Uganke Roperjeve resonance

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- Motivacija: Zakaj je Roperjeva resonanca (N(1440)) nekaj posebnega?
- Energija
- Sipalne amplitude
- Elektroprodukcija pionov

Model

Mezoni so linearno sklopljeni s kvarki

$$H = H_{\mathrm{quark}} + \int \mathrm{d}k \sum_{lmt} \left\{ \omega_k\, a^\dagger_{lmt}(k) a_{lmt}(k) + \left[V_{lmt}(k) a_{lmt}(k) + V_{lmt}(k)^\dagger\, a^\dagger_{lmt}(k) \right] \right\}$$

$V_{lmt}(k)$ odgovoren tudi za radialne ekscitaticije kvarkov, npr. $1s \rightarrow 2s$ prehode.

Konstrukcija matrike K (Chew-Low)

$$K^{JT}_{\pi N \pi N}(k, k_0) = -\pi \sqrt{\frac{\omega_k E_N}{k\, W}} \langle \Psi^N_{JT}(W) || V(k) || \Psi_N \rangle \, .$$

"principal-value" (PV) stanja:

$$|\Psi^N_{JT}(W)\rangle = \sqrt{\frac{\omega_0 E_N}{k_0 W}} \left\{ \left[a^\dagger(k_0) |\Psi_N\rangle \right]^{JT} - \frac{\mathcal{P}}{H - W} \left[V(k_0) |\Psi_N\rangle \right]^{JT} \right\},$$

Normalizacija

$$\langle \Psi^{\mathrm{P}}_\alpha(W) | \Psi^{\mathrm{P}}_\beta(W') \rangle = \delta(W - W') \delta_{\alpha\beta} (1 + \mathbf{K}^2)_{\alpha\alpha} \, .$$

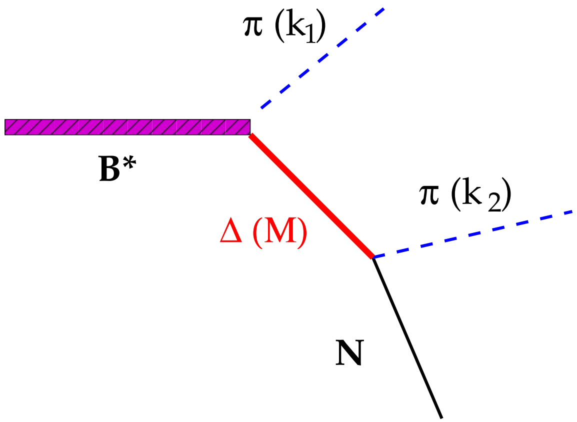
Ortonormirana stanja

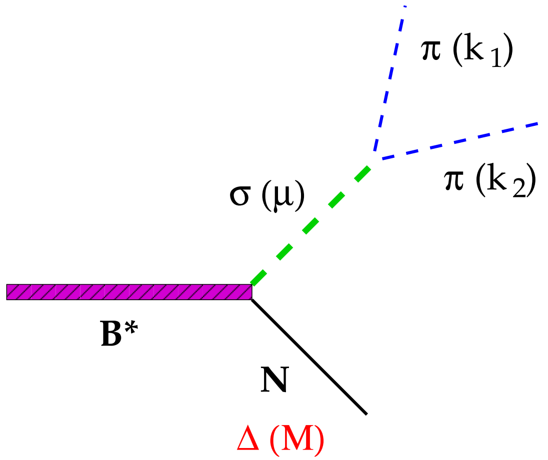
$$|\widetilde{\Psi}^\alpha(W)'\rangle = \sum_\beta \left[\mathbf{1} + \mathbf{K}^2 \right]^{-1/2}{}_{\beta,\alpha} |\Psi^\beta(W)\rangle$$

Predpostavka o dvo-pionskih razpadih

Kaskadni razpad:

$$\pi N \to B^* \to \pi\Delta \to \pi\pi N \qquad \pi N \to B^* \to \sigma N \to (2\pi)^{I=0}_{T=0} N$$





$$\omega_1 = W - E = \frac{W^2 - M^2 + m_\pi^2}{2W}, \qquad \omega_\mu = W - E_N = \frac{W^2 - M_N^2 + \mu^2}{2W},$$

$$k_1 = \sqrt{\omega_1^2 - m_\pi^2}, \qquad E = \sqrt{M^2 + k_1^2} \qquad k_\mu = \sqrt{\omega_\mu^2 - m_\mu^2} \qquad E_N = \sqrt{M_N^2 + k_\mu^2}.$$

$$M_N + m_\pi < M < W - m_\pi \qquad 2m_\pi < \mu < W - M_N$$

Konstrukcija večkanalne matrike K

PV stanja v kanalu $\pi\Delta$

$$|\Psi^{\Delta}_{JT}(W,M)\rangle = \sqrt{\frac{\omega_1 E}{k_1 W}}\left\{\left[a^\dagger(k_1)|\widetilde{\Psi}_\Delta(M)\rangle\right]^{JT} - \frac{\mathcal{P}}{H-E}\left[V(k_1)|\widetilde{\Psi}_\Delta(M)\rangle\right]^{JT}\right\}.$$

Vmesno stanje Δ:

$$\langle\widetilde{\Psi}_\Delta(M)|\widetilde{\Psi}_\Delta(M')\rangle = \delta(M-M')\,.$$

$$\begin{aligned}|\widetilde{\Psi}_\Delta(M)\rangle \;&\approx\; w_\Delta(M)\left\{|\Phi_\Delta\rangle - \int\frac{\mathrm{d}k\;\mathcal{V}_{N\Delta}(k,k_2)}{\omega_k + E_N(k) - M}\,[a^\dagger(k)|\Phi_N\rangle]^{\frac{3}{2}\frac{3}{2}}\right.\\ &\qquad\left. - \int\frac{\mathrm{d}k\;\mathcal{V}_{\Delta\Delta}(k)}{\omega_k + E_\Delta(k) - M}\,[a^\dagger(k)|\Phi_\Delta\rangle]^{\frac{3}{2}\frac{3}{2}}\right\} + \ldots\\ w_\Delta(M)^2 \;&\approx\; \frac{1}{\pi}\frac{\frac{1}{2}\Gamma_\Delta}{(M_\Delta - M)^2 + (\frac{1}{2}\Gamma_\Delta)^2}\end{aligned}$$

Nastavek za kanalna stanja

kanal πN:

$$|\Psi_{JT}^{N}(W)\rangle = \sqrt{\frac{\omega_0 E_N(k_0)}{k_0 W}} \Big\{ \sum_B c_B^N(W)|\Phi_B\rangle + \boxed{[a^\dagger(k_0)|\Psi_N(k_0)\rangle]^{JT}}$$
$$+ \int \frac{\mathrm{d}k \quad \chi_{JT}^{N}(k,k_0)}{\omega_k + E_N(k) - W}[a^\dagger(k)|\Psi_N(k)\rangle]^{JT} + \int \mathrm{d}M' \int \frac{\mathrm{d}k\ \chi_{JT}^{\Delta N}(k,k_0,M')}{\omega_k + E'(k) - W}[a^\dagger(k)|\widetilde{\Psi}_\Delta(M')\rangle]^{JT} \Big\},$$

kanal $\pi\Delta(M)$

$$|\Psi_{JT}^{\Delta}(W,M)\rangle = \sqrt{\frac{\omega_1 E(k_1)}{k_1 W}} \Big\{ \sum_B c_B^\Delta(W,M)|\Phi_B\rangle + \boxed{[a^\dagger(k_1)|\widetilde{\Psi}_\Delta(M)\rangle]^{JT}}$$
$$+ \int \frac{\mathrm{d}k\ \chi_{JT}^{N\Delta}(k,k_1,M)}{\omega_k + E_N(k) - W}[a^\dagger(k)|\Psi_N(k)\rangle]^{JT} + \int \mathrm{d}M' \int \frac{\mathrm{d}k\ \chi_{JT}^{\Delta}(k,k_1,M',M)}{\omega_k + E'(k) - W}[a^\dagger(k)|\widetilde{\Psi}_\Delta(M')\rangle]^{JT} \Big\}.$$

Nad eno-pionskim pragom: $K_{NN}(W) = \pi \dfrac{\omega_0 E_N(k_0)}{k_0 W}\chi_{JT}^{N}(k_0,k_0)\,,$

Nad dvopionskim pragom:

$$K_{\Delta N}(W,M) = \pi\sqrt{\frac{\omega_0 E_N(k_0)\omega_1 E(k_1)}{k_0 k_1 W^2}}\,\chi_{JT}^{\Delta N}(k_1,k_0,M)\,,$$
$$K_{N\Delta}(W,M) = \pi\sqrt{\frac{\omega_0 E_N(k_0)\omega_1 E(k_1)}{k_0 k_1 W^2}}\,\chi_{JT}^{N\Delta}(k_0,k_1,M)\,,$$
$$K_{\Delta\Delta}(W,M',M) = \pi\sqrt{\frac{\omega_1 E(k_1)\omega_1' E(k_1')}{k_1 k_1' W^2}}\,\chi_{JT}^{\Delta}(k_1',k_1,M',M)\,.$$

Integralna enačba za matriko K

(Lippmann-Schwingerjeve enačba)

$$\begin{aligned}
\chi^{N}_{JT}(k,k_0) &= -\sum_B c^N_B(W) V_{NB}(k) + \mathcal{K}^{NN}(k,k_0) + \int \mathrm{d}k' \frac{\mathcal{K}^{NN}(k,k')\chi^{N}_{JT}(k',k_0)}{\omega'_k + E_N(k') - W} + \int \mathrm{d}k' \frac{\mathcal{K}^{N\Delta}_{M_\Delta}(k,k')\hat{\chi}^{\Delta N}_{JT}(k',k_0)}{\omega'_k + E_\Delta(k') - W} \\
\hat{\chi}^{\Delta}_{JT}(k,k_1) &= -\sum_B \hat{c}^\Delta_B(W,M)\, V^{M'}_{\Delta B}(k) + \mathcal{K}^{\Delta\Delta}_{M'M}(k,k_1) + \int \mathrm{d}k' \frac{\mathcal{K}^{\Delta\Delta}_{M'M_\Delta}(k,k')\hat{\chi}^{\Delta}_{JT}(k',k_1)}{\omega'_k + E_\Delta(k') - W} + \int \mathrm{d}k' \frac{\mathcal{K}^{\Delta N}_{M'}(k,k')\hat{\chi}^{N\Delta}_{JT}(k',k_1)}{\omega'_k + E_N(k') - W} \\
\hat{\chi}^{\Delta N}_{JT}(k,k_0) &= -\sum_B c^N_B(W) V^{m}_{\Delta B}(k) + \mathcal{K}^{\Delta N}_{M}(k,k_0) + \int \mathrm{d}k' \frac{\mathcal{K}^{\Delta N}_{M}(k,k')\chi^{N}_{JT}(k',k_0)}{\omega'_k + E_N(k') - W} + \int \mathrm{d}k' \frac{\mathcal{K}^{\Delta\Delta}_{MM_\Delta}(k,k')\hat{\chi}^{\Delta N}_{JT}(k',k_0)}{\omega'_k + E_\Delta(k') - W} \\
\hat{\chi}^{N\Delta}_{JT}(k,k_1) &= -\sum_B \hat{c}^\Delta_B(W,M) V_{NB}(k) + \mathcal{K}^{N\Delta}_{M}(k,k_1) + \int \mathrm{d}k' \frac{\mathcal{K}^{N\Delta}_{M_\Delta}(k,k')\chi^{\Delta}_{JT}(k',k_1)}{\omega'_k + E_\Delta(k') - W} + \int \mathrm{d}k' \frac{\mathcal{K}^{NN}(k,k')\hat{\chi}^{N\Delta}_{JT}(k',k_1)}{\omega'_k + E_N(k') - W}
\end{aligned}$$

$$\begin{aligned}
(W - M^0_B) c^N_B(W) &= V_{NB}(k_0) + \int \mathrm{d}k \frac{\hat{\chi}^{\Delta N}_{JT}(k,k_0)\, V_{\Delta B}(k)}{\omega_k + E_\Delta(k) - W} + \int \mathrm{d}k \frac{\chi^{N}_{JT}(k,k_0)\, V_{NB}(k)}{\omega_k + E_N(k) - W} \\
(W - M^0_B) \hat{c}^\Delta_B(W,M) &= V_{\Delta B}(k_1) + \int \mathrm{d}k \frac{\chi^{N\Delta}_{JT}(k,k_1)\, V_{NB}(k)}{\omega_k + E_N(k) - W} + \int \mathrm{d}k \frac{\hat{\chi}^{\Delta}_{JT}(k,k_1)\, V_{\Delta B}(k)}{\omega_k + E_\Delta(k) - W}
\end{aligned}$$

Določitev polov matrike K

Enačba za koeficiente $c^H_{\mathcal{R}'}$

$$\sum_{\mathcal{R}'} A_{\mathcal{R}\mathcal{R}'}(W) c^H_{\mathcal{R}'}(W, m_H) = \mathcal{V}^M_{H\mathcal{R}}(k_H)\,,$$

$$\mathbf{U}\mathbf{A}\mathbf{U}^T = \mathbf{D}\,, \qquad \mathbf{D} = \begin{bmatrix} Z_{\mathcal{R}}(W)(W - M_{\mathcal{R}}) & 0 & 0 \\ 0 & Z_{\mathcal{R}'}(W)(W - M_{\mathcal{R}'}) & 0 \\ 0 & 0 & Z_{\mathcal{R}''}(W)(W - M_{\mathcal{R}''}) \end{bmatrix}$$

$$\widetilde{\mathcal{V}}_{H\mathcal{R}} = \sum_{\mathcal{R}'} u_{\mathcal{R}\mathcal{R}'} \mathcal{V}_{H\mathcal{R}'}\,, \qquad \tilde{c}^H_{\mathcal{R}} = \frac{\widetilde{\mathcal{V}}_{H\mathcal{R}}}{Z_{\mathcal{R}}(W)(W - M_{\mathcal{R}})}\,.$$

$$\chi^{H'H} = -\sum_{\mathcal{R}} \widetilde{\mathcal{V}}_{H\mathcal{R}} \frac{1}{Z_{\mathcal{R}}(W)(W - M_{\mathcal{R}})} \widetilde{\mathcal{V}}_{H'\mathcal{R}}$$

Reševanje enačbe v aproksimaciji separabilnih jeder

$$\frac{1}{\omega_k + \omega'_k - \omega_0 + E_B(\bar{k}) - E_N(k_0)} \approx \frac{\omega_0 + E_B(\bar{k}) - E_N(k_0)}{(\omega_k + E_B(\bar{k}) - E_N(k_0))(\omega'_k + E_B(\bar{k}) - E_N(k_0))}$$

$$\bar{k}^2 \approx \langle(\boldsymbol{k}_0 + \boldsymbol{k}_1)^2\rangle \approx k_0^2 + k_1^2\,, \qquad E_B(\bar{k}) + E_N(k_0) - \omega_0 \approx 2M_B$$

$$\begin{aligned}
\mathcal{K}^{NN}(k,k') &= \sum_i f_{NN}^{B_i} \frac{M_{Bi}}{E_N} (\omega_0 + \varepsilon_i^N) \frac{\mathcal{V}_{B_iN}(k')\mathcal{V}_{B_iN}(k)}{(\omega'_k + \varepsilon_i^N)(\omega_k + \varepsilon_i^N)} \\
\mathcal{K}^{N\Delta}_{M}(k,k') &= \sum_i f_{N\Delta}^{B_i} \frac{M_{Bi}}{E} (\omega_1 + \varepsilon_i^N) \frac{\mathcal{V}_{B_iN}(k')\mathcal{V}_{B_i\Delta}(k)}{(\omega'_k + \varepsilon_i^N)(\omega_k + \varepsilon_i^\Delta(M))} = \mathcal{K}^{\Delta N}_{M}(k',k) \\
\mathcal{K}^{\Delta\Delta}_{M'M}(k,k') &= \sum_i f_{\Delta\Delta}^{B_i} \frac{M_{Bi}}{E'} (\omega'_1 + \varepsilon_i^\Delta(M)) \frac{\mathcal{V}_{B_i\Delta}(k)}{(\omega_k + \varepsilon_i^\Delta(M))} \frac{\mathcal{V}_{B_i\Delta}(k')}{(\omega'_k + \varepsilon_i^\Delta(M'))}
\end{aligned}$$

$$\varepsilon_i^N = \frac{M_{Bi}^2 - M_N^2 - m_\pi^2}{2E_N}\,, \qquad \varepsilon_i^\Delta(M) = \frac{M_{Bi}^2 - M^2 - m_\pi^2}{2E}\,,$$

Rešitev za matriko K:

$$K_{hh'} = K_{hh'}(\text{reson}) + K_{hh'}(\text{ozadje}) = \pi\mathcal{N}_H\mathcal{N}_{h'}\left\{\sum_B \frac{\mathcal{V}_{hB}\mathcal{V}_{h'B}}{(M_B - W)} + \mathcal{D}_{hh'}\right\}$$

Konstrukcija sipalne matrike (Heitlerjeva enačba)

$$
\begin{aligned}
T_{NN}(W) &= K_{NN}(W) + \mathrm{i}\left[T_{NN}(W)K_{NN}(W) + \int_{M_N+m_\pi}^{W-m_\pi}\mathrm{d}M\, T_{N\Delta}(W,M)K_{\Delta N}(W,M)\right] \\
&\quad + \mathrm{i}\int_{2m_\pi}^{W-M_N}\mathrm{d}\mu\, T_{N\sigma}(W,\mu)K_{\sigma N}(W,\mu)\,, \\
T_{N\Delta}(W,M) &= K_{N\Delta}(W,M) + \mathrm{i}\left[T_{NN}(W)K_{N\Delta}(W,M) + \int_{M_N+m_\pi}^{W-m_\pi}\mathrm{d}M'\, T_{N\Delta}(W,M')K_{\Delta\Delta}(W,M',M)\right] \\
&\quad + \mathrm{i}\int_{2m_\pi}^{W-M_N}\mathrm{d}\mu\, T_{N\sigma}(\mu)K_{\sigma\Delta}(\mu,M)\,, \\
T_{N\sigma}(\mu) &= K_{N\sigma}(\mu) + \mathrm{i}T_{NN}K_{N\sigma}(\mu) \\
&\quad + \mathrm{i}\int_{M_N+m_\pi}^{W-m_\pi}\mathrm{d}M\, T_{N\Delta}(M)K_{\Delta\sigma}(M,\mu) \\
&\quad + \mathrm{i}\int_{2m_\pi}^{W-M_N}\mathrm{d}\mu'\, T_{N\sigma}(\mu')K_{\sigma\sigma}(\mu',\mu)\,.
\end{aligned}
$$

Fazni premik δ in neelastičnost η: $S = 1 + 2\mathrm{i}T_{NN}(W) = \eta(W)\mathrm{e}^{2\mathrm{i}\delta(W)}$.

Rezultati v modelu oblačne vreče

$$\langle \Phi_{B'} || V(k) || \Phi_B \rangle = r_q\, v(k)\, \langle J_{B'}, T_{B'} = J_{B'} || \sum_{i=1}^{3} \sigma_m^i \tau_t^i || J_B, T_B = J_B \rangle$$

$$v(k) = \frac{1}{2f} \frac{k^2}{\sqrt{12\pi^2\omega_k}} \frac{\omega_{\mathrm{MIT}}}{\omega_{\mathrm{MIT}} - 1} \frac{j_1(kR_{\mathrm{bag}})}{kR_{\mathrm{bag}}} .$$

$$r_q = \begin{cases} 1 & \text{za} \quad B = B' = (1s)^3 \\ r_\omega = \left[\frac{\omega^1_{\mathrm{MIT}}(\omega^0_{\mathrm{MIT}} - 1)}{\omega^0_{\mathrm{MIT}}(\omega^1_{\mathrm{MIT}} - 1)} \right]^{1/2} = 0.457 & \text{za} \quad B = (1s)^3, B' = (1s)^2(2s)^1 \\ \frac{2}{3} + r_\omega^2 & \text{za} \quad B = B' = (1s)^2(2s)^1 \end{cases}$$

$R_{\mathrm{bag}} = 0.83$ fm, $f = 76$ MeV

podobni rezultati za $0.75 \text{ fm} < R_{\mathrm{bag}} < 1.0 \text{ fm}$

Prosti parametri: gole mase resonančnih stanj

$$M_R = 1510 \text{ MeV}, \quad M_\Delta = 1232 \text{ MeV}, \quad M_{\Delta^*} = 1770 \text{ MeV}$$

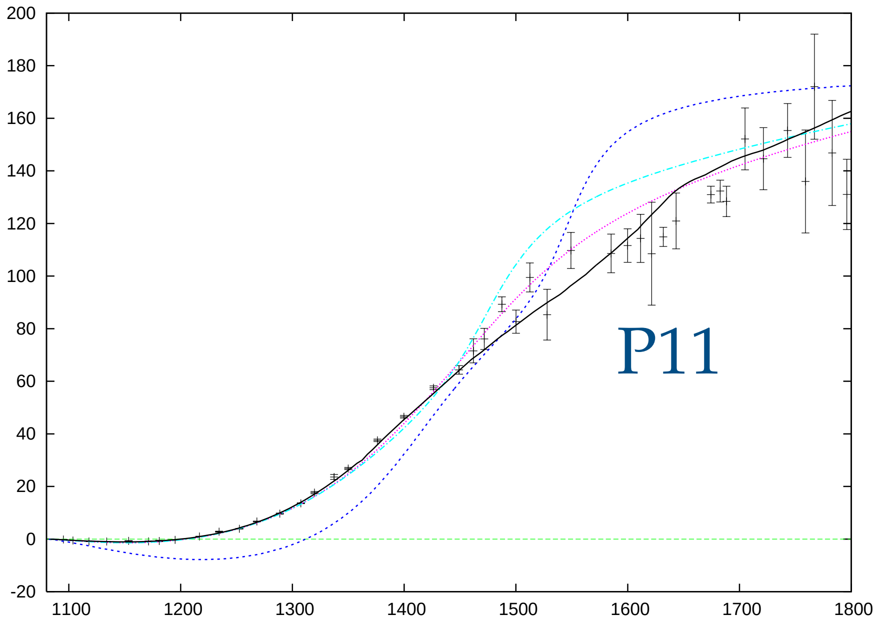
Parametri kanala σN: $G_\sigma = 0.8\,, \quad m_\sigma = 450 \text{ MeV}\,, \quad \Gamma_\sigma = 550 \text{ MeV}$

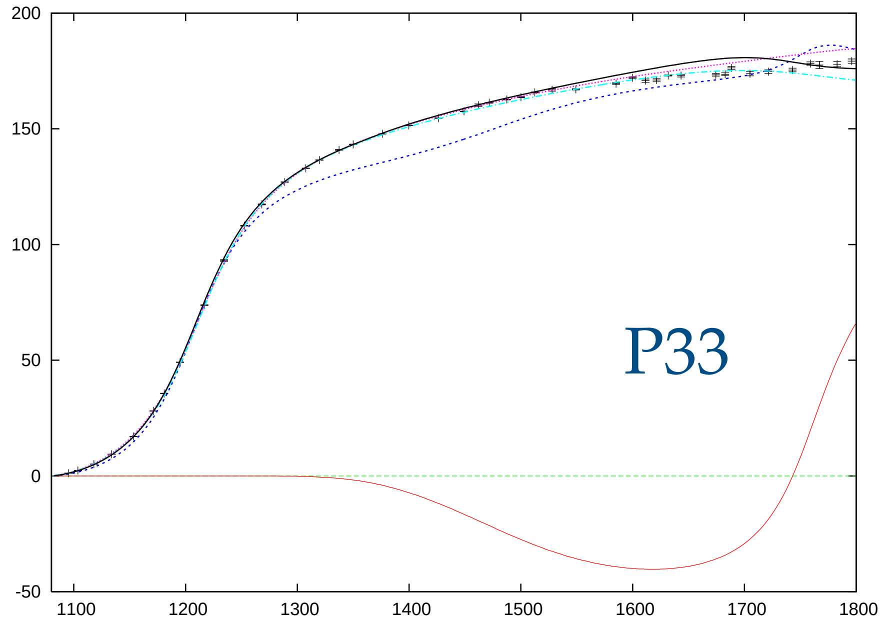
Rezutati

... samo kanala πN in $\pi\Delta$

... dodan kanal σN ($\sigma\Delta$):

Fazni premik

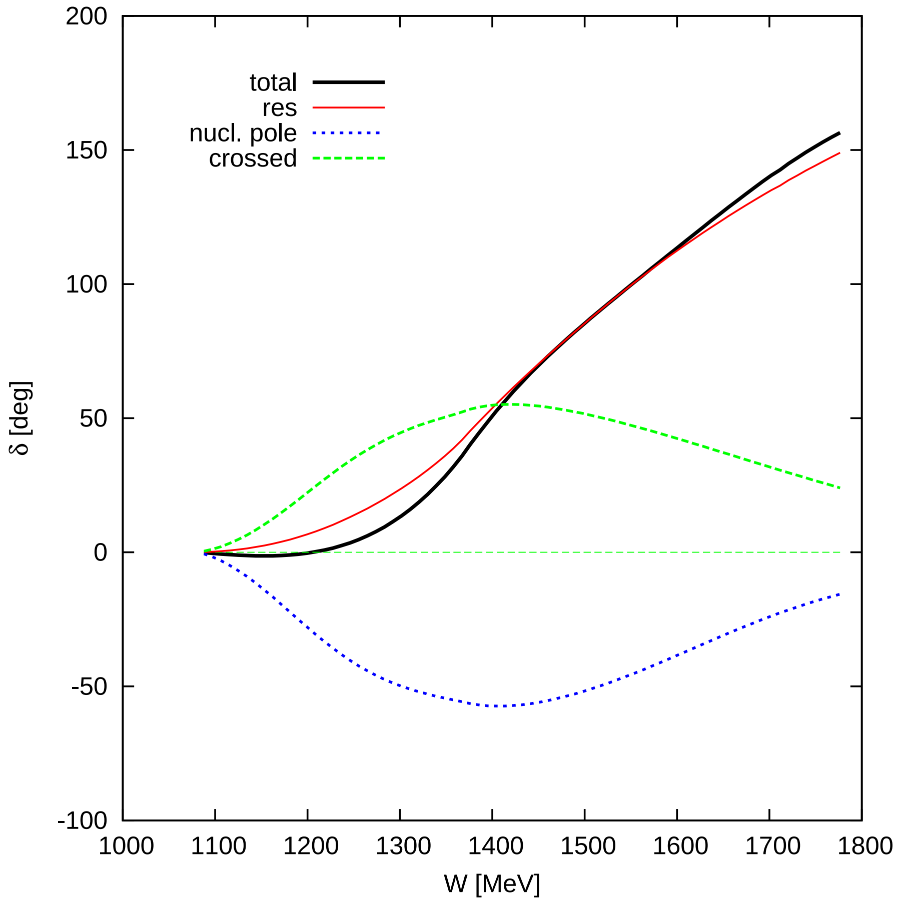


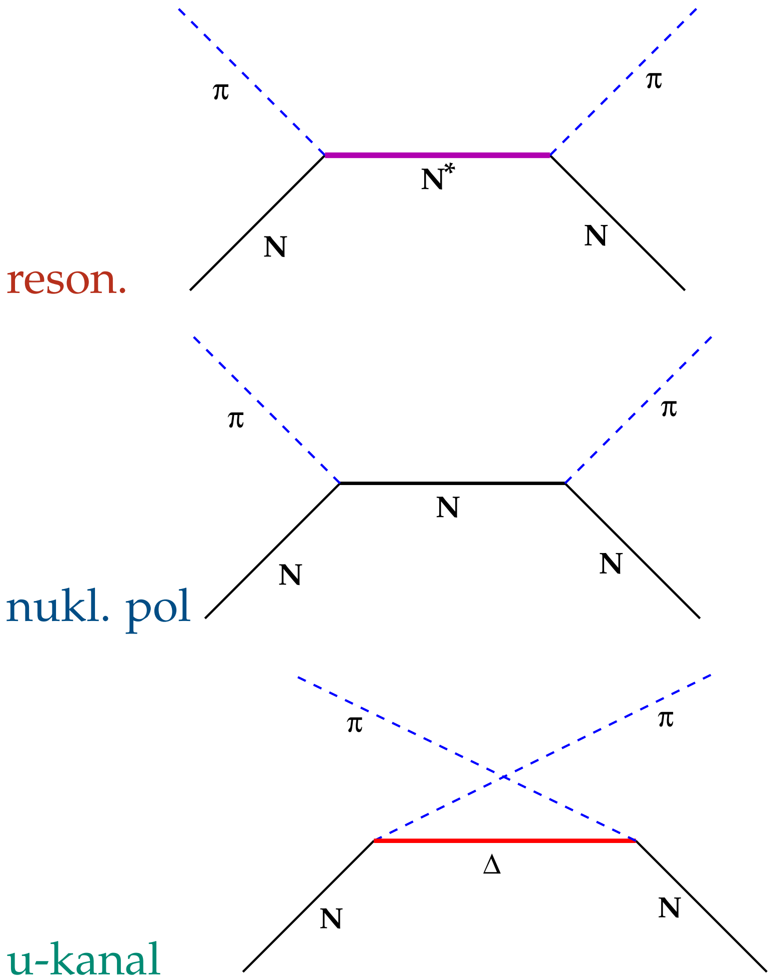


$$\frac{g_{\pi NR}}{g_{\pi NR}^{\mathrm{quark}}} = 1.0\,, \quad \frac{g_{\pi N\Delta}}{g_{\pi N\Delta}^{\mathrm{quark}}} = 1.0$$

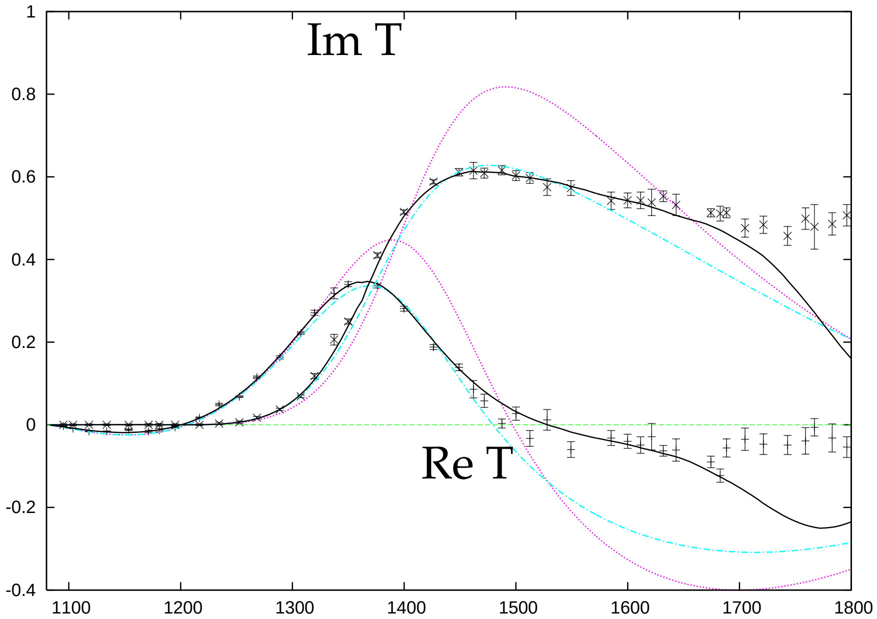
$$\frac{g_{\pi NR}}{g_{\pi NR}^{\mathrm{quark}}} = 1.0 \qquad \frac{g_{\pi N\Delta}}{g_{\pi N\Delta}^{\mathrm{quark}}} = 1.0$$

Prispevek resonančnega dela in prispevek ozadja k sipalnemu premiku v P11 valu

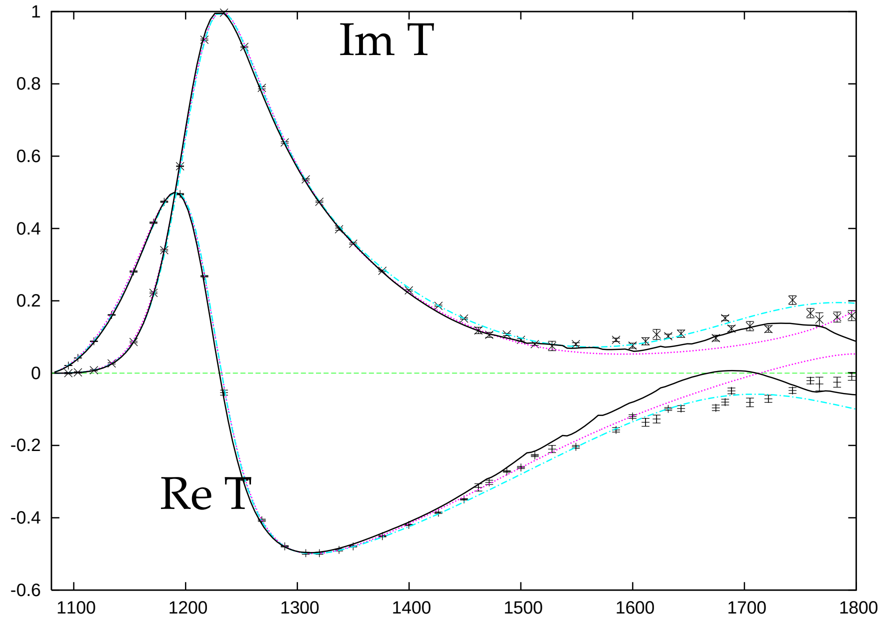




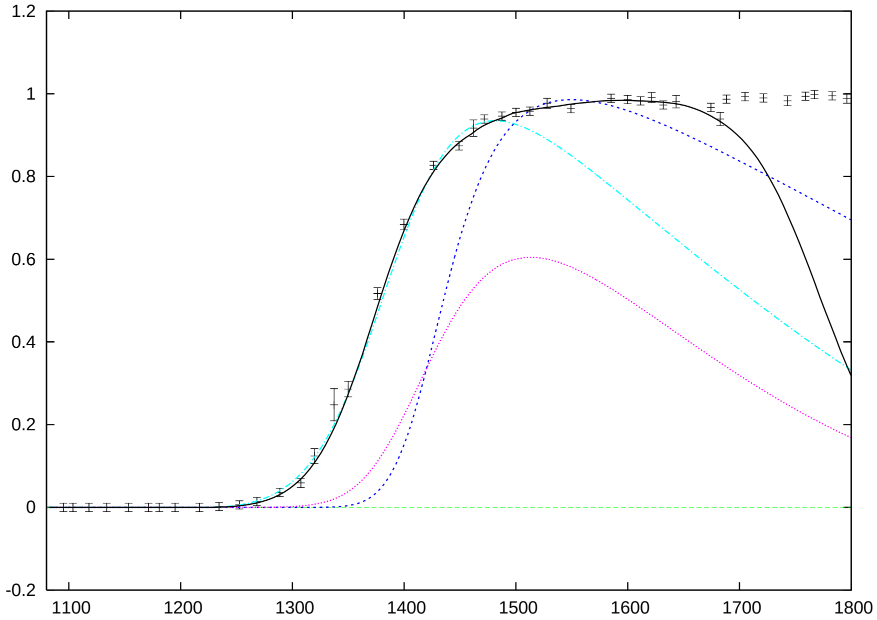
Realni in imaginarni del sipalne amplitude



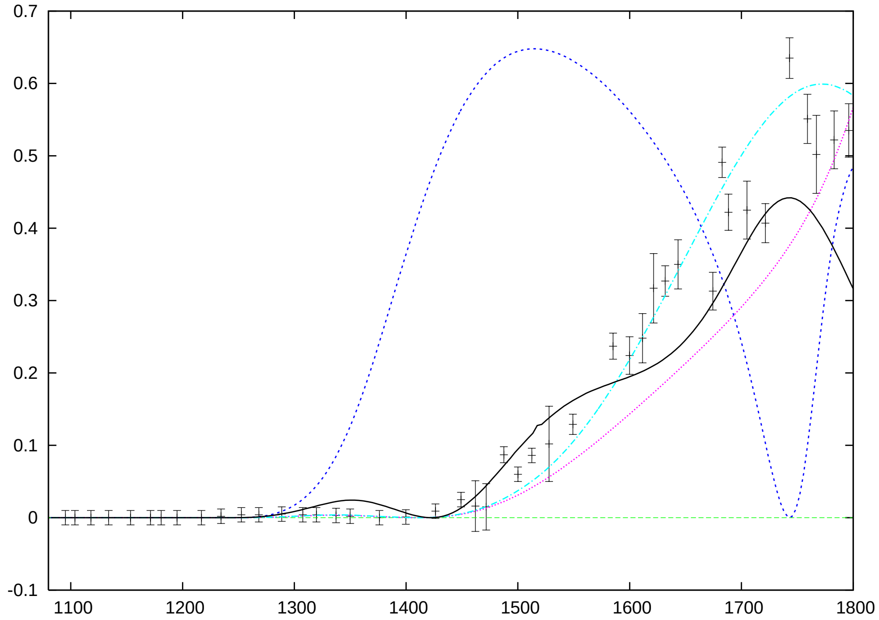
P11



P33



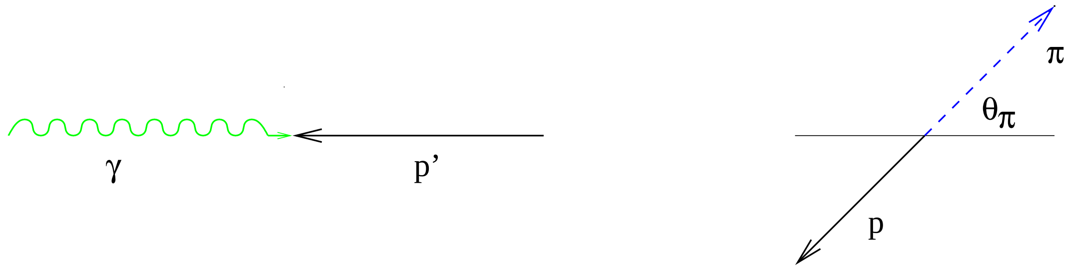
P11



P33

neelastičnost

Elektroprodukcija pionov



V matriki K se pojavi nov kanal, γN.
Ker ke EM interakcija bistveno šibkejša od močne, predpostavimo

$$K_{\gamma N\,\gamma N} \ll K_{\gamma N\,\pi N} \ll K_{\pi N\,\pi N}$$

(in podobno v ostalih kanalih). Heitlerjeva enačba za elektroprodukcijske amplitude se v tem primeru poenostavi:

$$\begin{aligned}\mathcal{M}_N(W) &= \mathcal{M}_N^K(W) + \mathrm{i}\Big[T_{\pi N\pi N}(W)\mathcal{M}_N^K(W) + \overline{T}_{\pi N\pi\Delta}(W,\bar{M})\overline{\mathcal{M}}_\Delta^K(W,\bar{M}) \\ &\quad + \overline{T}_{\pi N\sigma N}(W,\bar{\mu}_\sigma)\overline{\mathcal{M}}_\sigma^K(W,\bar{\mu}_\sigma)\Big]\end{aligned}$$

Matrika T za elektroprodukcijo je povezan z elektroprodukcijsko amplitudo z

$$T_{\gamma N\pi N}^{(JT)} = \mathrm{i}\pi\,\frac{1}{\sqrt{2\pi}^3}\sum_m \sqrt{k_0 k_\gamma}\,\mathcal{M}_N(W,M_J,M_T,t,k_\gamma,\mu)\,Y_{1m}(\hat{r})\,C_{\frac{1}{2}m_s 1m}^{JM_J} C_{\frac{1}{2}\frac{1}{2}1t}^{TM_T}$$

Izvrednotiti moramo:

$$\mathcal{M}_N^K(W) = -\sqrt{\frac{\omega_\gamma}{k_0}}\langle\Psi_{N*}^N(W)|\tilde{V}_\gamma(\mu,\boldsymbol{k}_\gamma)|\Phi_N\rangle\,,$$

$$\mathcal{M}_\Delta^K(W,M) = -\sqrt{\frac{\omega_\gamma}{k_0}}\langle\Psi_{N*}^\Delta(W,M)|\tilde{V}_\gamma(\mu,\boldsymbol{k}_\gamma)|\Phi_N\rangle\,,$$

$$\mathcal{M}_\sigma^K(W,\mu_\sigma) = -\sqrt{\frac{\omega_\gamma}{k_0}}\langle\Psi_{N*}^\sigma(W,\mu_\sigma)|\tilde{V}_\gamma(\mu,\boldsymbol{k}_\gamma)|\Phi_N\rangle$$

$$V_\gamma(\mu,\boldsymbol{k}_\gamma) = \frac{1}{\sqrt{2\pi}^3}\tilde{V}_\gamma(\mu,\boldsymbol{k}_\gamma)\,,\qquad \tilde{V}_\gamma(\mu,\boldsymbol{k}_\gamma) = \frac{\mathrm{e}_0}{\sqrt{2\omega_\gamma}}\int \mathrm{d}\boldsymbol{r}\,\boldsymbol{\varepsilon}_\mu\cdot\boldsymbol{j}(\boldsymbol{r})\mathrm{e}^{\mathrm{i}\boldsymbol{k}_\gamma\cdot\boldsymbol{r}}$$

Razbitje amplitude v resonančni del in ozadje:

Resonančni del vsebuje pol pri energiji resonance.
Matrične elemente lahko zapišmo v obliki:

$$\mathcal{M}_H^K = \sqrt{\frac{\omega_\gamma E_N^\gamma}{k_0 W}}\, g(W)\, K_{NH}\, \langle \Psi_{N*}^{(\mathrm{res})}(W) | \tilde{V}_\gamma | \Psi_N \rangle + \mathcal{M}_H^{K\,(\mathrm{non})} \qquad H = N, \Delta, \sigma$$

$$\begin{aligned}\mathcal{M}_H^{K\,(\mathrm{non})} &= -\sqrt{\frac{\omega_\gamma E_N^\gamma}{k_0 W}} \Big\{ g(W)\, K_{NH}^{(\mathrm{bg})}\, \langle \Psi_{N^*}^{(\mathrm{res})}(W) | \tilde{V}_\gamma | \Psi_N \rangle \\ &\quad + \sqrt{\frac{\omega_H E_H}{k_H W}} \left[c_N^H \langle \Psi_{N^*}^{(\mathrm{n.p.})} | \tilde{V}_\gamma | \Psi_N \rangle + \langle \Psi_{N*}^{H\,(\mathrm{dir})} | \tilde{V}_\gamma | \Psi_N \rangle \right] \Big\}\end{aligned}$$

Potem lahko zapišemo

$$\mathcal{M}_N^{(\mathrm{res})} = \sqrt{\frac{\omega_\gamma E_N^\gamma}{k_0 W}}\, g(W)\, \langle \Psi_{N*}^{(\mathrm{res})}(W) | \tilde{V}_\gamma | \Psi_N \rangle\, T_{\pi N \pi N} = \sqrt{\frac{\omega_\gamma E_N^\gamma}{k_0 W}}\, g(W)\, A_{N*}\, T_{\pi N \pi N}$$

pri čemer neresonančni del (ozadje) zadošča enačbi, ki ne vsebuje več polov:

$$\mathcal{M}_N^{(\mathrm{non})} = \mathcal{M}_N^{K\,(\mathrm{non})} + \mathrm{i}\left[T_{\pi N \pi N} \mathcal{M}_N^{K\,(\mathrm{non})} + \overline{T}_{\pi N \pi \Delta} \overline{\mathcal{M}}_\Delta^{K\,(\mathrm{non})} + \overline{T}_{\pi N \sigma N} \overline{\mathcal{M}}_\sigma^{K\,(\mathrm{non})} \right].$$

Pri tem je A_{N*} elektro-excitacijska amplituda za tvorbo resonančnega stanja (EM magnetni oblikovni faktor).

$$A_{N*} \equiv \langle \Psi_{N*}^{(\mathrm{res})}(W) | \tilde{V}_\gamma | \Psi_N \rangle$$

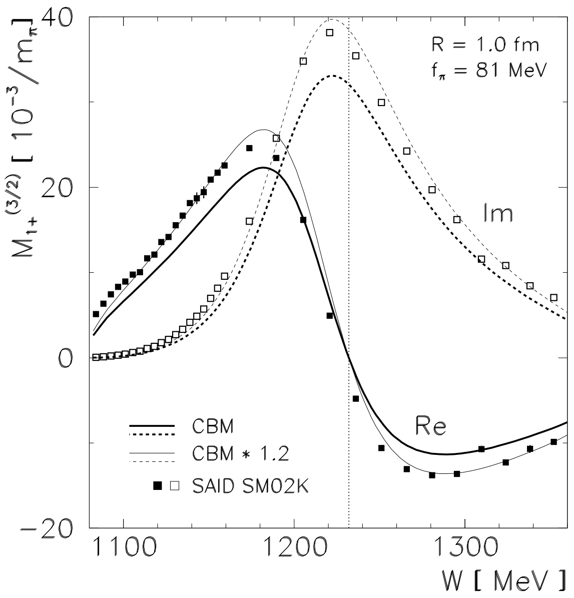
resonančno stanje ima obliko:

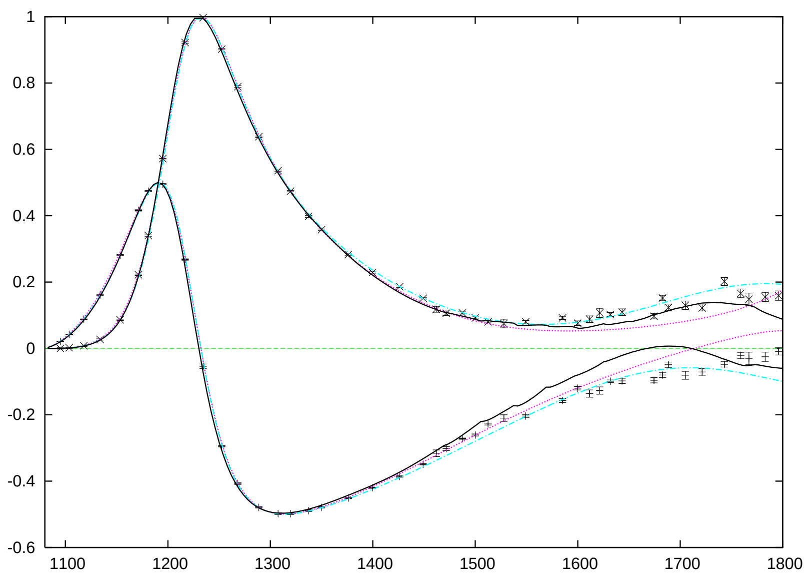
$$\begin{aligned} |\Psi_{N*}^{(\mathrm{res})}(W)\rangle \;&=\; z_{N*} \left\{ |\Phi_{N*}\rangle - \int \frac{\mathrm{d}k\, \mathcal{V}_{NN^*}(k)}{\omega_k + E_N(k) - M} \, [a^\dagger(k) |\Psi_N\rangle]^{JT} \right. \\ &\left. - \int \frac{\mathrm{d}k\, \mathcal{V}_{\Delta N^*}^{M_\Delta}(k)}{\omega_k + E_\Delta(k) - M} \, [a^\dagger(k) |\widehat{\Psi}_\Delta(M_\Delta)\rangle]^{JT} \right\} + \ldots \end{aligned}$$

P33 fotoprodukcijske amplitude v področju resonance $\Delta(1232)$

$N + \gamma \longrightarrow N + \pi$

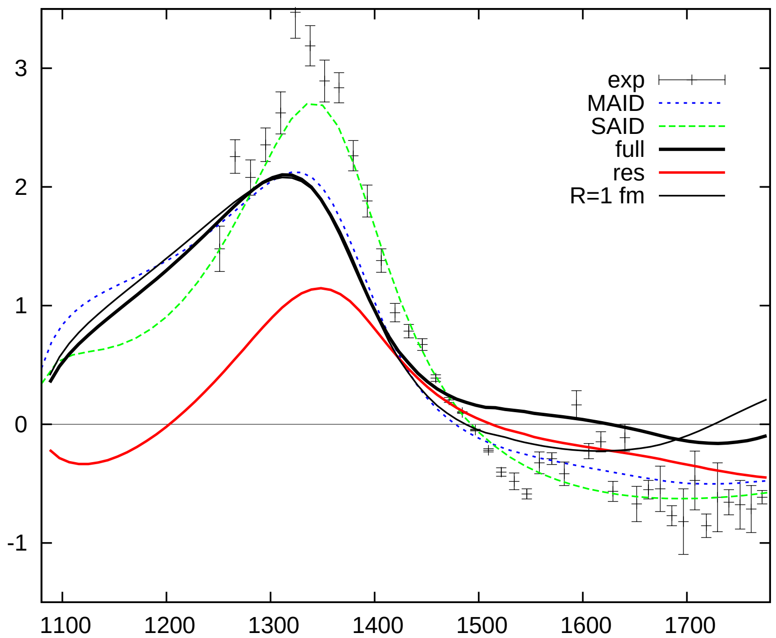
- dominira resonančni prispevek
- prispevek pionskega oblaka je primerljiv s prispevkom kvarkov

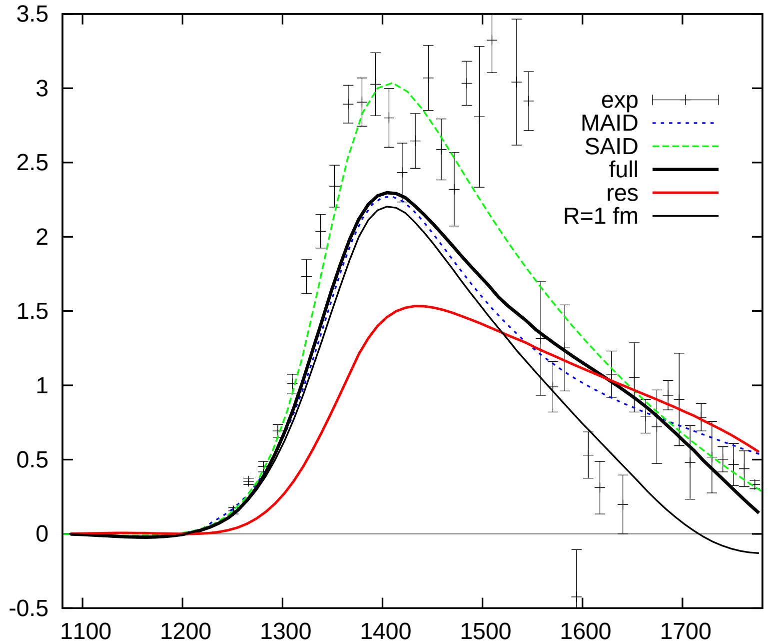




Fotoprodukcijske amplitude v območju $N(1440)$

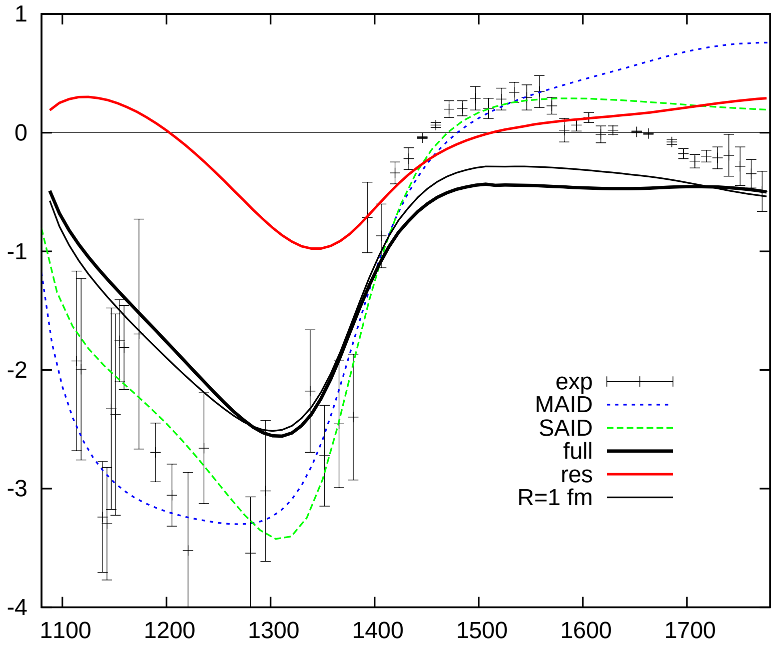
$$p + \gamma \rightarrow p + \pi^0$$



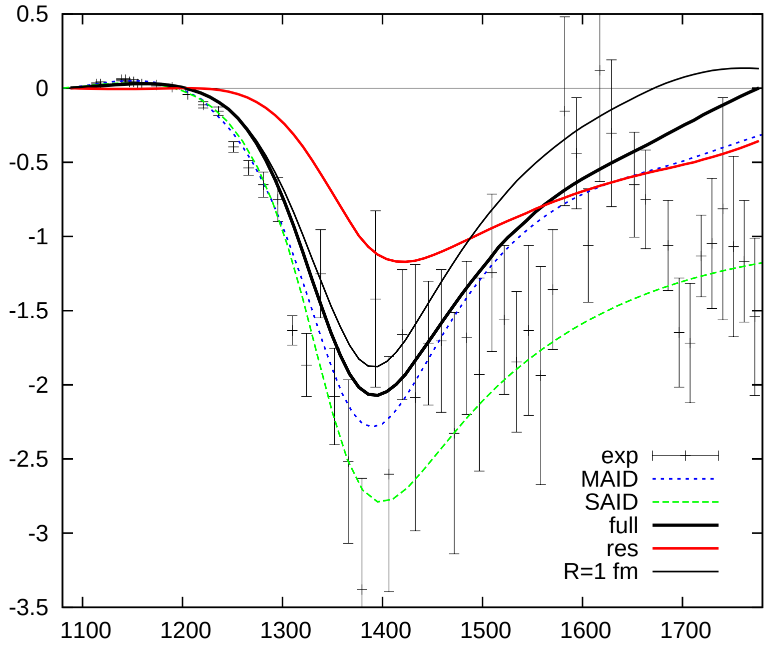


Re $M^p_{1-}(1/2)$ Im $M^p_{1-}(1/2)$

$n + \gamma \to n + \pi^0$

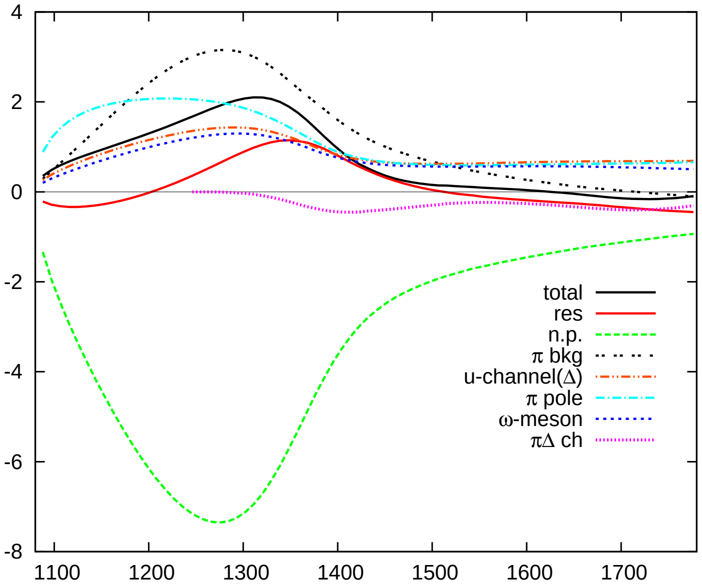


Re $M^n_{1-}(1/2)$

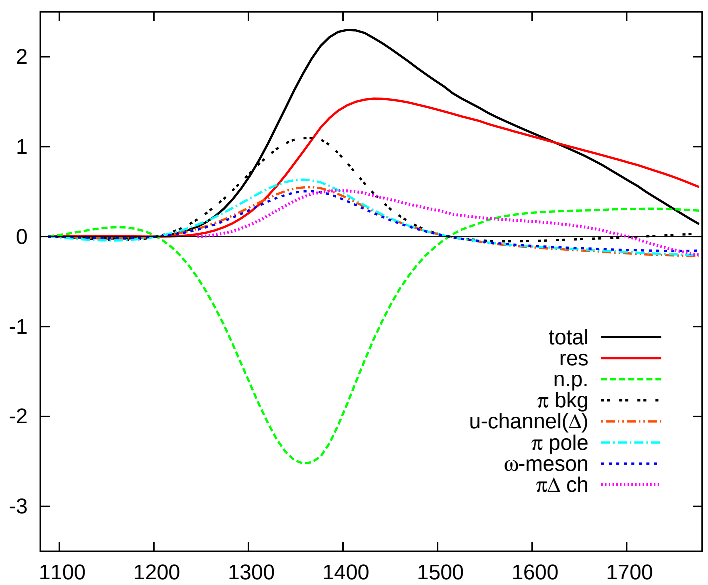


Im $M^n_{1-}(1/2)$

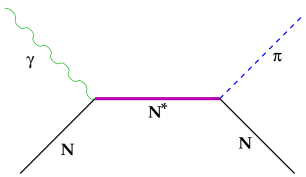
Različni prispevki k amplitudi



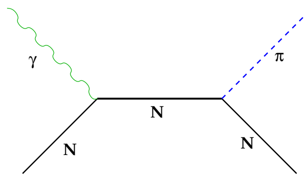
$\mathrm{Re}\, M_{1-}^{p}(1/2)$



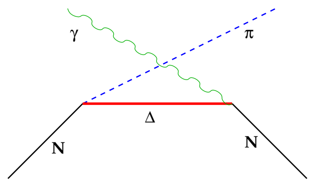
$\mathrm{Im}\, M_{1-}^{p}(1/2)$



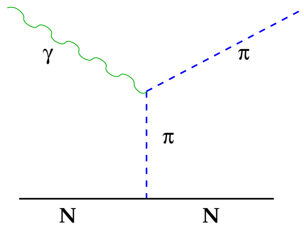
resonant



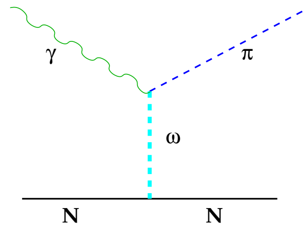
nucleon pole



crossed



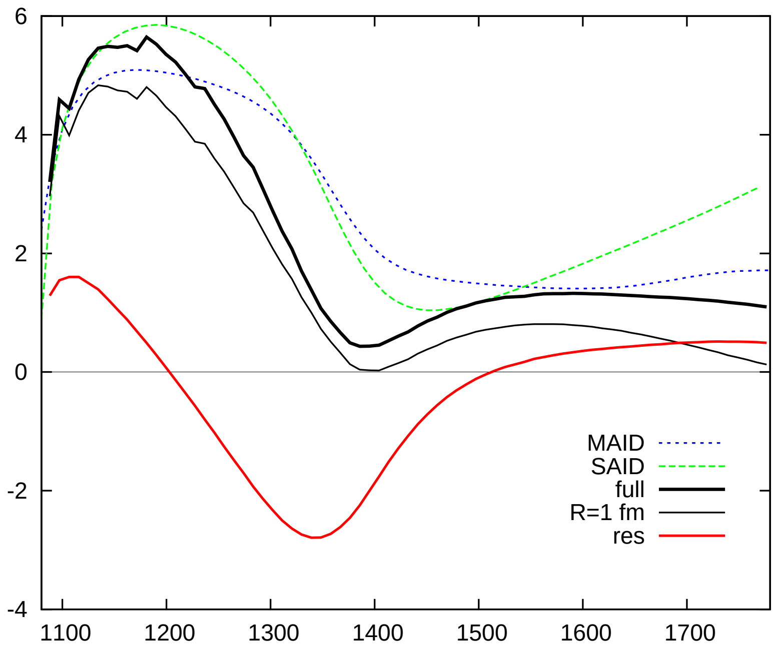
pion pole

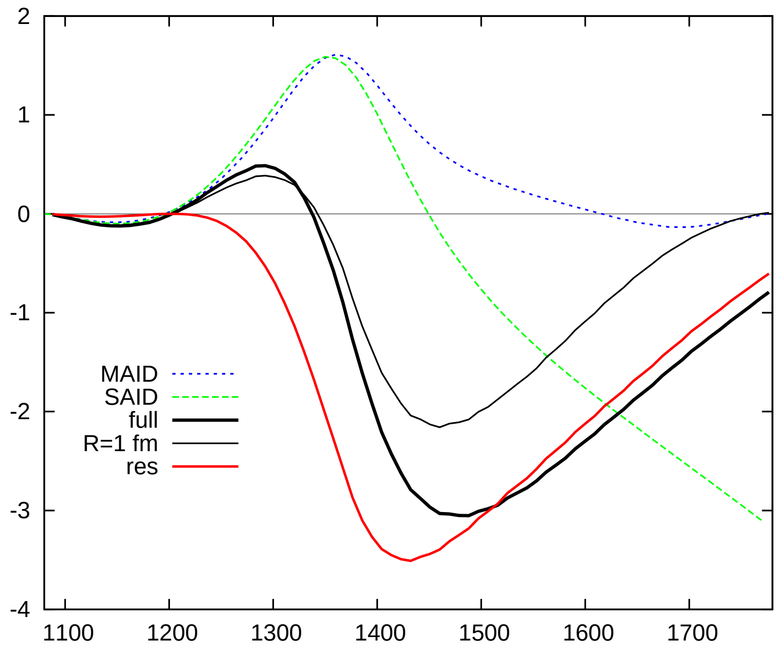


omega meson

Skalarne fotoprodukcijske amplituda

$p + \gamma \rightarrow p + \pi^0$

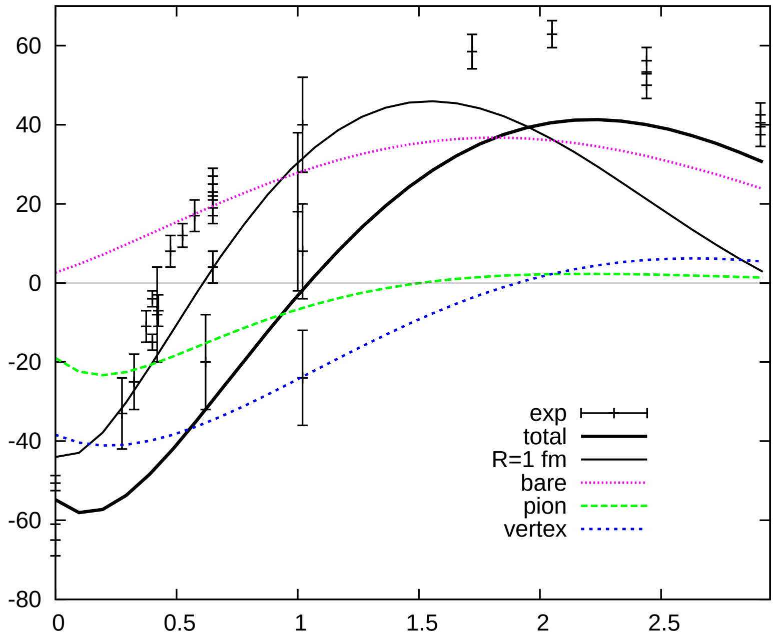




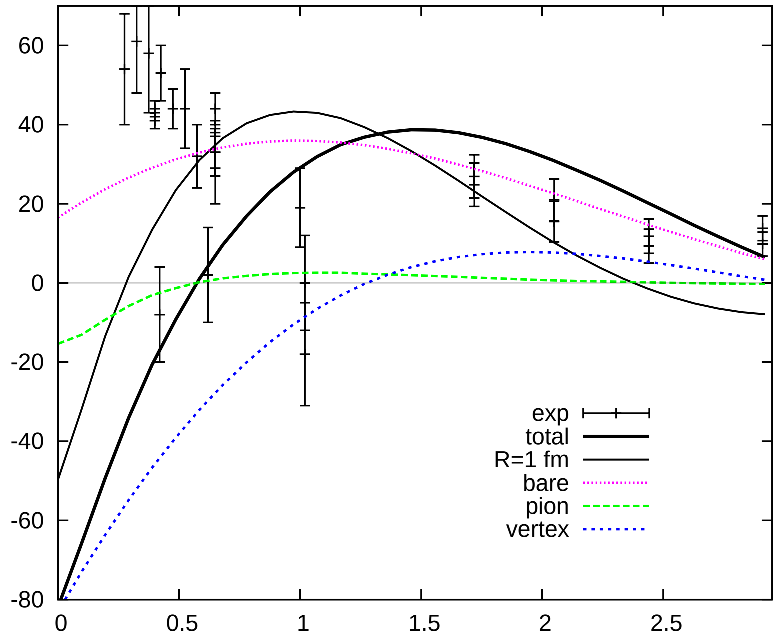
Re $S^p_{1-}(1/2)$ Im $S^p_{1-}(1/2)$

Elektro-excitacijske amplitude za Roperja

$p + \gamma \to N(1440)$



$^pA_{1/2}$



$^pS_{1/2}$

Rezime

- Metoda večkanalne matrike K omogoča:
  - konsistentno vključitev večdelčnih kvarkovskih stanj v račun sipalnih in elektroprodukcijskih amplitud,
  - enolično ločitev prispevkov na resonančni del, na prispevek ozadja in prispevek sosednjih resonanc,
  - enoten okvir računanja sipalnih in elektroprodukcijskih amplitud,
  - enolično določitev elektroekscitacijskih amplitud (oblikovnih faktorjev) resonance.

- Rezultati v parcialnih valovih P33 in P11 (Roper):
  - sipalne amplitude v območju od praga do $\sim$ 1700 je mogoče razložiti s pomočjo kanalov πN, $\pi\Delta$, σN, $(\pi N(1440), \sigma\Delta)$,
  - kvarkovski modeli dajejo premajhne vrednosti $g_{\pi\Delta}$ in $g_{\pi N^*}$, metoda vodi do znatnega ojačanja preko renormalizacije verteksa in vpliva sosednjih resonanc,
  - model dobro napove vse glavne zančilnosti za $M_{1-}(W, Q^2)$, eksotične prostostne stopnje niso potrebne,
  - nejasna eksperimentalna situacija za S_{1-}, velike razlike med modelskimi računi,
  - pomemben prispevek pionskega oblaka posebno pri $Q^2 \to 0$.