General Minimal Flavor Violation

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based on Kagan, Perez, Volansky, JZ, 0903.1794

Outline

- motivation
- formalism
- phenomenological consequences
- conclusions

Why flavor physics?

- can be very predictive/constraining
  - $\Gamma(K_L \to \mu^+\mu^-)/\Gamma(K^+ \to \mu^+\nu) \Rightarrow$ predict. of c quark
  - CPV in K decay $\Rightarrow 3^{\rm rd}$ generation
  - $\Delta m_{B_d} \Rightarrow m_t$ prediction
  - FCNCs suppressed in SM (GIM)
  - only one weak phase
  - $\cdots$
- probes higher scales/contrains NP models
- guides model building (gauge mediation, ...)

SM flavor puzzle(s)

- quark flavor sector well measured
  - 10 parameters, exhibit a hierarchy ($\lambda = 0.22$)

$$\begin{array}{ccc} y_t \sim 1 & y_c \sim \lambda^3 & y_u \sim \lambda^7 \\ y_b \sim \lambda^2 & y_s \sim \lambda^5 & y_d \sim \lambda^6 \\ V_{us} \sim \lambda & V_{cb} \sim \lambda^2 & V_{ub} \sim \lambda^3 \end{array}$$

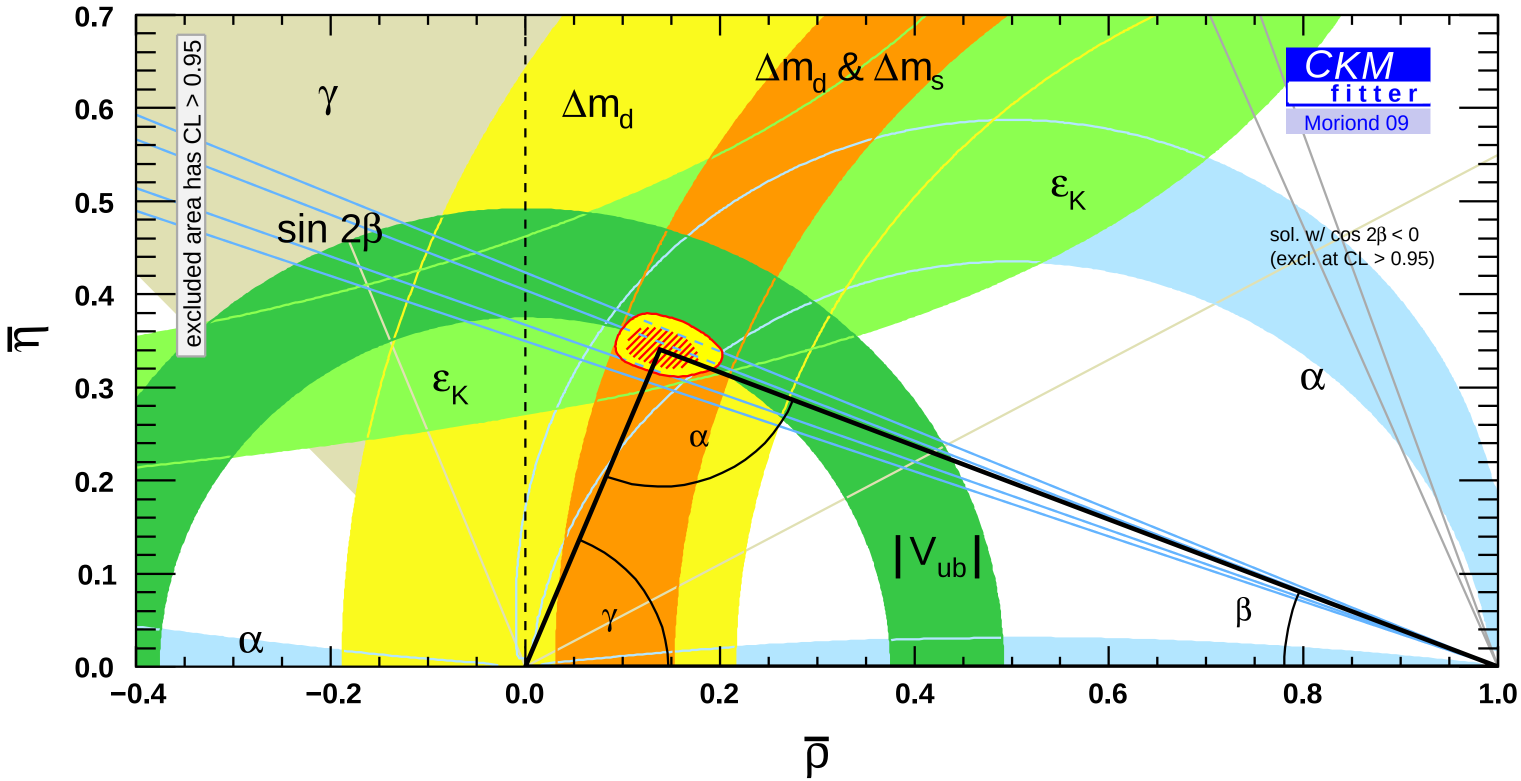
+ $O(1)$ weak phase

  - lepton sector less well measured, different hierarchies
- SM flavor puzzle: why this structure?
- the answer may well not be related to TeV scale

NP flavor puzzle

- new physics expected at TeV scale
  - hierarchy problem
  - dark matter
- but generic flavor structure of TeV NP violates low energy flavor constraints $\Rightarrow$ **NP flavor puzzle**

Present constraints



- nontrivial agreement between different transitions
  - loop vs. tree, B vs. K sector
- KM mechanism dominant source of CPV
  - $\Rightarrow$ move to precision tests

$\Delta F = 2$ processes/NP flavor puzzle

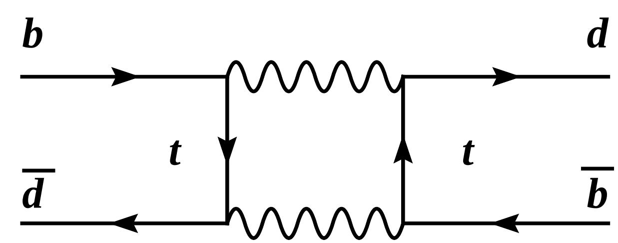
- new physics $O(1)$ (maximally) flavor violating

$$\mathcal{H}_{\rm eff} = \left(\frac{G_F^2 m_W^2}{8\pi^2}\left(V_{ti}^* V_{tj}\right)^2 C_0 + \frac{C_{\rm NP}}{\Lambda_{\rm NP}^2}\right)\left[\bar{d}_i \gamma_\mu (1-\gamma_5) d_j\right]^2$$

- measurements exclude $O(1)$ corrections to mixing

$K - \bar{K}$ mix.:	$\big(\underbrace{V_{ts}^*}_{\sim\lambda^2}\underbrace{V_{td}}_{\sim\lambda^3}\big)^2 \frac{1}{\Lambda_{\rm MFV}^2} > \frac{C_{\rm NP}}{\Lambda_{\rm NP}^2} \Rightarrow$	$\Lambda_{\rm NP} \gtrsim 10^4$ TeV
$B_d - \bar{B}_d$ mix.:	$\big(\underbrace{V_{tb}^*}_{\sim 1}\underbrace{V_{td}}_{\sim\lambda^3}\big)^2 \frac{1}{\Lambda_{\rm MFV}^2} > \frac{C_{\rm NP}}{\Lambda_{\rm NP}^2} \Rightarrow$	$\Lambda_{\rm NP} \gtrsim 5 \cdot 10^2$ TeV
$B_s - \bar{B}_s$ mix.:	$\big(\underbrace{V_{tb}^*}_{\sim 1}\underbrace{V_{ts}}_{\sim\lambda^2}\big)^2 \frac{1}{\Lambda_{\rm MFV}^2} > \frac{C_{\rm NP}}{\Lambda_{\rm NP}^2} \Rightarrow$	$\Lambda_{\rm NP} \sim 10^2$ TeV

with $\Lambda_{\rm MFV} = \sqrt{8}\pi / G_F m_W \sim 6$ TeV



MFV

D'Ambrosio, Giudice, Isidori, Strumia, 2002

- quark sector formally inv. under $U(3)_Q \times U(3)_u \times U(3)_d$ if the Yukawas promoted to spurions

$$Y'_{u,d} = V_Q Y_{u,d} V^\dagger_{u,d}$$

- this constrains possible FV structures, example in $\bar{Q}Q$:
  - allowed: $\bar{Q}(Y_u Y_u^\dagger)^n Q$, not allowed: $\bar{Q}Y_d^\dagger(Y_u Y_u^\dagger)^n Q$
- sometimes additional assumptions included
  - CP only violated by Yukawas
  - only SM 4-quark ops

Buras *et al*, 2000

- no such assumptions here
- for simplicity work in large $\tan\beta$ limit

Motivation

two questions

- Y_u, Y_d have $O(1)$ eigenvalues $y_{t,b}$, why are we able to expand $\bar{Q} f(\epsilon_u Y_u, \epsilon_d Y_d) Q$?
  - if $\epsilon_{u,d} \ll 1$: series truncates after first few terms $\Rightarrow$ **Linear MFV** $\Rightarrow$ expansion in $Y_{u,d}$
  - if $\epsilon_{u,d} = O(1)$: higher terms important $\Rightarrow$ **Nonlinear MFV** $\Rightarrow$ need to reorganize expansion
- can we distinguish LMFV vs. NLMFV?
  - interesting since $\epsilon_{u,d} \propto \log(\mu_W/\Lambda_F) \Rightarrow$ could give a handle on physics at higher scales (with caveats)

NLMFV

- NLMFV: need to keep higher orders in $Y_{u,d}$
- but $Y_{u,d}$ are almost aligned $\Rightarrow$ the off-diagonal elements in $V_{\rm CKM}$ allow for an expansion
- in orig. MFV formulation equiv. to

$$\Delta^n = \Delta, \qquad \Delta = V_{\rm CKM} {\rm diag}\,(0,0,1) V_{\rm CKM}^\dagger$$

Formalism

- inspired by nonlinear sigma model
- $\mathcal{G}^{\rm SM}$ broken by $y_{t,b}$ to $\mathcal{H}^{\rm SM} = U(2)_Q \times U(2)_u \times U(2)_d \times U(1)_3$, we mod out broken symm. genrs. of $\mathcal{G}^{\rm SM}/\mathcal{H}^{\rm SM}$

$$Y_{u,d} = e^{i\hat{\rho}_Q} e^{\pm i\hat{\chi}/2} \tilde{Y}_{u,d} e^{-i\hat{\rho}_{u,d}}, \qquad \tilde{Y}_{u,d} = \begin{pmatrix} \phi_{u,d} & 0 \\ 0 & y_{t,b} \end{pmatrix}$$

- ρ_i spurion "Goldstone bosons", can be set to zero
- χ the misalignement spurion in down quark basis

$$\hat{\chi} = \begin{pmatrix} 0 & \chi \\ \chi^\dagger & 0 \end{pmatrix}, \quad \chi^\dagger \to i(V_{td}, V_{ts}), \quad \phi_u \to V_{\rm CKM}^{(2)\dagger} \, \mathrm{diag}\left(\frac{m_u}{m_t}, \frac{m_c}{m_t}\right)$$

- the bilinears are invariant under $\mathcal{H}^{\rm SM}$

$$\chi' = U_Q^{2\times 2}\chi, \qquad \phi'_{u,d} = U_Q^{2\times 2}\phi_{u,d} U_{u,d}^{2\times 2\dagger}$$

Comparing with usual MFV notation

- MFV LL example ($\Delta_{ij}^{k} = y_k^2 V_{ki}^* V_{kj}$ with $i \neq j$)

$$\bar{Q}\left[a_1 Y_u Y_u^\dagger + a_2 (Y_u Y_u^\dagger)^2\right]Q + \left[b_2\,(\bar{Q} Y_u Y_u^\dagger Y_d Y_d^\dagger)Q + h.c.\right] + \cdots =$$

$$\bar{d}_L^i\left[(a_1 + a_2 y_t^2)\Delta_{ij}^t + a_1 \Delta_{ij}^c\right] d_L^j + \left[b_2 y_b^2\,\bar{d}_L^i \Delta_{ib}^t b_L + h.c.\right]$$

  - LMFV: $a_1 \gg a_2, b_2$, NLMFV: $a_1 \sim a_2 \sim b_2$
  - $a_{1,2}$ are real, b_2 can be complex

- in GMFV notation

$$\left(c_b \overline{d_L^{(2)}} \chi b_L + h.c\right) + c_t \overline{d_L^{(2)}} \chi \chi^\dagger d_L^{(2)} + c_c \overline{d_L^{(2)}} \phi_u \phi_u^\dagger d_L^{(2)}$$

- LO: $c_b \simeq (a_1 y_t^2 + a_2 y_t^4 + b_2 y_b^2)$, $c_t \simeq a_1 y_t^2 + a_2 y_t^4$, $c_c \simeq a_1$

CP violation

- assuming MFV, new CPV effects significant only if new flavor-diag. CP sources
- the CKM generated flavor-diag. phase $\chi^\dagger[\phi_u^\dagger\phi_u, \phi_d^\dagger\phi_d]\chi$ small
- if new CP sources present:
  - $\overline{d_L^{(2)}}\chi\sigma_{\mu\nu}b_R F^{\mu\nu}$ would give enhanced and correlated CPV in $B \to X_s\gamma$ and $B \to X_d\gamma$
  - to ϵ_K can contribute $(\overline{d_L^{(2)}}\chi\chi^\dagger\phi_d d_R^{(2)})^2$: 50% contrib. to ϵ_K corresp. to $\Lambda \approx 0.8$ TeV
  - contributions to both $B-\bar{B}$ and $B_s-\bar{B}_s$

B_d, B_s mixing

- 2 classes of non-hermitian $\Delta B = 2$ effective operators
  - class-1 (no $d_R^{(2)}$): $(\overline{d_L^{(2)}}\chi b_{L,R})^2$,...
  - class-2 (with $d_R^{(2)}$): $(\overline{d_R^{(2)}}\phi_d^\dagger \chi b_L)\,(\overline{d_L^{(2)}}\chi b_R)$,...
- class-2 contribs. only to $B_s - \bar{B}_s$ mixing (up to m_d/m_s)
- $SU(3)_F$ breaking in $B_{d,s} - \bar{B}_{d,s}$ bag parameters small $\Rightarrow$

  same NP phase shift in $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$
- class-1 contribs. dominate if Λ comparable for all ops.
- sizable CPV in B_s system requires class-2 contribs.
- barring cancelations

  NP CPV in $B_s - \bar{B}_s$ mix. $>$ NP CPV in $B_d - \bar{B}_d$ mixing

Up-quark sector

- up-quark mass basis: $\phi_d = V_{CKM}^{(2)\dagger} \mathrm{diag}\,(m_d, m_s)/m_b$ and $\chi = i(V_{ub}, V_{cb})$
- in NLMFV new contribs. can be greatly enhanced
- top FCNCs
  - in SM $BR(t \to cX) \sim \mathcal{O}(10^{-12})$
  - in NLMFV $\overline{\tilde{u}^{(2)}} \chi t$ can lead to $BR(t \to cX) \sim \mathcal{O}(10^{-5})$
- enhancements for CPV in $D - \bar{D}$ mixing
  - relevant operators: $(\overline{\tilde{u}}_L^{(2)} \chi \chi^\dagger u_L)^2$, $(\overline{\tilde{u}}_L^{(2)} \chi \chi^\dagger u_L)(\overline{\tilde{u}}_L^{(2)} \phi_d \phi_d^\dagger u_L)$
  - resulting CP violation in mixing $\arg(M_{12}/\Gamma_{12}) = O(5\%)\,(1\ \mathrm{TeV}/\Lambda)^2\,(\sin 2\gamma, \sin\gamma)$

Conclusions

- introduced a new formalism for contruction of operators obeying MFV assumptions
- phenomenological consequences: correlations between B_s and B_d mixing, enhanced CPV in $B \to X_{s,d}\gamma$, enhanced FCNCs in up-quark observables (top FCNCs, $D - \bar{D}$ mixing)

Backup slides

CP violation

- assuming MFV, new CPV effects significant only if new flavor-diag. CP sources
- if $U(2)_L$ exact $\Rightarrow$ KM phase can be removed $\Rightarrow$ CP violation only from flavor-diagonal phases
- spurions that break $U(2)_L$ are $\phi_{u,d}$ and χ
- χ contains the CKM phase
- any additional contributions are suppressed by at least $[\phi_u^\dagger \phi_u, \phi_d^\dagger \phi_d] \sim (m_s/m_b)^2 (m_c/m_t)^2 \sin\theta_C \sim 10^{-9}$