The flavour puzzle and accidental symmetries

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based on: Ferretti, King, R hep-ph/0609047
Outline

* The flavour puzzle in the SM
* Approaches to the flavour puzzle
* A new approach
  * Basic idea
  * Vus and LR symmetry
  * s/b vs c/t and Pati–Salam
  * Neutrinos
  * A model
The flavour puzzle in the SM

* 3 families, or $U(3)^5$ symmetry of the fermion gauge lagrangian

![Flavour Puzzle Diagram]

- **family number**
  - (horizontal)
  - not understood
- **gauge irreps**
  - (vertical)
  - well understood

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The flavour puzzle in the SM

- 3 families, or $U(3)^5$ symmetry of the fermion gauge lagrangian
- Pattern of $U(3)^5$ breaking from Yukawa sector (most SM pars)

$$\mathcal{L}^{\text{flavor}}_{\text{SM}} = \lambda_{ij}^E e_i^c L_j H^+ + \lambda_{ij}^D d_i^c Q_j H^+ + \lambda_{ij}^U u_i^c Q_j H + \text{h.c.}$$
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  - CKM mixing angles $\ll 1$

\[ Q = m_t \]
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    - different hierarchies: $U \gg D, E$
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![Flavour spectrum](image)
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    - 2 PMNS mixing angles $O(\pi/4)$, 1 small
    - $\theta_{23} = O(\pi/4)$ & $|\Delta m^2|_{12} \ll |\Delta m^2|_{23}$, $\theta_{12} \neq \pi/4$
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The flavour puzzle in SM extensions

* The peculiar SM flavour structure \((m_1, m_2 \ll \langle H \rangle, |V_{td}|, |V_{ts}| \ll 1)\) allows the SM to pass the FCNC test

\[
\mathcal{L}(Q \ll \langle H \rangle) \supset \frac{\bar{s}d\bar{s}d}{\Lambda_f^2}, \quad \frac{1}{\Lambda_f^2} \sim \frac{1}{(10^3 \text{ TeV})^2}
\]

\[
\left( \frac{1}{\Lambda_f^2} \right)_{\text{SM}} \sim \frac{g^4}{(4\pi)^2} \times (V_{su_i}^\dagger V_{ui d})(V_{su_j}^\dagger V_{uj d}) \cdot \frac{f(\frac{m_{u_i}^2}{M_W^2}, \frac{m_{u_j}^2}{M_W^2})}{M_W^2}
\]

* The unknown physics at the SM cutoff should not provide new generic flavour structure

\[
\left( \frac{1}{\Lambda_f^2} \right)_{\text{NP}} \sim \frac{1}{\Lambda_{\text{NP}}^2}
\]

• e.g. in the MSSM \(\delta_{12} \ll 1\), or \((\tilde{m}_d^2 - \tilde{m}_s^2)/\tilde{m}^2\) and \(|W_{31}|, |W_{32}| \ll 1\)

* Is the origin of the peculiar structure in SM and the MSSM/NP the same?
(Some) approaches to the flavour puzzle

* “Flavour symmetries” acting on family indexes (subgroup of $U(3)^5$)
  * symmetric limit: only $O(1)$ Yukawas possibly allowed: $\lambda_t$ ($\lambda_b \lambda_T$)
    - e.g. $t^c$, $q_3$, $h$ neutral under a $U(1)$: $Y_{33} t^c q_3 h$ is allowed
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- spontaneous breaking of $U(1)$ by SM singlets $\phi$ at high scale
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  * breaking communicated to SM fermions by heavy messengers ($M = \text{mass}$)
    - at $E \ll M$

\[
\begin{align*}
  & t^c \quad M \quad Q \quad \bar{Q} \\
  & h \quad \phi \\
  & \frac{\phi}{M} t^c q_2 h
\end{align*}
\]
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“Flavour symmetries” acting on family indexes (subgroup of U(3)\(^5\))

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  - at \(E \ll M\)

- gauge/global, continuous/discrete, abelian/non-abelian
Extra-dimension mechanisms

• flavour symmetry breaking with boundary conditions
• localized fermions
**Extra-dimension mechanisms**

- flavour symmetry breaking with boundary conditions
- localized fermions
  - e.g. in RS-type models:

\[
\lambda_{ij} \propto e^{(1-c_i-c_j)\pi k R}
\]

\[
m_H \sim M_5 e^{-\pi k R}
\]

\[
k = \text{curvature}
\]

\[
\psi_i(y) \propto e^{(1/2-c_i)ky}
\]

\[
c = \text{bulk mass in } k \text{ units}
\]
A new (economical) approach

The knowledge (or existence) of special horizontal dynamics is not required to explain the fermion hierarchical pattern.

The pattern follows from the relative lightness of one set of heavy fields and is associated to the breaking of the gauge group (Pati-Salam).

Chiral symmetries acting on family indexes “protecting” the mass of the lighter families emerge in this context as accidental symmetries.
Large $\theta_{23}$ (from $m_\nu$) and normal hierarchy

\[ m_{i,j}^\nu = -\lambda_i \lambda_j \langle h_u \rangle^2 / M_c \]

Exchange of a single $N^c$

Focus on “23” block:

- Single $N^c$: $m_3 \gg m_2 = 0$ hierarchy (m_2 \neq 0 from subdominant contributions)
- $\lambda_2 \approx \lambda_3$: $\tan \theta_{23} \approx 1$ large $\theta_{23}$ (barring charged lepton rotation)
  (for generic $\lambda_i$’s)
Charged fermions

Exchange of a single family of messengers
(no Yukawas at ren. level)

- Single messenger: $m_t \gg m_c = 0 \rightarrow \text{hierarchy}$ (whatever $\lambda_i$ & $\alpha_j$); $m_2 \neq 0$ from subdominant contributions
- $\lambda_i$ & $\alpha_j = O(1)$: large angles in CKM? No:
- No need of constraints on couplings, family symmetry [see also: Barr hep-ph/0106241]
$m_2 \neq 0$
\( m_2 \neq 0 \)

* Neglect H and 1st family for now (will turn out not to contribute)
m_2 \neq 0

Neglect H and 1^{st} family for now (will turn out not to contribute)

\[
Y_{ij}^U = \lambda_i^{Q_u} \alpha_j^Q \frac{\langle \phi \rangle}{M_Q} + \alpha_i^U \lambda_j^{U_q} \frac{\langle \phi \rangle}{M_U}
\]

\[
Y_{ij}^D = \lambda_i^{Q_d} \alpha_j^Q \frac{\langle \phi \rangle}{M_Q} + \alpha_i^D \lambda_j^{D_q} \frac{\langle \phi \rangle}{M_D}.
\]
m_2 \neq 0

Neglect H and 1^{st} family for now (will turn out not to contribute)

\[ Y_{ij}^U = \lambda_i^{Qu} \alpha_j^Q \frac{\langle \phi \rangle}{M_Q} + \alpha_i^U \lambda_j^{Uq} \frac{\langle \phi \rangle}{M_U} \]

\[ Y_{ij}^D = \lambda_i^{Qd} \alpha_j^Q \frac{\langle \phi \rangle}{M_Q} + \alpha_i^D \lambda_j^{Dq} \frac{\langle \phi \rangle}{M_D} . \]

m_2 \ll m_3: one term must dominate
\[ m_2 \neq 0 \]

Neglect H and 1\textsuperscript{st} family for now (will turn out not to contribute)

\[ Y_{ij}^U = \lambda_i^{Q u} \alpha_j^Q \langle \phi \rangle_{MQ} + \alpha_i^U \lambda_j^{U q} \langle \phi \rangle_{MU} \]

\[ Y_{ij}^D = \lambda_i^{Q d} \alpha_j^Q \langle \phi \rangle_{MQ} + \alpha_i^D \lambda_j^{D q} \langle \phi \rangle_{MD} \]

\[ m_2 \ll m_3: \text{one term must dominate} \]

\[ |V_{cb}| \ll 1: M_Q \ll M_U, M_D \text{ (left-handed dominance)} \]
Early comments

\[ Y^U \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon_U & \epsilon_U \\ 0 & \epsilon_U & 1 \end{pmatrix}, \quad Y^D \propto \begin{pmatrix} 0 & 0 & 0 \\ \epsilon_D & \epsilon_D & \epsilon_D \\ \epsilon_D & \epsilon_D & 1 \end{pmatrix} \]

with a proper basis choice up to \( O(1) \) coefficients

\[
\epsilon_U = \frac{M_Q}{M_U} \\
\epsilon_D = \frac{M_Q}{M_D}
\]

\* \( m_1 = 0 \): first family “protected” by accidental flavour symmetry \( U(1)_1 \)

\* \( m_2 \ll m_3 \) in terms of \( M_Q \ll M_{U,D} \) (horizontal from vertical)

• in the effective theory below \( M_{U,D} \), the second family mass is protected by an accidental symmetry \( U(1)_2 \)

\* \( m_s/m_b \sim |V_{cb}| \)

\( U(1)'s \) and RS-type: \( Y \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon_1 \epsilon_2 & \epsilon_2 \\ 0 & \epsilon_1 & 1 \end{pmatrix} \) up to \( O(1) \) coefficients

\[ \Rightarrow m_s/m_b \sim \epsilon_2 |V_{cb}| \]
$V_{us}$ and $\text{SU}(2)_R$

\[ Y^U \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon_U & \epsilon_U \\ 0 & \epsilon_U & 1 \end{pmatrix} \]

\[ Y^D \propto \begin{pmatrix} 0 & 0 & 0 \\ \epsilon_D & \epsilon_D & \epsilon_D \\ \epsilon_D & \epsilon_D & 1 \end{pmatrix} \]

$|V_{us}| \sim 1$ unless $(\lambda^U_{iq})_i \approx (\lambda^D_{iq})_i \rightarrow \text{SU}(2)_R$ (or mild fine-tuning)
$V_{us}$ and $SU(2)_R$

$Y^U \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon_U & \epsilon_U \\ 0 & \epsilon_U & 1 \end{pmatrix}$

$Y^D \propto \begin{pmatrix} 0 & 0 & 0 \\ \epsilon_D & \epsilon_D & \epsilon_D \\ \epsilon_D & \epsilon_D & 1 \end{pmatrix}$

$|V_{us}| \sim 1$ unless $(\lambda^{Uq})_i \approx (\lambda^{Dq})_i \rightarrow SU(2)_R$ (or mild fine-tuning)

- $G_{LR} = SU(2)_L \times SU(2)_R \times SU(3)_c \times U(1)_{B-L}$
- $q^c_i = \left( \begin{array}{c} u^c_i \\ d^c_i \end{array} \right)$, $Q^c = \left( \begin{array}{c} U^c \\ D^c \end{array} \right)$, $l^c_i = \left( \begin{array}{c} n^c_i \\ e^c_i \end{array} \right)$, $L^c = \left( \begin{array}{c} N^c \\ E^c \end{array} \right)$, $h = \left( \begin{array}{c} h_u \\ h_d \end{array} \right)$
- $\lambda^c_i$ $q_i$ $Q^c$ $h \Rightarrow (\lambda^{Uq})_i = (\lambda^{Dq})_i = \lambda^c_i$
m_c/m_t \text{ vs } m_s/m_b \text{ and } SU(4)_c

\begin{align*}
Y^U & \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon_U & \epsilon_U \\ 0 & \epsilon_U & 1 \end{pmatrix} \\
Y^D & \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon_D & \epsilon_D \\ 0 & \epsilon_D & 1 \end{pmatrix}
\end{align*}

\begin{itemize}
  \item m_c/m_t \sim m_s/m_b \text{ unless } M_U \gg M_D
  
  \item Unbroken SU(2)_R: M_c \bar{Q}^c Q^c \Rightarrow M_U = M_D = M_c \Rightarrow m_c/m_t = m_s/m_b
  
  \item SU(3)_c \rightarrow SU(4)_c: SU(4)_c \text{ adjoint couples } SU(2)_R \text{ breaking to } Q^c

  \begin{itemize}
    \item (SU(2)_R \text{ breaking by } \tilde{L}^c, \tilde{\bar{L}}^c \text{ scalars with vevs along } \tilde{\mathcal{N}}^c, \tilde{\bar{\mathcal{N}}}^c \text{ (standard)})
  \end{itemize}
  
  \item Then m_c/m_t \ll m_s/m_b \text{ does not require a new ad hoc scale}
\end{itemize}
Neutrinos

**Prediction:** $N^c(\lambda_3^c l_3 + \lambda_2^c l_2) h_u$ (with charged leptons (almost) diagonal)

**Takes care of $\theta_{23}$ + normal hierarchy if $N^c$ dominates the seesaw:**

$$\tan \theta_{23} \sim \frac{\lambda_2^c}{\lambda_3^c} \quad |\Delta m^2_{12}| \ll |\Delta m^2_{23}|$$

**$n^c_i$ also contribute to the seesaw:** $\lambda_3 n^c_i L h_u$

**Must give mass to 2 massless linear combinations:** $\eta_{ki} s_k f_i^c F'_c$, $s_k = (111++)$

**Note: no new mass scale ($n^c_i$ and $N^c$ at the same scale, but $N^c$ dominates)***

$$m_3 = \rho_\nu \frac{v_{EW}^2}{2 s_{23}^2 M_c}, \quad m_{1,2} \approx 0$$

$$M_c \approx 0.6 \cdot 10^{15} \text{ GeV} \rho_\nu, \quad \rho_\nu = O(1)$$
The model

Below the cutoff $\Lambda$ (will turn out to be $> 10^{16-17}$ GeV):

$\bullet$ $G = G_{PS} = SU(2)_L \times SU(2)_R \times SU(4)_c + Z_2$ and SUSY (with $R_P$)

$\bullet$ Field content:

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$F = (L \ Q_1 \ Q_2 \ Q_3)$, $F^c = (L^c \ Q_1^c \ Q_2^c \ Q_3^c)$, $\Sigma = (A \ T \ \bar{T} \ G)$

$\bullet$ Assumptions:

- Heavy masses through $\langle X_c \rangle$, $\langle F'_c \rangle$ (PS breaking)
- $Z_2$ breaking (along $B-L$) at $v \ll M_c$ ($T_{3R}$, $N'_c$)

$M_c \sim V_c \equiv \langle \tilde{N}'_c \rangle$
$v \equiv \langle \phi \rangle$
$\epsilon \equiv v/M_c \ll 1$
The most general ren. superpotential gives rise to the full pattern of II and III family charged fermion masses and mixing

\[*\]

\[ W_{\text{ren}} = \lambda_i f_i^c F h + \lambda_i^c f_i F^c h + \lambda_{ij}^H f_i^c f_j H + \alpha_i \phi f_i \bar{F} + \alpha_i^c \phi f_i^c \bar{F}^c \]

\[ + M \bar{F} F + M_c \bar{F}^c F^c + \gamma M \Sigma^2 + \sigma_c \bar{F}' \Sigma F^c + \sigma_c \bar{F}'^c \Sigma F' + \eta' F' F' H + \bar{\eta}' \bar{F}' F' H + \ldots \]

gives rise to the full pattern of II and III family charged fermion masses and mixing

\[*\]

D, E sector:

\[ Y^D = \begin{pmatrix}
0 & 0 & 0 \\
0 & \alpha_2^c \lambda_2^c \epsilon/3 & \alpha_3^c \lambda_3^c \epsilon/3 \\
0 & \alpha_3^c \lambda_2^c \epsilon/3 & -s \lambda_3
\end{pmatrix} \quad Y^E = - \begin{pmatrix}
0 & 0 & 0 \\
0 & \alpha_2^c \lambda_2^c \epsilon & \alpha_3^c \lambda_3^c \epsilon \\
0 & \alpha_3^c \lambda_2^c \epsilon & s \lambda_3
\end{pmatrix} \]

- \( m_b = m_{\text{tau}} \)
- \( 3m_s = m_{\text{mu}} \)
- \( |V_{\text{cb}}| \sim m_s/m_b \)
- \( \epsilon = 0.06 \times O(1) \)

\( \epsilon \equiv v/M_c \ll 1 \)
\( \tan \theta \equiv \alpha_3 v/M = O(1) \)
\( s = \sin \theta \)
U sector: \[ Y^U = - \begin{pmatrix} 0 & 0 & 0 \\ 0 & (4/9)\alpha_2^c\lambda_2^c\rho_u \epsilon^2 & (4/9)\alpha_2^c\lambda_3^c\rho_u \epsilon^2 \\ 0 & (4/9)\alpha_3^c\lambda_2^c\rho_u \epsilon^2 & s\lambda_3^c \end{pmatrix} \]

Notation: \( \rho \) denotes a combination of \( O(1) \) parameters

\( \epsilon^2: \) \( Y_c \) from \( \lambda_i^c U^c q_i h_u \) with \( U^c \rightarrow (U^c)_{\text{light}} \)

where the \( (U^c)_{\text{light}} \) component of \( U^c \) is suppressed by

- \( v/M_c = \epsilon \)
- \( M/V_c \sim \epsilon \)

2 hierarchies in terms of 1
To summarize (charged fermions)

Assumptions:

- Field content below $\Lambda$
- Messenger masses along $T_{3R}, \tilde{N}^c$
- $Z_2$ breaking at lower scale ($\phi$) along B-L

Results:

- $m_s \ll m_b, m_{\mu} \ll m_{\tau}$
- $|V_{cb}| \sim m_s/m_b$
- $(m_{\tau}/m_b)_M \approx 1, (m_{\mu}/m_s)_M \approx 3$
- $m_c/m_t \ll m_s/m_b$
- $m_{u,d,e}$ further suppressed (no $Hh\phi$)
- $\theta_{23}$ + normal hierarchy in neutrino sector: easy
- $M_c \approx 0.6 \cdot 10^{15}$ GeV $\rho_\nu, \rho_\nu = \mathcal{O}(1)$ ($N^c = FN$)
The first family

• Masses and mixings of the first family originate above the cutoff $\Lambda$ (not specified) and can be parameterized by NR operators

• There exists a choice of NR operators accounting for 1st family masses and mixings (some should be forbidden, e.g. $f^c_i f^f_j \Phi h, F' c F' \Phi h \rightarrow m_u$ too large):

  - $\bar{F}'_c F' \Phi h$ induces $h-H$ mixing in the down (but not up) sector, thus communicating the $U(1)_1$ breaking from $f^c_i f^f_j H$ to the light D, E sectors (but not to the U sector)

  - The up quark mass is consistent with an effect from $M_{Pl}$

  - $M_c/\Lambda \sim m_2/m_3$ (determines $\Lambda$ up to $O(1)$)

  - $\theta_{13} = \frac{m_2}{m_3} \frac{\tan \theta_{12}}{1 + \tan^2 \theta_{12}} \frac{a + b}{a - b} + \ldots$

  - $|V_{us}| \sim M_c/\Lambda \sim m_2/m_3$
A problem

\[ Y^D = \begin{pmatrix} \rho_h \lambda_{11}^H \epsilon' & \rho_h \lambda_{12}^H \epsilon' & \rho_h \lambda_{13}^H c \epsilon' \\ \rho_h \lambda_{21}^H \epsilon' & \alpha_2^c \lambda_2^c / 3 & \alpha_2^c \lambda_3^c c \epsilon / 3 \\ \rho_h \lambda_{31}^H \epsilon' & \alpha_3^c \lambda_2^c / 3 & -s \lambda_3 \end{pmatrix} \]

\[ Y^E = \begin{pmatrix} \rho_h \lambda_{11}^H \epsilon' & \rho_h \lambda_{12}^H \epsilon' & \rho_h \lambda_{13}^H c \epsilon' \\ \rho_h \lambda_{21}^H \epsilon' & -\alpha_2^c \lambda_2^c \epsilon & -\alpha_2^c \lambda_3^c c \epsilon \\ \rho_h \lambda_{31}^H \epsilon' & -\alpha_3^c \lambda_2^c \epsilon & -s \lambda_3 \end{pmatrix} \]

gives \( m_e \approx m_d, |V_{us}| \approx m_d/m_s \) unless

\[ \lambda_{11}^H / \lambda_{12,21}^H < \sqrt{m_d/m_s}/3 \sim 0.08 \]

\[ \text{Then: } 3m_e \approx m_d, |V_{us}| \approx (m_d/m_s)^{1/2} \text{ (at the price of a fine-tuning } > O(10)) \]
Supersymmetry breaking

- The sfermion mass spectrum depends on whether $E < M_c$ or $E > M_c$
- In the absence of flavour symmetries, a GMSB-like mechanism is the natural option
- If $E \geq M_c$: large RGE-induced FCNCs in the “23” sector of right-handed fermions

- Flavour messengers = SUSY-breaking messengers?
- $Z_2$-breaking = SUSY-breaking (through $G_{PS}$ adjoint or fundamental)?
- Peculiar sfermion spectrum?
Summary

* Neutrinos might suggest that the peculiar hierarchical pattern of charged fermion masses and mixings (up to $O(1)$ factors) does not require special horizontal structure.

* In the context of an economical PS model we obtained:
  - the full pattern of II and III family masses and mixings with mild assumptions on the content of the effective theory below a cutoff.
  - the full pattern of I family masses and mixings in terms of a selection of NR operators + $FT > O(10)$.

* Other features:
  - extended see-saw with dominance from extra singlet neutrinos.
  - see-saw fields = FN fields.
  - new supersymmetry breaking options?