

# Yukawa sector in $SO(10)$

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# Outline

- GUTs and neutrino mass
- Intermediate scales
- The see-saw and the Yukawa sector
- Non supersymmetric models
- The minimal SUSY model
- Departing from the minimal

# GUTs and neutrino mass

$SO(10)$ : all fermions in  $\underline{16}$  representation

$SU(5)$  fermions: in  $\underline{5}$  and  $\underline{10}$  representations

$\Rightarrow \nu_R$  is a singlet

- adding a singlet to the theory gives a lot of new parameters
- $SU(5)$  breaks directly to  $SU(3) \times SU(2) \times U(1)$ 
  - no intermediate scales

... and  $m_\nu$  calls for intermediate scales

# The (B-L) breaking scale

Best idea for small  $m_\nu$ : the see-saw mechanism

give  $\nu_R$  a mass by breaking B-L  
at a large scale  $M_R$

$$\langle \Delta \rangle \nu_R^T i \sigma_2 \nu_R$$

$$\langle \Delta \rangle = M_R$$

$$m_\nu = \frac{M_W^2}{M_R}$$

$$m_\nu \sim 0.01 eV$$

$$M_R \sim 10^{13} GeV$$

An intermediate scale would be convenient  
(not indispensable)

## SUSY: ONE-STEP UNIFICATION

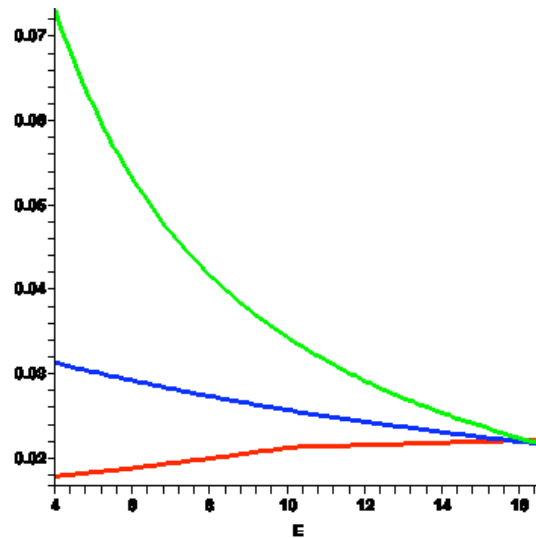
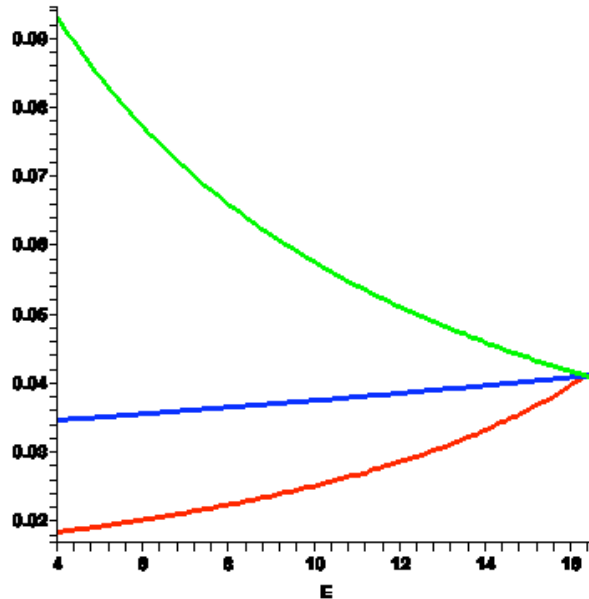
$$\frac{1}{\alpha_i(M_W)} = \frac{1}{\alpha_U} - \frac{b_i}{2\pi} \ln(M_G/M_W)$$

$$M_G \sim 10^{16} \text{ GeV}$$

## NON-SUSY: INTERMEDIATE SCALE

$$\frac{1}{\alpha_i(M_W)} = \frac{1}{\alpha_U} - \frac{b_i}{2\pi} \ln(M_R/M_W) - \frac{b'_i}{2\pi} \ln(M_G/M_R)$$

$M_G$  determined by the particle content



# SO(10) symmetry

Many possible  
intermediate scales

$$SO(10)$$

$$M_X \Downarrow \langle p \rangle$$

GUT scale

$$SU(4)_C \times SU(2)_L \times SU(2)_R$$

$$M_{PS} \Downarrow \langle a \rangle$$

$$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$M_R \Downarrow \langle \sigma \rangle$$

see-saw scale

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

# three types of see-saw

## TYPE I (renormalizable version)

- An  $SU(2)_R$  triplet with  $(B - L) = 2$  gets a vev at a large scale  $M_R$

$\langle \Delta^c \rangle \Rightarrow \nu^c$  mass  $\sim M_R$   
gives a mass to the right-handed neutrino

- At EW scale, neutrino gets a Dirac mass

$m_D$

$$\begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \rightsquigarrow m_\nu \sim \frac{m_D^2}{M_R} \sim \frac{M_W^2}{M_R}$$

## TYPE II

In Left-Right theories, terms like:

$$\Delta H^2 \Delta^c + m_\Delta \Delta^2$$

$H$  : bidoublet

$\Delta$  : Left-handed triplet

$\Delta^c$  : Right-handed triplet

Provide a small vev for the Left-handed triplet after EW breaking

$$\langle \Delta \rangle \sim \frac{\langle H \rangle^2 \langle \Delta^c \rangle}{m_\Delta^2} \sim \frac{M_W^2}{M_R} \quad \text{Mass for } \nu \text{ from } L^T \tau_2 \langle \Delta \rangle L$$

vev of  $\Delta^c$  induces a small vev for  $\Delta$  after EW breaking

In SUSY SO(10), triplets are in 126:

mixing with 54 or 210 can give such terms in the potential.

**TWO TYPES OF SEE-SAW ARE OF SAME MAGNITUDE:  
BUT VERY DIFFERENT PARAMETERS INVOLVED**



# Yukawa sector

Pati-Salam  
fourth color:

$$U = \begin{pmatrix} u \\ u \\ u \\ \nu \end{pmatrix} \quad D = \begin{pmatrix} d \\ d \\ d \\ e \end{pmatrix} \dots$$

$$\text{SO}(10): \quad \Psi_{16} = \begin{pmatrix} U \\ D \\ D^c \\ U^c \end{pmatrix}$$

- All fermions in one (spinorial) representation
- Couple to:

$$\Psi C \Gamma^a \Psi H_a \quad \underline{10}$$

$$\Psi C \Gamma^a \Gamma^b \Gamma^c \Psi D_{abc} \quad \underline{120} \text{ (antisym.)}$$

$$\Psi C \Gamma^a \Gamma^b \Gamma^c \Gamma^d \Gamma^e \Psi \Sigma_{abcde} \quad \underline{126}$$

# SU(4)<sub>C</sub> × SU(2)<sub>L</sub> × SU(2)<sub>R</sub> Decomposition

$$H_{10} = (6, 1, 1) + (1, 2, 2)$$

$$D_{120} = (\bar{10}, 1, 1) + (10, 1, 1) + (6, 3, 1) + (6, 1, 3) + (1, 2, 2) + (15, 2, 2)$$

$$\bar{\Sigma}_{126} = (\underbrace{10, 1, 3}_{\Delta_R}) + (\underbrace{\bar{10}, 3, 1}_{\Delta_L}) + (6, 1, 1) + (15, 2, 2)$$

- 126 can give type I and type II see-saw
- (15,2,2) in 126 can contain the SM Higgs
- is 126 enough for all fermion masses ? **no..**

## One doublet is not enough:

*Lazarides, Shafi Wetterich 1981*

*Clark, Kuo Nakagawa 1982*

$$M_U = y_{10} \langle 1, 2, 2 \rangle_{10}^u + y_{126} \langle 15, 2, 2 \rangle_{126}^u$$

$$M_D = y_{10} \langle 1, 2, 2 \rangle_{10}^d + y_{126} \langle 15, 2, 2 \rangle_{126}^d$$

$$M_E = y_{10} \langle 1, 2, 2 \rangle_{10}^d - 3 y_{126} \langle 15, 2, 2 \rangle_{126}^d$$

- only 10:  $m_d = m_l$
  - only 126:  $3 m_d = m_l$
  - 126 required for neutrino mass - but what else?
- is there a difference between choosing 10 or 120 ?

} at the GUT scale,  
for all generations

**Notice: same question for SUSY or non-SUSY models**

# non-susy: $126 + 10$

*Bajc, A.M, Vissani, Senjanovic 2005*

(2nd and 3rd generations only)

$$M_U = y_{10} \langle 1, 2, 2 \rangle_{10}^u + y_{126} \langle 15, 2, 2 \rangle_{126}^u$$

$$M_D = y_{10} \langle 1, 2, 2 \rangle_{10}^d + y_{126} \langle 15, 2, 2 \rangle_{126}^d$$

$$M_E = y_{10} \langle 1, 2, 2 \rangle_{10}^d - 3 y_{126} \langle 15, 2, 2 \rangle_{126}^d$$

$$M_{\nu_D} = y_{10} \langle 1, 2, 2 \rangle_{10}^u - 3 y_{126} \langle 15, 2, 2 \rangle_{126}^u$$

$$M_{\nu_L} = y_{126} \langle \overline{10}, 3, 1 \rangle_{126}^d$$

$$M_{\nu_R} = y_{126} \langle 10, 1, 3 \rangle_{126}^d$$

see-saw, type I and II:

$$M_N = -M_{\nu_D} M_{\nu_R}^{-1} M_{\nu_D} + M_{\nu_L}$$

approx.

$$\theta_q = V_{cb} = 0$$

$$\frac{\langle 2, 2, 1 \rangle_{10}^u}{\langle 2, 2, 1 \rangle_{10}^d} = \frac{m_c(m_\tau - m_b) - m_t(m_\mu - m_s)}{m_s m_\tau - m_\mu m_b} \approx \frac{m_t}{m_b}$$

- real 10:  $m_t = m_b$
- **need a complex 10** - PQ symmetry  $\rightarrow$  axion as Dark Matter

$$10^{10} \text{GeV} \leq M_{PQ} \leq 10^{13} \text{GeV}$$

$$16 \rightarrow e^{i\alpha} 16$$

$$10 \rightarrow e^{-2i\alpha} 10$$

$$\overline{126} \rightarrow e^{-2i\alpha} \overline{126}$$

Breaks PQ at the right-handed  
neutrino mass scale...

But cannot break completely: combination

$$U(1)_{PQ}, U(1)_{B-L}, T_{3R}$$

remains

# SUSY or not: 126 + 10

*Bajc, Vissani, Senjanovic 2002*

$$M_D = y_{10} \langle 1, 2, 2 \rangle_{10}^d + y_{126} \langle 15, 2, 2 \rangle_{126}^d$$

$$M_E = y_{10} \langle 1, 2, 2 \rangle_{10}^d - 3 y_{126} \langle 15, 2, 2 \rangle_{126}^d$$

Type II see-saw:  $M_N = M_{\nu_L} = y_{126} \langle 10, 1, 3 \rangle_{126}^d$

$$\left| \begin{array}{l} \theta_D = 0 \text{ (small mixing in } M_D) \\ m_s = m_\mu = 0 \end{array} \right. \quad M_N \propto \begin{pmatrix} 0 & 0 \\ 0 & m_b - m_\tau \end{pmatrix}$$

unless  $m_b = m_\tau$ , neutrino mixing vanishes

large  $\theta_{atm} \leftrightarrow b - \tau$  unification

Full 3-gen. analysis:  
- connection still true  
 $\theta_{13}$  close to exp. limit

*Matsuda, Koide, Fukuyama, Nishiura 2002*

*Goh, Mohapatra, Ng, 2003*

# non-susy: 126 + 120

(2nd and 3rd generations only)

$$M_U = y_{120} (\langle 1, 2, 2 \rangle_{120}^u + \langle 15, 2, 2 \rangle_{120}^u) + y_{126} \langle 15, 2, 2 \rangle_{126}^u$$

$$M_D = y_{120} (\langle 1, 2, 2 \rangle_{120}^d + \langle 15, 2, 2 \rangle_{120}^d) + y_{126} \langle 15, 2, 2 \rangle_{126}^d$$

$$M_E = y_{120} (\langle 1, 2, 2 \rangle_{120}^d - 3 \langle 15, 2, 2 \rangle_{120}^d) - 3 y_{126} \langle 15, 2, 2 \rangle_{126}^d$$

$$M_{\nu_D} = y_{120} (\langle 1, 2, 2 \rangle_{120}^u - 3 \langle 15, 2, 2 \rangle_{120}^u) - 3 y_{126} \langle 15, 2, 2 \rangle_{126}^u$$

$y_{120}$  antisymmetric

$$\frac{\langle 1, 2, 2 \rangle_{120}^u + \langle 15, 2, 2 \rangle_{120}^u}{\langle 1, 2, 2 \rangle_{120}^d + \langle 15, 2, 2 \rangle_{120}^d} \sim \frac{m_t}{m_b}$$

- real 120:  $m_t = m_b$
- complex 120: interesting connections with neutrino masses and mixings

# SUSY or not: 126 + 120

*Bajc, A.M., Vissani, Senjanovic 2005*

Most general charged  
fermion matrix:

(2nd and 3rd generations only)

$$M_f = \mu_f \begin{pmatrix} \sin^2 \theta & i \sin \theta \cos \theta + i \epsilon_f \\ -i \sin \theta \cos \theta - i \epsilon_f & \cos^2 \theta \end{pmatrix}$$

$$|\epsilon_f| \propto m_2^f / m_3^f \ll 1$$

to leading order in  $|\epsilon_f|$

$$\begin{aligned} |\mu_f| &= m_3^f \\ \sin 2\theta |\epsilon_f| &= m_2^f / m_3^f \end{aligned}$$

- neutrino masses
- relation  $m_b, m_\tau$
- quark mixing  $V_{cb}$



- neutrino masses

$$M_N^{II} \propto Y_{126} \propto \begin{pmatrix} \sin^2 \theta & \\ & \cos^2 \theta \end{pmatrix}$$

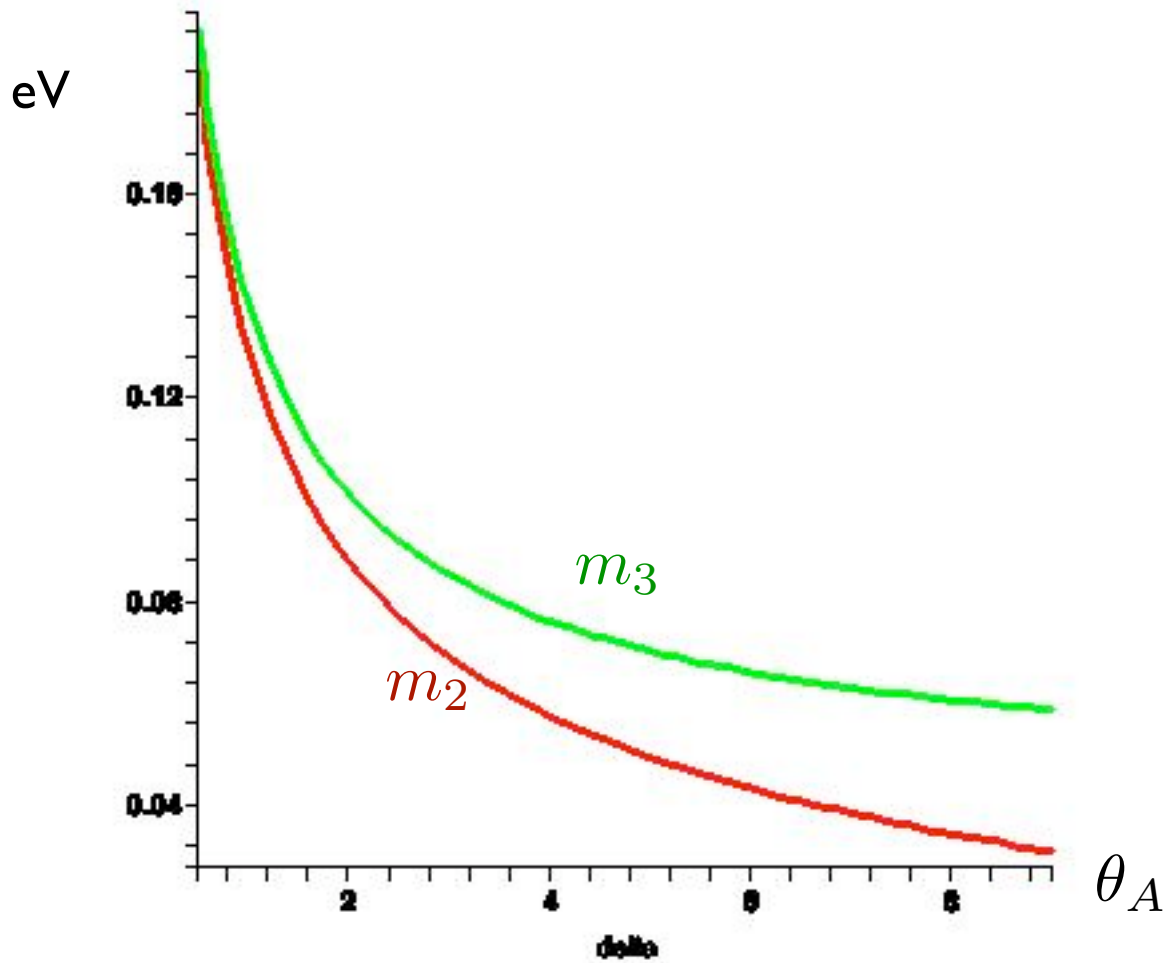
$$M_N^I \propto (Y_{126} + cY_{120})^T Y_{126}^{-1} (Y_{126} + cY_{120}) \propto \begin{pmatrix} \sin^2 \theta & \\ & \cos^2 \theta \end{pmatrix}$$

So  $\theta \sim \theta_A$  to leading order in  $|\epsilon_E|$

and for neutrino masses:

$$\frac{m_3^2 - m_2^2}{m_3^2 + m_2^2} = \frac{\cos 2\theta_A}{1 - \sin^2 2\theta_A/2} + \mathcal{O}(|\epsilon|)$$

▶ large  $\theta_A$  gives degenerate neutrinos



99% CL

$$36^\circ < \theta_{23} < 54^\circ$$

central value

$$|\Delta m_{23}^2| = (2.5 \pm 0.2) 10^{-3} \text{ eV}^2$$

•relation  $m_b, m_\tau$  at the GUT scale

$$\frac{m_\tau}{m_b} = 3 + 3 \sin 2\theta_A \operatorname{Re}[\epsilon_E - \epsilon_D] + \mathcal{O}(|\epsilon^2|)$$

▶ but  $m_\tau \sim 2m_b$  .... include

- running (large Yukawa coupling)
- correction of higher order in  $\epsilon$
- 3 generations

- quark mixing

$$|V_{cb}| = |\Re(\xi) - i \cos 2\theta_A \Im(\xi)| + \mathcal{O}(|\epsilon^2|)$$

$$\xi = \cos 2\theta_A (\epsilon_D - \epsilon_U)$$

since:  $\sin 2\theta |\epsilon_f| = m_2^f / m_3^f$

$$\sin 2\theta_A \simeq 1 \quad \cos 2\theta_A \simeq 0$$

then:  $\epsilon_D \simeq \frac{m_s}{m_b} \gg \frac{m_c}{m_t} \simeq \epsilon_U \quad \Rightarrow \quad |V_{cb}| \simeq \cos 2\theta_A \frac{m_s}{m_b}$

- ▶ large neutrino mixing implies small quark mixing
- ▶ even too small...

$$|V_{cb}| \sim 1/25$$

$$m_s/m_b < 1/20$$

$$\cos 2\theta_A < 1/3$$

# 126 + 10 or 126 + 120 ?

- In non-supersymmetric models, both possible in principle
  - ▶ 10 and 120 need to be complex
  - ▶ can have a PQ symmetry - axion as DM
- SUSY requires 126 + 10 for  $m_b = m_\tau$
- Type II (even Type I) see-saw can give relations between neutrino and charged fermions masses and mixings
- Detailed models can be even more predictive: symmetry breaking and unification constraints
- 10 + 120 ? radiative see-saw - works for split-SUSY

# Unification: non-SUSY

*Deshpande, Keith, Pal 1993*

$$m_\nu \geq m_t^2/M_R \quad \longrightarrow \quad M_R \geq 10^{13} \text{ GeV}$$

$$\log(M_R/\text{GeV})$$

I :	$SO(10) \xrightarrow{210} \{2_L 2_R 4_C\} \xrightarrow{45} \{2_L 2_R 1_X 3_c\} \xrightarrow{h} \{2_L 1_Y 3_c\}$	8.2-10.6
II :	$SO(10) \xrightarrow{54} \{2_L 2_R 4_C P\} \xrightarrow{210} \{2_L 2_R 1_X 3_c P\} \xrightarrow{h} \{2_L 1_Y 3_c\}$	8.6 - 13.6
III :	$SO(10) \xrightarrow{54} \{2_L 2_R 4_C P\} \xrightarrow{45} \{2_L 2_R 1_X 3_c\} \xrightarrow{h} \{2_L 1_Y 3_c\}$	8.0 - 13.6
IV :	$SO(10) \xrightarrow{54} \{2_L 2_R 1_X 3_c P\} \xrightarrow{210} \{2_L 2_R 1_X 3_c\} \xrightarrow{h} \{2_L 1_Y 3_c\}$	8.2-10.8
V :	$SO(10) \xrightarrow{210} \{2_L 2_R 4_C\} \xrightarrow{45} \{2_L 1_R 4_C\} \xrightarrow{h} \{2_L 1_Y 3_c\}$	11.0-11.2
VI :	$SO(10) \xrightarrow{54} \{2_L 2_R 4_C P\} \xrightarrow{45} \{2_L 1_R 4_C\} \xrightarrow{h} \{2_L 1_Y 3_c\}$	12.2 - 13.6
VII :	$SO(10) \xrightarrow{54} \{2_L 2_R 4_C P\} \xrightarrow{210} \{2_L 2_R 4_C\} \xrightarrow{h} \{2_L 1_Y 3_c\}$	11.3 - 13.6
VIII :	$SO(10) \xrightarrow{45} \{2_L 2_R 1_X 3_c\} \xrightarrow{45} \{2_L 1_R 1_X 3_c\} \xrightarrow{h} \{2_L 1_Y 3_c\}$	2.0-7.7
IX :	$SO(10) \xrightarrow{54} \{2_L 2_R 1_X 3_c P\} \xrightarrow{45} \{2_L 1_R 1_X 3_c\} \xrightarrow{h} \{2_L 1_Y 3_c\}$	2.0-10.0
X :	$SO(10) \xrightarrow{210} \{2_L 2_R 4_C\} \xrightarrow{210} \{2_L 1_R 1_X 3_c\} \xrightarrow{h} \{2_L 1_Y 3_c\}$	-
XI :	$SO(10) \xrightarrow{54} \{2_L 2_R 4_C P\} \xrightarrow{210} \{2_L 1_R 1_X 3_c\} \xrightarrow{h} \{2_L 1_Y 3_c\}$	2.0-13.5
XII :	$SO(10) \xrightarrow{45} \{2_L 1_R 4_C\} \xrightarrow{45} \{2_L 1_R 1_X 3_c\} \xrightarrow{h} \{2_L 1_Y 3_c\}$	2.0-5.3

# Unification: non-SUSY

*Deshpande, Keith, Pal 1993*

$$m_\nu \geq m_t^2/M_R \implies M_R \geq 10^{13} \text{ GeV}$$

$\log(M_R/\text{GeV})$

I:	$SO(10) \xrightarrow{210} \{2_L 2_R 4_C\} \xrightarrow{45} \{2_L 2_R 1_X 3_C\} \xrightarrow{h} \{2_L 1_Y 3_C\}$	8.2 - 10.6
II:	$SO(10) \xrightarrow{54} \{2_L 2_R 4_C P\} \xrightarrow{210} \{2_L 2_R 1_X 3_C P\} \xrightarrow{h} \{2_L 1_Y 3_C\}$	8.6 - 13.6
III:	$SO(10) \xrightarrow{54} \{2_L 2_R 4_C P\} \xrightarrow{45} \{2_L 2_R 1_X 3_C\} \xrightarrow{h} \{2_L 1_Y 3_C\}$	8.0 - 13.6
IV:	$SO(10) \xrightarrow{54} \{2_L 2_R 1_Y 3_C P\} \xrightarrow{210} \{2_L 2_R 1_Y 3_C\} \xrightarrow{h} \{2_L 1_Y 3_C\}$	8.2 - 10.9
V:	$SO(10) \xrightarrow{210} \{2_L 2_R 4_C\} \xrightarrow{45} \{2_L 1_R 4_C\} \xrightarrow{h} \{2_L 1_Y 3_C\}$	11.0 - 11.2
VI:	$SO(10) \xrightarrow{54} \{2_L 2_R 4_C P\} \xrightarrow{45} \{2_L 1_R 4_C\} \xrightarrow{h} \{2_L 1_Y 3_C\}$	12.2 - 13.6
VII:	$SO(10) \xrightarrow{54} \{2_L 2_R 4_C P\} \xrightarrow{210} \{2_L 2_R 4_C\} \xrightarrow{h} \{2_L 1_Y 3_C\}$	11.3 - 13.6
VIII:	$SO(10) \xrightarrow{10} \{2_L 1_R 1_Y 3_C\} \xrightarrow{10} \{2_L 1_R 1_Y 3_C\} \xrightarrow{10} \{2_L 1_Y 3_C\}$	20.77
IX:	$SO(10) \xrightarrow{54} \{2_L 2_R 1_Y 3_C P\} \xrightarrow{45} \{2_L 1_R 1_Y 3_C\} \xrightarrow{h} \{2_L 1_Y 3_C\}$	20.100
X:	$SO(10) \xrightarrow{210} \{2_L 2_R 4_C\} \xrightarrow{210} \{2_L 1_R 1_Y 3_C\} \xrightarrow{h} \{2_L 1_Y 3_C\}$	20.100
XI:	$SO(10) \xrightarrow{54} \{2_L 2_R 4_C P\} \xrightarrow{210} \{2_L 1_R 1_Y 3_C\} \xrightarrow{h} \{2_L 1_Y 3_C\}$	20.125
XII:	$SO(10) \xrightarrow{10} \{2_L 1_R 4_C\} \xrightarrow{10} \{2_L 1_R 1_Y 3_C\} \xrightarrow{10} \{2_L 1_Y 3_C\}$	20.53

# Unification: SUSY

- One-step: no intermediate scales
  - ▶  $m_\nu \propto M_W^2/M_{GUT}$  can be too small
- Potentials very constrained: no survival principle
  - ▶ calculate all the masses
- See-saw + SUSY = MSSM with R-parity
  - ▶ R-parity is in the center of SO(10)
  - ▶ R-parity  $\equiv$  Matter parity =  $(-1)^{3(B-L)}$
  - ▶ See-saw: break (B-L) with a (B-L)-even field in order to give  $\tilde{a}_R$  mass

*Aulakh, A.M, Rasin,  
Senjanovic 1998*

...get R-parity preserved  
and the stable LSP is a DM candidate



# What is the minimal renormalizable SUSY- GUT ?

- Based on SO(10)
- With a see-saw for neutrino mass:  $126 + \overline{126}$
- Yukawa sector:  $10 + \overline{126}$  needed: the light Higgs must be a combination of doublets in 10 and  $\overline{126}$

▶ need a mixing  $\langle \Phi \rangle H_{10} \overline{\Sigma}_{126}$  can use 210

- Symmetry breaking down to LR  
( $126, \overline{126}$  break down to MSSM)

*Babu, Mohapatra, 1993*

$$\Phi_{210}, H_{10}, \overline{\Sigma}_{126}, \Sigma_{126}$$

▶ 210 can do that too

# minimal SO(10)

*Clark, Kuo, Nakagawa, 1982*

*Aulakh, Bajc, A.M, Vissani, Senjanovic, 2003*

$$\Psi_{16}, H_{10}, \Sigma_{126}, \bar{\Sigma}_{\bar{126}}, \Phi_{210}$$

$$W_H = m_\Phi \Phi^2 + m_\Sigma \Sigma \bar{\Sigma} + \lambda \Phi^3 + \eta \Phi \Sigma \bar{\Sigma} + m_H H^2 + \Phi H (\alpha \Sigma + \bar{\alpha} \bar{\Sigma}) \\ + y_{10} \Psi C \Gamma \Psi H + y_{126} \Psi C \Gamma^5 \Psi \bar{\Sigma}$$

- 26 real parameters: same as MSSM
- light Higgs made up of 126, 10 and 210 doublets
  - ▶ rich enough Yukawa structure
- Type I and II see-saw
  - ▶ possibility of connecting large  $\theta_A$  with  $b$ -unification
- symmetry can be broken down to MSSM (+R-parity)
  - ▶ stable LSP

# symmetry breaking

$$SO(10)$$

$$M_X \Downarrow \langle p \rangle \quad \text{in } 210$$

$$SU(4)_C \times SU(2)_L \times SU(2)_R$$

$$M_{PS} \Downarrow \langle a \rangle \quad \text{in } 210$$

$$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$M_R \Downarrow \langle \sigma \rangle \quad \text{in } 126$$

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

# symmetry breaking

$$H \equiv 10 = (6, 1, 1) + (1, 2, 2)$$

$$\begin{aligned} \Phi \equiv 210 &= (15, 1, 1) + (1, 1, 1) + (15, 1, 3) \\ &+ (15, 3, 1) + (6, 2, 2) + (10, 2, 2) + (\overline{10}, 2, 2) \end{aligned}$$

$$\Sigma \equiv 126 = (\overline{10}, 1, 3) + (10, 3, 1) + (6, 1, 1) + (15, 2, 2)$$

$$\overline{\Sigma} \equiv \overline{126} = (10, 1, 3) + (\overline{10}, 3, 1) + (6, 1, 1) + (15, 2, 2)$$

SM singlets:

$$\text{vev} \sim M_{GUT}$$

type II see-saw:

$$\text{vev} \sim M_W^2 / M_{GUT}$$

doublets:

$$\text{vev} \sim M_W$$

- Find the symmetry breaking conditions
- Calculate masses for all states
- Find the composition of the light Higgs doublets

*Bajc, A.M, Vissani, Senjanovic 2004*

*Aulakh, Girdaar, 2004*

*Fukuyama, et. al. 2004*

# An overconstrained model

After fine-tune of the SM Higgs mass:  
8 parameters left in the heavy Higgs sector

$$m, \alpha, \bar{\alpha}, |\lambda|, |\eta|, \phi = \arg \lambda = -\arg \eta$$

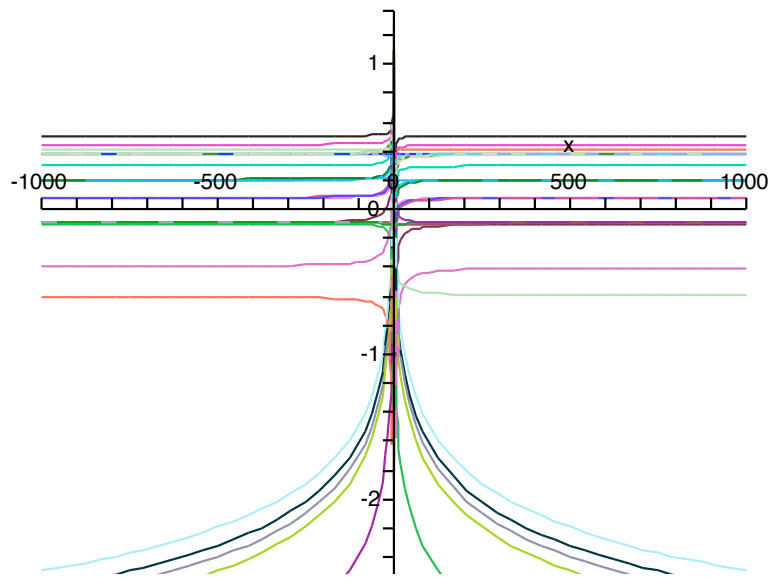
$$x = \Re(x) + i\Im(x)$$

↑  
ratio of masses

Vevs and masses of all states have form:

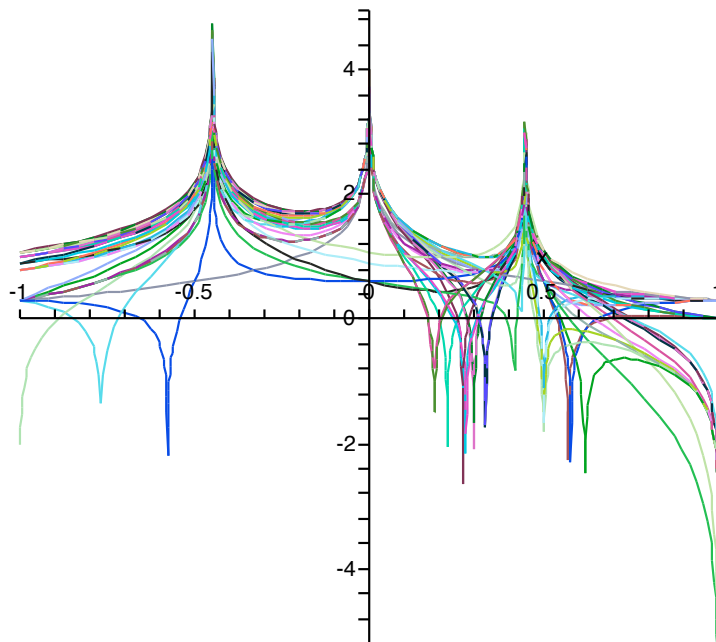
$$\sim \frac{m}{\lambda} f(x)$$
$$\frac{m}{\sqrt{\lambda\eta}} f(x)$$

- variation with parameters quite smooth, with  $x$  non-trivial



$$\text{Log}[M_i/10^{16}]$$

$$x \rightarrow \infty$$



$$x \text{ real} < 1$$

- Light states spoil unification:  
keep  $x < 1$

# Fermion mass fitting

- The light Higgs is a combination **no longer arbitrary**

$$H_{u,d} = r_{u,d}^{10} H_{u,d}^{10} + r_{u,d}^{\overline{126}} H_{u,d}^{\overline{126}} + r_{u,d}^{126} H_{u,d}^{126} + r_{u,d}^{210} H_{u,d}^{210}$$

►  $r_{u,d}^{\mathbf{I}}$  known functions of the parameters

- Assume type II see-saw

$$m_\nu = y_{126} v_\Delta \quad v_\Delta = \frac{(\alpha r_u^{10} + \sqrt{6}\eta r_u^{\overline{126}}) r_u^{210}}{m_\Delta}$$

► neutrino mass depends on the **same** parameters

# General analysis (type I and II)

*Bertolini, Frigerio, Malinsky, 2005-2006*

*Aulakh, Garg, Girdaar, 2005-2006*

*Mohapatra, Goh, Ng, Dutta, Mimura...*

- Do the complete fit with all fermion masses and all parameters

- Parameter space for type I and type II getting smaller

- Include unification constrains, threshold effects - even worse

*Babu, Macesanu*

*Wang, Yang*

**too small** neutrino mass: model seems to be ruled out !



# Summary

- SO(10): ideal framework for small neutrino mass
- Models can provide connections between fermion masses and mixings, for example
  - ▶  $b - \tau$  unification  $\longleftrightarrow$  large  $\theta_{atm}$  (10+126)
  - ▶ large neutrino  $\longleftrightarrow$  small quark mixings (120+126)
  - ▶ large  $\theta_{atm}$   $\longleftrightarrow$  degenerate neutrinos (120+126)
- Non supersymmetric models are alive and well
- Minimal SUSY GUT is in trouble
  - \* lack of intermediate scales
- Next-to-minimal SUSY GUT may not be predictive ...
  - \* but work is in progress