Effective field theory and the problem of motion in General Relativity

Rafael A. Porto
Carnegie Mellon University

Overview

- The Problem of motion. PN expansion.
- Divergences. EFT for extended objects.
- NR 2-body problem in GR, matching into NRGR.
- Extension to spinning bodies.
- Power counting, LO & finite size effects and effacement.
- New results at 3PN. Algebraic structure.
- Conclusions and much more...

The problem of motion

- The size scale (finite extension) $k \sim 1/r_s$
- The orbit scale (Internal problem) $k \sim 1/r$
- The radiation scale (External problem) $k \sim v/r$

PN expansion

- Point particle, slow motion approximation.

$$T^{\mu\nu} = m \int d\tau v^{\mu}(\tau) v^{\nu}(\tau) \delta(x^{\alpha} - x^{\alpha}(\tau)) \qquad v \ll 1$$

- Weak gravity. Pertubatively solve Einstein eqs.

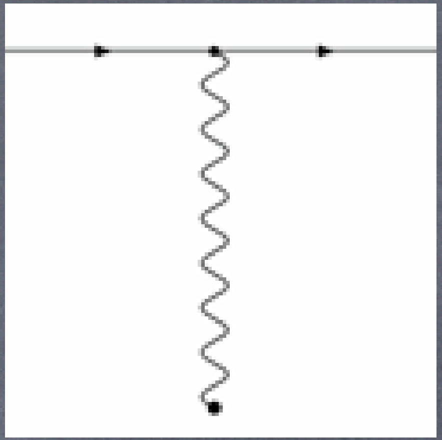
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad \partial^2 h_{\mu\nu} = -16\pi(T^{\mu\nu} + \tau^{\mu\nu})$$

$$\tau^{\mu\nu} = \tau_1^{\mu\nu}[h,h] + \tau_2^{\mu\nu}[h,h,h] + ...$$

- At 3PN the expansion blows up (iterative Green functions give rise to divergent integrals).
- Unsystematic regularization, ‘non-traditional’ dim. reg. methods. Unclear whether can be extended to higher orders.
- Decoupling of the internal structure not fully understood.
- From the point of view of EFT, decoupling naturally leads to a tower of gravitational theories with a systematic power counting scheme: Feynman diagrams.
- Consistent to all order. Divergences are handled by standard procedures.

Feynman Diagrams. Intuitive idea.

$$\partial^2 h^{(0)}_{\mu\nu} = -16\pi T_{\mu\nu} \to h^{(0)}_{\mu\nu} \sim \frac{m}{\mathbf{k}^2}\delta(k^0)e^{i\mathbf{k}\cdot\mathbf{x}} \to h^{(0)} \sim \frac{1}{r}$$

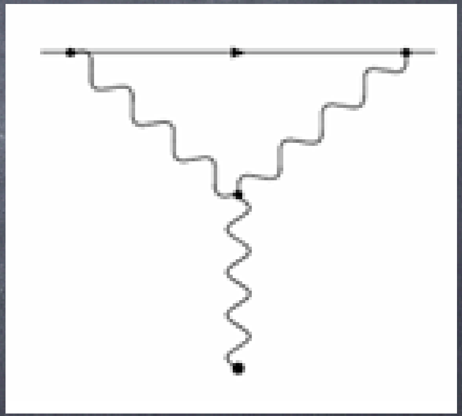


Propagator

$$-\frac{i}{\mathbf{k}^2}$$

mass Insertion

$$-i\frac{m}{2m_{Pl}}\delta(k\cdot v)v^\mu v^\nu$$



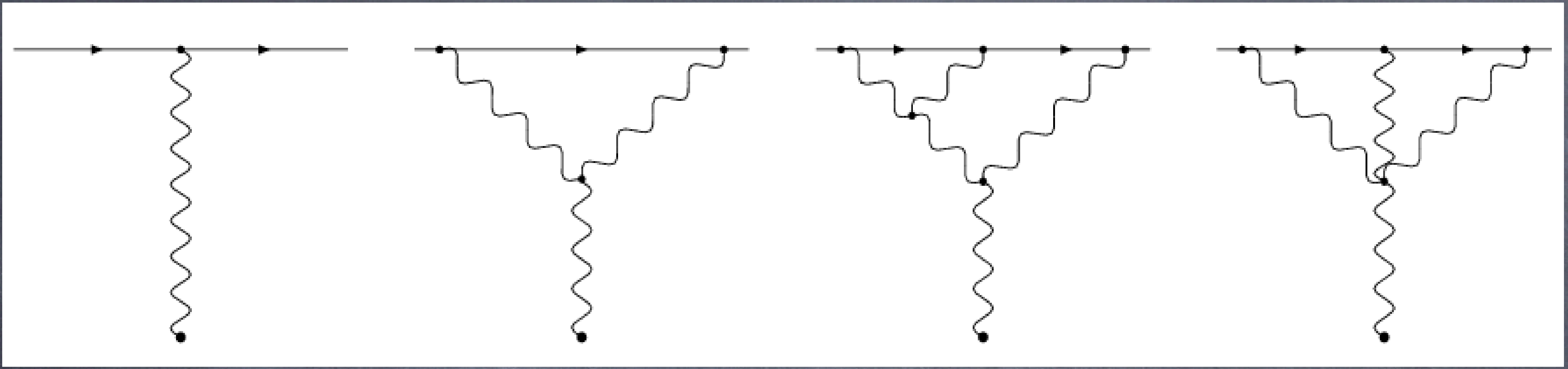
Toy model $\tau_1[h^{(0)}, h^{(0)}]$ $\tau_1 \sim gh^2$

The first iteration is finite

$$h^{(1)} \sim gm^2 \int_{\mathbf{q},\mathbf{p}} \frac{1}{\mathbf{k}^2}\frac{1}{\mathbf{p}^2}\frac{1}{\mathbf{q}^2}\delta(k^0)\delta(\mathbf{q}+\mathbf{p}-\mathbf{k})$$

vertex coupling $\sim g$

General Procedure



$$exp[iS_{eff}] = \int \mathcal{D}H exp[iS[g+H, x_a] + iS_{GF}]$$

$$h_{\mu\nu} = \frac{i}{k^2} P_{\mu\nu;\alpha\beta} \; \frac{i}{m_{Pl}} \frac{\delta S_{eff}[g]}{\delta h_{\alpha\beta}}\Bigg|_{h=0}.$$

Regularization & Renormalization.

$$T^{\mu\nu}_{(3)} \sim \delta(k^0) I_0(\mathbf{k}) \left[\frac{1}{16}(1-2\epsilon)(k^2\eta^{\mu\nu} - k^\mu k^\nu) - \frac{1}{8}\left(1 - \frac{21}{2}\epsilon\right) k^2 v^\mu v^\nu\right]$$

$$I_0(\mathbf{k}) = \int \frac{d^{d-1}\mathbf{p}}{(2\pi)^{d-1}} \frac{d^{d-1}\mathbf{k}}{(2\pi)^{d-1}} \frac{1}{\mathbf{q^2 p^2 (q+p+k)^2}} = \frac{1}{32\pi^2}\left[\frac{1}{2\epsilon} + ln(4\pi) - \gamma + 3 - ln\frac{\mathbf{k}^2}{\mu^2}\right] + O(\epsilon)$$

We need to regularize the theory, e.g. add counter terms.

$$\frac{1}{m_{Pl}} \int \frac{d^4k}{(2\pi)^4} h_{\mu\nu}(-k) T^{\mu\nu}_{ct}(k)$$

$$T^{\mu\nu}_{ct}(k) = (2\pi)\delta(k^0)\left[c_R(\eta^{\mu\nu}k^2 - k^\mu k^\nu) + 1/2 c_V k^2 v^\mu v^\nu\right]$$

Consistent with Ward identities (gauge invariance).

$$k_\mu T^{\mu\nu}_{ct} = 0$$

EFT for extended non-spinning objects

(I. Rothstein & W. Goldberger Phys Rev D73, 104029, 2006)

Main Idea: Insertion of non minimal terms in the action to handle divergences and thus account for the finite size of the constituents (decoupling). Unknown parameters describing the internal structure are fixed by *matching* observables in the one-particle sector such as scattering amplitudes.

$$S_{pp} = -\sum_a \int d\tau_a \left(m_a + c_R^{(a)} R(x_a) + c_V^{(a)} R_{\mu\nu} v_a^\mu v_a^\nu + ... \right)$$

Other possible extra terms $\int d\tau R^2, \ \int d\tau R_{\mu\nu} R^{\mu\nu}, \ etc$

Later on spin will give us more degrees of freedom to play with

NR 2-body problem in GR: Matching into NRGR

We need to treat one scale at the time: separate the orbit scale (internal) from the radiation scale (external). In order to do that we need manifest power counting in the PN expansion parameter.

- Split the metric in modes $$h_{\mu\nu} = \bar{h}_{\mu\nu} + H_{\mu\nu}$$
- Potential modes $$\partial_i H_{\mu\nu} \sim \frac{1}{r} H_{\mu\nu} \quad \partial_0 H_{\mu\nu} \sim \frac{v}{r} H_{\mu\nu}$$

Off-shell modes responsible for Coulumb-like interaction (1/r)

- Radiation modes $$\partial_\alpha \bar{h}_{\mu\nu} \sim \frac{v}{r} \bar{h}_{\mu\nu}$$

Long wave-length radiation (on-shell) modes

Introduce into the action:

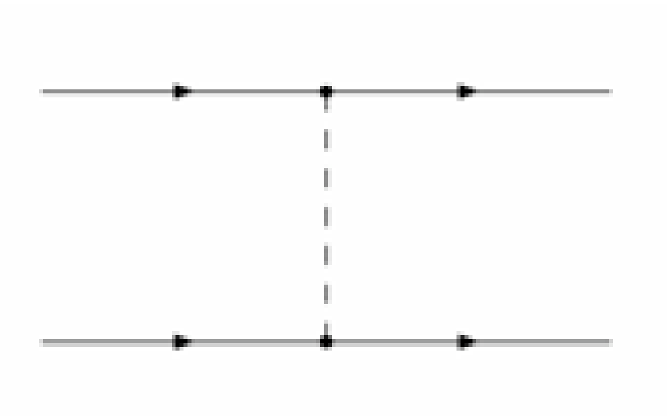
$$\mathcal{L}_{H^2} - \frac{1}{2}\int_{\mathbf{k}}\left[\mathbf{k}^2 H_{\mathbf{k}\mu\nu}H^{\mu\nu}_{-\mathbf{k}} - \frac{\mathbf{k}^2}{2}H_{\mathbf{k}}H_{-\mathbf{k}} - \partial_0 H_{\mathbf{k}\mu\nu}\partial_0 H^{\mu\nu}_{-\mathbf{k}} + \frac{1}{2}\partial_0 H_{\mathbf{k}}\partial_0 H_{-\mathbf{k}}\right]$$

With $H^{\mu\nu}(x) = \int_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} H^{\mu\nu}_{\mathbf{k}}(x^0) \;\rightarrow \partial_\mu \sim \frac{v}{r}$

Which leads to the propagator $\langle H_{\mathbf{k}\mu\nu}(x^0) H_{\mathbf{q}\alpha\beta}(0)\rangle = -(2\pi)^3\delta^3(\mathbf{k}+\mathbf{q})\frac{i}{\mathbf{k}^2}\delta(x_0)P_{\mu\nu;\alpha\beta}$

Virial theorem $\frac{m}{m_{Pl}} \sim \sqrt{Lv} \quad \rightarrow \quad H_{\mathbf{k}} d^3\mathbf{k} \sim \frac{m_{Pl}v^2}{\sqrt{L}}$

The LO contribution to the effective action



$$\frac{im_1 m_2}{8m_{Pl}^2}\int dt_1 dt_2 \delta(t_1 - t_2)\int_{\mathbf{k}}\frac{1}{\mathbf{k}^2}e^{-i\mathbf{k}\cdot(\mathbf{x}_1-\mathbf{x}_2)}$$

$$\sim i\int dt \frac{G_N m_1 m_2}{|\mathbf{x}_1(t) - \mathbf{x}_2(t)|} \quad \sim i[(dt)(d^3\mathbf{k})(m/m_{Pl})H]^2 \sim mvr$$

Lesson:

By integrating out the potential modes one gets an ET of pp coupled to radiation modes (treated as a background field). In the process we will obtain the effective gravitational potential for the motion of the system (Real part of S_eff), and calculate the power spectrum (Imaginary part of S_eff). By treating the bodies as external sources we thus solve the problems of treating gravity as an ET where the masses involved are larger than the Planck mass. The perturbative expansion is in powers of Lv^n. It is possible to show quantum (loop) effects are suppressed by 1/L.

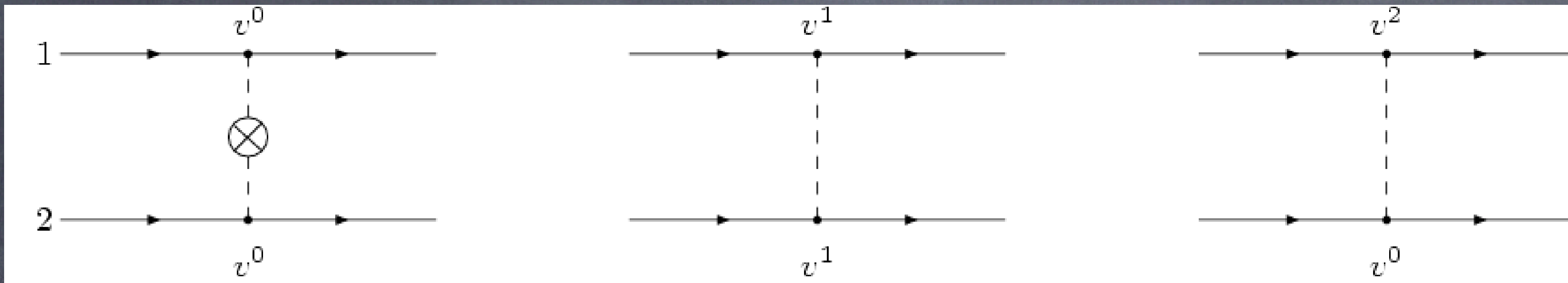
Typical diagram $m^{N_m} m_{Pl}^{2(N_g-P)} r^{N_m+2N_g-2P}$

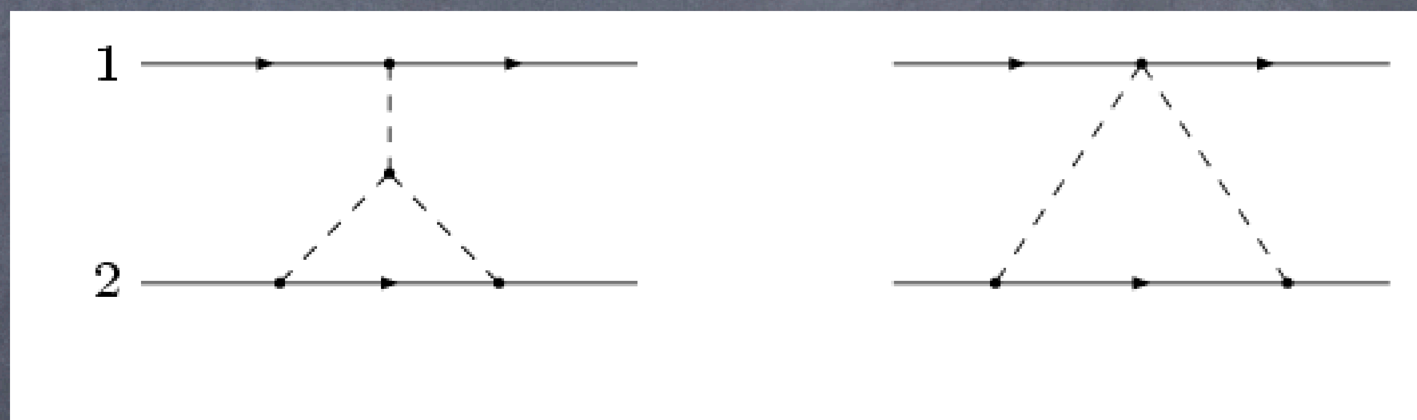
Power counting

$\mathbf{k}$	$H^{\mathbf{k}}_{\mu\nu}$	$\bar{h}_{\mu\nu}$	m/m_{Pl}
$1/r$	$r^2 v^{1/2}$	v/r	$\sqrt{Lv}$

NLO (2PN) EIH action

(For a toy model, see RAP & R. Sturani gr-qc/070106. Proc. Les Houches summer school)

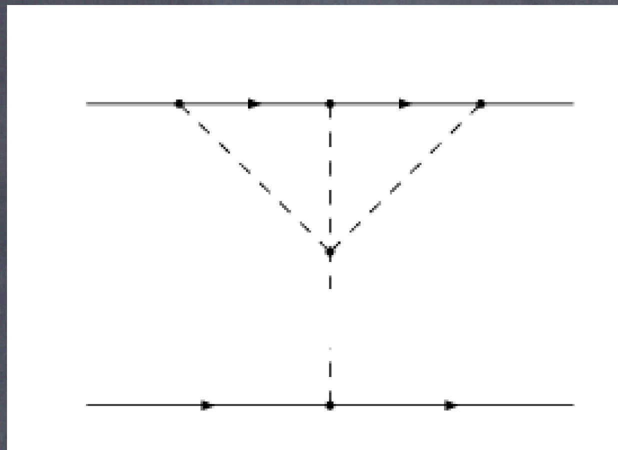




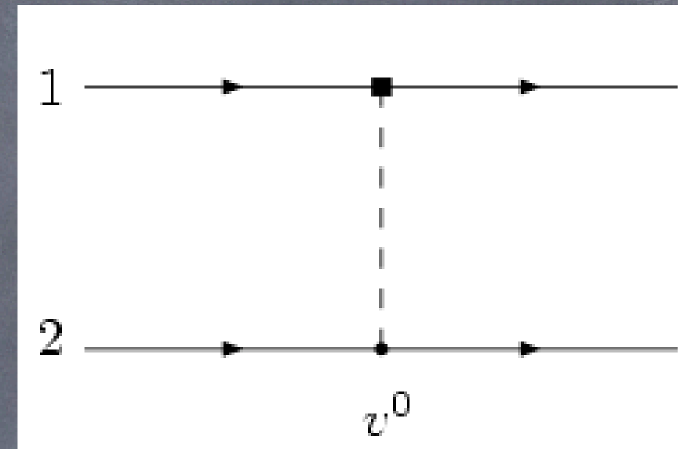
$$G(p) \propto \frac{1}{p_0^2 - \vec{p}^2} \approx -\frac{1}{\vec{p}^2} - \frac{p_0^2}{\vec{p}^4} + \ldots$$

$$L_{EIH} = \frac{1}{8}\sum_a m_a \mathbf{v}_a^4 + \frac{G_N m_1 m_2}{2|\mathbf{x}_1 - \mathbf{x}_2|}\left[3(\mathbf{v}_1^2 + \mathbf{v}_2^2) - 7(\mathbf{v}_1 \cdot \mathbf{v}_2) - \frac{(\mathbf{v}_1 \cdot \mathbf{x}_{12})(\mathbf{v}_2 \cdot \mathbf{x}_{12})}{|\mathbf{x}_1 - \mathbf{x}_2|^2}\right] - \frac{G_N^2 m_1 m_2 (m_1 + m_2)}{2|\mathbf{x}_1 - \mathbf{x}_2|^2}$$

Order v^6: Divergences & Effacement



$\longrightarrow$



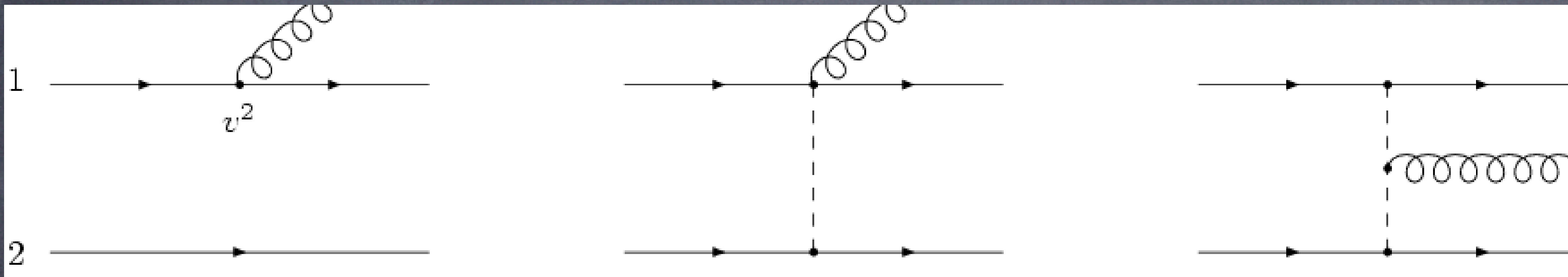
As we showed, this diagram in the potential diverges and renormalizes c_R and c_V. However, these coefficients can be washed away by field redefinition (conformal transformation).

$$\delta g_{\mu\nu} = -\int d\tau \frac{\delta(x - x(\tau))}{\sqrt{g}} \left((c_R + \frac{1}{2} c_V) g_{\mu\nu} + c_V v_\mu v_\nu \right)$$

New divergences do not show up until O(v^10) (Riemann^2 terms). The Effacement is thus proven up to 5PN for non-spinning bodies.

Coupling to Radiation

To retain manifest power counting we need to multipole expand around a reference point X

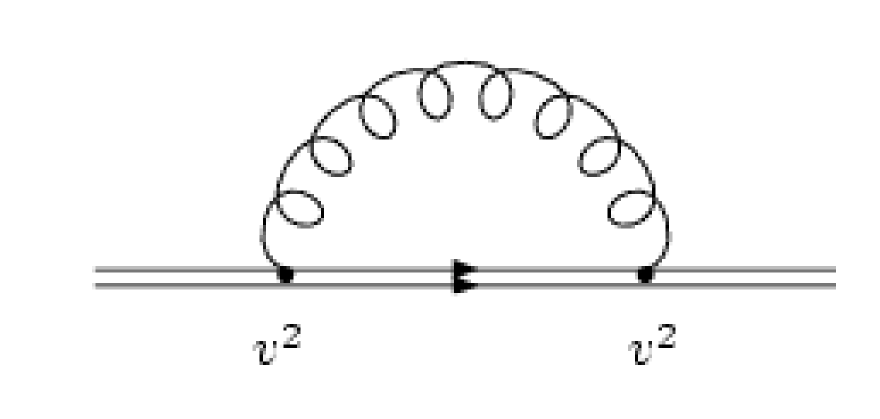


$$\bar{h}_{\mu\nu}(x^0,\mathbf{x}) = \bar{h}_{\mu\nu}(x^0,\mathbf{X}) + \delta\mathbf{x}_i\partial_i\bar{h}_{\mu\nu}(x^0,\mathbf{X}) + \frac{1}{2}\delta\mathbf{x}_i\delta\mathbf{x}_j\partial_i\partial_j\bar{h}_{\mu\nu}(x^0,\mathbf{X}) + \cdots$$

$$L = \frac{1}{2}\sum_a m_a\mathbf{v}_a^2 + \frac{G_N m_1 m_2}{|\mathbf{x}_1-\mathbf{x}_2|} - \frac{m}{2m_{Pl}}\bar{h}_{00} - \frac{\bar{h}_{00}}{2m_{Pl}}\left[\frac{1}{2}\sum_a m_a\mathbf{v}_a^2 - \frac{G_N m_1 m_2}{|\mathbf{x}_1-\mathbf{x}_2|}\right]$$
$$-\frac{1}{2m_{Pl}}\epsilon_{ijk}\mathbf{L}_k\partial_j\bar{h}_{0i} + \frac{1}{2m_{Pl}}\sum_a m_a\mathbf{x}_{ai}\mathbf{x}_{aj}R_{0i0j},$$

Gauge invariant Lagrangian of pp coupled to radiation modes. Conserved quantities do not radiate, only the quadruple piece will serve as a source.

Quadrupole Formula



$$Q_{ij} = \sum_a m_a \left(\mathbf{x}_{ai}\mathbf{x}_{aj} - \frac{1}{3}\delta_{ij}\mathbf{x}_a^2 \right)$$

$$\frac{1}{T} Im S_{eff}[x_a] = \frac{1}{2}\int dE d\Omega \frac{d^2\Gamma}{dE d\Omega}$$

$$Im S_{eff} = -\frac{1}{80 m_{Pl}^2}\int_{\mathbf{k}} \frac{1}{2|\mathbf{k}|}\mathbf{k}^4 |Q_{ij}(|\mathbf{k}|)^2 \rightarrow \frac{dE}{dt} = -\frac{G_N}{5}\langle \frac{d^3 Q_{ij}}{dt^3}\frac{d^3 Q_{ij}}{dt^3}\rangle$$

Spinning up in NRGR

(RAP, Phys Rev D73, 104031 (2006))

Rotational degrees of freedom $\longrightarrow$ tetrad e^a_J in the worldline

Introduce the generalized angular velocity $\longrightarrow$ $\Omega^{ab} = \eta^{IJ} e^b_I \frac{De^a_J}{D\lambda}$

Define spin and momentum as

$$\delta \mathcal{L} = -\frac{1}{2}(p^\mu \delta x_\mu + S^{ab}\delta\Omega_{ab})$$

by RPI the spin part of the action reads:

$$S_{spin} = -\int \frac{1}{2} S_i^{ab} \Omega^i_{ab}(x_i) d\lambda_i$$

Equations of motion

$$\delta S = -\int d\tau S^{\alpha\beta}\delta\Omega_{\alpha\beta} = -\int d\tau \left(-\frac{DS^{\alpha\beta}}{D\tau}e_{K\alpha} - \frac{De_{K\alpha}}{D\tau}S^{\alpha\beta} + S^{\rho\nu}\frac{De_{J\nu}}{D\tau}e_{\rho K}e^{J\beta}\right)\delta e^K_\beta .$$

$$\frac{DS^{\mu\nu}}{D\tau} = S^{\mu\lambda}\Omega_\lambda{}^\nu - \Omega^\mu{}_\lambda S^{\lambda\nu} = p^\mu u^\nu - u^\mu p^\nu$$

In a locally flat frame

$$\frac{Dp_\gamma}{D\tau} = \frac{1}{2}S^{\alpha\beta}\left(\Gamma_{\alpha\beta\sigma,\gamma} - \Gamma_{\alpha\beta\gamma,\sigma}\right)u^\sigma = -\frac{1}{2}R_{\gamma\sigma\alpha\beta}S^{\alpha\beta}u^\sigma$$

These are the Papapetrou equations

Constraints (SSC)

Covariant choice

$$V^\mu = S^{\mu\nu} p_\nu = 0 \to S^{\mu\nu} S^\star_{\mu\nu} \sim 0$$

Gauge fixing

$$e^0 - p/m \sim 0$$

Non-canonical Dirac algebra

$$[x^i_a, x^j_a]_{db} = \frac{S^{ji}_a}{m^2_a}$$

$$[x^k_a, S^{ij}_a]_{db} = \frac{1}{m_a}(S^{ki}_a v^j_a - S^{kj}_a v^i_a)$$

Choice of CM, Newton-Wigner SSC

$$mS^{0\mu} = S^{\mu\nu}p_\nu \to S^{i0} = \frac{1}{2}v^j S^{ij} + O(v^4)$$

Coordinate transformation which leads to a canonical algebra (at LO)

$$\vec{S}_a \to (1 - \frac{1}{2}\vec{v}_a^2)\vec{S}_a + \frac{1}{2}\vec{v}_a(\vec{v}_a \cdot \vec{S}_a)$$

$$\vec{x}_a \to \vec{x}_a - \frac{1}{2m_a}\vec{S}_a \times \vec{v}_a$$

Power counting and spin-gravity coupling

$$S^{ab}p_b = 0 \rightarrow S^{k0} = S^{kl}u^l + ...$$

Spin will be a sub-leading effect. It will now change the previous power counting rules.

$$S = I\omega = I\frac{v_{rot}}{r_s} \sim m v_{rot} r_s < m r_s \sim Lv$$

spin one-graviton vertex coupling

$$\eta^{IJ}e^I_\mu e^J_\nu = \eta_{\mu\nu} + \frac{h_{\mu\nu}}{m_{Pl}} \rightarrow \delta e^I_a = \frac{1}{2m_{Pl}} h_{ab}\, e^{bI}$$

$$\longrightarrow \qquad -\frac{1}{2}S^{ab}\delta\Omega_{ab} = \frac{m_{Pl}}{2} h_{\alpha\gamma,\beta} S^{\alpha\beta}_L u^\gamma$$

Feynman Rules

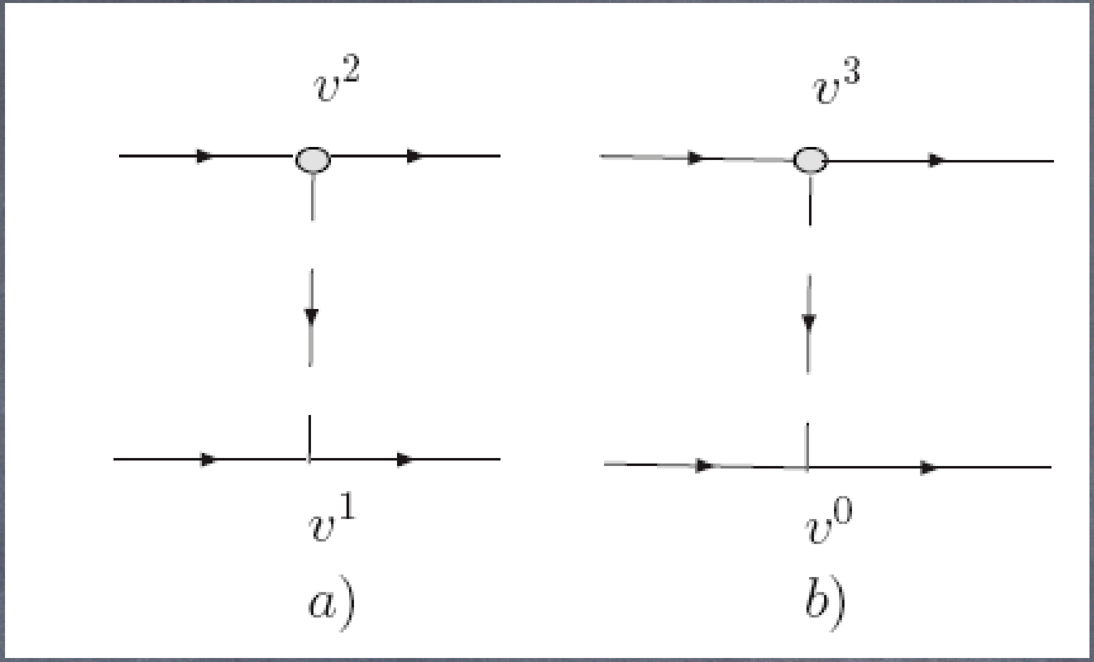
$$L_{1PN}^{NRGR} = \frac{1}{2m_p} H_{i0,k} S^{ik},$$

$$L_{1.5PN}^{NRGR} = \frac{1}{2m_p} \left(H_{ij,k} S^{ik} u^j + H_{00,k} S^{0k} \right),$$

$$L_{2PN}^{NRGR} = \frac{1}{2m_p} \left(H_{0j,k} S^{0k} u^j + H_{i0,0} S^{i0} \right)$$

$$+ \frac{1}{4m_p^2} S^{ij} \left(H_j^{\lambda} H_{0\lambda,i} - H_j^k H_{0i,k} \right).$$

1.5PN Spin-Orbit



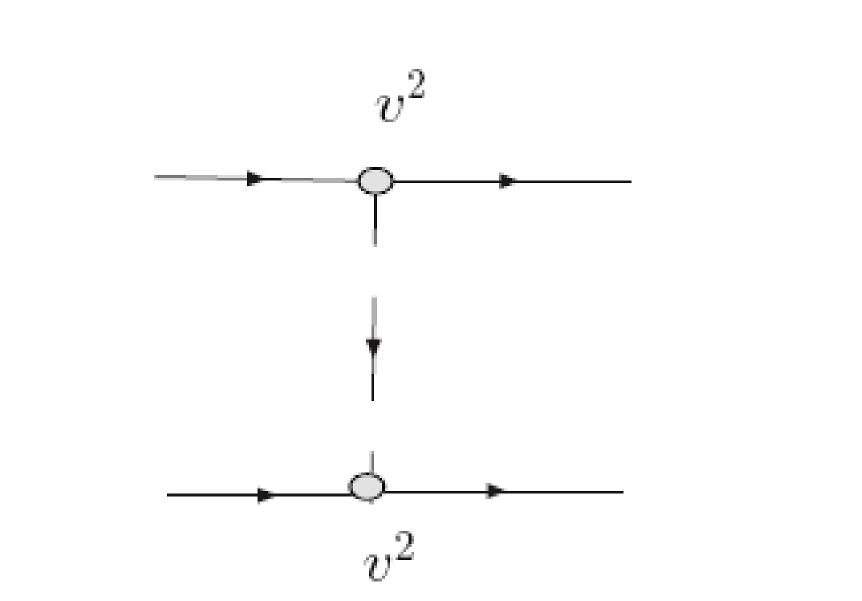
in the covariant SSC

$$V_{SO} = \frac{2G_N}{r^2}\mu(\vec{n}\times\vec{v})\cdot\left(\left(1+\frac{m_1}{m_2}\right)\vec{S_2}+\left(1+\frac{m_2}{m_1}\right)\vec{S_1}\right)$$

in the NW SSC

$$\bar{V}_{SO} = \frac{2G_N}{r^2}\mu(\vec{n}\times\vec{v})\cdot\left(\left(1+\frac{3m_1}{4m_2}\right)\vec{S_2}+\left(1+\frac{3m_2}{4m_1}\right)\vec{S_1}\right)$$

Spin-Spin at 2PN (SSC independent at LO)



$$V_{SS} = -\frac{G_N}{r^3}\left(\vec{S}_1 \cdot \vec{S}_2 - 3\vec{S}_1 \cdot \vec{n}\vec{S}_2 \cdot \vec{n}\right)$$

$$V_{SO}dt \sim Lv^3 \quad V_{SS}dt \sim Lv^4$$

Divergences with spin insertion & finite size effects

Tidal effects have a non trivial running and do not contribute to the one point function. For spin insertions they start out formally at 3PN, and 5PN for maximally rotating bodies through higher dimensional operators such as

$$D^2 R_{\mu\nu\alpha\beta} S^{\mu\rho} S^{\alpha}_{\rho} u^{\nu} u^{\beta}$$

$$\mu \frac{dC^2_D}{d\mu} \sim \frac{m}{m^4_{Pl}}$$

$$c_{D^2}(\partial^4 H_{\mathbf{k}} d^3\mathbf{k}) S^2 d\tau \sim \sqrt{L} v^{6+2s}$$

Self induced effects do not get renormalized and show up at leading order

$$R_{\mu\nu\alpha\beta} S^{\mu\sigma} S^{\alpha}_{\sigma} u^{\nu} u^{\beta} \rightarrow V_{S^2O} = C_{RS^2} \frac{1}{r^3} \left(3(\vec{S}\cdot\vec{S})^2 - \vec{S}\cdot\vec{S} \right)$$

$$C_{RS^2} \sim \frac{m}{r_s^2 m^4_{Pl}} \sim \frac{1}{m}$$

Naive power counting

$$\frac{1}{m_p^{n_s+n_m-1}}\left(\frac{m}{m_p}\right)^{n_m} L^{n_s}\frac{v^{sn_s}}{m_p^{n_s}}\frac{v^2 m_p}{\sqrt{L}}\frac{r}{r^d v}\sim\sqrt{L}v^{2d}$$

$$d = 2n_s + n_m - 1 \qquad \tilde{d}\equiv d-2\geq 2$$

$\tilde{d}=0\rightarrow n_s=1,\ n_m=1$ No logarithmic divergences

$\tilde{d}=1\rightarrow n_s=2(1),\ n_m=0(2)$ No logarithmic divergences

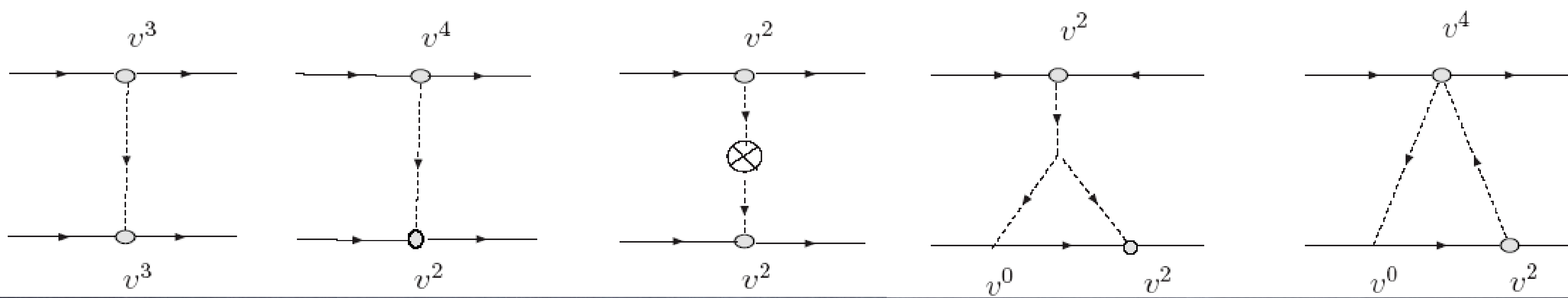
$\tilde{d}=2\rightarrow n_s=2(1),\ n_m=1(3)$ Finite size effects. Notice naive power counting breaks down due to the cutoff scale r_s

At higher orders we encounter tidal effects, they first start at v^10 and the effacement is proven

The Hyperfine EIH potential at 3PN in the Newton-Wigner SSC

(RAP & I. Rothstein, Phys Rev Lett97, 021101 (2006))

one graviton exchange — Non-linear terms



$$
\begin{aligned}
V^{ss}_{3PN} \;=\; & -\frac{G_N}{2r^3}\Bigg[\vec{S}_1\cdot\vec{S}_2\left(\frac{3}{2}\vec{v}_1\cdot\vec{v}_2 - 3\vec{v}_1\cdot\vec{n}\vec{v}_2\cdot\vec{n} - \left(\vec{v}_1^2+\vec{v}_2^2\right)\right) - \vec{S}_1\cdot\vec{v}_1\vec{S}_2\cdot\vec{v}_2 - \frac{3}{2}\vec{S}_1\cdot\vec{v}_2\vec{S}_2\cdot\vec{v}_1 + \vec{S}_1\cdot\vec{v}_2\vec{S}_2\cdot\vec{v}_2 \\
& + \vec{S}_2\cdot\vec{v}_1\vec{S}_1\cdot\vec{v}_1 + 3\vec{S}_1\cdot\vec{n}\vec{S}_2\cdot\vec{n}\left(\vec{v}_1\cdot\vec{v}_2 + 5\vec{v}_1\cdot\vec{n}\vec{v}_2\cdot\vec{n}\right) - 3\vec{S}_1\cdot\vec{v}_1\vec{S}_2\cdot\vec{n}\vec{v}_2\cdot\vec{n} - 3\vec{S}_2\cdot\vec{v}_2\vec{S}_1\cdot\vec{n}\vec{v}_1\cdot\vec{n} \\
& + 3(\vec{v}_2\times\vec{S}_1)\cdot\vec{n}(\vec{v}_2\times\vec{S}_2)\cdot\vec{n} + 3(\vec{v}_1\times\vec{S}_1)\cdot\vec{n}(\vec{v}_1\times\vec{S}_2)\cdot\vec{n} - \frac{3}{2}(\vec{v}_1\times\vec{S}_1)\cdot\vec{n}(\vec{v}_2\times\vec{S}_2)\cdot\vec{n} \\
& - 6(\vec{v}_1\times\vec{S}_2)\cdot\vec{n}(\vec{v}_2\times\vec{S}_1)\cdot\vec{n}\Bigg] + \frac{3G_N^2(m_1+m_2)}{r^4}\left(\vec{S}_1\cdot\vec{S}_2 - 3\vec{S}_1\cdot\vec{n}\vec{S}_2\cdot\vec{n}\right),
\end{aligned}
$$

Includes first non-linear corrections to the spin-spin interaction

The SS Equations of Motion up to 3PN.

(RAP, gr-qc/0701105. Proc. MG11)

Before the SSC we have the algebra

$$\{x^\mu, \mathcal{P}_\alpha\} = \delta^\mu_\alpha, \quad \{x^\mu, p_\alpha\} = \delta^\mu_\alpha$$

$$\{\mathcal{P}^\alpha, \mathcal{P}^\beta\} = 0, \quad \{x^\mu, x^\nu\} = 0, \quad \{p^\alpha, p^\beta\} = \frac{1}{2} R^{\alpha\beta}{}_{ab} S^{ab}_L$$

$$\{x^\mu, S^{ab}_L\} = 0, \quad \{p^\alpha, S^{ab}_L\} = 0, \quad \{\mathcal{P}^\alpha, S^{ab}_L\} = 0$$

$$\{S^{ab}_L, S^{cd}_L\} = \eta^{ac} S^{bd}_L + \eta^{bd} S^{ac}_L - \eta^{ad} S^{bc}_L - \eta^{bc} S^{ad}_L$$

where $$\mathcal{P}^\mu = p^\mu - \frac{1}{2} \omega^\mu_{ab} S^{ab}_L$$

with ω^μ_{ab} the spin coefficients $\left(\mathcal{L}_{spin} \sim -\frac{1}{2} u_\mu \omega^\mu_{ab} S^{ab}_L \right)$

After the SSC is imposed the Dirac algebra emerges

$$\{x^i, \mathcal{P}_j\} = \delta^i_j + ..., \quad \{x^i, x^j\} = 0 + ...$$

$$\{\mathcal{P}^i, \mathcal{P}^j\} = 0 + ..., \quad \{x^i, S^i_L\} = 0 + ...$$

$$\{\mathcal{P}^j, S^i_L\} = 0 + ..., \quad \{S^i_L, S^j_L\} = \epsilon^{ijk} S^k_L$$

'...' represents a set of higher dimensional curvature x Spin terms. In principle we should worry about this terms. However, by power counting it is possible to show they do not play a role until 4PN.

Intuitive argument: To get a correction to the S_1*S_2 EOM coming from the algebra one needs to consider the S_2 piece of the SO Hamiltonian (1.5PN). The EOM are unaltered at LO so the algebra correction starts at 2.5PN

next to Intuitive argument:

Let us consider for instance the bracket $\{x^i, x^j\}$

This commutator receives corrections scaling as $\sim Rx^2 \frac{S}{m^2} + \ldots$

Remember in the covariant SSC this bracket goes like (at LO)

$$\{x^i, x^j\} = \frac{S^{ji}}{m^2}$$

Which accounts for a 1.5PN shift in the EOM.

The extra piece scales (at LO) as $\partial^2 h_{00} x^2$

In the weak gravity approx. $h_{00} \sim v^2$ and the algebra corrections effectively start at 2.5PN

Yet another approach: the Routhian in the covariant SSC

$$\mathcal{R} = -\sum_q \left(\int m_q \sqrt{u_q^2} d\lambda_q + \int \frac{1}{2} S_{Lq}^{ab} \omega_{ab\mu} u_q^\mu - \frac{1}{2m_q} R_{deab}(x_q) S_{Lq}^{cd} S_{Lq}^{ab} u_q^e u_c^q \, d\lambda_q \right)$$

$$\frac{\delta \mathcal{R}}{\delta x^\mu} = 0, \quad \frac{dS_L^{ab}}{d\tau} = \{S_L^{ab}, \mathcal{R}\}$$

To obtain PN corrections one calculates R perturbatively, and only imposes the SSC after the EOM are obtained from the algebra.

$$\{S^i, S^{j0}\} = \epsilon^{ijk} S^{0k} = v^i S^j - v^j S^i + \ldots,$$

For instance, the spin dynamics from the SO Hamiltonian reads

$$\frac{d\vec{S}_1}{dt} = 2\left(1 + \frac{m_2}{m_1}\right) \frac{\mu G_N}{r^2} (\vec{n} \times \vec{v}) \times \vec{S}_1 - \frac{m_2 G_N}{r^2} (\vec{S}_1 \times \vec{n}) \times \vec{v}_1$$

it agrees with the known result after $\vec{S}_1 \to (1 - \frac{1}{2}\vec{v}_1^2)\vec{S}_1 + \frac{1}{2}\vec{v}_1(\vec{v}_1 \cdot \vec{S}_1).$

Conclusions

- The problem of motion reduced to a tower of EFTs.
- World-line operators encoding finite size structure. Tidal effects start formally at 3PN and 5PN for maximally rotating compact objects. Self induced effects show up already at 2PN.
- Systematic method to calculate to all orders in the PN expansion. Textbook renormalization. No ambiguities. Divergences absorbed into short distance parameters. Matching.
- NRGR for spinning bodies. New results at 3PN.
- Absorption, self-force, finite size and spin radiation easy to handle with EFT techniques. Also applicable to other kinematical scenarios (large-small mass ratio).
- Work in progress....