$B \to V\gamma$ at NNLO in SCET

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Based on work with Ahmed Ali (DESY) and Christoph Greub (Bern)

Outline

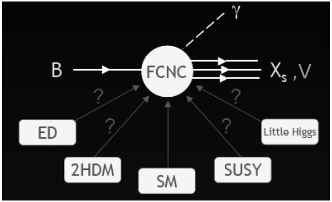
- Motivation
- QCD Factorization and SCET as applied to $B \to V\gamma$
- Matching calculations and hard-scattering kernels
- Numerical results and comparison with experiment

Radiative B decays

Example: $b \to s\gamma$ transition

$B \to X_s\gamma$ = inclusive radiative decay

$B \to K^*\gamma$ = exclusive radiative decay



FCNCs are loop suppressed in Standard Model

- $B \to X_s\gamma$ "standard candle" for new physics

Motivation for studying $B \to V\gamma$

Inclusive $B \to X_s\gamma$ has received much attention

- Calculable using OPE and heavy-quark expansion
- Branching fraction known to NNLO in perturbation theory (Misiak +16 others 2006)
- Also known to NNLO with cut on photon energy (Becher, Neubert 2006)

Exclusive $B \to V\gamma$ decays ($V = K^*, \rho, \omega, \dots$) also useful

- Exclusive $b \to d\gamma$ ($V = \rho, \omega$) will be well measured at LHC
- Provide independent checks on shape of unitarity triangle
- Calculable in QCD factorization approach

Idea of QCD factorization

$$65856 = 12 \times 56 \times 98$$

QCD factorization and today's talk

QCD Factorization:

Branching fraction obtained as a series in $(\alpha_s, \Lambda_{\rm QCD}/m_b) \ll 1$

$$\mathcal{B}(\bar{B} \to V\gamma) = \mathcal{B}^{\rm LO} \left| t^{\rm I} \zeta_{V_\perp} + t^{\rm II} \star \phi_\perp^V \star \phi_+^B + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right) \ldots \right|^2$$

- $\zeta_{V_\perp}$ and $\phi^{B,V}$ are non-perturbative (but universal)
- Hard-scattering kernels $t^{\rm I}$ and $t^{\rm II}$ are perturbative

$$t^{\rm I} = 1 + \mathcal{O}(\alpha_s) + \ldots \qquad t^{\rm II} = \mathcal{O}(\alpha_s) + \ldots$$

- The kernels $t^{\rm I}$, $t^{\rm II}$ known at NLO ($\mathcal{O}(\alpha_s)$) for some time

Today's talk: Hard-scattering kernels at NNLO ($\mathcal{O}(\alpha_s^2)$)

Why higher-orders?

QCD factorization is limited by:

- Hadronic uncertainties, especially in $\zeta_{V_\perp}$
- Power corrections (although hard to quantify)

Why bother with higher-order perturbative corrections?

1) Practical reasons

- In some ratios hadronic uncertainties tend to cancel
- NLO for branching fractions is LO for CP asymmetries

2) Theoretical reasons

- Check factorization at NNLO
- Study connection between QCDF and SCET (next slide)

What is SCET and why use it?

What: SCET = soft-collinear effective theory
(Bauer, Pirjol, Fleming, Stewart 2000)

Why: Allows to discuss factorization in EFT language

- hard-scattering kernels = matching (Wilson) coefficients
- non-perturbative functions = hadronic matrix elements

Advantages of SCET approach:

- Mass scales $m_b^2 \gg m_b\Lambda_{\rm QCD} \gg \Lambda_{\rm QCD}^2$ are clearly separated
- RG evolution of matching functions “resums” large logs
- All-orders factorization proof possible
  (Becher, Hill, Neubert 2005)

Outline

- Motivation
- **QCD Factorization and SCET as applied to $B \to V\gamma$**
- Matching calculations and hard-scattering kernels
- Numerical results and comparison with experiment

Outline of factorization discussion

1) QCD factorization

- Introduce effective weak Hamiltonian
- QCD factorization formula for matrix elements of weak Hamiltonian

2) Derivation of QCD factorization formula with SCET

Effective weak Hamiltonian

QCD effects at $\mu \sim m_b$ described by effective weak Hamiltonian:

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pq}^* V_{pb} \left[C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3}^{8} C_i Q_i \right]; \quad (q = s, d)$$

- The C_i are Wilson coefficients depending on M_W, M_Z, m_t
- The Q_i are operators built from QCD and photon fields

The Wilson coefficients are known in RG-improved perturbation theory to NNLO (α_s^2) (Csakon, Gorbahn, Haisch, Misiak, others ...)

This part is the same for inclusive and exclusive decays

Operators in effective weak Hamiltonian

Example: $b \to s\gamma$

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{ps}^* V_{pb} \left[C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3}^{8} C_i Q_i \right]$$

Most important operators for $b \to s\gamma$:

$$\begin{aligned} Q_1^p &= (\bar{s}\, p)_{V-A} (\bar{p}\, b)_{V-A} \qquad (p = u, c) \\ Q_7 &= -\frac{e\, \overline{m}_b(\mu)}{8\pi^2} \bar{s}\, \sigma^{\mu\nu} [1 + \gamma_5]\, b F_{\mu\nu} \\ Q_8 &= -\frac{g\, \overline{m}_b(\mu)}{8\pi^2} \bar{s}\, \sigma^{\mu\nu} [1 + \gamma_5]\, T^a\, b G^a_{\mu\nu} \end{aligned}$$

For $b \to d\gamma$ replace $s \to d$

Hadronic matrix elements of weak Hamiltonian

Branching fraction:

$$\mathcal{B}(B \to V\gamma) = \frac{\tau_B m_B}{4\pi}\left(1 - \frac{m_V^2}{m_B^2}\right)|\mathcal{A}|^2$$

Amplitude:

$$\mathcal{A} \sim \langle V\gamma|\mathcal{H}_{\rm eff}|\bar{B}\rangle \sim \sum_i \langle V\gamma|Q_i|\bar{B}\rangle$$

Main challenge: evaluate $\langle V\gamma|Q_i|\bar{B}\rangle$ = hadronic matrix elements

- QCD factorization is a method for doing this

QCD factorization formula I

Hadronic matrix elements factorize in heavy-quark limit

(Ali, Parkhomenko; Beneke, Feldmann, Seidel; Bosch, Buchalla 2001)

$$\langle V\gamma\,|Q_i|\,\bar{B}\rangle = F^{B\to V_\perp}\,T_i^{\mathrm{I}} + \int d\omega\,du\,\phi_+^B(\omega)\,\phi_\perp^V(u)\,T_i^{\mathrm{II}}(\omega,u) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)$$

Non-perturbative pieces: (from QCD sum rules, lattice)

- $F^{B\to V_\perp}$ is a hadronic form factor in QCD
- $\phi^{B,V_\perp}$ are light-cone distributions amplitudes (LCDAs)

Perturbative pieces: (as a series in α_s)

- T_i^{I} are "vertex corrections"
- T_i^{II} are "hard spectator corrections"

QCD factorization formula II

The QCD form factor itself obeys a factorization formula
(Beneke, Feldmann 2001)

$$F^{B\to V_\perp} = T^{\mathrm{I}}_{V_\perp}\zeta_{V_\perp} + \int d\omega\, du\, \phi^B_+(\omega)\,\phi^V_\perp(u)\,T^{\mathrm{II}}_{V_\perp}(\omega,u) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)$$

- the soft function $\zeta_{V_\perp}$ is purely non-perturbative
- **Equivalent** forms of factorization formula

$$\langle V\gamma\,|Q_i|\,\bar{B}\rangle = F^{B\to V_\perp}\,T^{\mathrm{I}}_i + \int d\omega\, du\, \phi^B_+(\omega)\,\phi^V_\perp(u)\,T^{\mathrm{II}}_i(\omega,u)$$

$$\langle V\gamma\,|Q_i|\,\bar{B}\rangle = \zeta_{V_\perp}\,t^{\mathrm{I}}_i + \int d\omega\, du\, \phi^B_+(\omega)\,\phi^V_\perp(u)\,t^{\mathrm{II}}_i(\omega,u)$$

Second form more useful for SCET

The hard-scattering kernels and factorization

Interested in hadronic matrix elements

$$\langle Q_i \rangle_{\text{had}} = \left\langle V\gamma \,|Q_i|\, \bar{B} \right\rangle$$

Instead calculate partonic matrix elements (Feynman Diagrams)

$$\langle Q_i \rangle_{\text{part}} = \langle (q\bar{q}')\gamma \,|Q_i|\, (b\bar{q}') \rangle$$

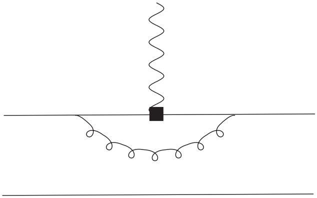
If the partonic matrix element satisfies (to all orders)

$$\langle Q_i \rangle_{\text{part}} = \zeta_{V_\perp,\text{part}}\, t_i^{\text{I}} + \int d\omega\, du\, \phi^B_{+,\text{part}}(\omega)\, \phi^V_{\perp,\text{part}}(u)\, t_i^{\text{II}}(\omega, u)$$

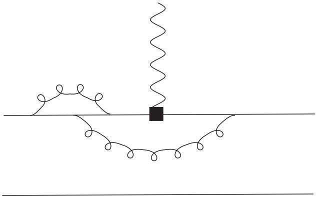
Then assume

$$\langle Q_i \rangle_{\text{had}} = \zeta_{V_\perp}\, t_i^{\text{I}} + \int d\omega\, du\, \phi^B_{+}(\omega)\, \phi^V_{\perp}(u)\, t_i^{\text{II}}(\omega, u)$$

Vertex corrections



$\zeta^{(0)}_{V_\perp} t^{\mathrm{I}(1)}$

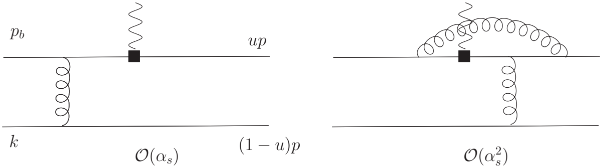


$\zeta^{(0)}_{V_\perp} t^{\mathrm{I}(2)}$

These are virtual corrections to matrix elements in $B \to X_s\gamma$

- QCD graphs almost completely known at NNLO
- Can obtain t^{I} to same accuracy

Hard spectator corrections



Complications:

1) Integrals depend on two different perturbative scales:

$$(2p \cdot p_b \sim m_b^2) \quad \gg \quad (2p \cdot k \sim m_b \Lambda_{\rm QCD} \sim (1.5\,{\rm GeV})^2)$$

"hard" "hard collinear"

- Large perturbative logs of form $\ln(m_b/\Lambda_{\rm QCD})$ in $t_i^{\rm II}$
  $\Rightarrow$ Need resummation

2) Individual graphs can contribute to both $t^{\rm I}$ and $t^{\rm II}$

Dealing with both points easiest in SCET

SCET factorization formula

In soft-collinear effective theory:

$$\langle V\gamma\,|Q_i|\,\bar{B}\rangle = \Delta_i C^A \zeta_{V_\perp} + \int d\omega\, du\, \phi_+^B(\omega)\,\phi_\perp^V(u)\, t_i^{\mathrm{II}}(\omega,u)$$

Spectator term is subfactorized:

$$t_i^{\mathrm{II}}(u,\omega) = \int_0^1 d\tau \Delta_i C^{B1}(\tau) j_\perp(\tau,u,\omega) \equiv \Delta_i C^{B1} \star j_\perp$$

- $\Delta_i C^i$ contain physics at the hard scale m_b^2
- $j_\perp$ contains physics at the jet scale $m_b \Lambda_{\mathrm{QCD}}$
- $\zeta_{V_\perp}, \phi^i$ are matrix elements of SCET operators
  $\Rightarrow$ Distinguish vertex and spectator terms at operator level
  $\Rightarrow$ Resum logs with RG evolution

Two-step matching: $\mathrm{QCD} \rightarrow \mathrm{SCET_I} \rightarrow \mathrm{SCET_{II}}$

$$m_b \gg \sqrt{m_b \Lambda_{\mathrm{QCD}}} \gg \Lambda_{\mathrm{QCD}}$$

QCD $\longrightarrow$	$\mathrm{SCET_I}$	$\mathrm{SCET_I} \longrightarrow$	$\mathrm{SCET_{II}}$	
$\mu_h \sim m_b$		$\mu_i \sim \sqrt{m_b \Lambda_{\mathrm{QCD}}}$		$\mu_f \gtrsim \Lambda_{\mathrm{QCD}}$
match $\longrightarrow$	RG evol. $\longrightarrow$	match $\longrightarrow$	RG evol.	
$\Delta_i C^{B1}(\mu_h) \times$	$U_{\mathrm{I}}(\mu_h, \mu_i) \times$	$j_\perp(\mu_i) \times$	$U_{\mathrm{II}}(\mu_i, \mu_f) \times$	$\{\phi_i(\mu_f)\}$

- $\Delta_i C^{B1}(\mu_h)$ and $j_\perp(\mu_i)$ are free of large logarithms
- RG evolution of matching functions is resummation

First matching step: $\mathrm{QCD} \to \mathrm{SCET_I}$

Match the operators Q_i onto $\mathrm{SCET_I}$:

$$Q_i \to \Delta_i C^A J^A + \Delta_i C^{B1} \star J^{B1} + \Delta_i C^{B2} \star J^{B2} + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)$$

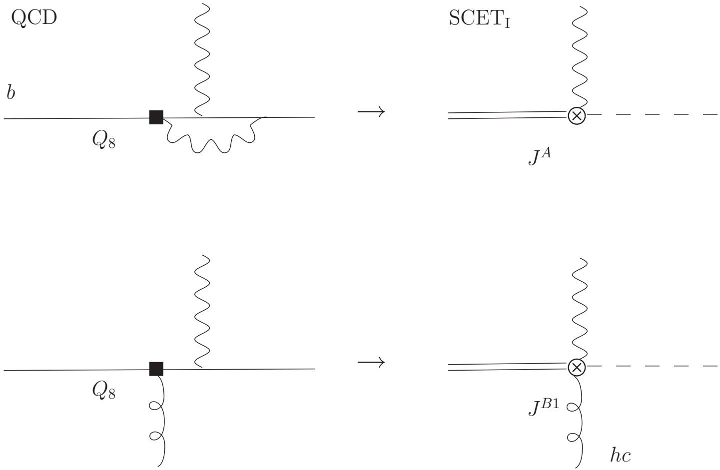
J^i are current operators in $\mathrm{SCET_I}$:

$$\begin{aligned} J^A &= \bar{\chi}_{hc}\, \not{\epsilon}_\perp (1+\gamma_5) h_v \\ J^{B1} &= \bar{\chi}_{hc}\, \not{\epsilon}_\perp \not{A}_{hc\perp} (1+\gamma_5) h_v \\ J^{B2} &= \bar{\chi}_{hc}\, \not{A}_{hc\perp} \not{\epsilon}_\perp (1+\gamma_5) h_v \end{aligned}$$

Important points about “$\mathrm{SCET_I}$”

- Fluctuations at m_b are integrated out and encoded in the $\Delta_i C^i$
- Matrix elements of $\mathrm{SCET_I}$ operators depend on the hard-collinear scale $m_b \Lambda_{\mathrm{QCD}}$ and hadronic scale Λ_{QCD}

Example: $\mathrm{QCD} \rightarrow \mathrm{SCET_I}$ for $Q_8 \sim \bar{b}G^{\mu\nu}s$



Integrating out $m_b\Lambda_{\text{QCD}}$: $\text{SCET}_{\text{I}} \to \text{SCET}_{\text{II}}$

Hard-collinear (intermediate) scale:

$$m_b\Lambda_{\text{QCD}} \sim (1.5\,\text{GeV})^2 = \text{perturbative}$$

Would like to integrate this out ($\text{SCET}_{\text{I}} \to \text{SCET}_{\text{II}}$)

$$J^i \to j_i(m_b\Lambda_{\text{QCD}}) \star O^{i,\text{SCET}_{\text{II}}}(\Lambda_{\text{QCD}})$$

- For J^{Bi} will do this and define hard-spectator term
- For J^A can't do this because convolution diverges

But: $\langle V\gamma|J^A|\bar{B}\rangle \sim \zeta_{V_\perp}$

- J^A maps onto vertex term

The vertex corrections in SCET

From previous slide

$$Q_i \to \Delta_i C^A J^A + \dots$$

Matrix element of J^A defines the soft function:

$$\langle V\gamma | J^A | \bar{B} \rangle \sim \zeta_{V_\perp}$$

Therefore

$$\langle V\gamma | Q_i | \bar{B} \rangle = \Delta_i C^A \, \zeta_{V_\perp} + \dots$$

and

$$\Delta_i C^A = t_i^{\mathrm{I}}$$

SCET matching coefficient = hard-scattering kernel
SCET matrix element = non-perturbative function

Second matching step: $\mathrm{SCET_I} \to \mathrm{SCET_{II}}$

$$Q_i \to \cdots + \Delta_i C^{B1} \star J^{B1} + \Delta_i C^{B2} \star J^{B2}$$

Can further match J^{Bi} onto 4-quark operators in $\mathrm{SCET_{II}}$

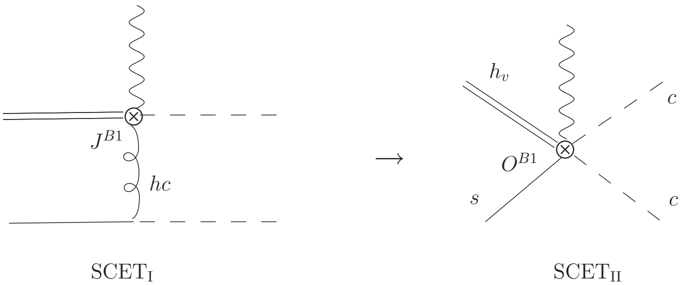
$$J^{B1} \to \int du \int d\omega\, j_\perp\left(\tau, u, \frac{m_b\omega}{\mu^2}\right) O^{B1}(u,\omega)$$

$$J^{B2} \to \int du \int d\omega\, j_\parallel\left(\tau, u, \frac{m_b\omega}{\mu^2}\right) O^{B2}(u,\omega)$$

Important points about "$\mathrm{SCET_{II}}$"

- Scale $m_b\Lambda_{\mathrm{QCD}}$ is integrated out and encoded in jet functions j_i
- Matrix elements of the $\mathrm{SCET_{II}}$ operators depend Λ_{QCD}
- The matrix elements of the $\mathrm{SCET_{II}}$ operators factorize into soft and collinear parts (no $\mathcal{L}_{\mathrm{eff}}^{s+c}$)

$\mathrm{SCET_I} \to \mathrm{SCET_{II}}$



LCDAs:

$$\Phi^B_{\alpha\beta}(\tilde{\omega}) = \int \frac{dt}{2\pi} e^{it\tilde{\omega}} \langle 0 \left| \bar{q}'_{s\beta}(tn_-) \, [tn_-, 0] \, h_{v\alpha}(0) \right| \bar{B} \rangle$$

$$\Phi^V_{\gamma\delta}(u) = \int \frac{ds}{2\pi} e^{-isun_+p} \langle V(p) \left| \bar{\xi}_{c,\delta}(sn_+) \, [sn_+, 0] \, \xi'_{c,\gamma}(0) \right| 0 \rangle$$

The hard spectator term in SCET

Hadronic matrix elements of $\mathrm{SCET_{II}}$ operators:

$$\begin{aligned}\langle V\gamma|O^{B1}|\bar{B}\rangle &\sim \phi_+^B(\omega)\phi_\perp^V(u)\\ \langle V\gamma|O^{B2}|\bar{B}\rangle &= 0\end{aligned}$$

- ϕ_+^B is matrix element of soft (HQET) operator
- $\phi_\perp^V$ is a matrix element of a collinear operator
- Proving that soft and collinear sectors factorize is complicated (Becher, Hill, Neubert 2005)

Put together to define spectator term in SCET

$$\begin{aligned}\langle V\gamma|(\Delta_i C^{B1}\star J^{B1})|\bar{B}\rangle &= (\Delta_i C^{B1}\star j_\perp)\star\phi^B\star\phi_\perp^V\\ &\equiv t_i^{\mathrm{II}}\star\phi^B\star\phi_\perp^V\end{aligned}$$

RG evolution from μ_h to μ_i

$$\Delta_i C^{B1}(\mu_h) \times U_{\rm I}(\mu_h, \mu_i) \times j_\perp(\mu_i) \times U_{\rm II}(\mu_i, \mu_f) \times \{\phi_i(\mu_f)\}$$

The RG-improved hard coefficients read

(Becher, Hill, Lee, Neubert 2004)

$$\Delta_i C^{B1}(u, \mu_i) = \left(\frac{m_b}{\mu_h}\right)^{a(\mu_h,\mu_i)} e^{S(\mu_h,\mu_i)} \int_0^1 dv\, U_\perp(u, v, \mu_h, \mu_i) \Delta_i C^{B1}(v, \mu_h)$$

The evolution factor $U_\perp$ obeys integro-differential equation

$$\mu \frac{d}{d\mu} U_\perp(u, v, \mu_h, \mu) = \int_0^1 dy\, \gamma_\perp(y, u) U_\perp(y, v, \mu_h, \mu_i)$$

The solution is found **numerically**

RG evolution from μ_f to μ_i

$$\Delta_i C^{B1}(\mu_h) \times U_{\rm I}(\mu_h, \mu_i) \times j_\perp(\mu_i) \times U_{\rm II}(\mu_i, \mu_f) \times \{\phi_+^B(\mu_f)\phi_\perp^V(\mu_f)\}$$

The evolution factor $U_{\rm II}(\mu_i, \mu_f)$ is product of:

- Evolution of $\phi_\perp^V$ (Brodsky-Lepage kernel)
- Evolution of ϕ_+^B (Neubert, Lange 2003)

Effect of this resummation on branching ratios is small

- No details today (no time)

SCET factorization summary

SCET factorization formula:

$$\langle V\gamma | Q_i | \bar{B} \rangle = \Delta_i C^A \zeta_{V_\perp} + \left(\Delta C^{B1} \star j_\perp\right) \star \phi_\perp^V \star \phi_+^B$$

- Physics at hard scale m_b in the ΔC^i
- Physics at hard-collinear scale $m_b \Lambda_{\mathrm{QCD}}$ in $j_\perp$
- SCET matching coefficients are the hard-scattering kernels

$$\begin{aligned} t_i^{\mathrm{I}} &= \Delta_i C^A \\ t_i^{\mathrm{II}} &= \Delta_i C^{B1} \star j_\perp \end{aligned}$$

- $\zeta_{V_\perp}, \phi_\perp^V, \phi_+^B$ are matrix elements in SCET
- RG evolution is resummation

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- **Matching calculations and hard-scattering kernels**
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Matching Calculations I: Vertex Corrections

$$\langle V\gamma|Q_i|\bar{B}\rangle = \Delta_i C^A \zeta_{V_\perp} + \left(\Delta C^{B1} \star j_\perp\right) \star \phi_\perp^V \star \phi_+^B$$

Strategy for the matching calculations

$$Q_i \to \Delta_i C^A J^A + \Delta_i C^{B1} \star J^{B1} + \Delta_i C^{B2} \star J^{B2}$$

To find matching coefficients:

- Equate renormalized Green's functions in QCD and SCET

Matching coefficients independent of external states.

Simplest:

- Partonic matrix elements (Feynman diagrams)
- Use dim. reg. (scaleless integrals vanish)
- On-shell (many loop diagrams in SCET vanish in dim. reg.)
- To extract $\Delta_i C^A$ choose states that don't overlap with J^{Bi}

Vertex corrections

$$Q_i \to \Delta_i C^A J^A + \Delta_i C^{B1} \star J^{B1} + \dots$$

$$J^A = \bar{\chi}_{hc} \not{\epsilon}_\perp (1+\gamma_5) h_v; \qquad J^{B1} = \bar{\chi}_{hc} \not{\epsilon}_\perp \not{\mathcal{A}}_{hc_\perp} (1+\gamma_5) h_v$$

Observation: The B-type current has an extra gluon field $\mathcal{A}_{hc_\perp}$

- To match onto J^A use partonic states with **no external gluons**

$$\langle Q_i \rangle \equiv \langle q(p)\gamma(q)|Q_i|b(p_b)\rangle$$

- For on-shell matching the only kinematic invariants $\sim m_b^2$

$$p_b^2 = m_b^2; \qquad p^2 = 0; \qquad 2p \cdot p_b = m_b^2$$

Matching conditions for vertex corrections

Matching condition:

$$\langle Q_i \rangle_{\text{QCD,ren}} = \langle Q_i \rangle_{\text{SCET,ren}}$$

QCD matrix element:

- $\langle Q_i \rangle_{\text{QCD,ren}}$ are the virtual corrections to inclusive $B \to X_s \gamma$
- Can take them from known calculations

SCET matrix element:

- For on-shell matching no hard-collinear scale
- SCET_I loop integrals are scaleless and vanish in dim. reg.

$$\Rightarrow \langle Q_i \rangle_{\text{SCET,ren}} = \Delta_i C^A Z_J \langle J^A_{\text{tree}} \rangle$$

- Z_J is the renormalization factor for the J^A operator

Matching conditions for vertex corrections: II

Write the QCD amplitude as

$$\langle Q_i \rangle_{\text{QCD,ren}} \equiv D_i \langle Q_{7,\text{tree}} \rangle$$

In dim. reg. ($d = 4 - 2\epsilon$) the matching condition is

$$\Delta_i C^A(m_b, m_c, \mu) = \Delta_7 C^{A(0)} \lim_{\epsilon \to 0} Z_J^{-1}(\epsilon, m_b, \mu)\, D_i(\epsilon, m_b, m_c, \mu)$$

- Tree level coefficient

$$\Delta_7 C^{A(0)} = -\frac{e\, \overline{m}_b\, 2E_\gamma}{4\pi^2} \approx -\frac{e\, \overline{m}_b\, m_b}{4\pi^2}$$

- The renormalization factor Z_J determined order by order by requiring that $\Delta_i C^A$ finite as $\epsilon \to 0$

Results for vertex corrections

The coefficients are obtained as a series in α_s:

$$\Delta_i C^A = \Delta_7 C^{A(0)} \left[\delta_{i7} + \frac{\alpha_s(\mu)}{4\pi} \Delta_i C^{A(1)} + \left(\frac{\alpha_s(\mu)}{4\pi} \right)^2 \Delta_i C^{A(2)} \right]$$

- For Q_7 and Q_8 we obtained exact results to NNLO (α_s^2)
- For Q_1 we obtained the NNLO results only in large-β_0 limit

Checks on results:

- No IR poles in matching coefficients **(factorization)**
- J^A also appears in SCET treatment of $B \to X_s \gamma$ with cut on E_γ
  $\Rightarrow$ Checked Q_7 with Becher and Neubert 2006 (results agree)

The coefficients for Q_7 and Q_8 to NNLO

$(L = \ln \mu/m_b)$

$$
\begin{aligned}
\Delta_7 C^{A(1)} &= C_F \left[-2L^2 - 7L - 6.8225\right], \\
\Delta_7 C^{A(2)} &= C_F^2 \left(2L^4 + 14L^3 + 38.1449L^2 + 56.14711L + 7.8159\right) \\
&\quad + C_F C_A \left(-4.8889L^3 - 33.9758L^2 - 92.3415L - 83.8866\right) \\
&\quad + C_F n_l \left(0.8889L^3 + 6.8889L^2 + 19.9050L + 23.8254\right) \\
&\quad + C_F n_h \left(-1.3333L^2 + 2.8889L - 0.810288\right)
\end{aligned}
$$

$$
\begin{aligned}
\Delta_8 C^{A(1)} &= C_F \left[2.6667L + 1.4734 + 2.0944i\right], \\
\Delta_8 C^{A(2)} &= -C_F^2 \left[5.3333L^3 + 32.2802L^2 + 50.9612L + 1.8875\right. \\
&\quad \left. + i(4.1888L^2 + 31.4159L + 29.8299)\right] \\
&\quad + C_F C_A \left[15.1111L^2 + 31.6617L + 2.3833 + i(23.7365L + 28.0745)\right] \\
&\quad - C_F n_l \left[1.7778L^2 + 4.0386L + 1.7170 + i(2.7925L + 4.4215)\right] \\
&\quad + C_F n_h \left[1.7778L^2 - 2.0741L + 0.8829\right]
\end{aligned}
$$

The coefficients for Q_1 to NNLO

$$\Delta_1 C^{A(1)} = \frac{m_b}{\overline{\overline{m}}_b} C_F \left[-3.8519 L + r^{(1)} \left(\frac{m_c^2}{m_b^2} \right) \right]$$

$$\Delta_1 C^{A(2)} = -\frac{3\beta_0}{2} \frac{m_b}{\overline{\overline{m}}_b} C_F \left[2.47 L^2 + l^{(2)} \left(\frac{m_c^2}{m_b^2} \right) L + r^{(2)} \left(\frac{m_c^2}{m_b^2} \right) \right]$$

- $r^{(i)}$ and $l^{(2)}$ calculated as an expansion in m_c^2/m_b^2
- NNLO result is only known in large-β_0 limit ($n_f \to -3\beta_0/2$)
- Deviations from large-β_0 limit can be important (discussed later)

Matching Calculations II: Hard spectator corrections

$$\langle V\gamma\,|Q_i|\,\bar{B}\rangle = \Delta_i C^A \zeta_{V_\perp} + \left(\Delta C^{B1} \star j_\perp\right) \star \phi_\perp^V \star \phi_+^B$$

Plan:

- Review lowest-order (α_s) results
- Explain structure of α_s^2 results
- Explain our calculation of α_s^2 corrections from Q_8

The structure of t^{II} at $\mathcal{O}(\alpha_s)$

The leading contributions are $\mathcal{O}(\alpha_s)$:

$$t_i^{II(0)}(u,\omega) = \int_0^1 d\tau \, \Delta_i C^{B1(0)}(\tau) j_\perp^{(0)}(\tau,u,\omega)$$

The hard coefficients $\Delta_i C^{B1}$ depend on the operator

$$\Delta_7 C^{B1(0)}(\tau) = \frac{e\overline{m}_b}{4\pi^2}, \quad \Delta_8 C^{B1(0)}(\tau) = \frac{1-\tau}{\tau}\frac{1}{3}\frac{e\overline{m}_b}{4\pi^2}$$

$$\Delta_1 C^{B1(0)}(\tau) = \frac{1}{3}\frac{e\overline{m}_b}{4\pi^2} f\left(\frac{m_c^2}{m_b^2},\tau\right)$$

The jet function $j_\perp$ is universal

$$j_\perp^{(0)}(\tau,u,\omega) = -\frac{4\pi C_F\alpha_s}{N_c}\frac{1}{m_b\omega\bar{u}}\delta(\tau-u)$$

The structure of t^{II} at $\mathcal{O}(\alpha_s^2)$

The $\mathcal{O}(\alpha_s^2)$ corrections take the form

$$t_i^{\mathrm{II}(1)}(u,\omega) = \Delta_i C^{B1(1)} \star j_\perp^{(0)} + \Delta_i C^{B1(0)} \star j_\perp^{(1)}$$

Status of $\mathcal{O}(\alpha_s^2)$ corrections:

- The one-loop jet function $j_\perp^{(1)}$ known
  (Beneke and Yang, Becher and Hill 2004)
- The one-loop hard coefficient $\Delta_7 C^{B1(1)}$ known
  (Becher, Hill, Neubert 2005)
- The hard coefficient $\Delta_8 C^{B1(1)}$ known **(our work)**
- $\Delta_1 C^{B1(1)}$ remains unknown (requires two loops)

The calculation of t_8^{II} at $\mathcal{O}(\alpha_s^2)$

$$t_8^{\mathrm{II}(1)} = \Delta_8 C^{B1(1)} \star j_\perp^{(0)} + \Delta_8 C^{B1(0)} \star j_\perp^{(1)}$$

Calculate t_8^{II} directly in QCD factorization, but:

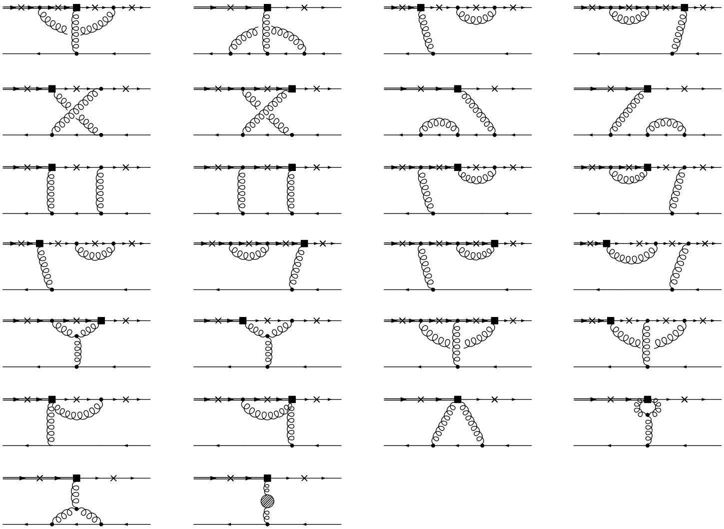
- Evaluate partonic matrix elements using "method of regions" (Beneke and Smirnov 1997)
- Show t_8^{II} depends on only hard and hard-collinear regions. The correspondence with SCET is:

$$\begin{aligned} A_{h,\mathrm{fin}}^{(1)} &= \Delta_8 C^{B1(1)} \star j_\perp^{(0)} \\ A_{hc,\mathrm{fin}}^{(1)} &= \Delta_8 C^{B1(0)} \star j_\perp^{(1)} \end{aligned}$$

Since $j_\perp^{(0)}$ is a δ-function, get $\Delta_8 C^{B1(1)}(\tau)$ from the convolution

One loop graphs for Q_8

($\times$ = photon emission)



Calculating t_8^{II} in QCD factorization

Isolate the one-loop graphs whose Dirac structure matches O^{B1}

$$\mathcal{A}_8^{(1)} = A^{(1)} \left(\not{\epsilon}_\perp \gamma_{\nu_\perp} \otimes \gamma_\perp^\nu \right) + \ldots$$

UV renormalized amplitude:

$$A^{(1)} \quad \to \quad A^{(1)} + A^{(1)}_{\mathrm{c.t.}} = A^{(1)} + \left(Z_\alpha^{(1)} + Z_m^{(1)} + Z_{88}^{(1)} - \frac{u}{\bar{u}} \frac{Z_{87}^{(1)}}{Q_d} \right) A^{(0)}$$

Extract t_8^{II} from renormalized amplitude:

$$\phi^{b\bar{q}'(0)} \star t_8^{\mathrm{II}(1)} \star \phi^{q\bar{q}'(0)} = A^{(1)} + A^{(1)}_{\mathrm{c.t.}} - \phi^{b\bar{q}'(1)} \star t_8^{\mathrm{II}(0)} \star \phi^{q\bar{q}'(0)} - \phi^{b\bar{q}'(0)} \star t_8^{\mathrm{II}(0)} \star \phi^{q\bar{q}'(1)}$$

IR poles in amplitude subtracted by renormalized LCDAs:

- t_8^{II} is free of IR physics
- **Factorization**

Results for t_8^{II}

$$\Delta_8 C^{B1(1)} \star j_\perp^{(0)} = A^{(1)}_{\mathrm{h,fin}} = \frac{\alpha_s}{4\pi}[C_F h_F + C_A h_A] t_8^{II(0)}$$

$(L_{hc} = \ln(m_b\omega/\mu^2))$

$$\begin{aligned}
h_A = {} & \left(4 - 2\ln u - \frac{2\ln u}{\bar u^2} + 2\ln\bar u\right) L_{hc} + i\pi\left(1 - \frac{1}{\bar u} - \frac{2\ln u}{\bar u^2}\right) + 2 - \frac{\pi^2}{3} + \frac{3}{\bar u} \\
& - \frac{\ln\bar u}{\bar u} + \left(-1 + \frac{2}{\bar u} - \frac{1}{\bar u^2}\right)\ln u + \left(1 - \frac{3}{2\bar u} - \frac{1}{2(2-\bar u)}\right)\ln\bar u \ln u - \ln^2\bar u \\
& + \left(1 + \frac{1}{\bar u^2}\right)\ln^2 u + \left(2 - \frac{1}{4\bar u} - \frac{1}{2\bar u^2} - \frac{1}{4(2-\bar u)}\right)\mathrm{Li}_2(\bar u) \\
& + \left(-\frac{5}{2\bar u} + \frac{3}{\bar u^2} - \frac{1}{2(2-\bar u)}\right) g(\bar u) + \left(-\frac{1}{2\bar u} + \frac{1}{2(2-\bar u)}\right) h(\bar u) \\
h_F = {} & \ldots
\end{aligned}$$

with

$$\begin{aligned}
g(u) &= \int_0^1 dy\, \frac{\ln[1 - uy(1-y)]}{y} = \ldots \\
h(u) &= \int_0^1 dy\, \frac{\ln[1 - uy(1-y)]}{1 - uy} = \ldots
\end{aligned}$$

Outline

- Motivation
- QCD Factorization and SCET as applied to $B \to V\gamma$
- Matching calculations and hard-scattering kernels
- **Numerical results and comparison with experiment**

The vertex and hard spectator amplitudes

The branching fraction for $B \to K^*\gamma$ decays is

$$\mathcal{B}(B \to K^*\gamma) = \frac{\tau_B m_B}{4\pi} \left(1 - \frac{m_{K^*}^2}{m_B^2}\right) |\mathcal{A}_{\rm v} + \mathcal{A}_{\rm hs}|^2$$

The vertex (v) and hard-spectator (hs) amplitudes are

$$\begin{aligned} \mathcal{A}_{\rm v} &= \frac{G_F}{\sqrt{2}} V_{cs}^* V_{cb} \sum_i C_i(\mu_{\rm QCD})\, \Delta_i C^A(m_b, \mu_{\rm QCD}, \mu) \zeta_{K_\perp^*}(\mu) \\ \mathcal{A}_{\rm hs} &= \frac{G_F}{\sqrt{2}} V_{cs}^* V_{cb} \sum_i C_i(\mu_{\rm QCD})\, t_i^{II}(\mu_{\rm QCD}, \mu) \star \left(\phi_B \star \phi_{K_\perp^*}\right)(\mu) \end{aligned}$$

- A_v and A_{hs} are separately RG invariant
- Can study their contribution to the amplitudes separately

Vertex corrections to NNLO

The ratio of NNLO to LO is:

$$\frac{\mathcal{A}_{\mathrm{v}}^{\mathrm{NNLO}}}{\mathcal{A}_{\mathrm{v}}^{\mathrm{LO}}} = 1 + (0.096 + 0.057i)\,[\alpha_s] + (-0.007 + 0.030i)\,\left[\alpha_s^2\right]$$

In terms of individual contributions

$$1 + \Big((0.264 + 0.034i)\,[Q_1] - (0.184)\,[Q_7] + (0.016 + 0.023i)\,[Q_8]\Big)\,[\alpha_s]$$
$$+ \Big((0.073 + 0.022i)\,[Q_1] - (0.081)\,[Q_7] + (0.002 + 0.008i)\,[Q_8]\Big)\,[\alpha_s^2]$$

- NNLO correction is small because of large cancellation between Q_1 and Q_7
- That Q_1 is only large-β_0 limit result can be significant (See branching fractions)

Hard spectator corrections to NNLO

Total corrections:

$$\frac{\mathcal{A}_{\rm hs}^{\rm NNLO}}{\mathcal{A}_{\rm V}^{\rm LO}} = (0.11 + 0.05i)\,[\alpha_s] + (0.03 + 0.01i)\,[\alpha_s^2]$$

In terms of individual operators:

$$= \Big((0.023 + 0.046i)\,[Q_1] + 0.074\,[Q_7] + 0.010\,[Q_8]\Big)\,[\alpha_s]$$
$$+ \Big((0.004 + 0.003i)\,[Q_1] + 0.025\,[Q_7] + 0.003 + 0.005i)\,[Q_8]\Big)\,[\alpha_s^2]$$

$([Q_1] = \Delta_1 C^{B1(0)} \star j_\perp^{(1)})$

- The NNLO corrections are individually small
  $\Rightarrow$ Errors associated this term are small
  $\Rightarrow$ Resummation effects $\sim$ 10% (but stabilize μ-dependence)

Determining $\zeta_{V_\perp}$ from the QCD form factor

Results depend strongly on $\zeta_{V_\perp}$. To determine it:

- Require that $\langle V\gamma | Q_7 | \bar{B} \rangle \propto F^{B\to V_\perp}$
- Use the QCD factorization formula for $F^{B\to V_\perp}$
- Use recent sum rule results $F^{B\to V_\perp} = 0.31 \pm 0.4$
  (Ball, Jones, Zwicky '05)

At NNLO:

$$\zeta_{V_\perp}(\mu = m_b) \simeq 0.35 \pm 0.05$$

The vertex corrections dominate the spectator ones:

$$\frac{F^{B\to V_\perp}}{\zeta_{V_\perp}} = \left(1 - 0.15[\alpha_s] - 0.06[\alpha_s^2]\right)[\mathrm{v}] + \left(0.07[\alpha_s] + 0.03[\alpha_s^2]\right)[\mathrm{hs}]$$

Branching Fractions at NNLO

Results at NNLO in units of 10^{-5}

$$
\begin{aligned}
\mathcal{B}(B^+ \to K^{*+}\gamma) &= 4.6 \pm 1.2\,[\zeta_{K^*}] \pm 0.4\,[m_c] \pm 0.2\,[\lambda_B] \pm 0.1\,[\mu] \\
\mathcal{B}(B^0 \to K^{*0}\gamma) &= 4.3 \pm 1.1\,[\zeta_{K^*}] \pm 0.4\,[m_c] \pm 0.2\,[\lambda_B] \pm 0.1\,[\mu] \\
\mathcal{B}(B_s \to \phi\gamma) &= 4.3 \pm 1.1\,[\zeta_{\phi}] \pm 0.3\,[m_c] \pm 0.3\,[\lambda_B] \pm 0.1\,[\mu]
\end{aligned}
$$

Matching not complete because of Q_1:

- 100% uncertainty to NNLO vertex correction in large-β_0 limit: $\Rightarrow \Delta\mathcal{B} \approx \pm 0.5$
- 100% uncertainty to NLO hard-spectator correction: $\Rightarrow \Delta\mathcal{B} \approx \pm 0.1$
- Results beyond large-β_0 limit would reduce errors (but 3 loops)

Comparison with experiment

Compared to current experimental numbers (HFAG, LP 2007):

$$\frac{\mathcal{B}(B^+ \to K^{*+}\gamma)_{\text{SM,NNLO}}}{\mathcal{B}(B^+ \to K^{*+}\gamma)_{\text{expt}}} = 1.1 \pm 0.35\,[\text{theory}] \pm 0.07\,[\text{expt.}]$$

$$\frac{\mathcal{B}(B^0 \to K^{*0}\gamma)_{\text{SM,NNLO}}}{\mathcal{B}(B^0 \to K^{*0}\gamma)_{\text{expt}}} = 1.1 \pm 0.35\,[\text{theory}] \pm 0.06\,[\text{expt.}]$$

$$\frac{\mathcal{B}(B_s \to \phi\gamma)_{\text{SM,NNLO}}}{\mathcal{B}(B_s \to \phi\gamma)_{\text{expt}}} = 0.8 \pm 0.2\,[\text{theory}] \pm 0.3\,[\text{expt.}]$$

- Theory errors about 30%
- Dominant error is in $\zeta_{V_\perp}$

Limitations of QCD factorization approach to $B \to V\gamma$

1) Power corrections in $\Lambda_{\rm QCD}/m_b$:

- Some power corrections factorize, some don't
- While SCET has potential to deal with power corrections, no serious attempt so far
- Treatment of these may rely on QCD sum rules (Ball, Zwicky, Jones 2007)

2) Hadronic uncertainties:

- The branching fractions are very sensitive to $\zeta_{V_\perp}$
- The soft function $\zeta_{V_\perp}$ has large uncertainties

$$\zeta_{V_\perp}(\mu = m_b) \simeq 0.35 \pm 0.05$$

Theory errors in branching fractions stuck at 20-30% until these improve.

Conclusions

$B \to V\gamma$ decays are interesting and will probe $b \to d\gamma$.

The branching fractions obey a QCD factorization formula.

Improving it requires:

- **NNLO perturbative corrections**
- More precise knowledge of $\zeta_{V_\perp}$
- Treatment of power corrections

We obtained most of the NNLO perturbative corrections.

The other points must be addressed for precise branching fractions.

But: can look for ratios where hadronic uncertainties drop out