/2,0) \oplus (0,3/2)$ and $(1/2,1) \oplus (1,1/2)$. The preceding representations are the only ones which contain no states with isospin greater than $3/2$. Such $I > 3/2$ baryon states have never been observed. If the first two cases were realized in nature, then the spectra of highly excited nucleons and deltas would consist of parity doublets. However, the energy of the parity doublet in the nucleon spectrum *a-priori* would not coincide with the energy of the doublet with the same spin in the delta spectrum. This is because these doublets

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would belong to different representations of $SU(2)_L \times SU(2)_R$. On the other hand, if $(1/2,1)\oplus(1,1/2)$ were realized, then the highly lying states in N and Δ spectrum would consists of multiplets that contain one N parity doublet and one Δ parity doublet with the same spin and are degenerate in mass. We show that available spectroscopic data for nonstrange baryons in the $\sim$ 2 GeV region is consistent with excited baryons approximately falling into $(1/2,1) \oplus (1,1/2)$ representation of $SU(2)_L \times SU(2)_R$ with approximate degeneracy between positive and negative parity N and Δ resonances of the same spin.

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Coupled channel formulation of decays in the point form Goldstone boson exchange model ⋆

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In the Goldstone Boson Exchange (GBE) model [1] the hyperfine interaction is generated by a constituent quark-meson vertex. Combined with a "semi relativistic" kinetic energy and linear confinement, eigenvectors of the three quark Hamiltonian have been calculated [2]; the eigenvalues give a good fit for the low-lying nucleon and strange baryon spectrum. But the spectrum is a point spectrum, so that excited states have no widths; it is desirable to augment the GBE model to include strong and electromagnetic decays.

The GBE Hamiltonian can be reinterpreted as a mass operator in point form relativistic quantum mechanics [3]. The goal of this note is to show how the mass operator on the three quark space can be enlarged to a mass operator on the direct sum of a three quark plus three quark and meson space, with transitions (off diagonal mass operator) between the two spaces produced by the original quark-meson vertex.

To show how the matrix mass operator is constructed it will prove useful to review some features of the point form. Recall that in the point form of relativistic quantum mechanics all interactions are in the four-momentum operator, which must satisfy the point form equations:

$$[P_\mu,\ P_\nu] = 0 \tag{1}$$

$$U_\Lambda P_\mu U_\Lambda^{-1} = (\Lambda^{-1})^\nu_\mu P_\nu\ , \tag{2}$$

where U_Λ is a unitary operator representing the Lorentz transformation Λ. These equations are simply the Poincaré commutations relations with global rather than infinitesimal Lorentz generators. The mass operator is given by $M = \sqrt{P \cdot P}$ and must have a spectrum that is bounded from below.

Multiparticle states such as the three quark states do not have nice transformation properties under Lorentz transformations; in particular the Wigner rotation of each particle is different, so that the spins and orbital angular momentum cannot be coupled together. Velocity states are multiparticle states in their overall

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center of momentum, boosted by a four-velocity v:

$$|v, \mathbf{k_i}, \mu_i >: = U_{B(v)}(|k_1, \mu_1 > ...|k_n, \mu_n >) \tag{3}$$

$$= \sum (|p_1, \sigma_1 > ...|p_n, \sigma_n >)\pi D^j_{\sigma_i, \mu_i}(R_{W_i}), \tag{4}$$

where $p_i = B(v)k_i$, $\sum \mathbf{k_i} = \mathbf{0}$, and $R_{W_i} = B^{-1}(p_i/m)B(v)B(k_i/m)$. Under Lorentz transformations, using the definition, Eq.3, velocity states transform as

$$U_\Lambda|v, \mathbf{k_i}, \mu_i > = |\Lambda v, R_W \mathbf{k_i}, \mu_i^{'} > \pi D^j_{\mu_i^{'}, \mu_i}(R_W); \tag{5}$$

that is, the Wigner rotation $R_W = B^{-1}(\Lambda v)\Lambda B(v)$ is the same in all the arguments of the D functions, and is the same Wigner rotation multiplying all the internal momentum vectors, $\mathbf{k_i}$. That means all the spins as well as orbital angular momenta can be coupled together exactly as is done nonrelativistically [4]. From the relation between external and internal momenta, it also follows that

$$M_{free}|v, \mathbf{k_i}, \mu_i > = \sum \sqrt{m_i^2 + \mathbf{k_i}^2}|v, \mathbf{k_i}, \mu_i > \tag{6}$$

$$V_\mu|v, \mathbf{k_i}, \mu_i > = v_\mu|v, \mathbf{k_i}, \mu_i >, \tag{7}$$

where M_{free} is the free mass operator, and V_μ is the free four-velocity operator. Then the free four-momentum operator is $M_{free}V_\mu$ and it is the free mass operator that is perturbed to introduce dynamics in the relativistic system.

This so-called Bakamjian-Thomas procedure [5] is implemented in the point form by writing $P_\mu = MV_\mu$, where now M is the sum of free and interacting mass operators, $M = M_{free} + M_I$. M takes the place of the Hamiltonian in nonrelativistic quantum mechanics and it is not hard to show that if M commutes with the four-velocity operator the point form Eq.1 is satisfied, while if M commutes with Lorentz transformations, the point form Eq.2 is satisfies.

Consider now a mass operator on the direct sum space of the form

$$M = \begin{bmatrix} M_{3Q} & V^\dagger \\ V & M_{3Q-M} \end{bmatrix} \tag{8}$$

where $M_{3Q} = M_{fr} + M_{conf}$ but does not include the hyperfine interaction. V is the mass operator generated by the meson-quark vertex [6],

$$< v, \mathbf{k_i}, \mu_i|V|v, \mathbf{k_i^{'}}, \mu_i^{'} > = < v, \mathbf{k_i}, \mu_i|\mathcal{L}_I(0)|v, \mathbf{k_i^{'}}, \mu_i^{'} > f(\Delta m) \tag{9}$$

where Δm is $m - m^{'}$ and m (respectively $m^{'}$) is the mass of the velocity state, as given in Eq.6. The interaction Lagrangian couples the quark to the meson and $f(\Delta m)$ is a form factor which is determined by the hyperfine potential.

Then the GBE mass operator can be written as $M_{GBE} = M_{3Q} + VM_\pi{}^{-1}V^\dagger$ where the last term gives the hyperfine mass operator. M_π is the meson propagator while $M_{3Q-M} = \sqrt{M^2_{GBE} + k^2} + \sqrt{m_\pi^2 + k^2}$.

The goal now is to reduce the coupled channel problem to one involving only the 3Q space, such that M_{GBE} has another term added to it which accounts

for the decays of the excited states:

$$M|\Psi> = m|\Psi> \tag{10}$$

$$M_{3Q}|\Psi_{3Q}> + V^\dagger|\Psi_{3Q-M}> = m|\Psi_{3Q}>$$

$$V|\Psi_{3Q}> + M_{3Q-M}|\Psi_{3Q-M}> = m|\Psi_{3Q-M}>$$

$$|\Psi_{3Q-M}> = \frac{1}{m - M_{3Q-M}} V|\Psi_{3Q}>$$

$$(M_{3Q} + V^\dagger \frac{1}{m - M_{3Q-M}} V)|\Psi_{3Q}> = m|\Psi_{3Q}> \tag{11}$$

$$(M_{GBE} + V^\dagger (\frac{1}{m - M_{3Q-M}} - \frac{1}{M_\pi})V)|\Psi_{3Q}> = m|\Psi_{3Q}> \tag{12}$$

The new term added on to M_{GBE} represents the effect of the coupled channel; it contains the difference between the propagator from the four-particle space and the meson propagator. Since this last term is a difference between two operators, it is hopefully a small correction to the dominant term. If that is the case it should be possible to use perturbation theory to calculate the level shifts and widths of the excited states, which would then be small corrections to the levels given by the GBE mass operator itself. On the other hand if the correction is large, perturbation theory cannot be used. But then also the good fits to experimental data obtained by just the GBE mass operator will be significantly modified by adding on the new term so that it is no longer clear that any improvement results.

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Quasilinearization method and its application to physical problems ⋆

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The general properties of the quasilinearization method [1–3], particularly its fast convergence, monotonicity and numerical stability are analyzed and illustrated on different physical problems. The method approximates a solution of a nonlinear differential equation by treating nonlinear terms as a perturbation about linear ones, and is not based, unlike perturbation theories, on the existence of some kind of a small parameter. Each approximation of the method sums many orders of the perturbation theory. The method provides accurate and stable answers for any coupling strengths, including for super singular potentials for which each term of the perturbation theory diverges and the perturbation expansion does not exist even for a very small couplings.

In order to further analyze and highlight the power and features of the quasilinearization method (QLM), we have made [2] numerical computations on the nonlinear ordinary first order differential equations for the S-wave scattering length $a_0 = a(\infty)$ and phase shifts δ_0, respectively, obtained in the variable phase approach [4]. We have considered different singular and nonsingular, attractive and repulsive potentials, namely Yukawa, Pöschl-Teller and Newton potentials, and have compared the results obtained by the quasilinearization method with the exact solutions.

It is shown also [3] that the quasilinearization method gives excellent results when applied to different nonlinear ordinary differential equations in physics, such as the Blasius, Duffing, Lane-Emden and Thomas-Fermi equations. The first few quasilinear iterations already provide extremely accurate and numerically stable answers.

Our conclusions can be formulated as follows:

i) The QLM treats the nonlinear terms as a perturbation about the linear ones [1] and is not based, unlike perturbation theories, on the existence of some kind of small parameter. As a result, as we see on our examples, it is able to handle, unlike the perturbation theory, large values of the coupling constant.
ii) The method provides very accurate and numerically stable and fast convergent answers for any values of the coupling constant giving the accuracy of

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at least five significant figures required in this work. Already the first few iterations provide precise answers for small and intermediate values of the coupling constant. The number of iterations necessary to reach a given precision only moderately increases for larger values of the coupling constants.

iii) The method provides very accurate and numerically stable answers also for any potential strength in the case of super singular potentials for which each term of the perturbation theory is infinite and the perturbation treatment is not possible even for a very small coupling.

In view of all this, since most equations of physics, from classical mechanics to quantum field theory, are either not linear or could be transformed to a nonlinear form, the quasilinearization method may turn out to be extremely useful and in many cases more advantageous than the perturbation theory or its different modifications, like expansion in inverse powers of the coupling constant, the 1/N expansion, etc.

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On the structure of the pion: A QCD–inspired point of view

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Abstract. The effective interaction between a quark and an anti-quark as obtained previously with by the method of iterated resolvents is replaced by the so called up-down-model and applied to flavor off-diagonal mesons including the pion. The only free parameters are those of canonical quantum chromo-dynamics (QCD), particularly the coupling constant and the masses of the quarks.

The so obtained light-cone wave function can be used to calculate the pion's form factor, particularly its mean-square radius can be computed analytically. The results allow for the exciting conclusion that the pion is built by highly relativistic constituents, in strong contrast to composite systems like atoms or nuclei with non-relativistic constituents.

1 Introduction

One of the most urgent problems in contemporary physics is to compute the structure of hadrons in terms of their constituents, based on a covariant theory such as QCD.

Among the hadrons the pion is the most mysterious particle. I have proposed an oversimplified model, the $\uparrow\downarrow$-model, which has many drawbacks but the virtue of being inspired by QCD and of having the same number of parameters one expects in a full theory: namely the 6 flavor quark masses, the strong coupling constant (7) and one additional scale parameter (8) originating in the murky depth of renormalization theory.

The model is QCD-inspired by virtue of the fact, that it is based on the full light-cone Hamiltonian as obtained from the QCD-Lagrangian in the light-cone gauge, with zero-modes disregarded. In consequence, the pion is treated on the same footing as all other pseudo-scalar and pseudo-vector mesons.

The model should be contrasted to Lattice Gauge Calculations, see for example [1]. It is not generally known that LGC's have considerable uncertainty to extrapolate their results down to such light mesons as a pion. It is also not generally known that lattice gauge calculations get *always strict and linear confinement* even for QED, where we know the ionization threshold. The 'breaking of the string', or in a more physical language, the ionization threshold is one of the hot topics at the lattice conferences [2]. Moreover, in order to get the size of the pion, thus the form factor, another generation of computers is required, as well as physicists to run them.

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The model should be contrasted also to phenomenological approaches. They usually do not address to get the pion. For the heavy mesons, as well for light baryons [3], where they are so successful, phenomenological model have quite many parameters, in any case more that the above canonical ones. A detailed comparison and systematic discussion of the bulky literature can however be postponed, until we are ready to solve the full Eq.(11).

The model should be contrasted, finally, to Nambu-Jona-Lasinio-like models which are so successful in accounting for isospin-aspects. I cannot quote the huge body of literature but mention in passing that the NJL-models are not renormalizable, that NJL has no relation to QCD, and that NJL deals mostly with the very light mesons. There is no way to treat the heavy flavors, see also [4].

2 Motivation

The light-cone approach to the bound-state problem in gauge theory [5] aims at solving $H_{LC}|\Psi\rangle = M^2|\Psi\rangle$. If one disregards possible zero modes and works in the light-cone gauge, the (light-cone) Hamiltonian H_{LC} is a well defined Fock-space operator and given in [5]. Its eigenvalues are the invariant mass-squares M^2 of physical particles associated with the eigenstates $|\Psi\rangle$. In general, they are superpositions of all possible Fock states with its many-particle configurations. For a meson, for example, holds

$$\begin{aligned}|\Psi_{\rm meson}\rangle = &\sum_i \Psi_{q\bar q}(x_i,\mathbf{k}_{\perp_i},\lambda_i)|q\bar q\rangle \quad + \sum_i \Psi_{gg}(x_i,\mathbf{k}_{\perp_i},\lambda_i)|gg\rangle \\ &+ \sum_i \Psi_{q\bar q g}(x_i,\mathbf{k}_{\perp_i},\lambda_i)|q\bar q g\rangle + \sum_i \Psi_{q\bar q q\bar q}(x_i,\mathbf{k}_{\perp_i},\lambda_i)|q\bar q q\bar q\rangle + \dots .\end{aligned}$$

If all wave functions like $\Psi_{q\bar q}$ or Ψ_{gg} are available, one can analyze hadronic structure in terms of quarks and gluons [5].

For example, one can calculate the space-like form factor of a hadron quite straightforwardly. As illustrated in Fig. 1, it is just a sum of overlap integrals analogous to the corresponding non-relativistic formula [5]:

$$F(q^2) = \sum_{n,\lambda_i}\sum_a e_a \int \overline{\prod_i} \frac{dx_i\, d^2\mathbf{k}_{\perp i}}{16\pi^3}\, \psi_n^{(\Lambda)*}(x_i,\boldsymbol{\ell}_{\perp i},\lambda_i)\, \psi_n^{(\Lambda)}(x_i,\mathbf{k}_{\perp i},\lambda_i). \quad (1)$$

Here e_a is the charge of the struck quark, $\Lambda^2 \gg \mathbf{q}_\perp^2$, and

$$\boldsymbol{\ell}_{\perp i} \equiv \begin{cases} \mathbf{k}_{\perp i} - x_i\mathbf{q}_\perp + \mathbf{q}_\perp & \text{for the struck quark} \\ \mathbf{k}_{\perp i} - x_i\mathbf{q}_\perp & \text{for all other partons,}\end{cases}$$

with $\mathbf{q}_\perp^2 = Q^2 = -q^2$. All of the various form factors of hadrons with spin can be obtained by computing the matrix element of the plus current between states of different initial and final hadron helicities.

3 The method of iterated resolvents

Because of the inherent divergences in a gauge field theory, the QCD-Hamiltonian in 3+1 dimensions must be regulated from the outset. One of the few practical ways is vertex regularization [5,6], where every Hamiltonian matrix element,

particularly those of the vertex interaction (the Dirac interaction proper), is multiplied with a convergence-enforcing momentum-dependent function. It can be viewed as a form factor [5]. The precise form of this function is unimportant here, as long as it is a function of a cut-off scale (Λ).

Perhaps one can attack the problem of diagonalizing the (light-cone) Hamiltonian H_{LC} by the discretized light cone quantization (DLCQ), see for example [7]. But, alternatively, it might be better to reduce the many-body problem behind a field theory to an effective one-body problem. The derivation of the effective interaction becomes then the key issue. By definition, an effective Hamiltonian acts only in the lowest sector of the theory (here: in the Fock space of one quark and one anti-quark). And, again by definition, it has the same eigenvalue spectrum as the full problem. I have derived such an effective interaction by the method of iterated resolvents [6], that is by systematically expressing the higher Fock-space wave functions as functionals of the lower ones. In doing so the Fock-space is not truncated and all Lagrangian symmetries are preserved. The projections of the eigenstates onto the higher Fock spaces can be retrieved systematically from the $q\bar{q}$-projection, with explicit formulas given in [8].

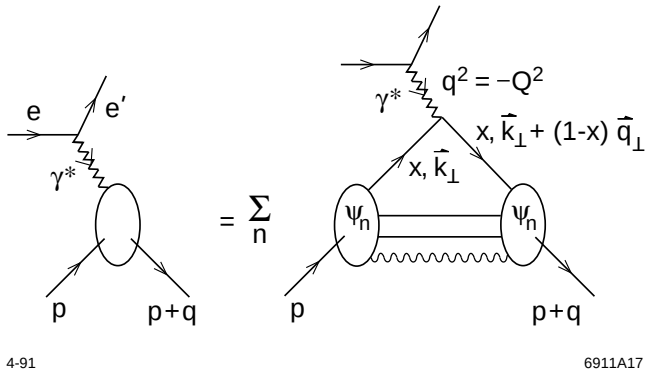

Fig. 1. Calculation of the form factor of a bound state from the convolution of light-cone Fock amplitudes. The result is exact if one sums over all ψ_n.

Let me sketch the method briefly. Details may be found in [6,8]. DLCQ with its periodic boundary conditions has the advantage that the LC-Hamiltonian is a matrix with a finite number of Fock-space sectors, which we denumerate by n, with $1 < n \leq N$. The so called harmonic resolution $K = LP^+/(2\pi)$ acts as a natural cut-off of the particle number. As shown in Figure 2, $K = 3$ allows for $N = 8$, and $K = 4$ for $N = 13$ Fock-space sectors, for example. The Hamiltonian matrix is sparse: Most of the matrix elements are zero, particularly if one includes only the vertex interaction V. For n sectors, the eigenvalue problem in terms of block matrices reads

$$\sum_{j=1}^{n} \langle i|H_n(\omega)|j\rangle\langle j|\Psi(\omega)\rangle = E(\omega)\ \langle i|\Psi(\omega)\rangle, \qquad \text{for } i = 1, 2, \ldots, n. \tag{2}$$

n	Sector	1 $q\bar{q}$	2 gg	3 $q\bar{q}\,g$	4 $q\bar{q}\,q\bar{q}$	5 $gg\,g$	6 $q\bar{q}\,gg$	7 $q\bar{q}\,q\bar{q}\,g$	8 $q\bar{q}\,q\bar{q}\,q\bar{q}$	9 $gg\,gg$	10 $q\bar{q}\,ggg$	11 $q\bar{q}\,q\bar{q}\,gg$	12 $q\bar{q}\,q\bar{q}\,q\bar{q}\,g$	13 $q\bar{q}\,q\bar{q}\,q\bar{q}\,q\bar{q}$
1	$q\bar{q}$		·		·	·	·	·	·	·	·	·	·	·
2	gg	·			·		·	·	·	·	·	·	·	·
3	$q\bar{q}\,g$					·		·	·	·	·	·	·	·
4	$q\bar{q}\,q\bar{q}$	·	·			·	·		·	·	·	·	·	·
5	$gg\,g$	·		·	·			·	·		·	·	·	·
6	$q\bar{q}\,gg$	·	·		·				·	·		·	·	·
7	$q\bar{q}\,q\bar{q}\,g$	·	·	·		·				·	·		·	·
8	$q\bar{q}\,q\bar{q}\,q\bar{q}$	·	·	·	·	·	·			·	·	·		·
9	$gg\,gg$	·	·	·	·		·	·	·			·	·	·
10	$q\bar{q}\,ggg$	·	·	·	·	·			·				·	·
11	$q\bar{q}\,q\bar{q}\,gg$	·	·	·	·	·	·		·	·				·
12	$q\bar{q}\,q\bar{q}\,q\bar{q}\,g$	·	·	·	·	·	·	·		·	·			
13	$q\bar{q}\,q\bar{q}\,q\bar{q}\,q\bar{q}$	·	·	·	·	·	·	·	·	·	·	·		

Fig. 2. The Hamiltonian matrix for a meson. The matrix elements are represented by energy diagrams. Only vertex diagrams V are shown. Zero matrices are marked by a dot $(\cdot)$.

I can always invert the quadratic block matrix of the Hamitonian in the last sector to define the n-space resolvent G_n, that is

$$G_n(\omega) = \frac{1}{\omega - H_n(\omega)}. \tag{3}$$

Using G_n, I can express the projection of the eigenfunction in the last sector by

$$\langle n|\Psi(\omega)\rangle = G_n(\omega) \sum_{j=1}^{n-1} \langle n|H_n(\omega)|j\rangle \, \langle j|\Psi(\omega)\rangle, \tag{4}$$

and substitute it in Eq.(2). I then get an effective Hamiltonian where the number is sectors is diminished by 1:

$$H_{n-1}(\omega) = H_n(\omega) + H_n(\omega) G_n(\omega) H_n(\omega). \tag{5}$$

This is a recursion relation, which can be repeated until one arrives at the $q\bar{q}$-space. The fixed point equation $E(\omega) = \omega$ determines then all eigenvalues.

For the block matrix structure as in Figure 2, with its many zero matrices, the reduction is particularly easy and transparent. For $K = 3$ one has the following sequence of effective interactions:

$$H_8 = T_8, \quad H_7 = T_7 + VG_8V, \quad H_6 = T_6 + VG_7V, \quad H_5 = T_5 + VG_6V. \tag{6}$$

The remaining ones get more complicated, *i.e.*

$$H_4 = T_4 + VG_7V + VG_7VG_6VG_7V, \tag{7}$$
$$H_3 = T_3 + VG_6V + VG_6VG_5VG_6V + VG_4V, \tag{8}$$
$$H_2 = T_2 + VG_3V + VG_5V, \tag{9}$$
$$H_1 = T_1 + VG_3V + VG_3VG_2VG_3V. \tag{10}$$

For $K = 4$, the effective interactions in Eq.(6) are different, see for example [8], but it is quite remarkable, that they are the same for the remainder, particularly Eq.(10). In fact, the effective interactions in sectors 1-4 are independent of K: The *continuum limit* $K \to \infty$ *is then trivial*, and will be taken in the sequel.

In the continuum limit, the effective interaction in the $q\bar{q}$-space has thus two contributions: A flavor-conserving piece $U_{\mathit{eff-conser}} = VG_3V$ and a flavor-changing piece $U_{\mathit{eff-change}} = VG_3VG_2VG_3V$. The latter cannot get active in flavor-off-diagonal mesons. Notice that these expressions are an exact result.

4 The eigenvalue equation in the $q\bar{q}$-space

After some approximations [6], the effective one-body equation for flavor off-diagonal mesons (mesons with a different flavor for quark and anti-quark), becomes quite simple:

$$M^2\langle x, \mathbf{k}_\perp; \lambda_1, \lambda_2|\psi\rangle = \left[\frac{\overline{m}_1^2 + \mathbf{k}_\perp^2}{x} + \frac{\overline{m}_2^2 + \mathbf{k}_\perp^2}{1-x}\right]\langle x, \mathbf{k}_\perp; \lambda_1, \lambda_2|\psi\rangle \tag{11}$$
$$-\frac{1}{3\pi^2}\sum_{\lambda_q', \lambda_2'}\int \frac{dx' d^2\mathbf{k}_\perp'\, R(x', \mathbf{k}_\perp'; \Lambda)}{\sqrt{x(1-x)x'(1-x')}}\frac{\overline{\alpha}}{Q^2}\langle \lambda_1, \lambda_2|S|\lambda_1', \lambda_2'\rangle\, \langle x', \mathbf{k}_\perp'; \lambda_1', \lambda_2'|\psi\rangle.$$

Here, M^2 is the eigenvalue of the invariant-mass squared. The associated eigenfunction $\psi \equiv \Psi_{q\bar{q}}$ is the probability amplitude $\langle x, \mathbf{k}_\perp; \lambda_1, \lambda_2|\psi\rangle$ for finding a quark with momentum fraction x, transversal momentum $\mathbf{k}_\perp$ and helicity λ_1, and correspondingly the anti-quark with $1-x$, $-\mathbf{k}_\perp$ and λ_2. The $\overline{m}_1$ and $\overline{m}_2$ are (effective) quark masses and $\overline{\alpha}$ is the (effective) coupling constant. The mean Feynman-momentum transfer of the quarks is denoted by

$$Q^2 \equiv Q^2(x, \mathbf{k}_\perp; x', \mathbf{k}_\perp') = -\frac{1}{2}\left[(k_1 - k_1')^2 + (k_2 - k_2')^2\right], \tag{12}$$

and the spinor factor $S = S(x, \mathbf{k}_\perp; x', \mathbf{k}_\perp')$ by

$$\langle \lambda_1, \lambda_2|S|\lambda_1', \lambda_2'\rangle = [\overline{u}(k_1, \lambda_1)\gamma^\mu u(k_1', \lambda_1')]\ [\overline{v}(k_2', \lambda_2')\gamma_\mu v(k_2, \lambda_2)]\,. \tag{13}$$

The regulator function $R(x', \mathbf{k}_\perp'; \Lambda)$ restricts the range of integration as function of some mass scale Λ. I happen to choose here a soft cut-off (see below), in contrast to the previous sharp cut-off [9]. Note that Eq.(11) is a fully relativistic equation. I have derived the same effective interaction also with the method of Hamiltonian flow equations, see [10].

The effective quark masses $\overline{m}_1$ and $\overline{m}_2$ and the effective coupling constant $\overline{\alpha}$ depend, in general, on Λ. In the spirit of renormalization theory they are renormalization constants, subject to be determined by experiment, and hence-forward will be denoted by m_1, m_2, and α, respectively. In next-to-lowest order of approximation the coupling constant becomes a function of the momentum transfer, $\overline{\alpha} \longrightarrow \overline{\alpha}(Q;\Lambda)$, with the explicit expression given in [6].

5 The $\uparrow\downarrow$-model and its renormalization

It might be to early for solving Eq.(11) numerically in full glory like in Ref.[9]. Rather should I try to dismantle the equation of all irrelevant details, and develop a simple model.

The quarks are at relative rest, when $\mathbf{k}_\perp = 0$ and $x = \overline{x} \equiv m_1/(m_1+m_2)$. For very small deviations from these equilibrium values the spinor matrix is proportional to the unit matrix, with

$$\langle \lambda_1, \lambda_2 | S | \lambda_1' \lambda_2' \rangle \sim 4 m_1 m_2 \; \delta_{\lambda_1,\lambda_1'} \; \delta_{\lambda_2,\lambda_2'}, \tag{14}$$

for details see [10]. For very large deviations, particularly for $\mathbf{k}_\perp'^2 \gg \mathbf{k}_\perp^2$, holds

$$Q^2 \simeq \mathbf{k}_\perp'^2, \qquad \text{and} \qquad \langle \uparrow\downarrow | S | \uparrow\downarrow \rangle \simeq 2\mathbf{k}_\perp'^2. \tag{15}$$

Both extremes are combined in the "$\uparrow\downarrow$-model" [10]:

$$\frac{S}{Q^2} \equiv \frac{4 m_1 m_2}{Q^2} + 2 \Longrightarrow \frac{4 m_1 m_2}{Q^2} + 2R(\Lambda, Q), \quad \text{with} \quad R(\Lambda, Q) = \frac{\Lambda^2}{\Lambda^2 + Q^2}. \tag{16}$$

It interpolates between two extremes: For small momentum transfer, the '2' generated by the hyperfine interaction is unimportant and the dominant Coulomb aspects of the first term prevail. For large momentum transfers the Coulomb aspects are unimportant and the hyperfine interaction dominates.

The model over-emphasizes many aspects: It neglects the momentum dependence of the Dirac spinors and thus the spin-orbit interaction; it also neglects the momentum dependence of the spin-spin interaction. But the 2 creates havoc: Its Fourier transform is a Dirac-delta function with all its consequences in a bound-state equation.

Here is an interesting point: One is familiar with field theoretic divergences like the effective masses and the effective coupling constant. One is used less to "divergences" residing in a finite number 2. They must be regulated also, and renormalized.

In consequence I replace Eq.(11) by

$$\begin{aligned} M^2 \psi(x, \mathbf{k}_\perp) &= \left[\frac{m_1^2 + \mathbf{k}_\perp^2}{x} + \frac{m_2^2 + \mathbf{k}_\perp^2}{1-x} \right] \psi(x, \mathbf{k}_\perp) \\ &- \frac{\alpha}{3\pi^2} \int \frac{dx' d^2\mathbf{k}_\perp'}{\sqrt{x(1-x)x'(1-x')}} \left(\frac{4 m_1 m_2}{Q^2} + \frac{2\Lambda^2}{\Lambda^2 + Q^2} \right) \psi(x', \mathbf{k}_\perp'), \end{aligned} \tag{17}$$

where $\psi(x, \mathbf{k}_\perp) \equiv \langle x, \mathbf{k}_\perp; \uparrow, \downarrow | \psi \rangle$.

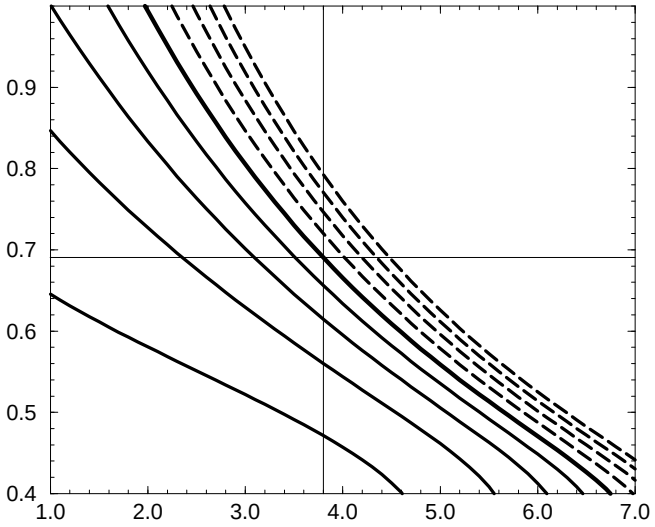

Fig. 3. Nine contours $\alpha_n(\Lambda)$ are plotted versus Λ/Δ, from bottom to top with $n = 4, 3, \cdots, -3, -4$. The contours are obtained by $M_0^2(\alpha, \Lambda) = n\Delta^2 + M_\pi^2$. The thick contour $n = 0$ refers to the pion with $M_0^2 = M_\pi^2$. Mass unit is $\Delta = 350$ MeV.

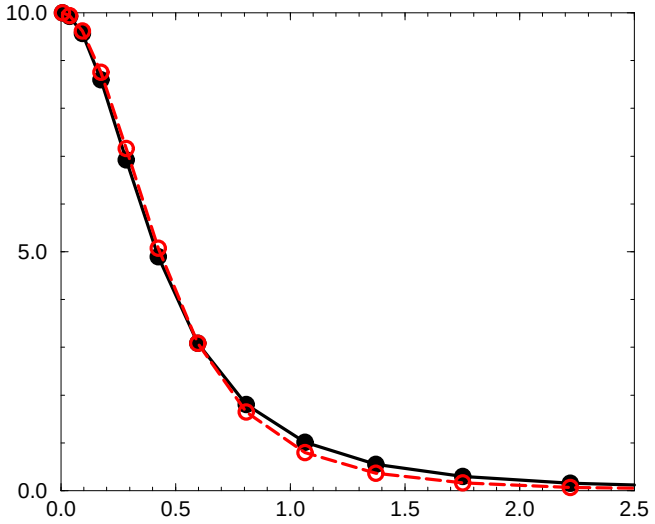

Fig. 4. The pion wave function $\Phi(p)$ is plotted versus $p/(1.338m)$ in an arbitrary normalization. The filled circles indicate the numerical results, the open circles the analytical function $\Phi_a(p)$.

For equal quark masses $m_1 = m_2 = m$, the eigenvalues depend now on three parameters, the canonical α and m, and the regularization scale Λ. The dependence can be quite strong as seen in Figure 3. There, the lowest mass-squared eigenvalue is plotted versus α and Λ for the fixed quark mass $m = 406$ MeV.

Since Λ is an unphysical parameter, its impact must be removed by renormalization. Recently, much progress was made on this question [11,12]: Adding

	$\overline{u}$	$\overline{d}$	$\overline{s}$	$\overline{c}$	$\overline{b}$
u		768	871	2030	5418
d	140		871	2030	5418
s	494	494		2124	5510
c	1865	1865	1929		6580
b	5279	5279	5338	6114	

Table 1. The calculated mass eigenvalues in MeV. Those for singlet-1s states are given in the lower, those for singlet-2s states in the upper triangle.

to $R(\Lambda, Q)$ a counterterm $C(\Lambda, Q)$ and requiring that the sum $\widetilde{R}(\Lambda, Q) = R(\Lambda, Q) + C(\Lambda, Q)$, and thus $M^2(\Lambda; \alpha, m)$, be independent of Λ, determines $C(\Lambda, Q)$. One remains with $\widetilde{R}(\Lambda, Q) = \mu^2/(\mu^2 + Q^2)$. In line with renormalization theory, one then can go to the limit $\Lambda \longrightarrow \infty$. In turn, μ becomes one of the parameters of the theory to be determined by experiment.

6 Determining the canonical parameters

The theory has seven canonical parameters which have to be determined by experiment: α, μ and the 5 flavor masses m_f (if we disregard the top). How can we determine them?

The problem is not completely trivial. Let me restrict first to the light flavors. With $m_u = m_d = m$, one has 3 parameters, and in consequence needs 3 experimental data. The pion mass $M_\pi = 140$ MeV and the rho mass $M_\rho = 768$ MeV do not suffice. One needs a third datum, the mass of an exited pion, for example.

Since the mass of the excited pion $\pi^\pm$ is not known with sufficient experimental precision, and since the $\uparrow\downarrow$-model might be to crude a model to begin with, I choose here $m_u = m_d = 406$ MeV and $M_{\pi^*} = M_\rho = 768$ MeV, for no good reason other than convenience. These assumptions are less stringent than they sound, by two reasons. First, the rho has a mass less than $2m$ and should be a true bound state. Second, the Yukawa potential in Eq.(17) acts like a Dirac-delta function in pairing theory for example: it pulls down essentially one state, the pion, but leaves the other states unchanged.

We thus remain with the two parameters α and μ. Each of the two equations, $M_0^2(\alpha, \mu) = M_\pi^2$ and $M_1^2(\alpha, \mu) = M_{\pi^*}^2$ determine a function $\alpha(\mu)$. Their intersection point determines the required solution, which is $\alpha = 0.761$ and $\mu = 1.15$ GeV [11]. These differ marginally from the previous analysis [10], with $\mu = 1.33$ GeV, for which Figure 3 yields $\alpha = 0.6904$. Once I have the up and down mass, the strange, charm and bottom quark mass can be determined by reproducing the masses of the $K,^-$ D^0 and $B,^-$ respectively. The parameters in the $\uparrow\downarrow$-model can thus be taken as

$$\begin{array}{lllllll} \alpha & & \mu & m_u = m_d & m_s & m_c & m_b \\ 0.6904 & & 1.33\ \text{GeV} & 406\ \text{MeV} & 508\ \text{MeV} & 1666\ \text{MeV} & 5054\ \text{MeV}. \end{array} \qquad \text{(I)}$$

	$\overline{u}$	$\overline{d}$	$\overline{s}$	$\overline{c}$	$\overline{b}$
u		768	892	2007	5325
d	140		896	2010	5325
s	494	498		2110	—
c	1865	1869	1969		—
b	5278	5279	5375	—	

Table 2. Empirical masses of the flavor-off-diagonal physical mesons in MeV. Vector mesons are given in the upper, scalar mesons in the lower triangle.

	$M_{f\bar{f}}$	M	M_{exp}
π^0	140	140	135
η	140	485	549
η'	661	958	958
η_c	2870	2915	2980
η_b	8922	8935	—

Table 3. Flavor-diagonal mass eigenvalues in the FM-model for pseudo-scalar mesons with the parameter $a = (491\ \mathrm{MeV})^2$.

	$M_{f\bar{f}}$	M	M_{exp}
ρ^0	768	768	768
ω	768	832	782
Φ	973	1019	1019
J/Ψ	3231	3242	3097
Υ	9822	9825	9460

Table 4. Flavor-diagonal mass eigenvalues in the FM-model for pseudo-vector mesons with the parameter $a = (255\ \mathrm{MeV})^2$.

7 The masses of the physical mesons

Solving Eq.(17) with the parameters of Eq.(I) generates the mass^2-eigenvalues of all flavor off-diagonal pseudo-scalar mesons. They are compiled in Table 1. The corresponding wave functions are also available, but not shown here. In view of the simplicity of the model, the agreement with the empirical values [13] in Table 2 is remarkable. The mass of the first excited states in Table 1 correlates astoundingly well with the experimental mass of the pseudo-vector mesons, as given in Table 2. Notice that all numbers in Tables 1 and 2 are rounded for convenience.

Since the $\uparrow\downarrow$-model in Eq.(17) does not expose confinement one should emphasize that the difference between the physical meson masses in Table 1 and the sum of the bare quark masses is larger than a pion mass. One could call this a kind of practical confinement.

What about the flavor diagonal mesons?– They cannot be a solution to Eqs.(11) or (17), since the flavor-changing piece of the full effective interaction can generate matrix elements between different flavors. Thus far the precise structure of the flavor changing part $U_{eff-change} = VG_3VG_2VG_3V$ has not been analyzed in detail, because it requires a considerable effort.

Rather, the following flavor-mixing model (FM-model [15]) has been investigated. In the FM-model, the full effective Hamiltonian including its flavor mixing is reduced to the lowest $f\bar{f}$-states, *i.e.* to

$$\langle f\bar{f}|H_{eff}|f'\bar{f}'\rangle = \langle\psi_{f\bar{f}}|T + VG_3V + VG_3VG_2VG_3V|\psi_{f'\bar{f}'}\rangle = M^2_{ff'}\,\delta_{ff'} + a. \quad (18)$$

Conceptually, it is important that $M^2_{ff'}$ is the eigenvalue of Eq.(17). The flavor-mixing matrix element $a = \langle\psi_{f\bar{f}}|VG_3VG_2VG_3V|\psi_{f'\bar{f}'}\rangle$ depends on the flavors and could be calculated with a solution of Eqs.(11) or (17). In the crude FM-model, however, it is treated as a flavor-independent parameter to be fixed by experiment. For 5 flavors one faces thus the numerical diagonalization of a 5×5 matrix.

The parameter a for pseudo-scalar mesons was fitted to the mass of the η', and for pseudo-vector mesons to the Φ, with the results compiled in Tables 3 and 4 . Three facts, however, one gets for free: First, the π^0 is degenerate in mass with $\pi^\pm$, as well as the ρ^0 with $\rho^\pm$. That they form isospin-triplets is a non-trivial aspect of QCD. Second, both the η–η' and the ω–Φ splitting are in the right bull park. Third, that the wave functions of the π^0, η and η' have very much SU(3)-character [15] is even less trivial from the point of view of QCD.

8 The wavefunction of the pion

For carrying out this programme in practice, I need an efficient tool for solving Eq.(17). Such one has been developed recently [10]. I outline in short the procedure for the special case $m_1 = m_2 = m$. I change integration variables from x to k_z by substituting

$$x(k_z) = \frac{1}{2} + \frac{k_z}{2\sqrt{m^2 + \mathbf{k}_\perp^2 + k_z^2}}, \qquad \psi(x,\mathbf{k}_\perp) = \frac{\sqrt{1 + (\mathbf{k}_\perp^2 + k_z^2)/m^2}}{\sqrt{x(1-x)}}\,\phi(k_z,\mathbf{k}_\perp). \quad (19)$$

The variables $(k_z,\mathbf{k}_\perp)$ are collected in a 3-vector $\mathbf{p}$ and Eq.(17) becomes

$$\left[M^2 - 4m^2 - 4\mathbf{p}^{\,2}\right]\phi(\mathbf{p}) = -\frac{4\alpha}{3\pi^2}\int d^3\mathbf{p}\,'\left(\frac{2m}{(\mathbf{p}-\mathbf{p}\,')^2} + \frac{1}{m}\frac{\mu^2}{\mu^2 + (\mathbf{p}-\mathbf{p}\,')^2}\right)\phi(\mathbf{p}\,'). \quad (20)$$

For the present purpose it suffices to restrict to spherically symmetric s-states $\phi(\mathbf{p}) = \phi(p)$ and to apply Gaussian quadratures with 16 points. On an alpha work station it takes a couple of micro-seconds to solve this equation for a particular case. The resulting numerical wavefunction $\phi(p)$ is displayed in Figure 4 and compared with

$$\Phi_a(p) = \mathcal{N}\left(1 + \frac{p^2}{p_a^2}\right)^{-2}, \qquad \text{with} \quad p_a = 1.338\ m. \quad (21)$$

Such an analytical form is convenient in many applications. For example, the light-cone wavefunction $\psi(x, \mathbf{k}_\perp)$ can be obtained in closed form by Eq.(19), *i.e.*

$$\psi(x, \mathbf{k}_\perp) = \frac{\mathcal{N}}{\sqrt{x(1-x)}} \frac{\left(1 + \dfrac{m^2\,(2x-1)^2 + \mathbf{k}_\perp^2}{4x(1-x)\,m^2}\right)^{\frac{1}{2}}}{\left(1 + \dfrac{m^2\,(2x-1)^2 + \mathbf{k}_\perp^2}{4x(1-x)\,p_a^2}\right)^{2}}. \tag{22}$$

I can use this to calculate the form factor from Eq.(1), and thus the exact root-mean-square radius [14], even in closed form with $\langle r^2\rangle = -6\ dF_2(Q^2)/dQ^2\big|_{Q^2=0}$:

$$\langle r^2\rangle = \frac{3}{4p_a^2}\,\frac{34 + 37s^2 - 41s^4 + 15s^6 + 3b(s)(-8 - 16s^2 + 21s^4 - 17s^6 + 5s^8)}{5(s^2-1)[4 - 4s^2 + 3s^4 + 3b(s)s^4(-2+s^2)]}, \tag{23}$$

with $s = m/p_a$ and the abbreviation $b(s) = \arctan(\sqrt{s^2-1})/\sqrt{s^2-1}$. The size of the $q\bar{q}$ wavefunction is thus $\langle r^2\rangle^{\frac{1}{2}} = 0.33$ fm, half as large as the empirical value $\langle r^2\rangle^{\frac{1}{2}}_{exp} = 0.67$ fm.

This completes the goal: I have a pion with the correct mass, and I have an analytic expression for its light-cone wave function. Eq.(22) could be used thus as a baseline for calculating the higher Fock-space amplitudes, as explained in [8].

It might well be that the wavefunction in Eq.(22) is consistent with Ashery's experiment [16].

9 Conclusions

The proposed pion of the $\uparrow\downarrow$-model is rather different from the pions in the literature. I have found no evidence that the vacuum condensates are important, but I conclude that the pion is describable by a QCD-inspired theory: The very large coupling constant in conjunction with a very strong hyperfine interaction makes it a ultra strongly bounded system of constituent quarks. More then 80 percent of the constituent quark mass is eaten up by binding effects. No other physical system has such a property.

The effective Bohr momentum of the constituents in the pion turns out as $p_a = 1.338\,m$, see Eq.(21). The mean momentum of the constituents is thus 40 percent larger than their mass, which means that they move highly relativistically quite in contrast with the constituents of atoms or nuclei. No wonder that potential models thus far have failed for the pion. One might mention that lattice gauge calculations use all the computer power in this world to generate the potential energy of the quarks and then one uses a *non-relativistic* Schrödinger equation to calculate the bound states.

All this is to be confronted with the present oversimplified $\uparrow\downarrow$-model, which however has the virtue to calculate the pion and other physical mesons by a covariant and relativistically correct theory. To the best of my knowledge there is no other model which can describe *all mesons* quantitatively from the π up to the Υ from a common point of view, which here is QCD.

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A three-body confinement force in constituent quark models ⋆

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Abstract. We discuss the role of a three-body colour confinement interaction introduced on algebraic grounds and present some of its implications for the spectra of baryons, tetraquarks and six-quark systems within a simple quark model.

In quark models two distinct types of three-body forces have been introduced so far, namely long-range, confinement forces, as e.g. in [1] or short-range forces, associated to the instanton 't Hooft interaction, as e.g. in [2].

Here we shall discuss the implications on baryon spectroscopy of the long-range confinement forces only. We are not presently concerned with the origin of these forces which is still controversial, as lattice calculations suggest [3]. We shall use an algebraic approach inspired by Ref. [4], based on the invariant operators of SU(3).

Let us consider the Hamiltonian

$$H = T + V_{2b} + V_{3b} \ , \tag{1}$$

where T is the kinetic energy and V_{2b} a 2-body confinement interaction of the form

$$V_{2b} = \sum_{i<j} V_{ij} \ (c_1 + \frac{4}{3} + F_i^a F_j^a) \ , \tag{2}$$

where $F_i^a = \frac{1}{2}\lambda_i^a$ is the colour charge operator of the quark i and c_1 an arbitrary constant which we set equal to 1 as in Ref. [4]. For simplicity we take

$$V_{ij} = \frac{1}{2} \ m\omega^2 \ (r_i - r_j)^2 \ . \tag{3}$$

V_{3b} is the 3-body confinement interaction

$$V_{3b} = V_{ijk} = \mathcal{V}_{ijk} \mathcal{C}_{ijk} \ , \tag{4}$$

with

$$\mathcal{V}_{ijk} = \frac{1}{2} \ c \ m\omega^2 \ [(r_i - r_j)^2 + (r_j - r_k)^2 + (r_k - r_i)^2] \ , \tag{5}$$

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where c is a strength parameter and $\mathcal{C}_{ijk}$ a colour operator of type

$$\mathcal{C}_{ijk} = d^{abc}\, F_i^a\, F_j^b\, F_k^c\ . \tag{6}$$

The coefficients d^{abc} are some real constants, symmetric under any permutation of indices (see e.g. Ref. [5], chapter 8). Performing integration in the colour space and expressing H in terms of the internal coordinates $\boldsymbol{\rho} = (\mathbf{r}_1 - \mathbf{r}_2)/\sqrt{2}$ and $\boldsymbol{\lambda} = (\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3)/\sqrt{6}$, we have:

$$H = 3m - \frac{\hbar^2}{2m}(\nabla_\rho^2 + \nabla_\lambda^2) + \frac{3}{2}m\omega^2\chi_i(\rho^2 + \lambda^2)\ , \tag{7}$$

with

$$\chi_i = \begin{cases} \dfrac{5}{3} + \dfrac{10}{9}c & \text{i=1 (singlet)} \\ \dfrac{13}{6} - \dfrac{5}{36}c & \text{i=8 (octet)} \\ \dfrac{8}{3} + \dfrac{1}{9}c & \text{i=10 (decuplet)} \end{cases} \tag{8}$$

In the expressions of χ_i ($i = 1, 8$ or 10), the first and second terms stem from the colour part of V_{2b} and V_{3b} respectively. The energies E_1, E_8 and E_{10} of the singlet, octet and decuplet states in a 3q system are

$$E_1 = 3m + 3\hbar\omega\sqrt{3\chi_1}\ , \tag{9}$$

$$E_8 = 3m + 4\hbar\omega\sqrt{3\chi_8}\ , \tag{10}$$

$$E_{10} = 3m + 5\hbar\omega\sqrt{3\chi_{10}}\ . \tag{11}$$

The baryon spectrum is correctly described provided [4]

$$-\frac{3}{2} < c < \frac{2}{5}\ . \tag{12}$$

The lower limit comes from the stability constraint $\chi_1 > 0$ and the upper limit from imposing the right sequence in the spectrum, i.e. from the requirement $\chi_8 > \chi_1$. The closer c is to the lower limit, the larger is the gap between the colour octet and singlet states. To see this, let us take c = -1.43. There is of course some arbitrariness in choosing m and $\hbar\omega$. As typical values for quark models we take $m = 0.340$ GeV and $\hbar\omega = 0.6$ GeV [6]. This implies an octet-singlet gap $\Delta E = E_8 - E_1 \approx 5.5$ GeV. For $c = 0$ (no three-body force) one would have $\Delta E = 3.5\hbar\omega \approx 2.1$ GeV. Therefore the gap is increased considerably by a three-body force with a strong negative strength. This is a desired feature for quark models with three valence quarks (no gluons). In the same way one can show that the decuplet state is located above the octet with quite a large gap for a limiting value of c.

Let us now consider tetraquarks, i.e. $q^2\overline{q}^2$ systems and denote the quarks by 1 and 2 and the antiquarks by 3 and 4. One can first form $q\overline{q}$ pairs which are next coupled to colour singlets. These states are either singlet-singlet states $|1_{13}1_{24}\rangle$ or octet-octet states $|8_{13}8_{24}\rangle$ (see Ref. [5], chapter 10). One can have a three-body interaction acting in a $q^2\overline{q}$ subsystem as

$$\overline{\mathcal{C}}_{ijk} = -d^{abc}\, F_i^a\, F_j^b\, \overline{F}_k^c\ , \tag{13}$$

or a three-body interaction acting in a $q\overline{q}^2$ subsystem as

$$\overline{\mathcal{C}}_{ijk} = d^{abc}\ F_i^a\ \overline{F}_j^b\ \overline{F}_k^c\ , \tag{14}$$

where

$$\overline{F}_i^a = -\frac{1}{2}\lambda_i^{a*}\ , \tag{15}$$

is the charge operator of an antiquark. The operators (13) or (14) between these tetraquark states have the same eigenvalue, which is calculated in two steps. First, one evaluates the eigenvalue between the states $|\bar{3}_{12}3_{34}\rangle$ and $|6_{12}\bar{6}_{34}\rangle$, where the two quarks couple either to a $\overline{3}$ or a 6 state and the antiquarks to a 3 or a $\overline{6}$ state. One obtains $-5/9$ for the $|\bar{3}_{12}3_{34}\rangle$ state and $5/18$ for $|6_{12}\bar{6}_{34}\rangle$. The physically relevant states $|1_{13}1_{24}>$ and $|8_{13}8_{24}>$ are then defined by the transformations (see e.g. Ref. [7]):

$$|1_{13}1_{24}>= \sqrt{\frac{1}{3}}|\overline{3}_{12}3_{34}> + \sqrt{\frac{2}{3}}|6_{12}\overline{6}_{34}>\ , \tag{16}$$

$$|8_{13}8_{24}>= -\sqrt{\frac{2}{3}}|\overline{3}_{12}3_{34}> + \sqrt{\frac{1}{3}}|6_{12}\overline{6}_{34}>\ . \tag{17}$$

Thus one gets:

$$\langle 1_{13}1_{24}|\overline{\mathcal{C}}_{123}|1_{13}1_{24}\rangle \propto [\frac{1}{3}(-\frac{5}{9}) + \frac{2}{3}\ \frac{5}{18}]\ c = 0\ , \tag{18}$$

and

$$\langle 8_{13}8_{24}|\overline{\mathcal{C}}_{123}|8_{13}8_{24}\rangle \propto [\frac{2}{3}(-\frac{5}{9}) + \frac{1}{3}\ \frac{5}{18}]\ c = -\frac{5}{18}\ c\ , \tag{19}$$

which shows that with a negative c one raises the expectation value of the octet-octet above the singlet-singlet state more than with $c = 0$. This implies that in the presence of a 3-body confinement interaction with $c < 0$ the coupling between octet-octet and singlet-singlet states due to a hyperfine splitting will be diminished, which amounts to make a ground state tetraquark less stable. This seems to be consistent with the experimental observation that no stable tetraquark system has been seen so far.

The q^6 systems are important for the NN problem. Here we discuss the sector IS = (01) or (10). It is well known that the physical NN state is a combination of three symmetry states containing the orbital configurations $[6]_O$ and $[42]_O$, as shown for example in [8]. In fact the three symmetry states allowed by the Pauli principle can be combined into the NN and $\Delta\Delta$ states and the unphysical hidden-colour CC state. The latter has an important role at short separation between the 3q clusters. Using 3-body fractional parentage coefficients, one can calculate the matrix elements of the three-body force (4)-(6) in the basis of the states $|NN>$,

$|\Delta\Delta>$ and $|CC>$. This gives rise to the following matrix:

	NN	$\Delta\Delta$	CC
NN	$\frac{28}{81}c$	$\frac{38\sqrt{5}}{405}c$	$\frac{38\sqrt{5}}{135}c$
$\Delta\Delta$	$\frac{38\sqrt{5}}{405}c$	$\frac{121}{405}c$	$\frac{76}{135}c$
CC	$\frac{38\sqrt{5}}{135}c$	$\frac{76}{135}c$	$\frac{9}{5}c$

(20)

The eigenvalues of this matrix are $E_1 = c/9, E_2 = c/9$ and $E_3 = 20c/9$. This shows that the effect of the 3-body colour confinement on NN and $\Delta\Delta$ is identical and rather small as compared to that on CC. In particular for a negative value of c, the spectrum of NN, $\Delta\Delta$ and CC lowers and shrinks. For a positive c, the situation is the other way round. This means that, for $c < 0$, V_{3b} itself brings some attraction and implies a stronger coupling of CC to NN and $\Delta\Delta$ due to a hyperfine interaction. This will lead to a reduced hard core repulsion in the NN potential.

In conclusion, a three-body confinement force can affect the spectrum of multiquark systems in a positive or negative way, depending on the strength c. In particular, if c is negative, the unphysical octet and decuplet states of a system of three quarks become well separated from the colour singlet states, which is a desired feature for models with three valence quarks only. In tetraquarks its role is also positive because it decouples the colour octet-octet state from the singlet-singlet one, the first being unphysical. For six-quark systems the role of the three-body confinement force with a negative strength is controversial. It increases the coupling of the hidden-colour CC states to the physical NN and $\Delta\Delta$ states. On one hand, this brings more attraction into the NN potential, which is useful to lower too high hard core potentials, but on the other hand this implies stronger Van der Waals forces. The latter is in contrast with the hopes of Ref. [4].

Details of the calculations can be found in Ref. [9]. One should notice that the present study is based on a simple harmonic oscillator confinement. It would be useful to extend it to a more realistic confinement. Also, for tetraquarks and six-quark systems the results are derived for compact configurations, i.e. for zero separation between the hadronic clusters, here of type $q\overline{q}$ or q^3. It would certainly be interesting to study non-zero separation distances (molecular type configurations).

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Few-nucleon reactions of astrophysical interest ⋆

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Abstract. The study of few-nucleon reactions at low energies has evolved into a mature area of research. We present a brief review of the methods and recent advances obtained by the Pisa-Jefferson Lab/ODU collaboration.

One of the most important applications of nuclear physics is to make precise model-independent predictions of rates of astrophysical reactions, with possibly quantitative estimates of "theoretical" uncertainties. This goal is particularly important for providing nuclear physics input to stellar models. In fact, for several of these reactions the rates are too small to be measured in laboratories and therefore their estimates can be obtained only by using a theoretical model.

The description of these processes requires the knowledge of the initial (bound) and final (in general, continuum) nuclear states and the use of electromagnetic (EM) and weak current operators constructed consistently with the interaction used to generate the wave functions. The nuclear EM and weak current models developed so far (see, for example, Refs. [2,3]) include one– and two–body operators. The EM two–body current is constructed consistently with the nucleon–nucleon interaction, in order to satisfy current conservation [4]. The vector part of the weak current is obtained from the isovector EM current via the CVC hypothesis. The axial part has been constrained by reproducing the experimental value of tritium beta decay. The present model of the current has been tested in numerous few–nucleon processes and it is thought to be quite realistic.

The $A = 3, 4$ bound and continuum wave functions have been calculated by expanding on a basis of correlated hyperspherical harmonic (CHH) functions [1]. Such a technique has been proven to be rather accurate. It should be emphasized that with the CHH technique the inclusion of the Coulomb potential, clearly very important in the energy range considered here, does not present any difficulty. Let us review briefly the CHH method for the trinucleon bound state. The wave function has been written as

$$\Psi_3 = \sum_{\alpha=1}^{N_c} \sum_{K=K_0}^{K_M} \frac{u_{\alpha K}(\rho)}{\rho^{5/2}} \sum_{ijk \text{ cyclic}} f_\alpha(r_{jk})\, {}^{(2)}\mathcal{P}_K^{\ell_\alpha, L_\alpha}(\phi_i)\mathcal{Y}_\alpha(jk,i) \quad , \qquad (1)$$

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where

$$\mathbf{x}_i = \mathbf{r}_j - \mathbf{r}_k \,, \qquad \mathbf{y}_i = (\mathbf{r}_j + \mathbf{r}_k - 2\,\mathbf{r}_i)/\sqrt{3} \,, \tag{2}$$

$$\rho = \sqrt{x_i^2 + y_i^2} \,, \qquad \cos\phi_i = x_i/\rho \,, \tag{3}$$

$\mathbf{r}_i$ denoting the position of particle i, and the variable ρ is the so–called hyperradius. The correlation functions $f_\alpha(r_{jk})$ have been introduced in order to accelerate the convergence, as explained below. The angle–spin–isospin functions $\mathcal{Y}_\alpha(jk,i)$ are defined as

$$\mathcal{Y}_\alpha(jk,i) = \{[Y_{l_\alpha}(\hat{\mathbf{x}}_i) \otimes Y_{L_\alpha}(\hat{\mathbf{y}}_i)]_{\Lambda_\alpha}[s_\alpha^{jk} \otimes s_\alpha^i]_{S_\alpha}\}_{JJ_z}[t_\alpha^{jk} \otimes t_\alpha^i]_{TT_z} \,. \tag{4}$$

Each α–channel is specified by the angular momenta ℓ_α, L_α coupled to give Λ_α, and by the spin (isospin) s_α^{jk} (t_α^{jk}) and s_α^i (t_α^i) of the pair jk and the third particle i, coupled to give S_α (T). The antisymmetrization of the wave function Ψ requires $\ell_\alpha + s_\alpha^{jk} + t_\alpha^{jk}$ be odd; in addition, $\ell_\alpha + L_\alpha$ must be even for positive parity states and odd for negative ones. The hyperspherical polynomials ${}^{(2)}P_K^{\ell_\alpha,L_\alpha}$ are given by [5]

$${}^{(2)}P_K^{\ell_\alpha,L_\alpha}(\phi_i) = N_n^{\ell_\alpha,L_\alpha}(\sin\phi_i)^{L_\alpha}(\cos\phi_i)^{\ell_\alpha} P_n^{L_\alpha+1/2,\ell_\alpha+1/2}(\cos 2\phi_i) \,, \tag{5}$$

where $N_n^{\ell_\alpha,L_\alpha}$ is a normalization factor and $P_n^{a,b}$ a Jacobi polynomial. The grand orbital quantum number is given by $K = \ell_\alpha + L_\alpha + 2n$, n being a non–negative integer. In Eq. (1), $K_0 = \ell_\alpha + L_\alpha$ is the minimum grand orbital quantum number and K_α is the maximum selected value.

In the limit $N_c \,,\ K_\alpha \to \infty$, the expansion basis used in Eq. (1) is complete. In a practical calculation, N_c and K_α are kept finite and increased until convergence in the binding energy is achieved. In this regard, it should be stressed the importance of the inclusion of the correlation functions $f_\alpha(r_{jk})$ in Eq. (1) for improving the convergence rate. The standard (uncorrelated) HH expansion is recovered if such functions are set equal to one. However, for potentials containing a strong repulsion at small distances, the rate of convergence of the HH expansion is found to be very slow. The role of the correlation function $f_\alpha(r)$ in Eq. (1) is therefore to accelerate the convergence of the expansion by improving the description of the system when two particles are close to each other. These correlation functions are obtained from solutions of suitable two–body zero–energy Schrödinger equations, with a technique outlined in ref. [1]. The present approach has been applied also to the the study of the ground state of the α particle and to the continuum states of $A = 3, 4$ nuclear systems.

Some of the computed S-factors are presented in Table 1. The effect of the two-body currents is seen to increase significantly as the number of particles involved in the reaction grows. The last column of the table shows the "theoretical uncertainty", namely the variance of the S-factor values obtained by choosing different realistic nuclear Hamiltonians H for calculating the bound and scattering wave functions. This choice is reflected also in the current. Indeed, the EM current $\mathbf{j}(\mathbf{q})$–more precisely, its longitudinal part–is constructed to satisfy the continuity

reaction	$S_1(0)$	$S_{1+2}(0)$	$\Delta S/S$
$p + p \to {}^2H + e^+ + \nu_e$	3.90×10^{-25}	3.95×10^{-25}	0.5%
$p + d \to {}^3He + \gamma$	1.32×10^{-7}	1.88×10^{-7}	2%
$p + {}^3He \to {}^4He + e^+ + \nu_e$	29.0×10^{-23}	9.64×10^{-23}	5%

Table 1. Zero energy S-factors (in Mev-barn) for various reactions. In the column labelled $S_1(0)$ ($S_{1+2}(0)$), the S-factors obtained including the one-body (one– and two–body) terms in the nuclear current are reported. In the last column, the "theoretical uncertainty" (obtained as explained in the text) is also reported.

equation $\mathbf{q}\cdot\mathbf{j}(\mathbf{q}) = [H, \rho(\mathbf{q})]$ and therefore depends on the nuclear interaction chosen to describe the nuclear states. The axial weak current too is related to H via PCAC and also because one of its parameter is fixed by reproducing the tritium beta decay rate within a given model Hamiltonian. These theoretical uncertainties are rather small, indicating that the dependence on the interaction model is rather weak.

Recently, another model for the transition current, obtained from an effective field theory (EFT) based on the chiral symmetry, has been developed [6]. Using this approach, the transition operators are organized according to a "systematic"power counting in the heavy baryon chiral perturbation theory. They have been worked out up to N^3LO: at this level the transition operators have one and two body terms and there appears one unknown parameter in the chiral Lagrangian, which has been fixed again by fitting the tritium β-decay rate [7].

With this model for the transition current, and using the nuclear wave functions calculated as above, the S-factors of the first and third reaction considered in Table 1 turn out to be rather close to those calculated here. For example, $S_{1+2}(0)$ for the reaction $p + p \to {}^2H + e^+ + \nu_e$ is 3.94×10^{-25} [7], only 0.25% smaller than the result reported in Table 1. This substantiates the confidence on the accuracy of the theoretical estimates of these S-factors. The extension to other processes, as muon capture on light nuclei, is in progress.

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Covariant electromagnetic and axial nucleon form factors in the Goldstone-boson-exchange constituent quark model ⋆

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Abstract. We show that within the Goldstone-boson-exchange constituent quark model not only the light and strange baryon spectra but also electromagnetic and axial nucleon form factors come out in rather good agreement with phenomenology. A consistent description of the electroweak nucleon form factors is essentially dependent on the inclusion of Lorentz boost effects which can be treated accurately in the point form approach to relativistic quantum mechanics.

1 GBE constituent quark model

A promising approach to low-energy hadrons consists in constituent-quark models (CQMs). Starting from rudimentary attempts more than two decades ago, one has constantly improved the description and gained a lot of insight into the properties of hadrons at low and intermediate energies. Evidently, CQMs can at most be effective models of quantum chromodynamics (QCD) in a domain where the fundamental theory is not (yet) accurately solvable. It appears that below a certain energy scale in the light-flavor sector the spontaneous breaking of chiral symmetry (SBχS) of QCD is responsible for the generation of constituent quarks as quasiparticles with dynamical masses much greater than the corresponding current-quark masses. Numerous theoretical as well as experimental evidences hint to a chirally broken phase of QCD.

If one understands the generation of constituent quarks as being caused by SBχS, one should at the same time also accept the other consequences of SBχS. According to that one is left with a residual $SU(3)_V$ symmetry associated with the existence of Goldstone bosons. It follows that one is dealing with new degrees of freedom in the light-flavor sector of QCD, namely with constituent quarks and Goldstone bosons instead of current quarks and gluons, which represent the

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original degrees of freedom in the QCD Lagrangian. Following this idea, already several years ago, Glozman and Riska have suggested a quark-quark hyperfine interaction in light and strange baryons which is mediated by Goldstone boson exchange (GBE) [1]. This interaction has a spin-flavor symmetry, which is obviously most appropriate for describing the spectra of light and strange baryons. Evidently, in addition to this hyperfine interaction one has to advocate some confining forces.

Here we specifically adhere to the version of the GBE CQM published in Ref. [2]. It comes with a mutual quark-quark interaction

$$V_{ij} = V_{conf} + V_{\chi}, \tag{1}$$

with a confinement potential in linear form

$$V_{conf}(r_{ij}) = V_0 + C r_{ij} \tag{2}$$

and the chiral interaction consisting of only the spin-spin part of the pseudoscalar meson exchange

$$V_{\chi}(\mathbf{r}_{ij}) = \left[\sum_{F=1}^{3} V_{\pi}(\mathbf{r}_{ij}) \lambda_i^F \lambda_j^F + \sum_{F=4}^{7} V_K(\mathbf{r}_{ij}) \lambda_i^F \lambda_j^F + V_{\eta}(\mathbf{r}_{ij}) \lambda_i^8 \lambda_j^8 + \frac{2}{3} V_{\eta'}(\mathbf{r}_{ij}) \right] \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j . \tag{3}$$

Here $\boldsymbol{\sigma}_i$ are the Pauli spin matrices and λ_i the Gell-Mann flavor matrices of the individual quarks. For all details of the parameterization of the $V_{\pi,K,\eta,\eta'}$ and the values of all parameters of the model we refer to the original paper Ref. [2].

The complete three-Q Hamiltonian of the model is then given by

$$H = H_0^{rel} + V \tag{4}$$

where H_0^{rel} is the relativistic kinetic-energy term

$$H_0^{rel} = \sum_{i=1}^{3} \sqrt{\mathbf{k}_i^2 + m_i^2}, \tag{5}$$

with m_i the masses and $\mathbf{k}_i$ the 3-momenta of the constituent quarks, which are restricted by $\sum_{i=1}^{3} \mathbf{k}_i = 0$.

The phenomenological spectra of the light and strange baryons are reasonably well described by the model. In Fig. 1 we show here just the spectra of N and Δ. The complete spectra including all the strange baryons can be found in Ref. [2].

2 Electromagnetic and axial nucleon form factors

For the calculation of electromagnetic and axial nucleon form factors one has to deal with boosts from the rest frame in which the nucleon wave function is originally obtained to a moving frame since at least one, the incoming or outgoing nucleons is moving in the scattering process. In any CQM the intrinsic movement of the constituent quarks inside a nucleon is highly relativistic making a

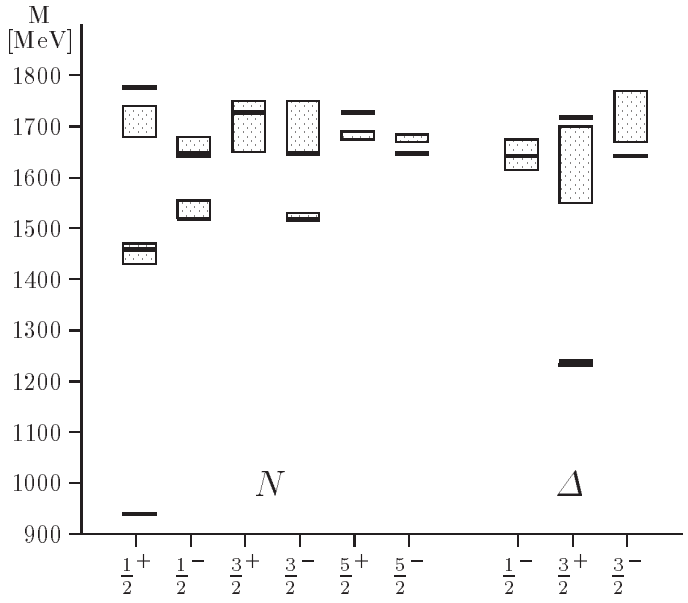

Fig. 1. N and Δ spectra of the GBE CQM. The bars are the calculated masses of states with angular momentum and parity J^P. The shaded boxes are the experimental masses.

nonrelativistic treatment of boosts obsolete. Even for very low momenta transferred to the nucleon from the external probe (electron or neutrino) constituent quarks with large momenta are involved.

One can incorporate relativity into a quantum theory with a finite number of degrees of freedom (as a CQM) by utilizing relativistic Hamiltonian dynamics (i.e. Poincaré-invariant quantum mechanics) [3]. From the various (unitarily equivalent) forms that are possible when defining the (kinematic) stability subgroup [4] the point form has some obvious advantages [5] in our studies. In this form the four-momentum operators P^μ contain all the dynamics. They commute with each other and can be diagonalized simultaneously. All other generators of the Poincaré group are not affected by interactions. In particular, the Lorentz generators are interaction-free and make the theory manifestly covariant. Through the introduction of so-called velocity states [6] we can carry out all necessary transformations of the momentum dependences in the wave functions and the relativistic quark spin rotations associated with boosting the nucleon state in an accurate manner.

In order to come from the semirelativistic Hamiltonian as given in section 1 to relativistic Hamiltonian dynamics one can apply the so-called Bakamjian-Thomas (BT) construction [7] where all dynamical generators of the Poincaré group are obtained from one auxiliary operator. This mass operator consists of a free part, which is just the relativistic kinetic energy, as given in Eq. (5), and an interaction term. The potential of the GBE CQM satisfies all conditions required for the relevant operators to fulfill the Poincaré algebra. Even if the potential term

is nonrelativistic one can thus arrive at a relativistic quantum theory by reinterpreting the Hamiltonian as a mass operator in a BT construction.

For the calculation of electromagnetic form factors it can be shown that the electromagnetic current operator can be written in such a way that it transforms as an irreducible tensor operator under the strongly interacting Poincaré group. Thus the electromagnetic form factors can be calculated as reduced matrix elements of such an irreducible tensor operator in the Breit frame. The same procedure can be applied to the axial current.

The current operator is assumed to be a single-particle current operator for point-like constituent quarks. This corresponds to a relativistic impulse approximation but specifically in point form. It is called the point-form spectator approximation (PFSA) because the impulse delivered to the nucleon is different from that delivered to the struck constituent quark. The electromagnetic current matrix elements have the usual form for a point-like Dirac particle, i.e.

$$\langle p_i', \lambda_i' | j^\mu | p_i, \lambda_i \rangle = e_i \bar{u}(p_i', \lambda_i') \gamma^\mu u(p_i, \lambda_i), \tag{6}$$

with $u(p_i, \lambda_i)$ the Dirac spinor of quark i with charge e_i, momentum p_i, and spin projection λ_i. Such a j^μ is not conserved a-priori but one can always construct a conserved current $\widetilde{j}^\mu = j^\mu - q^\mu (q \cdot j / q^2)$, with q the 4-momentum transfer. The new added term does not affect the $\mu = 0, 1, 2$ components which are used to calculate the form factors. The axial current matrix elements have the form

$$\langle p_i', \lambda_i' | A_a^\mu | p_i, \lambda_i \rangle = \bar{u}(p_i', \lambda_i') \left[g_A^q \gamma^\mu + \frac{2 f_\pi}{\widetilde{Q}^2 + m_\pi^2} g_{\pi q} \widetilde{q}^\mu \right] \gamma_5 \tfrac{1}{2} \tau_a u(p_i, \lambda_i), \tag{7}$$

where m_π is the pion mass, $f_\pi = 93.2$ MeV the pion decay constant, and $\widetilde{Q}^2 = -\widetilde{q}^2$, with $\widetilde{q} = p_i' - p_i$ the momentum transferred to a single quark. The quark axial charge is assumed to be $g_A^q = 1$, as for free bare fermions, and $g_{\pi q}$ is identified with the pion-quark coupling constant, with a numerical value as employed in the GBE CQM of Ref. [2].

Along this formalism we have calculated the complete set of electromagnetic and axial form factors, i.e. the electric and magnetic proton and neutron form factors as well as the axial and induced pseudoscalar nucleon form factors. The results have been published in a series of papers [9].

For example we show in Fig. 2 the ratios of the electric proton form factor to the dipole parameterization and to the magnetic proton form factor. The latter has recently been determined experimentally in a direct measurement at Jefferson Lab [8]. The data start to fall below one for momentum transfer squared $Q^2 \gtrsim 1$ GeV^2. Obviously this tendency is reproduced by the PFSA result though the agreement with the data is not perfect. However, in a completely nonrelativistic impulse approximation (NRIA) calculation the ratio would be 1 for all Q^2. In general the shapes of all electromagnetic form factors calculated in PFSA are in good agreement with the data up to $Q^2 \sim 1$ GeV^2. We mention, however, that for the magnetic form factors there are some small discrepancies from the data even at very low momentum transfers; of course, this affects also the magnetic moments.

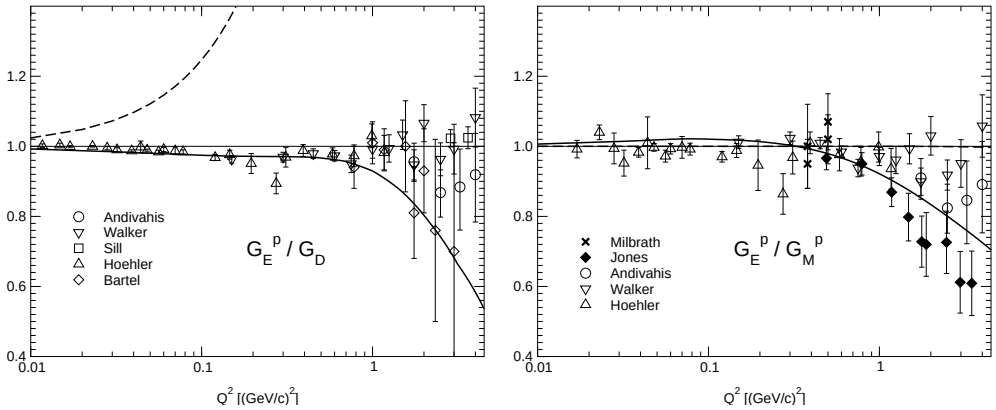

Fig. 2. Ratios of the proton electric form factor to the dipole form, $G_D(Q^2) = (1 + Q^2/(0.71\ \mathrm{GeV}^2))^{-2}$, (left) and to the proton magnetic form factor (right). The latter is normalized to 1 at $Q^2 = 0$. Solid lines: PFSA, dashed lines: NRIA. Experimental data as in Ref. [9].

The results for the axial and the induced pseudoscalar form factors are presented in Fig. 3. In case of the axial form factor we slightly underestimate the experimental values for the axial charge and the axial radius. The PFSA results are, however, much closer to the data than the ones from a completely nonrelativistic calculation. In case of the induced pseudoscalar form factor it is evident that the data can only be described by including the pion pole term in the axial current of the constituent quark. This is consistent with the basic assumption of the GBE CQM that pions couple to the constituent quarks.

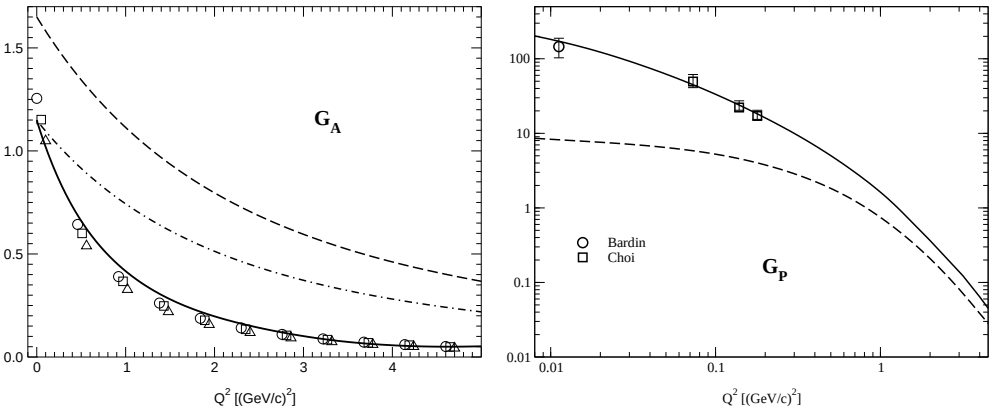

Fig. 3. Nucleon axial form factor (left) and induced pseudoscalar form factor (right). The solid lines are the PFSA predictions of the GBE CQM. In the left plot the dashed curve is the NRIA result and the dashed-dotted curve the result without Lorentz boosts but with a relativistic axial current. In the right plot the dashed curve is the PFSA result obtained without the pion-pole term in the axial current of the constituent quark. Experimental data as in Ref. [9]

In summary, the GBE CQM yields quite a consistent picture of the electroweak nucleon structure. The usage of a relativistic approach appears as most important. It is remarkable how the inclusion of relativistic effects in PFSA brings the theoretical predictions close to experiments.

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Excitation of non-quark degrees of freedom in N* ⋆

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Abstract. We study the contribution of glueball and σ-meson degrees of freedom to nucleon excited states in the framework of a chiral version of the chromodielectric model. We have found that these degrees of freedom considerably lower the energy of the Roper and may substantially weaken the electroexcitation amplitudes for the N(1440) and in particular for the N(1710).

Among the excited states of the nucleon the Roper resonance, N(1440), plays a rather special role since, due to its relatively low excitation energy, a simple picture in which one quark populates the 2s level does not work here. The relatively low energy of the N* has been explained by

- the residual interaction originating from chiral mesons exchange [1],
- allowing the confining potential to vibrate (e.g. in the MIT bag model [2], or in models with dynamically generated confinement, [3]),
- describing the N* as a pure sigma meson excitation rather than the excitation of the quark core [4],
- explicit excitations of glue-field [5].

We present a simple model, the chromodielectric model (CDM), which is particularly suitable to describe the interplay of glueball and meson excitations together with quark radial excitations. The model includes a chromodielectric field χ which assures quark dynamical confinement, and the chiral fields, σ and π. The Lagrangian takes the form [6]:

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi + \frac{g}{\chi}\bar{\psi}(\hat{\sigma} + i\boldsymbol{\tau}\cdot\hat{\boldsymbol{\pi}}\gamma_5)\psi + \mathcal{L}_{\sigma,\pi} + \mathcal{L}_\chi \ , \tag{1}$$

where

$$\mathcal{L}_\chi = \tfrac{1}{2}\partial_\mu\hat{\chi}\,\partial^\mu\hat{\chi} - \frac{1}{2}M^2\hat{\chi}^2 \ , \qquad \mathcal{L}_{\sigma,\pi} = \tfrac{1}{2}\partial_\mu\hat{\sigma}\partial^\mu\hat{\sigma} + \tfrac{1}{2}\partial_\mu\hat{\boldsymbol{\pi}}\cdot\partial^\mu\hat{\boldsymbol{\pi}} - \mathcal{U}(\hat{\boldsymbol{\pi}}^2 + \hat{\sigma}^2) \ , \tag{2}$$

and $\mathcal{U}$ is the usual Mexican hat potential. The model parameters $g = 0.03$ GeV and $M = 1.4$ GeV are chosen to reproduce best the ground state properties [7]; for m_σ we have taken $0.6\ \text{GeV} < m_\sigma < 1.2\ \text{GeV}$.

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The Roper resonance is described as a cluster of three quarks in radial-orbital configuration $(1s)^2\,(2s)^1$, surrounded by pion and σ-meson clouds and by a chromodielectric field. The fields oscillate together with quarks. The ansätze for the nucleon $|N\rangle$ and the Roper $|R'\rangle$ are assumed in the form of coherent states [8] on top of $(1s)^3$ and $(1s)^2\,(2s)^1$ configurations, respectively, projected onto subspace with good angular momentum and isospin. Different profiles are assumed for the Roper and the nucleon, and the boson fields are allowed to adapt to the quark configuration. The 'potential' breathes together with the quarks as illustrated in Fig. 1. The proper orthogonalization of both states is ensured by writing:

$$|R\rangle = \frac{1}{\sqrt{1-c^2}}(|R'\rangle - c|N\rangle)\ , \qquad c = \langle N|R'\rangle\ . \tag{3}$$

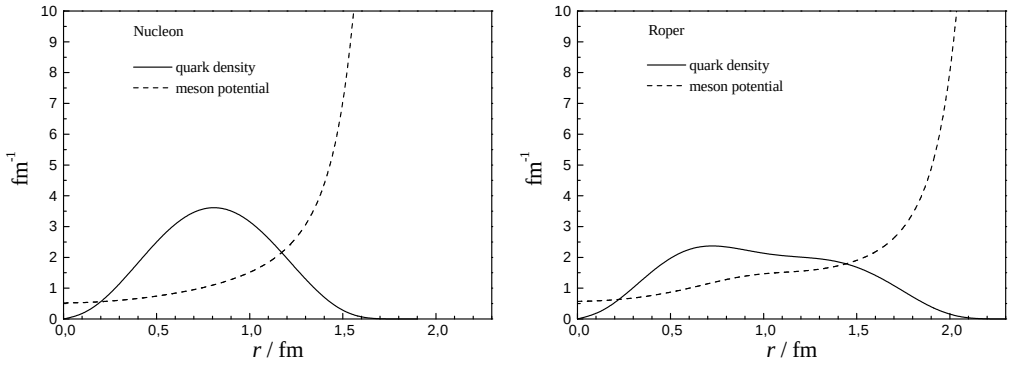

Fig. 1. The baryon densities (solid lines) and the effective potentials (dashed lines) generated by the self-consistently determined π, σ and χ fields in the nucleon and the Roper.

We investigate here another possible type of excitation in which the quarks remain in the ground state configuration $(1s)^3$ while the chromodielectric field and the σ-field oscillate. Such oscillations can be described by expanding the boson fields as small oscillations around their ground state values. For the σ-field we write:

$$\hat{\sigma}(\mathbf{r}) = \sum_n \frac{1}{\sqrt{2\varepsilon_n}}\,\varphi_n(r)\frac{1}{\sqrt{4\pi}}\left[\tilde{a}_n + \tilde{a}_n^\dagger\right] + \sigma(r)\ ,$$

where φ_n and ε_n satisfy the Klein-Gordon equation

$$\left(-\nabla^2 + m^2 + \frac{d^2V(\sigma(r))}{d\sigma(r)^2}\right)\varphi_n(r) = \varepsilon_n^2\varphi_n(r)\ . \tag{4}$$

A simple ansatz for the annihilation (creation) operator of the n-th mode is given by

$$\tilde{a}_n = \int dk\,\tilde{\varphi}_n(k)\left(\tilde{a}(k) - \sqrt{2\pi\omega_\sigma(k)}\,\eta(k)\right)\ , \qquad \tilde{a}_n|N\rangle = 0\ , \tag{5}$$

where $\eta(k)$ and $\tilde{\varphi}_n(k)$ are the Fourier transforms of $\sigma(r)$ and $\varphi_n(r)$, respectively, and $\omega_\sigma(k) = k^2 + m_\sigma^2$. The effective potential in (4) is

$$V_{\sigma\sigma}(r) = \frac{d^2V(\sigma(r))}{d\sigma(r)^2} = \lambda\left[C_2\phi(r)^2 + 3\sigma(r)(\sigma(r) + 2\sigma_v)\right]\ , \tag{6}$$

where σ is the fluctuating part of the full field and C_2 is a projection coefficient slightly smaller than unity. Similar expressions hold for the χ field with the effective potential

$$V_{\chi\chi}(r) = \frac{d^2V(\chi(r))}{d\chi(r)^2} = -\frac{3}{2\pi}\frac{g}{\chi(r)^3}\left[(\sigma(r)+\sigma_v)(u(r)^2 - v(r)^2) + 2\phi(r)u(r)v(r)\right] . \tag{7}$$

The effective potential turns out to be repulsive for the χ-field and attractive for the σ-field; in the latter case there exists at least one bound state with the energy ε_1 of typically 100 MeV below the σ-meson mass.

The ansatz for the Roper can now be simply extended as

$$|R^*\rangle = c_1|R\rangle + c_2\tilde{a}^\dagger_\sigma|N\rangle , \tag{8}$$

where $\tilde{a}^\dagger_\sigma$ is the creation operator for the lowest vibrational mode. The coefficients c_i and the energy are determined by solving the generalized eigenvalue problem in the 2×2 subspace. The solution with the lowest energy corresponds to the Roper, while the orthogonal combination to one of the higher excited states with nucleon quantum numbers, e.g., the $N(1710)$, provided the σ-meson mass is sufficiently small. The energy of the Roper is reduced (see Table 1), though the effect is small due to weak coupling between the state (3) and the lowest vibrational state with the energy ε_1. The reduction becomes more important if the mass of the σ-meson is decreased. The energy of the combination orthogonal to the ground state is close to $E_N + \varepsilon_1$ with σ-meson vibrational mode as the dominant component.

m_σ	E_N	2s–1s	ΔE_R	ΔE_{R*}	c_2	ε_1
1200	1269	446	354	353	0.05	1090
700	1249	477	367	364	0.12	590

Table 1. For two σ-masses we show the nucleon energy (E_N), the Roper-nucleon energy splittings calculated from the single particle energy difference (2s–1s), the state (3) (ΔE_R) and the state (8) (ΔE_{R*}). All energies are given in MeV.

The electromagnetic nucleon-Roper transition amplitudes as well as the transition amplitudes to higher excitations with nucleon quantum numbers represent an important test which may eventually distinguish between the models listed at the beginning. The transverse helicity amplitude is defined as

$$A_{1/2} = -\zeta\sqrt{\frac{2\pi\alpha}{k_W}}\int d^3\mathbf{r}\ \langle\tilde{R}_{+\frac{1}{2},M_T}|\mathbf{J}_{em}(\mathbf{r})\cdot\boldsymbol{\epsilon}_{+1}\, e^{i\mathbf{k}\cdot\mathbf{r}}|\tilde{N}_{-\frac{1}{2},M_T}\rangle \tag{9}$$

where k_W is the photon momentum at the photon point, and the scalar helicity amplitude as

$$S_{1/2} = \zeta\sqrt{\frac{2\pi\alpha}{k_W}}\int d\mathbf{r}\,\langle\tilde{R}_{+\frac{1}{2},M_T}|J^0_{em}(\mathbf{r})\ e^{i\mathbf{k}\cdot\mathbf{r}}|\tilde{N}_{+\frac{1}{2},M_T}\rangle . \tag{10}$$

Here J^μ_{em} is the EM current derived from the Lagrangian density (1):

$$J^\mu_{em} = \sum_{i=1}^{3} \overline{q}_i \gamma^\mu(i) \left(\frac{1}{6} + \frac{1}{2}\tau_0(i) \right) q_i + (\boldsymbol{\pi} \times \partial^\mu \boldsymbol{\pi})_0 \ . \tag{11}$$

The amplitudes (9) and (10) contain a phase factor, ζ, determined by the sign of the decay amplitude into the nucleon and the pion.

The new term in (8) *does not contribute* to the nucleon-Roper transition amplitudes. Namely, for an arbitrary EM transition operator $\hat{O}$ involving only quarks and pions we can write

$$\langle N|\tilde{a}_1 \hat{O}|N\rangle = \langle N|[\tilde{a}_1, \hat{O}]|N\rangle = 0 \ , \tag{12}$$

because of (5) and since the operators $\tilde{a}_n$ commute with $\hat{O}$.

A possible way to identify such a state would be to search for those excited states for which the amplitudes are strongly reduced compared to those calculated in a model with only quark degrees of freedom. There have been several attempts to calculate these amplitudes in various models, such as in chiral quark models [9,10], models with explicit gluon degrees of freedom [5] and relativistic versions of the constituent quark model [11]. Unfortunately, the present status of theoretical prediction is rather unclear because of a strong sensitivity of transition amplitudes on small variation of the Roper wave function. To understand the nature of the Roper remains a big challenge for theoreticians as well as for experimentalists.

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Constraints on effective constituent quark masses from phenomenology ⋆

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Abstract. From the assumption of a two-particle Hilbert space for mesons and from rather general properties of the effective quark-quark potential we constrain considerably the choice of effective constituent quark masses.

1 Nonrelativistic models

We consider the following form of the Hamiltonian for the quark-antiquark system

$$H = \frac{p^2}{2\mu} + V_0(r) + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 V_s(m_1, m_2; r),$$

where μ is the reduced mass of the system and m_1 and m_2 are the quark and antiquark masses.

We make rather general assumptions about the potential:

1. The central potential $V_0(r)$ is flavour independent
2. The central potential is monotonic function or r and satisfies the conditions for a positive Laplacian and concavity

$$\frac{d}{dr} r^2 \frac{dV_0}{dr} > 0 \quad \textit{and} \quad \frac{d^2 V_0}{dr^2} < 0.$$

3. The spin-spin potential V_s satisfies the condition that μV_s decreases with total mass of both quarks $M = m_1 + m_2$
4. The spin-spin potential for vector mesons is a monotonic function of r and has positive Laplacian

$$\frac{d}{dr} r^2 \frac{dV_s}{dr} > 0$$

In the family of potentials which satisfy the conditions 1. and 2. one can find the "QCD inspired" Coulomb-plus-linear potential and power law potential

$$V_0(r) = -\frac{\alpha}{r} + \beta r + U_0,$$
$$V_0(r) = A + B r^{\beta},$$

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while the conditions 3. and 4. are satisfied for example by

$$V_s(m_1, m_2; r) = \frac{\alpha}{m_1 m_2} \frac{e^{-r/r_0}}{r} \qquad \alpha, r_0 > 0.$$

From these assumptions one can obtain inequalities between quark masses and masses of ground states of pseudoscalar and vector mesons, which to some extent restrict masses of constituent quarks as shown in Fig(1).

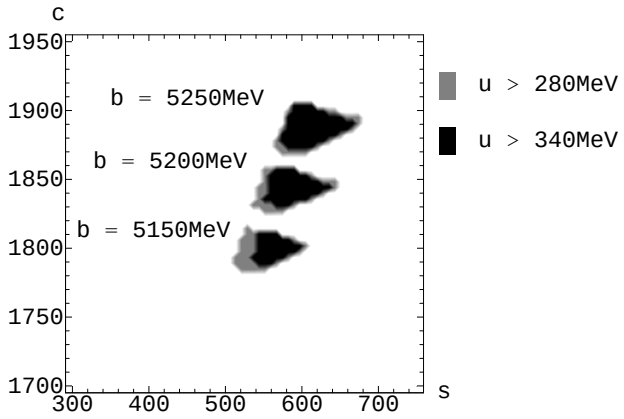

Fig. 1. Allowed mass region for strange and charmed quarks for different choices of light and bottom quarks.

2 Semirelativistic models

For heavy quark Q - light (or heavy) antiquark q pseudoscalar mesons we use the semirelativistic model with the Hamiltonian

$$H = \sqrt{p^2 + m_Q^2} + \sqrt{p^2 + m_q^2} + \frac{\boldsymbol{\sigma}_q \cdot \boldsymbol{\sigma}_Q}{m_q} F(m_Q) V_{ss} + V,$$

where we assume that the expectation value of $F(m_Q)V_{ss}/m_q$ is a monotonically decreasing function of m_q and that both V_{ss} and V are flavour (mass) independent.

The Hamiltonian for all vector mesons in our model has the general form

$$H = \sqrt{p^2 + m_1^2} + \sqrt{p^2 + m_2^2} + V(m_1, m_2),$$

where we demanded that the expectation value of $V(m_1, m_2)$ is a decreasing function of the quarks masses from where it follows that

$$\begin{aligned} E(K^*) - E(\rho) &< m_s - m_u, \\ E(D^*) - E(K^*) &< m_c - m_s, \\ E(B^*) - E(D^*) &< m_b - m_c. \end{aligned}$$

Using these assumptions we again obtained inequalities between masses of quarks and mesons which allowed us to constrain the masses of constituent quarks. In Fig(2) one can see that it is not possible to reproduce correctly the masses of ground state mesons with semirelativistic model if one takes mass of the charmed quark smaller then 1650 MeV. Then the mass of the bottom quark must be, according to upper inequalities, always larger then 4970 MeV.

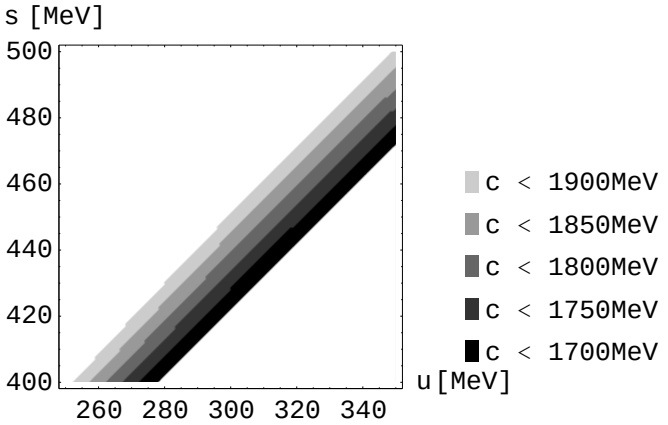

Fig. 2. Allowed mass regions for light and strange quarks for five different choices for mass of charm quark.

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Are there locally precise three-body wave functions? ⋆

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Abstract. This paper concentrates on two topics. First it presents cases which show that even in atomic physics, contrary to expectations, variational methods have problems on the 4-5th digits in expectation values which depend on the values of Ψ at the cusps, as opposed to the energy. Second, we compare the results of the direct method, CFHHM (Correlation function hyperspherical harmonic method), in atomic ionization calculations of the single and double ionization of the Helium atom and the Helium isoelectronic sequence for Z up to 10 and excited states up to $n = 5$. We calculate more n, Z dependencies than before; make predictions on corrections due to quasi-free mechanism using a new formula (several new light sources are becoming available).

1 Precision of the CFHHM wave function

Sophisticated variational methods nowadays try to overcome the fact that mathematically there is no reason to assume that a variational method would give accuracy for the expectation values comparable to that of the energy E. The motivations in this section is to show several examples where such variational calculation indeed break down.

CFHHM [1] is a direct solution of the Schrödinger equation by the separation of Ψ into the singular part (caused by the Coulomb interaction; this is specific to the atomic physics) and the smooth part, $\Psi = e^f\phi$. The Schrödinger equation is converted into the equation for $\chi = \rho^2\phi$ expanded into the hyperspherical harmonic basis (index ν; μ is connected with the global angular momentum):

$$\chi''_{\mu\nu} + \frac{1}{\rho}\chi'_{\mu\nu} + \left[2E - \frac{(2\mu+2)^2}{\rho^2}\right]\chi_{\mu\nu} = 2\sum_{\mu'\nu'} \overline{W}_{\mu\nu,\mu'\nu'}\chi_{\mu'\nu'}, \qquad (1)$$

where ρ is the hyperradius (a permutation-invariant measure of system size, given by a weighted sum of squares of the Jacobi coordinates), and W is the velocity-dependent potential, $\overline{W} = V - (\nabla f, \nabla) - \frac{1}{2}\nabla^2 f - \frac{1}{2}(\nabla f)^2 + \frac{2}{\rho^2}\frac{\partial f}{\partial \rho}$. The essential physical input to CFHHM is the correlation function which in general is nonlinear

$$f = \sum_{k=1}^{3}\left[a_k + (b_k - a_k)\exp\left(\frac{r_k}{n_k\langle r_k\rangle}\right)\right]r_k, \quad a_k = Z_iZ_j\frac{m_im_j}{m_i+m_j},$$

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where $\{i, j, k\}$ are a permutation of $\{1, 2, 3\}$, and Z_i and m_i are charges and masses of the particles. f but can be used in its linear form ($b_k = a_k$) for Helium and its isoelectronic sequence except H^-. Mathematically this function is an accelerator of the convergence but it also lowers the minimum μ where convergence starts; it reduces the number of HH required for a given precision by orders of magnitude; in addition, it can at the same time incorporate some asymptotic (clustering) properties. For example, the nonlinear correlation function for the positronium negative ion (Ps-) reduces the error of observables by two orders of magnitude while making the calculation even less time consuming.

The sticking probabilities (Table 1) in the muon-catalyzed fusion process are an example of CFHHM giving much smaller error margins than even the discrepancies between different variational calculations.

Table 1. Sticking probabilities ω_{nl} ($Q = 5.844$).

Method	K_m	1s	2s	4s	2p
CFHHM		0.6819(1)	0.0978	0.0126	0.0238
HCM (Abramov)	15	0.829 (?)			
	21	0.906 (?)			
	21	0.7001	0.1004	0.0130	0.0245[1)]
Var. (Hu)		0.6932	0.0992	0.0128	0.0241
Var. (Haywood)		0.6846			
Var. (Hu)		0.6817			
Kamimura		0.6842			
Var. (Hu)		0.6802	0.0975	0.0126	0.0237
Var. (recent)		0.6802-0.8422			

[1)] $Q = 5.846$.

In $e\mu^4$He we have a case where CFHHM has resolved high precision discrepancies. Even E converged faster than in a variational method (SVM), but the "local" expectation values definitely are better than the differences between two high-precision calculations by the same author:

$10^8\langle\delta(\mathbf{r}_{\mu\mathrm{He}})\rangle$	0.207 001 3**5**4 2(6)	CFHHM
	0.207 001 373 **6**10	Smith-Frolov 1995
	0.207 001 373 **4**3	Frolov 2000
$\langle\delta(\mathbf{r}_{e\mu})\rangle$	0.313 76**2** 07(7)	
	0.313 76**3** 0	
	0.313 76**0** 812	
$\langle\delta(\mathbf{r}_{e\mathrm{He}})\rangle$	0.320 63**3** 27(6)	
	0.320 62**6** 88	
	0.320 63**1** 162	

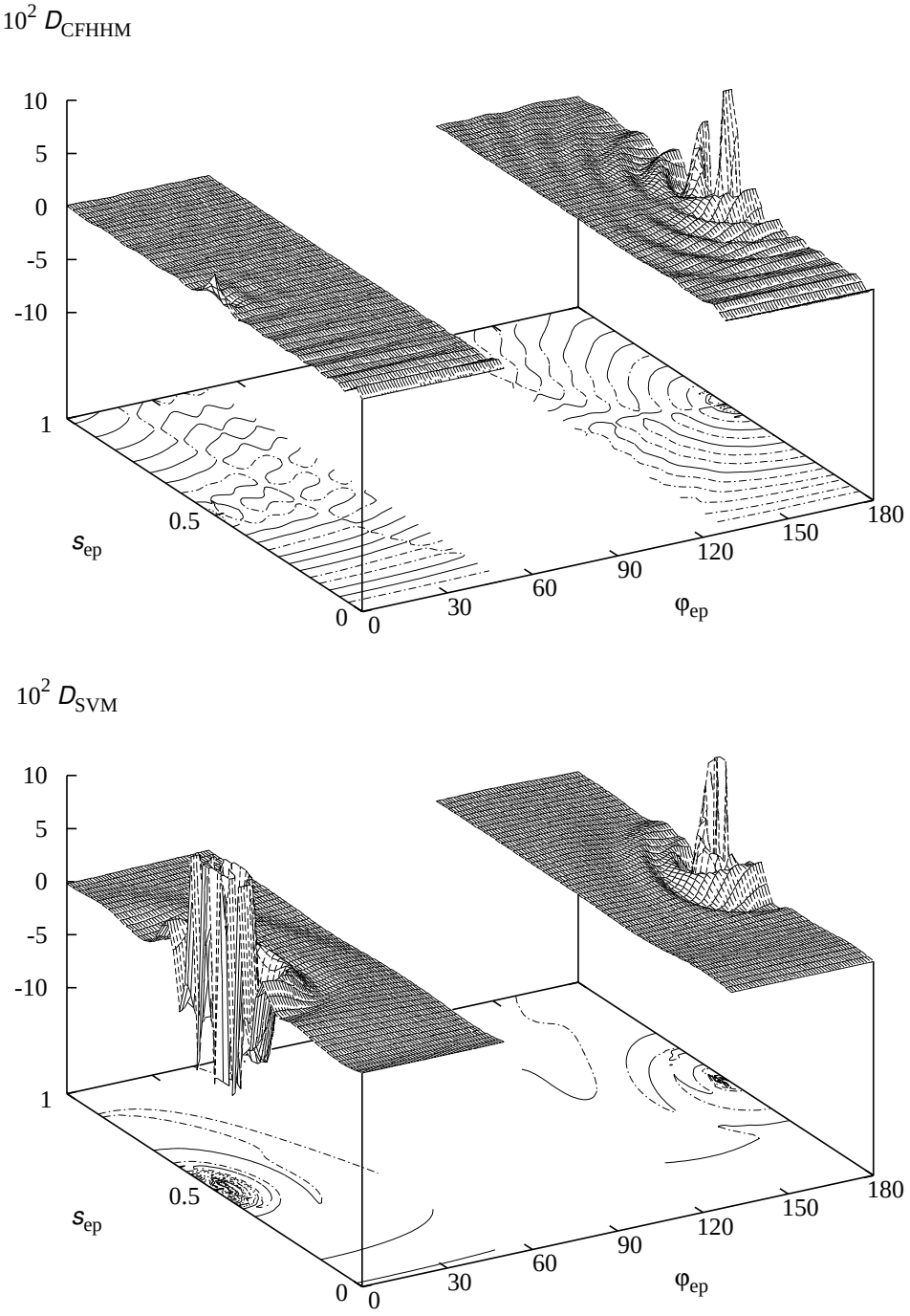

Fig. 1. $D = H\Psi/E\Psi - 1$ for Ps^-.

Bartlett (in 1935) suggested comparing the local energy, $D = H\Psi/E\Psi - 1$. Fig. 1 shows the comparison of SVM (Varga, Kukulin) and CFHHM for Ps- from Ref. [2]. While CFHHM is clearly better especially around the repulsive cusp which SVM avoids because it contributes little to E, CFHHM turns out to be "much worse" than SVM for almost all expectation values. However the δ operators are much better:

$\langle H \rangle$	0.262 005 069 5	CFHHM
	0.262 005 070 226	SVM
	0.262 005 070 232 965	EVE
$\langle r_{ee} \rangle$	8.548 5(2)	
	8.548 580 655 061	
	8.548 580 655 12	
$\langle \delta(\mathbf{r}_{ep}) \rangle$	0.020 733 14(6)*	
	0.020 731 048 976	
	0.020 733 198 0	
$\langle \delta(\mathbf{r}_{ee}) \rangle$	0.170 997(2)[-3]	
	0.171 112 600 741[-3]	
	0.170 996 99[-3]	

2 Ionization

The motivations of ionization calculations is to test CFHHM against systematic variational calculations by Forrey [3].

Experimentally it turns out that one electron takes away almost all energy (shake-off mechanism). Very soon (Byron et al.) it was also realized that in the early calculations the shake-off mechanism underestimates $\sigma^{++}(\omega)$ by a factor of 2, which indicates the importance of correlations in this three-body system; indeed, Helium is very strongly correlated. We shall calculate ratios of cross sections because they are independent of the photon energy at high (but nonrelativistic) photon energies (this just gets rid of the $\omega^{-7/2}$ factor). For precise calculations the dipole approximation (golden rule) is good but good initial three-body wave function is needed. This leads to the expressions for the double ionization cross section,

$$\sigma^{++}(\omega) \approx \frac{32\sqrt{2}Z^2\pi^2}{3c\omega^{7/2}} \left\{ \int \left| \Psi(\mathbf{0},\mathbf{s}) \right|^2 d\mathbf{s} - \sum_{\nu lm} \left| \int \Psi(\mathbf{0},\mathbf{s}) \psi_{\nu lm}(\mathbf{s}) d\mathbf{s} \right|^2 \right\} \qquad (2)$$

where $\psi_{\nu lm}(\mathbf{s})$ describes the unperturbed second electron in the field of the nucleus. (The cross section $\sigma^{+}(\omega)$ contains only the lowest integral, and $\sigma^{+*}(\omega)$ contains only the excitation integrals.) We start out with Ψ corresponding to reasonable E which need not be very precise (in CFHHM), but also should not be too imprecise (Table 2). Nevertheless, we end up with differences at 3rd – 4th digit (Table 3).

We anticipate several works with relevant data for experiments. For example, we obtain a 35 % quasi-free correction at 100 keV [4]. Rather small computational demands have been placed on CFHHM, but state-of-the-art results were

Table 2. Helium binding energy and R values: various methods.

Work	**Basis**	E	R
Present/	121	**2.9037243643**	**.01644**
CFHHM	441	**2.9037243765**	**.01644**
[3]		**2.903724377034**	**.01644**
Kheifets	7/MCHF	2.90181	.0167
Dalgarno	20	2.9037179	.0168

Table 3. $R = \sigma^{++}(\omega)/(\sigma^{+}(\omega) + \sigma^{+*}(\omega))|_{\omega\to\infty}$ values for the n^1S states in the Helium isoelectronic sequence (in percent), and the differences with Ref. [3].

n / Z	1	2	3	4	5
1	1.602				
2	**1.644**	0.903	0.369	0.169	0.088
3	0.85**5**	1.204	0.830	0.546	0.360
4	0.508	0.99**4**	0.849	0.677	0.530
5	0.334	0.768	0.728	0.643	0.553
6	0.23**5**	0.595	0.599	0.561	0.512
7	0.175	0.469	0.491	0.479	0.453
8	0.135	0.37**7**	0.406	0.406	0.395
9	0.107	0.309	0.339	0.346	0.344
10	0.087	0.258	0.287	0.297	0.299

improved. This calculation for the first time separates the three-body input from approximations like the dipole approximation. Higher excited states and QF corrections were calculated for the first time. We plan to extend this work to triplet states and Compton scattering.

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Unified quark-quark and quark-antiquark interactions for the meson and baryon sector

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Abstract. An adequate description of heavy dimesons (tetraquarks) requires a unified effective interaction which fits both light and heavy meson as well as light and heavy baryon spectra. Some aspects of the problem have been elucidated in lively discussions.

In the constituent quark model one needs effective quark masses and effective quark-quark interactions. There are two approaches to this input information. In the *first principle* (*ab initio*) calculations one would like to derive effective masses and interactions from QCD, at least approximately. We are, however, involved in the *practical (phenomenological)* approach to the constituent quark model: fit effective quark masses and effective interaction parameters so as to reproduce the known meson and baryon spectra, and then predict new states and explain dynamical processes! The usual restriction is to the two-particle Hilbert space for mesons and the three-particle Hilbert space for baryons. The question arises whether in this restricted space a good fit is possible at all, and whether it contains correct physics.

Since our main interest is in dimesons and dibaryons we need a unified interaction for all sectors. A possible unified picture of meson and baryon sectors assumes the one-gluon-exchange potential + linear confining potential. This model has had some success [1–4], but due to flavour independence and due to the colour structure of the spin-spin interaction it sacrifices several states in the baryon sector (for example Roper).

The one-Goldstone-boson-exchange model of Riska, Glozman, Plessas et al.[5,6] has a different flavour-spin structure and is more successful for light baryons, but it leaves it as an open question how to extend such a model to describe heavy baryons and mesons. Heavy baryons and mesons certainly need a spin-spin force of the one-gluon-exchange type since they do not feel pions.

The discussions at Bled was centered around five issues.

1. It has been suggested in the literature to simply take a combination of the meson-exchange and gluon-exchange effective interactions. The practical difficulty is to get enough spin-spin splitting for heavy mesons (OGE), enough

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lowering of Roper (OME), and still not to exceed the $N-\Delta$ splitting (OGE+OME). The conceptual difficulty, strongly advocated by Leonid Glozman, is, however, the coexistence of chirally broken and chirally symmetric phase. According to this doctrine, at the energy scale of light quarks, the chiral symmetry is spontaneously broken and the effective degrees of freedom are constituent quarks and light mesons (Goldstone bosons) – no explicit gluons – therefore OGE would make no sense. At the energy scale of heavy quarks the chiral symmetry plays no role and the effective degrees of freedom are quarks and gluons – perturbative QCD and OGE would make sense. It is then unclear what to use when there are both light and heavy quarks present in the system. The topic remained controversial. A compromise is needed, possible with a modified strength of OGE between a heavy and a light quark.

2. It has been suggested to extend the OME interaction between two quarks (such as has been successfully used by the Graz group) to the light quark-antiquark pair simply by the G-parity transformation. It has to be clarified, however, whether one should invent also relevant annihilation-creation graphs and how should one fit their strength. Such additional interaction would introduce many-quark many-antiquark configurations and the truncation to the one-quark one-antiquark space is questionable. Anyway, also the OGE interaction is not immune against this effect. Moreover, one should avoid double counting. Especially the description of pion using pion-exchange between quarks is very delicate.
3. The extended meson-exchange model (with vector meson exchange) offers improvements in dynamical processes of baryons (form factors) but it opens many new problems for the mesonic sector. For example, the G-parity transformed ω-exchange becomes strongly attractive at short distance; we are exploring good and bad consequences.
4. The NJL model is unified for the light meson and baryon sectors (containing u, d and/or s quarks). However, due to lack of confinement, and due to the need of a complicated cutoff, little can be calculated for excited mesons, and it is not easy to solve for baryons. Maybe NJL can at least inspire meaningful effective $q-q$ and $q-\bar{q}$ interactions [7].
5. It is of interest to verify as much as possible the "$V_{qq} = \frac{1}{2}V_{q\bar{q}}$" rule. It is implicit in OGE and reasonable fits with lowest experimental levels do not contradict it. A crucial test would be the comparison of charmonium (or botomium) with the QQq baryons as well as $QQ\bar{q}\bar{q}$ dimesons (tetraquarks) [8]; $Q = c, b$ and $q = u, d, s$. I would like to emphasise the paramount importance to search for these doubly-heavy baryons and dimesons, possibly at LHC.

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